LOAN VALUATION, A MODERN FINANCE PERSPECTIVE

by

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Abstract

The valuation of bank loans has attracted a lot of attention recently, and the case for marking-to-market loan portfolios has been made. In this note, five different valuation methods are evaluated vis–à-vis a benchmark, the Loan Arbitrage-Free Valuation. This model takes into account five variables that should be reflected in the economic value of a loan transaction, namely the probability of failure, the recovery rate, the tax implications, the level of debt and equity funding, and the risk premium demanded by the market.

Introduction

Many scholars have called for a wide application of marking-to-market rules for the valuation of loan portfolios of banks (Altman (1993$_a$), Kane (1983), White (1991)). The importance of loan valuation is fourfold. The first one is to provide better information on the solvency of a bank to investors, the holders of its deposits, bonds or shares. The second objective is to have a fairer application of international regulations on capital for which different accounting standards can create severe distortions (Scott and Iwahara, 1994). The third one is to protect the liability of the deposit insurance agency or of the lender of last resort by providing more precise and timely information on the capital cushion and on the eventual need for recapitalization. Finally, the last one is to provide a correct measurement framework to evaluate the performance of a lending unit. Opponents to a marked-to-market system have argued that, in the absence of a liquid market for loans (bank loans precisely exist because they solve asymmetric information problems), the valuation of loans would be costly and would entail a high degree of subjectivity (Berger, Kuester, O'Brien, 1989). The purpose of this note is to discuss the pros and cons of six different loan valuation methods. Five models will be compared to a benchmark, the Loan Arbitrage-Free valuation. The paper is structured as follows. In the first section, a model is developed to price a two year-to-maturity fixed rate loan. In the second section, we present and compare the six loan valuation methods .

Section One : A Loan Arbitrage-Free Pricing (LAFP) Model

Any model of loan pricing should take into account the probability of default, the recovery rate in case of default, the tax implications, the level of equity and debt funding, and the risk premia demanded by the market[1]. In related papers (Dermine, $1995_{a,b}$), we have argued that the risk-adjusted cost of equity used to finance a specific loan (say a loan to a chemical company) shall not be easily observable because there are no banks specialized in lending to one business sector only. The implication is that one cannot use the risk premium calculated from banks' shares traded on a stock market to infer an industry-specific risk premium. To solve this problem, we rely on the seminal work of Modigliani-Miller (1958) and propose a Loan Arbitrage-Free Pricing (LAFP) model which suggests to use the information from the traded corporate bond market. Instead of computing a risk-specific cost of equity, it proposes to evaluate separately the cash flows originating from the loan and the cash flows linked to the funding structure. Each set of cash flows will be evaluated at a specific risk-adjusted discount rate. The following example illustrates.

Consider a two year-to-maturity fixed rate loan funded by a two-year fixed rate interbank debt of 94 and equity of 6. The probabilities of default are defined as follows : 0 % in Year One, and 3 % in Year Two (with recovery of 60 in the default case because of the seniority of the loan)[2]. The relevant economic data are summarized as follows :

- **ECU M 100 two year-to-maturity fixed rate loan (interest paid at end of the year and principal at maturity)**
- **Corporate tax rate of 40 %**

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[1] **One should add the variable operating expenses.**

[2] **Probabilities of default can be inferred from a bank's own historical statistics, through inference from default observed on the corporate bond market (see Altman, 1993_b), or from option pricing-based model.**

- Expected return on one year- and two year-to-maturity, similar risk (zero coupon) corporate bond of 10 % and 10.2 %
- Fixed interbank rate of 10 % for the first year and 10 % for the second year
- Equity funding : 6 %
- Interbank funding : 94 %
- Probability of default in Year One : 0 %
- Probability of default in Year Two : 3 %
- Recovery of 60 in case of default.

The break-even loan interest rate, I, is such that the discounted value of expected cash flows is equal to the initial equity investment,

$$Equity = 6 = \frac{Ix(1-0.4)}{1.10} + \frac{0.97\ x[Ix(1-0.4)+100]\ +\ 0.03\ x[60+0.4x40]}{1.102^2} - \frac{94x0.10x(1-0.4)}{1.1} - \frac{94x0.10x(1-0.4)+94}{1.1x1.1}$$

The expected cash flow from the loan in Year Two includes two parts : The expected revenue in the case of non-default, and the recovery plus the tax shelter created by the losses in case of default. The break-even loan rate, I = 11.45 %, will capture implicitly the probability of default, the expected losses arising out of default, and the necessary *equity spread.* An equity spread arises because of the debt/equity funding structure and the fact that the cost of equity is not tax deductible[3].

[3]An alternative approach would be to discount the net after-tax expected cash flows accruing to shareholders at the leveraged-cost of equity. We have argued (Dermine, 1995$_{a,b}$) that the absence of risk-specific information on the leveraged cost of equity makes the LAFP approach easier to implement.

Section Two : Six Approaches to Loan Valuation

The problem of loan valuation arises one year later. The payment of interest has been made, and a methodology is needed to value the loan, the interbank debt, and the equity. The following parameters are assumed to apply at the end of the first year :

•Probability of bankruptcy in Year Two revised upward to 5 %
•The one year-to-maturity interbank rate is standing at 11 %
•The one year-to-maturity expected rate of return on corporate bond with similar risk is standing at 11.3 %.

With such parameters, one can verify (see appendix) that a one year-to-maturity loan priced according to the LAFP methodology will carry a contractual interest rate of 13.68 %. This rate is referred to as the current contractual rate on new loans with similar credit risk and maturity.

A proper valuation model should attempt to evaluate the loan transaction from a shareholders' perspective. Indeed, in case of default the loans are very likely to be sold to another bank, at a price reflecting its value for shareholders. The model should take into account :

- **The probability of bankruptcy**
- **The rate of recovery**
- **The tax implications**
- **The debt/equity funding structure[4]**
- **The relevant risk premium demanded by the market.**

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[4]One could argue that bank equity funding can be ignored as the asset could be sold directly to investors. While a valid argument for liquid assets, this is invalid for illiquid loans that are very likely to be repurchased only by banks that have a specific expertise in monitoring borrowers.

Six different valuation models are proposed. Five of them will be compared to the only method consistent with modern finance theory, the Loan Arbitrage-Free valuation method. The results are summarized in Table One.

Method 1 : 'Historical Accounting'

This (very much used) approach does not recognize any impairment of value as long as the contractual cash flow obligation is met (payment of principal or interest). Since interest has been paid at the end of the first year, equity is measured as the difference between the principal of the loan (100) and the face value of the debt (95) :

Equity = 100 - 94 = 6.

This approach is unsatisfactory as if ignores the expected future losses and the opportunity risk-adjusted return demanded by the markets.

Method 2 : 'Before-tax Contractual Opportunity Rate'

The second method suggests to evaluate the before-tax contractual cash flows at the before-tax contractual opportunity rate on a new loan with similar risk-maturity characteristics, 13.68 % for the loan, and 11 % for the funding.

$$Equity = \frac{11.45+100}{1.1368} - \frac{9.4+94}{1.11}$$

$$= 98.04 - 93.15 = 4.89$$

The equity is valued as the difference between the value of the loan (98.04) and the debt (93.15) at 4.89. The reduction in value of equity reflects the fact that the opportunity contractual rates on loans and deposits have increased. This approach is not satisfactory as it fails to take into account the expected cash flows, taxation, and the market's opportunity rates.

Method 3 : 'After-tax Contractual Opportunity Rate'

The third method suggests to evaluate the after-tax contractual cash flows at the after-tax contractual opportunity rate on a new loan with similar risk-maturity characteristics, 13.68 % x (1-0.4) for the loan, and 11 % x (1-0.4) for the funding.

$$Equity = \frac{11.45x(1-0.4)+100}{1+0.1368x0.6} - \frac{9.4x(1-0.4)+94}{1+0.11x0.6}$$

$$= 98.76 - 93.47 = 5.29$$

The equity is valued as the difference between the values of the loan (98.76) and debt (93.47) at 5.29. This approach is not satisfactory as it ignores expected cash flows and opportunity risk-adjusted discount rates.

Method 4 : Loan Arbitrage-Free Valuation

To evaluate the net value of the loan transaction a year later, one needs to compute the net value of the bank's asset, that is the value of the loan with a residual maturity of one year net of the value of the debt, the after-tax expected cash flows being discounted at the shareholders' opportunity risk-specific discount rate.

$$Equity = \frac{0.95 \; x \; [100+11.45x(1-0.4)] \; + \; 0.05x[60 \; +0.4x40]}{1.113}$$

$$- \frac{94x0.10x(1-0.4)+94}{1.11}$$

$$= 94.64 - 89.77 = 4.87$$

This valuation method is fully consistent with modern finance theory, taking properly into account the five variables that should be reflected in the economic value of a loan transaction, namely the probability of failure, the recovery rate, the tax implications, the level of debt and equity funding, and the risk premia demanded by the market.

Method 5 : The Opportunity Before-tax Internal Rate of Return

This method suggests to compute firstly the before-tax IRR on a new loan, and to apply the IRR to value the before-tax expected cash flows on the loan. The IRR on the new loan is computed as follows :

$$Loan = 100 = \frac{0.95 \; x \; (100+13.68) \; + \; 0.05x60}{1+IRR}$$

The before-tax IRR is equal to 11 %. This IRR is now used to value the before-tax expected cash flows from the loan :

$$Equity = \frac{0.95 \; x \; [100+11.45] \; + \; 0.05x[60 \;]}{1.11}$$

$$- \frac{94x0.10+94}{1.11}$$

$$= 98.08 - 93.15 = 4.94$$

This method is not satisfactory as it ignores some tax implications and the shareholders'opportunity discount rates.

Method 6 : The Opportunity After-tax Internal Rate of Return (IRR)

This method suggests to compute first the after-tax IRR on a new loan, and to apply the IRR to value the after-tax expected cash flows on the loan. The IRR on the new loan is computed as follows :

$$Loan = 100 = \frac{0.95 \; x \; (100+13.68x(1-0.4)) \; + \; 0.05x(60+0.4x40)}{1+IRR}$$

The after-tax IRR is equal to 6.6 %. This IRR is now used to compute the value of the loan, the debt being valued at its after-tax opportunity rate (11 %x(1-0.4) = 6.6 %) :

$$Equity = \frac{0.95\ x\ [100+11.45x(1-0.4)]\ +\ 0.05x[60+\ 0.4x40]}{1.066} - \frac{94x0.10x(1-0.4)+94}{1.066}$$

$$= 98.8 - 93.47 = 5.33$$

This method is incorrect as it does not value at the shareholders'risk-adjusted opportunity rates.

Six methods have been presented. The results are summarized in Table One. Only Method 4, the Loan-Arbitrage-Free Valuation allows to take properly into account the five variables that should be reflected in the economic value of a loan transaction, namely the probability of failure, the recovery rate, the tax implications, the level of debt and equity funding, and the risk premium demanded by the market. The last column of the Table shows the bias in valuation of equity vis-à-vis Method 4. If historical Method 1 can be ignored for the obvious reason of ignoring future expected losses and the current opportunity rates, one should note that the after-tax Methods 3 and 6 can yield large errors because they ignore the shareholders'opportunity rates. This issue is well noticed in investment theory (Brealey-Myers, 1988). In our example, Method 2, discounting the contractual before-tax flows at the current contractual rate on new loans, yields a result fairly close to the Loan-Arbitrage-Free valuation approach. But this result is not robust as

Method 2 fails to take into account properly all tax implications[5].

Method	Value of Loan	Value of Debt	Value of Equity	Overvaluation of Equity Relative to 'Method 4'
Method 1 'Historical'	100	94	6	23 %
Method 2 'Contractual Before-tax Loan Rate	98.04	93.15	4.89	+ 0.4 %
Method 3 'Contractual After-tax Loan Rate'	98.76	93.47	5.29	8.6 %
Method 4 LAF Valuation	94.64	89.77	4.87	-
Method 5 Before-tax IRR	98.08	93.15	4.94	+ 1.4 %
Method 6 After-tax IRR	98.8	93.47	5.33	+ 9.44 %

Table One : Six Loan Valuation Approaches, a Summary

[5]Indeed, we have shown (Dermine, 1987) that a large part of the value of a deep-discount asset is due to the ability to delay the payment of taxes on capital gains that are only taxed at maturity.

Summary

The valuation of loans has attracted a lot of attention recently, and the case of marking-to-market has been made. Five methods have been compared to a benchmark the Loan Arbitrage-Free Valuation model which takes properly into account five economic data that banks should take into account, that is the probability of failure, the recovery rate, the tax implications, the equity funding, and the risk premia demanded by the market. It does appear that other methods which ignore any of these variables can lead to significant valuation errors.

Appendix : The Current One Year-to-Maturity Opportunity Rate.

Keeping the same assumptions as those used in the paper, the break-even loan interest rate, I, is such that the discounted value of expected cash flows is equal to the equity investment,

$$Equity = 6 = \frac{0.95\ x[Ix(1-0.4)+100] + 0.05\ x[60+0.4x40]}{1.113} - \frac{94x0.10x(1-0.4)+94}{1.1}$$

The break-even rate, I, is 13.68 %.

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