

Ratings, Reviews, and the Marketing of New Products

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Abstract

In this paper, we study how user-generated content, in the form of average ratings or reviews about new products, impacts market dynamics and the firm's equilibrium actions. We construct a two-period model where a firm introduces a new product in the first period and needs to decide how extensively to advertise it and how much to charge for it. Consumers are homogeneous in their valuation for quality yet heterogeneous in their tastes, along a circle, for the product's positioning. In the basic model setup, all consumers in period 2 are aware of the product and we examine three possible information transfer structures from first- to second-generation consumers: no additional information (benchmark); average ratings (AR); and the distribution of ratings and taste locations (Reviews). We find that second period demand is sensitive to the information transferred. In the AR case, second-generation buyers are uniformly spread around the circle while in the Reviews case they are more tightly concentrated around the consumer location that best matches the product's positioning. Second, we show that in the AR case a firm will advertise to a smaller set of consumers in the first period relative to the benchmark case. This occurs because the firm tries to use advertising strategically to increase first period average ratings in an effort to bump up subsequent demand. Furthermore, if advertising is sufficiently cheap to execute, in the AR case the firm will price higher relative to the benchmark. Third, we find that as more information is transferred across generations there is a greater likelihood that average ratings will exhibit an increasing rather than decreasing pattern over time. This happens because with more information, i.e., with reviews rather than just average ratings, a second-generation consumer is better able to make purchase decisions based on product positioning. Fourth, we show that a firm's profits suffer when only average ratings are available, and these profits are lower than even the benchmark case. Total welfare is highest in the Reviews case, provided advertising is sufficiently cheap. We then examine an alternative setup whereby second-generation consumers can only be made aware of the product through social interactions with first-generation consumers who purchased. We find that the firm has a greater incentive to advertise in the first period, and this tendency is generally more pronounced as the degree of homophily increases, i.e., as social interactions are more likely to occur between consumers who share the same preferences. Moreover, average ratings are more often expected to display an increasing rather than decreasing pattern over time in this setting.

Keywords: Word-of-Mouth, Online Reviews, Product Ratings, Social Networks, Advertising, Quality, Pricing, User-generated content

1 Introduction

One of the hallmarks of the digital transformation witnessed over the past two decades is the fact that people are more easily and immediately connected to each other than ever before. While this has resulted in the rise of several social media platforms over which individuals share various daily experiences with friends, family members and colleagues, it has also had profound implications for businesses. In particular, consumers can use online tools to share impressions of the products and services they buy, often on the vendor's website or on e-commerce sites, and prospective buyers can examine these impressions and incorporate them into their own decision making. Consumer generated content can thus impact demand and hence firm fortunes.

For instance, if a new product receives an average rating of 4.5 (out of 5) based on the input from early buyers, we might expect later consumers to be more inclined to buy it compared to a product receiving an average rating of say 3.8. That said, customers might also take the time to write detailed reviews about the product and why they liked (or disliked) it given their specific preferences. If subsequent consumers read these reviews, they could arrive at a different conclusion regarding the product than had they relied solely on the average rating. Specifically, while some aspects of the product are appreciated by all consumers and can be summarized by a 'quality' level (e.g., the speed of a computer, the battery life of a smartphone, the miles per gallon of a car), other product aspects may fit the tastes of some consumers but not those of others (e.g., the exterior design of the computer, the screen size of the device, the plot line of a movie). As a result, a consumer might provide a low rating for a product because it does not fit their particular taste, despite its relatively high quality. If a later arriving consumer only observes the rating, without more detailed information on what drove it, s/he might be worried the product is of low quality and be deterred. But if this same consumer reads the full review and figures out that the reason for the low rating had to do with a product taste/fit issue that does not apply to them, they may in fact decide to make a purchase. From the firm's perspective, the question is whether the nature of the information transferred across consumer generations should affect any of its strategic actions and, if so, how?

In this paper, we examine the interplay between product assessments communicated by consumers and firms' strategic actions, namely advertising and pricing. We focus on the following research questions.

- Does a firm have a greater or lower incentive to expend on advertising when consumers have access to reviews vs. just average ratings? Does this incentive depend on the social interaction across generations?
- Does the information transferred across generations affect how aggressively a firm prices its product?
- What should we expect in terms of rating dynamics? Under what conditions will the pattern of ratings be increasing vs. decreasing over time?
- How does the type of information transferred across generations impact firm profits and consumer welfare? Specifically, should we expect profits to be lower while consumer surplus to be higher when more information is transferred?

To address these questions we construct a two-period model where a firm introduces a new product in the first period and needs to decide how extensively to advertise it and how much to charge for it. The product's intended positioning is ex-ante known to the firm whereas its quality is uncertain prior to market consumption. Consumers are homogeneous in their valuation for quality, yet heterogeneous in their tastes for the product's positioning (along a circle) and in their base valuation for consumption in the category. We examine three possible customer information transfer structures: a benchmark setting where no first-generation consumer evaluations are passed on; an Average Ratings (AR) setting where only the average ratings (corresponding to the average net consumption utility) of first-generation consumers who purchased are passed on; and a Reviews setting where the joint distribution of ratings and taste locations of first-generation consumers who purchased are passed on. We first solve a basic model where all second-generation consumers are aware of the product and later examine a setting where second-generation awareness only occurs through social interaction with previous buyers.

Our analysis of the basic model yields several interesting findings. First, we show that second period demand is the same in size regardless of whether only average ratings or reviews are transferred; yet the set of people who purchase is different. In the AR case, second-generation consumers are uniformly spread around the circle, while in the Reviews case they are more tightly concentrated around the consumer location that best matches the product’s positioning. Second, we show that in the AR case a firm will target a smaller set of consumers with advertising in the first period relative to the benchmark case (of no information transfer). This occurs because the firm in the AR case tries to use advertising strategically to increase first period average ratings in an effort to bump up consumer demand in the second period. Furthermore, if advertising is sufficiently cheap to execute, in the AR case the firm will be induced to price higher as it expects to sell to consumers with a higher valuation for the product. Third, we show that as more information is transferred, there is a greater likelihood that average ratings will exhibit an increasing rather than decreasing pattern over time. This happens because with more information, i.e., with Reviews rather than AR as input, a second-generation consumer is better able to make purchase decisions based on product positioning. This results in second-generation purchases, and subsequent ratings, by consumers with higher valuations on average. That said, even with Reviews we can characterize conditions for average ratings to decrease across generations. Surprisingly, such a declining pattern happens when first-generation consumers discover post consumption that the product is of relatively high quality. Finally, we show that a firm’s profits suffer when only average ratings are available, and these profits are even lower than the benchmark (no information case). Total welfare is the highest in the Reviews case, provided advertising is sufficiently cheap.

We then explore an alternative setting whereby awareness in the second period is contingent upon connecting with a consumer who purchased in the first period. Specifically, a second-generation consumer interacts with a sample of first-generation consumers whose location along the circle (i.e., their horizontal preference) is either identical to his/her own, thus reflecting homophily, or randomly drawn. In this setting, we find that the firm generally has a greater incentive to advertise in the first period than in the basic model and, interestingly, will cast the widest advertising net when no information is transferred across

generations (i.e., the benchmark case). Furthermore, the degree of homophily in social interactions plays a critical role: the more second-generation consumers expect to be paired with consumers like them in terms of location, the more broadly the firm tends to advertise in the first period. This is primarily true in the benchmark and AR cases because aware second-generation consumers don't know the product's exact position but presume they are close to it because homophily is high; and the firm tries to exploit this by advertising to more first-generation consumers who can serve as social contagion agents. This scenario, however, can break down in the Reviews case because aware consumers learn the exact product position. Lastly, we show that as long as there is some positive degree of homophily in cross-generation social interactions, we should expect greater chances for average ratings to exhibit an increasing pattern over time than in the basic model.

2 Literature Review

Our work contributes to the growing literature on the strategic implications of online consumer-generated evaluations of products and services. In particular, (Sun 2012) studies how average ratings and the variance of these ratings affects firm pricing and profits from a new product. In her model, all consumers are aware of the product and its location, which is fixed, but are uncertain about its quality and about the mismatch cost parameter along the horizontal dimension. She finds that a higher average rating, as expected, result in an increase in price and profits, yet greater variance can also yield higher profits when the average rating is low enough. Our work differs from (Sun 2012) in that we do not assume that consumers ex-ante know about the product's existence and hence the firm needs to engage in costly advertising. Furthermore, consumers in our model know the common mismatch cost parameter but are ex-ante uncertain about the product's position. We then examine how the nature of customer-generated evaluations (average ratings vs. full reviews that reveal the joint distribution of ratings and of buyers' locations) interacts with advertising and pricing in the first period. In (Mayzlin 2006), competing firms engage in anonymous self-promotion alongside truthful messages from early adopters at online review sites. Later adopters use this "mixture" of biased and unbiased WOM to inform their purchase decisions. She iden-

tifies conditions for online recommendations to be persuasive despite firms' manipulation attempts, with the lower quality firm promoting more aggressively. Our setting has only one firm and we assume that its promotional actions, in the form of targeted awareness advertising, are not confounded with the ratings and reviews from previous buyers. We further examine how our findings change when second period awareness depends on social interactions across generations. (Chen & Xie 2008) examine a seller's incentive to provide full or partial information regarding the two attributes of its product in the presence of consumer reviews. Their model assumes full awareness and differentiates between expert consumers, who can process the firm's advertising, vs. novices, who only use reviews from first period consumers to determine product match. They show that the firm's optimal strategy depends on the marginal cost of the product, how accurate consumer reviews are, and the size of the two segments. In our model, the new product is also defined along two dimensions, though in our case one is vertical and the other horizontal, and consumer heterogeneity is based on location along the circle. Moreover, we assume that for second-generation consumers information about the product can only be gleaned from user-generated input. Production costs do not play a role in our setup.

Our work is further related to (Godes 2017) who looks at how WOM may affect a firm's product quality decision. He finds that the optimal quality selected depends on whether WOM disproportionately impacts lower- or higher-taste consumers and on whether it serves to expand awareness or inform about quality (i.e., persuade). (Jiang & Yang 2018) also endogenize quality in a setting where second period consumers learn the true quality from first period consumers and the firm has private information regarding its marginal cost of quality provision (which it can signal through price). Their analysis reveals that a highly efficient firm (i.e., low marginal cost) may actually find it optimal to reduce product quality when its efficiency is known compared to when it is not. In our model, product quality is exogenous (though uncertain) and the focus is on how the nature of the information transferred between consumers (in the form of average ratings or full reviews) affects a firm's targeted advertising and pricing decisions.¹ Our model further includes a horizontal product

¹In (Godes 2017) the treatment of advertising is separate from that of WOM while we examine the interaction between them. We also note that in the extension, we micro-model social interactions rather than leaving them at a reduced form.

dimension that is absent in (Godes 2017) and (Jiang & Yang 2018).

Lastly, since an aspect of our analysis pertains to average rating dynamics our work is related to papers that have looked specifically at this issue. As expressed in (Godes & Silva 2012), and references therein, some researchers have found that average ratings tend to decrease over time while others have found the opposite. (Godes & Silva 2012) do a thorough empirical analysis that teases out temporal and sequential factors that may affect average ratings. Our theoretical exploration is sequential in nature, and reveals that the pattern of average ratings may depend on how much information is conveyed by consumers who purchased the product as well as on the discovered quality of the product; noting that higher quality products tend to generate lower subsequent average ratings.

To summarize, our work contributes to the literature on user-generated content and WOM along several dimensions. First, we examine the implications for firm behavior and market dynamics of different information structures that have not been looked at before at our level of detail in a single context, namely average ratings vs. full reviews. Second, we explore the interaction between consumer-generated information and the firm's targeted advertising. Third, we combine information from previous buyers in the form of ratings and reviews with the process of social interactions to generate awareness across generations. Fourth, we shed new light on the evolution of ratings and delineate conditions for various patterns to emerge.

The rest of the paper is organized as follows. In the next section we describe in detail the basic model setup. We then present the central findings from the analysis of this model and provide intuition for the firm's equilibrium behavior. Subsequently, we relax the assumption of automatic awareness in period 2 and assume it is a function of social interaction across generations. We end with concluding remarks and managerial implications. Most proofs are relegated to the Appendix.

3 Model

There are two generations of consumers, who live in periods $t = 1, 2$ respectively. We consider a Salop model such that in each generation there is a unit mass of consumers, spread uniformly on a circle of length 4. The firm has developed a new product that is

positioned at location y , which is uniformly drawn from $[0, 4]$. The location of the firm's new product is unobserved by consumers but known to the firm. The product is of quality q , drawn from a uniform distribution over $[0, 2]$ (independently of the draw of y). The quality q is unobservable to the firm, and becomes observable to consumers who purchase the product and try it, thus reflecting an experience good. We assume for simplicity that the firm produces this (indivisible) product at zero marginal cost.

The setting we have in mind is one where the firm knows what kind of product it is developing (or producing in the case of a creative good like a movie or show), which is why it knows the position the product best matches, but is unsure how “good” it is along the quality (vertical) dimension.² For example, a firm may produce a movie that is a thriller with comic elements of a particular humoristic style. It is uncertain, however, prior to its release and market consumption how strong the movie will be deemed with respect to various quality measures (acting, wardrobe, plot, directing, etc.). In other words, given the specific sub-genre (which the firm knows) whether the movie will be deemed a “hit” or a “dud”. Similar examples come to mind in the case of other innovation categories, such as new menus in restaurants, music (new singles/albums), and fashion, as well as in other categories where new product activity is prevalent like consumer electronics, cars, etc.³

To formalize the above characterization, let the utility of consumer i at location x that does not purchase the good be zero, and if she purchases the good her utility is

$$u_x = \alpha_i + \beta q - d(x, y) \tau - p,$$

where $d(x, y) \triangleq \min\{|x - y|, 4 - |x - y|\}$ is the shortest distance between her location x

²It is easy to extend the model to allow the product's expected quality to be related to some measure of ex-ante R&D effort. But as long as the firm remains uncertain about the outcome of this R&D in terms of the final appreciation by consumers (for example, its product quality falls within a range of possible values around a mean level that is a function of the R&D effort) our results will still carry through.

³Here are a few examples of products for which quality was revealed to be different from expectations. When the Samsung Galaxy Note 7 came out, many consumers knew the style and specs, which were clearly advertised. However, as more batteries malfunctioned and put users at risk, quality perceptions changed drastically. Similarly, Microsoft Surface, the first MS laptop, was launched with well advertised style and specs, yet a hardware bug, which prevented the laptop from going into sleep mode and leading to battery draining, was revealed after the launch. Additional examples include the Ford Escape, which surprised consumers for the better with respect to quality, and Windows 8 which was skipped by many Microsoft users after its performance was revealed to be unsatisfactory.

and the product's position y , τ is the disutility from consuming a product that is different from the consumer's ideal point (a.k.a., the transport cost), β is the marginal valuation for quality, and $\alpha_i \sim U[0, 1]$ is an idiosyncratic utility parameter known only to consumer i and reflecting his/her base valuation or attraction of products in the category. A consumer has no utility from purchasing more than one unit.

At the outset of the game, consumers are unaware of the new product's existence and the firm has the ability to engage in costly targeted advertising to inform a sub-set of them. The timeline of the model is as follows: In period $t = 1$, the firm chooses a price p and an advertising strategy consisting of an interval of length $4z$ centered around the product's position y .⁴ The generation 1 consumers that fall within the advertising interval learn about the existence of the product, its location (y) and price (p). The advertising cost is $A_y(z)$, such that $A_y(0) = 0$; $A'_y(z) > 0$; $A''_y(z) > 0$. Hence, advertising costs are assumed to be convex in the size of the group targeted. Consumers do not observe the strategy z , but learn whether they are inside the interval or not. Once advertising takes place, informed consumers make simultaneous purchasing decisions. Consumers who make a purchase experience the product and know their net utility from consumption.

In period 2, a fraction γ of generation 2 consumers become aware of the new product. This captures scenarios where through word-of-mouth (WOM) or exposure to press coverage some generation 2 consumers will learn about the existence of the product. For ease of exposition, in this section we solve for the case of $\gamma = 1$, i.e., all generation 2 consumers become aware, as it turns out that the results are qualitatively unchanged for any exogenous $0 < \gamma \leq 1$. In section 3.2, we micro-model the WOM process, which endogenously leads to only a subset of generation 2 consumers becoming aware of the product. Let aware generation 2 consumers observe the price p and a summary statistic I of information obtained from generation 1 consumers, and make simultaneous purchasing decisions.

We consider 3 options for the information transmission I :

⁴The ability to target advertising communications based on consumer characteristics and preferences is becoming increasingly more precise and common on digital platforms such as Facebook and Google. The assumption that the advertising interval is centered around y is without loss of generality. If we allow the firm to choose the center of the interval as well as its radius we find that it will always choose the product's position y . The proof (available from the authors upon request) follows a simple revealed preference argument.

1. Benchmark: generation 2 consumers only learn about the existence of the product.
2. Average Rating (AR): generation 2 consumers know about the existence of the product and observe the mean utility of first generation consumers.
3. Reviews: generation 2 consumers know about the existence of the product and observe the joint distribution of the utilities of first generation consumers and their locations.

To elaborate, we assume that given the visible consumption of the product by at least some generation 1 consumers that the baseline information transferred across generations is that the product exists and what its price is. When consumers in generation 1 also indicate a summary of their overall assessment of the product linked directly to their net utility from it, for example through an online rating system,⁵ we assume that generation 2 consumers have access to the mean utility through the average rating. Lastly, if generation 1 consumers who purchased the product not only provide a measure of their net utility but also write a review in which they discuss the fit of the product with their preference location, generation 2 consumers upon reading these reviews learn the utilities as well as locations of those who purchased the product.

In making its pricing and advertising decisions, the firm seeks to maximize total profits across generations with no discounting. To ensure interior solutions, we place an upper bound on the parameters τ and β (the location disutility parameter and the marginal sensitivity to quality respectively). These upper bounds imply that there is always some consumer (assuming he or she is made aware of the product) buying at the farthest location from the product and some consumer not buying the product at the closest location. In other words, the product is not ex-ante restricted to a niche population along the circle nor is it appealing to everyone at a given price. (See the Appendix for exact expressions).

⁵In many online systems, consumers can only choose from a discrete set of rating options, e.g., how many stars to give between 1 and 5. Yet the aggregated average rating presented to subsequent consumers is continuous, and one can show that a sufficiently large number of ratings will fairly accurately depict the true mean across the population of raters.

3.1 Analysis

For any given price and advertising level (p, z) , we can provide closed form solutions for the demand of each generation. Notably, we do not incorporate strategic behavior on the part of generation 1 consumers that is meant to influence the demand of generation 2 consumers.⁶ Hence, the demand of generation 1 consumers is independent of the information option I , and of the decisions of generation 2 consumers, and can be written as⁷

$$D_1 = z(1 + \beta - z\tau - p). \quad (1)$$

The mean utility, i.e., average rating, of purchasing consumers is, in turn

$$\mu_1(z) = \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)}. \quad (2)$$

An important observation is that $\frac{\partial \mu_1}{\partial z} < 0$ and therefore average ratings decrease in the size of the advertising interval. This is intuitive: increasing the advertising interval leads to an increase in the average distance between those consumers aware of the product and its position on the circle, which leads to a decrease in the average utility for any given quality q .

To analyze the behavior of generation 2, note that the three informational cases differ in the type of inferences that these subsequent consumers are able to make. In the benchmark case, generation 2 consumers only learn that the product exists and that its price is p , but remain unaware of q and y . In the AR case, given the average rating they observe, generation 2 consumers can in addition establish an estimate for q^e based on what they expect the size of the advertising interval to have been (but not the location of this interval).⁸ In particular,

⁶For example, one could argue that consumers derive positive utility from informing generation 2 consumers of their experiences, say due to altruistic motivations or a desire to build reputation as providers of reviews. We abstract away from these kind of considerations by assuming that all generation 1 consumers who purchased the product unconditionally provide the relevant input. We could relax that assumption by having some positive fraction of consumers who always provide the relevant information. Later we discuss the implications of our analysis for whether firms should encourage certain information transfers by consumers (e.g., via incentives or ease of platform use).

⁷See Proposition 8 and its proof in the Appendix for the derivation of expressions 1 and 2.

⁸Digital advertising methods today (such as search and Facebook ads) allow for very tailored messages

through the inverse function for the expression for $\mu_1(z)$ in (2), we have

$$q^e = -\frac{-6p - 6\mu_1 + 4(z^e)^2\tau^2 + 6p\mu_1 - 6z^e\tau - 3\beta^2 - 6\beta\mu_1 + 3p^2 + 6pz^e\tau + 6z^e\tau\mu_1 + 3}{6\beta - 6p\beta + 6\beta^2 - 6z^e\beta\tau},$$

where z^e captures consumers' expectations with respect to the size of the advertising interval z . In equilibrium, consumers will have correct expectations with respect to z . Therefore, as long as the firm chooses an advertising interval consistent with its equilibrium strategy, generation 2 consumers will infer quality correctly and $q^e = q$. Finally, in the Reviews case, generation 2 consumers observe the joint distribution of generations 1 consumers' utilities and locations, and are therefore able to accurately infer q and y .

We next turn to analyze the demand of generation 2 under the 3 informational cases.

3.1.1 Demand

The demand in period 2 depends on the inter-generation information transmission. We denote by D_2^0 , D_2^{AR} , D_2^R the demand of generation 2 consumers in the benchmark, Average Ratings, and Reviews cases, respectively. In Lemmas 2-4 in the Appendix we establish that the size of generation 2 demand in the benchmark, AR, and Reviews cases is

$$D_2^0(p) = 1 + \beta - \tau - p, \quad D_2^{AR}(p) = 1 + \beta q^e - \tau - p, \quad \text{and} \quad D_2^R(p) = 1 + \beta q - \tau - p, \quad (3)$$

respectively.

As can be gleaned from (3), the demand in period 2 differs across information structures. In particular, when neither ratings nor reviews are transmitted (benchmark case) the demand is independent of quality. However, when more information is transmitted, demand depends on either expected quality (AR case) or on actual quality (Reviews case). In equilibrium, as expectations are fulfilled, $z^e = z$ and $q^e = q$, and therefore the demand of generation 2

to be sent to consumers; yet who receives these messages is known only to the consumer (i.e., unknown to generation 2 consumers who then cannot infer location). Equivalently if consumers have correct expectations about the amount the firm spends on advertising they can establish the length of the advertising interval given the advertising cost function; but not where the set of targeted consumers are on the circle.

consumers in the AR case becomes

$$D_2^{AR}(p) = 1 + \beta q - \tau - p, \quad (4)$$

and hence the following relationships hold:

Lemma 1 *In equilibrium, $D_2^R(p) = D_2^{AR}(p)$, and if $q \gtrless 1$ then $D_2^R(p), D_2^{AR}(p) \gtrless D_2^0(p)$.*

Proof. Immediate from (3) and (4). ■

That is, holding price constant, realized generation 2 demand (in terms of size) is the same in the AR and Reviews cases. However, it is important to stress that, even if the prices are identical in both cases, the set of consumers who purchase will be different. In the AR case, purchasing consumers (who do not know the position of the product) will be uniformly spread around the circle, whereas in the Reviews case purchases will be centered around the product's position. Note further from Lemma 1 that since $E(q) = 1$, ex-ante expected demand in generation 2 is equal across the three informational cases. However, realized (ex-post) generation 2 demand in the benchmark case will be lower than that of the other two cases when $q > 1$, and the reverse is true when $q < 1$.

3.1.2 Marketing Decisions: Pricing and Advertising

The firm takes into account how demand plays out and chooses the price and advertising interval at the beginning of $t = 1$ to maximize its profits. In the benchmark case, the firm solves

$$\max_{z,p} \Pi = p \left(D_1(z, p) + E_q \left(D_2^0(p) \right) \right) - A_y(z)$$

or

$$\max_{z,p} \Pi = pz(1 + \beta - z\tau - p) + p(1 - \tau - p + \beta) - A_y(z).$$

Taking the first-order condition with respect to prices yields

$$p = \frac{1}{2z + 2} (z + \beta - \tau + z\beta - z^2\tau + 1), \quad (5)$$

and the respective first-order condition for the advertising interval is

$$p(1 + \beta - 2z\tau - p) = A'_y(z). \quad (6)$$

Equations (5) and (6) provide an implicit solution for (p, z) in the benchmark case. As we learned from Lemma 1, for any given advertising interval expected demand size (as a function of p) is identical across the informational cases. It is therefore not surprising that (5) remains unchanged in the different cases. On the other hand, the first-order condition with respect to the advertising interval does vary across information settings. In particular, in the AR case we have

$$p(1 + \beta - 2z\tau - p) + p\beta \frac{\partial E(q^e)}{\partial z} = A'_y(z). \quad (7)$$

Thus, the advertising-interval first order condition in the AR case introduces an additional term, $p\beta \frac{\partial E(q^e)}{\partial z}$, reflecting the fact that the firm takes into account that generation 2 consumers will form an expectation of quality based on the average rating they observe. Interestingly, the first order conditions are identical for the benchmark and Reviews settings. This is because generation 2 demand in both cases is independent of the advertising interval the firm selects. Intuitively, in the Reviews case generation 2 consumers extract all the information about the product (q and y) regardless of the size of the advertising interval (as long as $z > 0$), leading to an independence between the choice of first period interval and generation 2 demand. In the benchmark case, the reason for independence between the first period advertising interval and generation 2 consumption is the opposite: generation 2 consumers do not learn any information about the product regardless of the advertising interval selected.

The following proposition provides a comparison of the three cases in terms of the implications for the optimal pricing and advertising strategies.

Proposition 1 *The optimal price and advertising interval are the same in the Reviews and the benchmark cases, i.e., $p^0 = p^R$ and $z^0 = z^R$. The advertising interval is smaller in the AR case than in the benchmark and Reviews cases ($z^R, z^0 > z^{AR}$), and the price can either*

be higher or lower in the AR case than in the benchmark and Reviews cases. Specifically, there exist functions \bar{f} and \underline{f} with $\bar{f}(z) \geq \underline{f}(z)$, such that if for all $z \in \mathbb{R}^+$ $A_y(z) \geq \bar{f}(z)$ then the price in the AR case is lower than the price in the benchmark and Reviews cases ($p^{AR} < p^0 = p^R$), and if for all $z \in \mathbb{R}^+$ $A_y(z) \leq \underline{f}(z)$ the reverse is true ($p^{AR} > p^0 = p^R$).

Proof. Follows directly from Lemmas 5-7 in the Appendix. ■

Proposition 1 reveals that the firm will actually seek to advertise to a smaller set of consumers in the AR case than in the other two information settings. The reason is as follows. Generation 2 consumers use their belief with respect to z (i.e., z^e) to make an inference regarding q (i.e., q^e). The firm takes as given consumers' beliefs when making its advertising decision. For any given q , a decrease in z means that consumers whose ideal point is closer to the product's position will be the ones to buy. This, in turn, leads to an increase in the average ratings of generation 1, and thus to an increase in generations 2's estimate of q . Said differently, since generation 2 consumers do not observe the distribution of locations and ratings, they cannot fully separate out whether an increase in average ratings is due to the quality turning out to be high ex-post or due to consumers with better matching tastes rating the product. Hence, they will attribute part of an increase in AR to quality and this introduces an incentive for the firm to decrease z . This incentive is absent in the benchmark and Reviews cases as generation 2 demand is independent of the advertising interval chosen (as explained in connection with Proposition 1).

The intuition for the second part of the result regarding pricing in the AR case is more subtle, and relies on the following observation: given the targeted nature of the advertising strategy, the sales in period 1 are centered around the true position of the product (which is known to the firm). In the AR case, generation 2 consumers do not learn anything about the product positioning, and hence sales in period 2 are homogeneously spread around the circle. As a result, an increase in z has two confounding effects on the price. First, it increases overall demand in period 1 because more generation 1 consumers are aware of the product. Then, because awareness in period 1 is concentrated around y (as long as $z < 1$) given the interval the firm chooses to advertise to, the increase in the demand of generation 1 consumers relative to the demand of generation 2 consumers leads to an increase in the combined average willingness to pay. Second, an increase in z increases the average distance

from y of an aware generation 1 consumer, and therefore it lowers the average willingness to pay of generation 1 consumers. If the first effect dominates, the overall willingness to pay rises and the firm increases its price, whereas if the latter effect dominates the overall willingness to pay decreases and the firm decreases its price. It is left then to note that the first effect dominates when z^0 is small (which occurs when advertising is relatively costly, $A_y(z) \geq \bar{f}(z)$) and the latter dominates when z^0 is large (which occurs when advertising is relatively cheap, $A_y(z) \leq \underline{f}(z)$). The reason is that when z^0 is small, the added generation 1 consumers gained by reducing price are still relatively close to y , which is not the case when z^0 is large.

3.1.3 Average Rating Dynamics

Before turning to analyze the profit and welfare outcome properties of the different informational cases, we wish to point out one set of testable empirical implications: tracking the dynamics of average ratings across generations. We denote by $\mu_2^0, \mu_2^{AR}, \mu_2^R$ the average ratings of generation 2 in the benchmark, AR, and Reviews cases, respectively. From Lemmas 2-4 in the Appendix we know that:

$$\mu_2^0 = \frac{1 - p - \tau + (2q - 1)\beta}{2};$$

$$\mu_2^{AR} = \frac{1 + \beta q - \tau - p}{2};$$

and

$$\mu_2^R = \frac{3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 4\tau^2 - 6\tau + 3}{6(1 + \beta q - p - \tau)}.$$

The following result summarizes the rating dynamics in our model.

Proposition 2 *The rating dynamics in the three informational cases are:*

1. *Benchmark: the average rating of generation 2 consumers is always lower than the average rating of generation 1 consumers.*
2. *Average Ratings (AR): there exists a \bar{q}^{AR} ($\bar{q}^{AR} \leq 1$) such that the average rating of generation 2 consumers is lower than the average rating of generation 1 consumers if*

and only if $q > \bar{q}^{AR}$.

3. *Reviews:* there exists a \bar{q}^R such that the average rating of generation 2 consumers is lower than the average rating of generation 1 consumers if and only if $q > \bar{q}^R$. Moreover, $\bar{q}^R > \bar{q}^{AR}$.

Proof. Follows directly from Lemmas 8-10 in the Appendix. ■

Proposition 2 reveals that as more information is transferred across generations the more likely it is for average ratings to increase. In particular, when there is no consumption-related information passed on, per the benchmark case, we get that the average rating the product receives will unequivocally go down over time. This occurs because in the first period only generation 1 consumers close to the product's position were made aware of the product and hence derived relatively high net utility. In the second period, all generation 2 consumers are aware of the product and thus anyone along the circle with non-negative expected utility will purchase (as they do not know the location of those who were targeted in period 1 nor can they infer q). This results in many consumers with lower valuations purchasing in generation 2, some of whom may actually end up deriving negative utility ex-post. Hence the declining pattern for ratings in the benchmark case. At the other extreme, per the Reviews case, there can be a scenario where average ratings increase across generations and, somewhat surprisingly, this occurs when the quality of the product is actually revealed to be low. Here, generation 2 consumers know the location y and quality q of the product. This information allows those consumers who would get negative utility to avoid purchasing altogether. If the quality of the product turns out to be low enough ($q < \bar{q}^R$) then many generation 1 consumers who were made aware of the product would have purchased the good (given their ex-ante expectation that $q = 1$) only to be disappointed after their consumption (despite being located close to the product's position); and this dynamic will be reflected in low ratings in period 1 but higher ratings in period 2. In the AR case generation 2 consumers infer q , hence once again it is possible for ratings to increase. However, average ratings increase across generations for a smaller range of qualities when only average ratings are transferred for two reasons: a) generation 2 consumers cannot infer the positioning of the product (y) and thus still attribute some of the average rating to quality, b) the smaller

advertising interval in the AR case implies relatively high average ratings for generation 1 consumers.

3.1.4 Welfare

In this section we derive welfare implications. We begin by evaluating the profits commanded under each of the information transfer structures and show, somewhat counterintuitively, that more information passed across generations may hurt the firm. We then assess consumer surplus and conclude with the total welfare.

Profits Using the equilibrium values for the prices and advertising decisions in each case, we have

Proposition 3 *Expected firm profits are the same in the benchmark and Reviews cases, and lower in the Average Rating case. That is, $E(\Pi^0) = E(\Pi^R) \geq E(\Pi^{AR})$.*

Proof. See Appendix. ■

We first note that because in previous propositions we established that the firm sets the same prices and advertising intervals in the benchmark and Reviews cases, it is perhaps not surprising that the expected profit levels end up being the same, i.e., $E(\Pi^0) = E(\Pi^R)$ (of course depending on the realization of q and transfer of this information through the Reviews, the ex-post profit levels can differ). As for the AR case, it is easy to see that, in equilibrium, the profit function is the same as in the benchmark case. However, in the AR case the firm maximizes this function as if it can affect q^e , which introduces a distortion (the firm would have preferred to commit to the benchmark actions but instead optimally restricts the advertising interval), and this results in lower overall profits in the AR case.

Consumer surplus (CS) There are three forces affecting CS in the different cases. Specifically:

1. For a given informational case and advertising interval, a lower price increases CS.
2. For a given informational case and price, a larger advertising interval increases CS.

3. For a given price and advertising interval, when consumers have more information they make better consumption decisions and their surplus increases. Specifically,

- (a) If consumers know q , more of them are likely to buy when q is high and fewer when q is low, which generates higher welfare than buying solely based on the expected value of q .
- (b) If consumers know y , consumers that are closer to the product's position are more likely to buy than consumers that are farther away, which generates higher CS than buying based on the distance from an expected location for y .

Based on these observations we can state the following

Corollary 1 $CS^0 < CS^R$. Moreover, there exists a function \underline{f} such that if for all $z \in \mathbb{R}^+$, $A_y(z) \leq \underline{f}(z)$ then $CS^{AR} < CS^R$.

Recall that $p^0 = p^R$, $z^0 = z^R$, and that expected aggregate demand is identical in the Reviews and benchmark cases. Therefore, generation 1 consumers have the same CS in the Reviews and benchmark cases. Generation 2 consumers have better information in the Reviews than the benchmark case. Consequently, $CS^0 < CS^R$. For the second part of the proposition, recall from Proposition 1 that there exists a function \underline{f} such that if for all $z \in \mathbb{R}^+$, $A_y(z) \leq \underline{f}(z)$ the price under AR is higher than the price in the benchmark and Reviews cases. In that scenario then, the price is lower, the advertising interval is larger, and generation 2 consumers have more information in the Reviews relative to the AR case—thus satisfying all three conditions for greater CS outlined above.

Corollary 1 thus reveals that CS is always higher in the Reviews than in the benchmark case, and if advertising is sufficiently cheap, then CS in the AR case is lower than in the Reviews case.

Total welfare With the findings on profits and consumer surplus in hand, we can assess total welfare (denoted W). There are two forces determining the welfare in the different cases:

1. The maximum possible aggregate welfare (the social planner’s solution) is for everyone to get the product. This means that in all of the cases there is an under-provision of the product relative to the welfare maximizing provision. This is the standard monopoly result. This further implies that cases in which there is higher consumption levels generate greater welfare.
2. When consumers have more information, their consumption decisions are better (in the sense of net utility), and allocation is more efficient. This force was present in the CS analysis as well.

Given (1) and (2) and taking into account Proposition 3 and Corollary 1 we get the following:

Corollary 2 $W^0 < W^R$. Moreover, there exists a function \underline{f} such that if for all $z \in \mathbb{R}^+$, $A_y(z) \leq \underline{f}(z)$ then $W^{AR} < W^R$.

The corollary indicates that total welfare is always higher in the Reviews case than in the benchmark case, and if advertising is sufficiently cheap, then total welfare in the AR case is lower than in the Reviews case.

3.2 Awareness through Social Interaction

Recall that in the basic model setup presented in the previous section we assumed that all generation 2 consumers are aware of the product in period 2. This was meant to reflect settings whereby once a new product has been in the marketplace for a full period then, either through WOM, observational learning, or media coverage, generation 2 consumers are apprised of the product’s existence. As noted, the results from analyzing this setup are qualitatively unaffected if only an exogenous fraction of generation 2 consumers become aware of the product.

While this is a reasonable assumption, which allowed us to focus on the implications of various user-generated inputs available to generation 2 consumers (i.e., average ratings and/or full reviews from generation 1 customers), one wonders how micro-modeling the inter-generational awareness process might affect our findings and whether new insights can be

obtained in so doing. Furthermore, if indeed generation 2 awareness is contingent upon some form of social contagion from past customers, then one can expect demand in period 2 to be a direct function of the demand size in period 1; a feature that was absent in the basic setup. To explore these issues in greater depth, we now relax the assumption of “automatic” generation 2 awareness and let it depend on social interactions with generation 1 consumers.

Let generation 2 consumer j who is located at x interact with a sample of generation 1 consumers located according to the following distribution: with probability η these are a representative subset of consumers located at x , and with probability $(1 - \eta)$ they are a representative subset of consumers from a location drawn uniformly at random from around the circle. Consumer j knows the underlying process of sampling. If any sampled generation 1 consumer purchased the product, then consumer j becomes aware of it, otherwise she remains unaware. The parameter η ($0 < \eta \leq 1$) captures to what extent a generation 2 consumer is likely to be paired with a generation 1 consumer who is exactly like her in terms of horizontal tastes, in other words the degree of “homophily” between them. In particular, if $\eta = 1$, the consumer knows she is interacting with consumers that have the exact same location as she does, whereas if $\eta \rightarrow 0$, her interaction is with people she is (maximally) unsure about their preference along the circle.

Once a generation 2 consumer is aware of the product, we might consider two ways to model how she might gain more information about it. The first is to assume, as in the basic setup, that all generation 1 consumers who purchased the product provided some input about their experience, either in the form of average ratings or reviews (or a benchmark of no information), and all aware generation 2 consumers have access to this input. The second is to assume that an aware generation 2 consumer learns exclusively from the generation 1 consumers she is paired with, i.e., both awareness and any information about the consumption experience are transmitted through social interaction. We concentrate on presenting results from the former setting, as it allows a clean comparison to the basic model setup and bears on understanding the impact of various user-generated inputs on firm behavior.

For analytic tractability, we will assume that prices are exogenously given. This simplifies the analysis and allows focusing on the effects that micro-modeling awareness through inter-generational interaction has on the firm’s choice of advertising the interval and the resulting

welfare and ratings pattern. In the Technical Appendix, we provide numerical simulations showing that for a wide range of parameter values all our results qualitatively hold if pricing is endogenized.

It is easy to verify that the demand and ratings of generation 1 consumers carry over unchanged from the basic model of Section 3. Using superscript N to denote the setup with social network interactions across generations, we can therefore write

$$D_1^N = D_1 = z(1 + \beta - z\tau - p)$$

with the mean utility (i.e., average rating) of consumers who purchased the product

$$\mu_1^N(z) = \mu_1(z) = \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)}.$$

3.2.1 Generation 2 Demand

The demand in period 2 depends on the inter-generation transmission of information. We denote by D_2^{N-0} , D_2^{N-AR} , D_2^{N-R} generation 2 demand in the benchmark, Average Ratings (AR), and Reviews cases, respectively, and by z^e the advertising interval expected by generation 2 consumers. Proposition 9 in the Appendix establishes that $D_2^{N-0}(z) = z(1 - p + \beta - \tau + \tau\eta - z^e\tau\eta)$, $D_2^{N-AR}(z) = z(1 - p + q^e\beta - \tau + \tau\eta - z^e\tau\eta)$, and $D_2^{N-R}(z) = z(1 - p + q\beta - \tau + \tau\eta - z\tau\eta)$.

It is apparent that the demand in period 2 differs across information structures. In particular, when neither ratings nor reviews are transmitted (benchmark case) the demand is independent of quality. However, when more information is transmitted, demand depends on either expected quality (AR case) or on actual quality (Reviews case).

In equilibrium, as expectations are fulfilled, $z^e = z$ and $q^e = q$, and therefore

$$D_2^{N-0}(z) = z(1 - p + \beta - \tau + \tau\eta - z\tau\eta); \quad D_2^{N-AR}(z) = z(1 - p + q\beta - \tau + \tau\eta - z\tau\eta).$$

This implies that, similar to the basic model setting, the following relationships hold

Corollary 3 *In equilibrium, $D_2^{N-R}(z) = D_2^{N-AR}(z)$, and if $q \gtrless 1$ then $D_2^{N-R}(z), D_2^{N-AR}(z) \gtrless D_2^{N-0}(z)$.*

That is, holding the advertising interval constant, realized generation 2 demand (in terms of size) is the same in the AR and Reviews cases and has the same relation to the benchmark case, in terms of realized the quality, as in the basic model setup.

It is noteworthy that all the demand expressions have a multiplying factor of z , the advertising interval size. This is intuitive as only consumers in generation 1 that were targeted with advertising could have purchased the good and hence serve as a source of awareness for generation 2 consumers. Consequently, it should not be surprising that, under all information structures, the advertising interval the firm selects is larger in the social-interaction model studied here than in the basic model.⁹

It is further noteworthy that in all informational cases generation 2 demand depends on the homophily parameter η . This is because greater homophily implies higher chances of becoming aware of the product from someone who is more like you in terms of location, rather than someone randomly located along the circle, which in turn increases the likelihood of being close to the product's location.

3.2.2 Advertising

Solving for the optimal advertising interval in the model with social interactions, we can state the following dependence on the degree of homophily.

Proposition 4 *In the Benchmark and AR conditions, the advertising interval the firm selects is increasing in the level of homophily. In the Reviews condition, the advertising interval is increasing in the level of homophily if and only if $z^{N-R} < 1/2$.*

Proof. See Appendix. ■

The intuition for Proposition 4 is as follows. In the Benchmark and AR conditions, generation 2 consumers do not learn the exact location of the product, but rather draw

⁹Put differently, because demand in period 2 directly depends on the size of the demand from period 1, the firm has an incentive to advertise more extensively in period 1 to generate a sufficient number of “social contagion agents”.

an inference about it based on z^e and on the fact that they became aware of the product through the social interaction mechanism. As homophily gets stronger (η increases), then more individuals who are at a smaller than average distance from the product are made aware of the product (and also gain collective average ratings information in the AR case). Moreover, the aware generation 2 consumers infer their expected distance from the product's position based on z^e , which does not take into account the firm's increase of the interval of aware generation 1 consumers. Hence, from the firm's point of view, by enlarging the advertising interval it gains more generation 2 consumers who think they are closer to the product than they actually are.

Intuition for the effect of homophily in the Reviews condition is as follows: If $z^{N-R} < \frac{1}{2}$, for example because the cost of advertising is large, enlarging the advertising interval increases demand in generation 1 in locations that are still relatively close to the product's position. A strong degree of homophily, by definition, means that this action will increase awareness in generation 2 in locations that are closer than average to the product; which is beneficial to the firm. The opposite occurs, however, when $z^{N-R} > \frac{1}{2}$.

We now compare the advertising interval selected under the various information structures.

Proposition 5 *The advertising interval is the largest in the benchmark case and is smaller in the AR than in the Reviews case if homophily is sufficiently weak.*

Proof. See Appendix. ■

The reason that the advertising interval is largest in the benchmark condition is that in this case increasing the interval results in more aware consumers in period 2 who think they are closer to the product than they are (because they rely on z^e and not on z to form their expected distance from the product), hence the firm has a strong incentive to increase z . As for the second part of the proposition, our analysis from the basic model provides the intuition underlying the result that for weak homophily the advertising interval in the AR condition is smaller than in the Reviews condition. However, it is not immediate that this is the case for strong homophily, despite the fact that $\frac{dE_q(q^e)}{dz} < 0$. This is because an increase in the advertising interval in the AR case does not change consumers' expectations

with respect to their distance from the product, as these are based on z^e (and not z). If the firm then increases z in a strong homophily environment, generation 2 consumers relying on z^e believe, on average, that they are closer to the product than they really are.¹⁰ In the Reviews case, by contrast, consumers know the location of the product so this incentive to increase z is absent.

3.2.3 Average Rating Dynamics

Turning to assessing how the process of generating awareness through social interaction impacts average rating dynamics, we have

Proposition 6 *In the Benchmark case, for any level of homophily, generation 2 ratings are lower than generation 1 ratings. In the AR and Reviews cases, ratings increase over time for a larger range of qualities in the social-interaction setting compared to the basic model.*

Proof. See Appendix. ■

The intuition for the first part of Proposition 6 follows the same logic as in the basic model. For the second part, we begin by noting that in all informational cases (benchmark, AR, and Reviews), generation 2 ratings are higher in the social-interaction setting compared to the corresponding basic model. The reason is that with social interactions, and any positive level of homophily, the aware consumers in generation 2 are concentrated in closer proximity to the product's location than in the basic model. Furthermore, according to the observation made following Corollary 3, in all informational cases the advertising interval is larger in the social-interaction setting – and this implies that generation 1 consumers who are farther away from the product's location are made aware of the product compared to the basic model. Thus, generation 1 ratings are lower in the social-interaction setting.

3.2.4 Welfare

Profits The following proposition examines properties of the profits in the social-interaction setting and offers a sense of how they compare to profits in the basic model.

¹⁰This is true when taking the first order conditions, even though in equilibrium consumers should have correct expectations and we plug in $z^e = z$ when solving.

Proposition 7 *In the social-interaction setting, the highest expected profits are achieved in the Reviews condition, yet these profits are lower than those in the Reviews and Benchmark conditions of the basic model. That is, $E(\Pi^{N-0}), E(\Pi^{N-AR}) \leq E(\Pi^{N-R}) \leq E(\Pi^R) = E(\Pi^0)$.*

Proof. See Appendix. ■

In all three information transfer cases, the equilibrium expected profit functions take the same form. However, only in the Reviews condition is the firm maximizing the correct function with no distortions, leading to higher profits than in the Benchmark and AR conditions. To see why profits in the Reviews condition are greater in the basic vs. the social-interaction model, note that in the former setup all generation 2 consumers are aware of the product, whereas in the latter case awareness is more restricted. Given that aware generation 2 consumers otherwise have identical information in both models, the effect of awareness on profits dominates.

Consumer Surplus (CS) There are two forces affecting CS. Specifically:

1. For a given informational case, a larger advertising interval increases CS.
2. For a given advertising interval, when consumers have more information they make better consumption decisions and their surplus increases.

From these observations, it follows that if homophily is sufficiently weak then $CS^R \geq CS^{AR}$ and $W^R \geq W^{AR}$.

4 Concluding Remarks

In this paper, our goal has been to study how a firm's strategic marketing decisions depend on the type of consumer-generated content available to prospective buyers. In particular, we focused on whether reviews are available in addition to average ratings and developed a two-period model in which a firm introduces a new product and selects its advertising and pricing strategies. Consumers do not ex-ante know the quality of the product and a

sub-set of them become aware of the existence and positioning of the product when targeted by the firm’s advertising. Customers in the first period who purchased the product might be able to share their experience with consumers in the second period. We analyze three customer information transfer structures: a benchmark case where no consumer evaluations are passed on; an Average Rating case where second-generation consumers observe only the average net utility of those who purchased; and a Reviews case where second-generation consumers observe both the distribution of net utilities as well as the distribution of taste locations of those who purchased.

The basic model we analyzed is one where all second-generation consumers become aware of the product, reflecting settings where word of the product’s existence has spread (through the media, WOM, or observational learning). In this context, we first show that while the firm selects the same advertising and pricing strategy in the Reviews and the benchmark cases, its advertising scope in the AR case is more limited. The reason behind this result is that if customers expect the firm in the AR case to select the same advertising level as in the Reviews case, the firm then has an incentive to deviate and target a smaller segment of customers. This deviation would increase the average rating seen by second-generation consumers who will infer a higher quality product; presumably generating a higher profit for the firm. In addition, we find that the relative price the firm sets in the different informational cases depends on the cost to advertise. The analysis reveals that when advertising is relatively costly, the price set in the AR case is lower than that in the benchmark case, and the reverse is true when advertising is relatively cheap.

We further present several results regarding the dynamics of average ratings. As customer-generated assessments become more informative, the average rating is more likely to increase over time. This is because when reviews are available, a second-generation consumer learns about the product’s positioning and quality, and is thus better able to make purchase decisions based on the product’s positioning characteristics. In comparison, when only average ratings are available, such learning is impossible. In addition, we characterize conditions for average ratings to decrease across generations in the Reviews and Average Ratings cases. Interestingly, such a declining pattern happens when first-generation consumers discover that the product is of relatively high quality and their input prompts more generation 2

consumers, many of whom will have very low net positive utility, to purchase.

We further analyze the impact of the different information structures on profits, consumer surplus, and welfare. We find that relative to the benchmark case with no information, when consumers have access to average ratings this leads to lower expected firm profits, while access to reviews has no impact on expected profits. Consumer surplus and total welfare are higher in the Reviews than in the benchmark case and, if advertising is not too costly, they are also higher than in the AR case.

We separately examine an alternative setting where awareness in the second period is a function of social interactions across generations, while still differentiating between the three information structures (benchmark, Average Ratings, Reviews). Specifically, a second-generation consumer interacts with a sample of first-generation consumers whose location along the circle is either the same as her own, reflecting homophily, or randomly drawn. In this context, we find that the firm will cast the widest advertising net when no information is transferred across generations (i.e., the benchmark case). Furthermore, as the degree of homophily increases, i.e., more second-generation consumers expect to be paired with consumers like them, the firm tends to advertise more extensively in the first period. This is primarily true in the benchmark and AR cases because aware second-generation consumers don't know the product's exact position but presume they are close to it as homophily is strong. The firm tries to exploit this by targeting more consumers in the first period who can serve as social contagion agents. Lastly, with social interactions generating awareness we can expect greater chances for average ratings to exhibit an increasing pattern over time than in the basic model.

Our results have implications for firms, regulators, and platforms such as Amazon. First, firms are well advised to allow reviews on their webpages. Reviews do not diminish a firm's profits, whereas they increase consumer surplus. In particular, firms should strictly prefer, and thus encourage, consumers to leave detailed reviews, rather than only ratings. For a similar reason, regulators should allow and incentivize firms to create outlets for reviews and encourage prospective buyers to read them. Incentives may be required, because there is a positive externality generated when a firm introduces a system for reviews—consumer surplus is enhanced. The results also provide a rationale for market-maker platforms, such as

Amazon, investing extensively in review systems, including highlighting informative reviews, motivating consumers to review through reward schemes, and making reviews easily searchable. By doing so Amazon increases firms' profits and consumers' surplus, and is therefore able to attract many marketplace participants on both sides of the platform.

In terms of advertising strategy, the results have several implications that firms should heed to. A firm that enables reviews and expects consumers to read them, beyond just looking at average ratings, should expand its advertising program when launching a new product. And, if social interactions play a big role in generating awareness across generations, a firm may benefit from increasing its ad spend early on in the product's lifecycle. And this advice is especially relevant when "like-minded" consumers with a high degree of homophily are expected to connect with each other.

Another key observation emanating from our analysis is that a firm should not take a decreasing pattern of ratings as a necessarily bad sign. In fact, a higher quality product, which is likely to generate greater overall profits, can lead to such an outcome.

Although this paper analyzes the critical impact of user-generated content on a firm's strategic decisions and ratings dynamics, we recognize that we have focused our attention on a particular market setting. There are various ways to extend our analysis and enrich our understanding of the ramifications of consumers sharing their experiences with peers. One such extension could be that, instead of targeting advertising to consumers close to the position of its product, the firm can only select how much to expend on advertising and the higher the spend the more likely any consumer in period 1 may become aware of the product regardless of his or her location. In that extension, we find that the firm selects the same advertising level and price in all information transfer structures. In the benchmark and Reviews cases, we find that price is lower than the equivalent price under targeted advertising (our basic model set-up), while the number of customers reached can be higher or lower than with targeted ads. The main reason behind the lower price is that the demand is evenly spread along the circle with this type advertising and consumers evaluate the product as having an average degree of fit, whereas under targeted advertising the consumers are located near the product and use the exact product fit. As for the evolution of average ratings if no information is shared across generations, average rating remains constant. However, if a

later generation observes only the average rating of an earlier generation, the average ratings can go up or down. Finally, the average rating always goes up if consumers have access and read full reviews. Such an extension confirms that consumer-generated content might replace firm-initiated advertising and affects both pricing and advertising strategies. The impact on consumer surplus would be interesting to unveil as consumers are less informed about product fit/quality but also potentially face a lower price.

5 Appendix A: Deriving parameters for which our analysis is valid.

As explained the model set up, we wish to focus on interior solution for price and advertising. The required assumptions on the parameters τ and β are

Assumption 1 $1 - 2\tau > 2\beta$

Assumption 2 $1 > 3\beta$

The above assumptions are required to guarantee that the following holds in equilibrium:

1. $1 - 2\tau - p > 0$. Thus, there is always some consumer (assuming he or she is aware of the product) buying in the farthest location from the firm even if $q = 0$. In other words, the product is not restricted to a niche population along the circle but can appeal to a broad audience.
2. $2\beta - p < 0$. Thus, there is always someone not buying the highest quality in the closest location.

As can be seeing in the proofs in Appendix B, for any informational setup our analysis proceeds in 3 steps.

Step 1: Consumers' behaviors given (p, m) .

Step 2a: The firm's marketing strategy (z) given p and taking into account consumers' expected behavior.

Step 2b: The firm's pricing strategy given m and taking into account consumers' expected behavior.

In step 1, we focus on the case that $1 - 2\tau > p > 2\beta$. The mathematical restrictions that it puts on the analysis are the following: The optimization that we analyze for the firm's pricing decision allows the fractions of consumers who purchase the good in a certain location to go outside the $(0, 1)$ permissible interval. This should not be an issue if the firm chooses a price $p \in (2\beta, 1 - 2\tau)$ and an interval with $z < 1$. We next explain why.

1. As long as the firm's pricing decision is such that $2\beta - p < 0$, then the firm would not like to reduce the price even if we consider the fact that the demand fraction in a location cannot exceed 1. In fact, capping local demand at 1 reduces the firm's incentive to reduce price further.
2. As long as the firm's pricing decision is such that $1 - 2\tau - p > 0$ when $z < 1$, then the firm would not like to increase the price even if we consider the fact that the demand fraction in a location cannot go below 0. This is because the firm is not advertising in locations in which demand is expected to be zero (or below), so the firm was not affected by consequences for these locations when choosing its price.

While we do not obtain a closed form solution for the price and the advertising interval, we provide an implicit characterization for both. Our results are valid for any set of parameters for which, according to the characterization $1 - 2\tau > p > 2\beta$ and $z < 1$. For these parameters, the restriction that we put on our analysis in step 1 (and therefore of steps 2 and 3) is without loss of generality.

6 Appendix B: Proofs

Proposition 8 *The demand of generation 1 is*

$$D_1 = z(1 + \beta - z\tau - p).$$

The mean utility, i.e., average rating, of purchasing consumers is

$$\mu_1(z) = \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)}.$$

In particular we have, $\frac{\partial \mu_1}{\partial z} < 0$.

Proof. Uninformed consumers don't purchase. An informed consumer i who is located at x also

learns y and adopts iff

$$\begin{aligned}\alpha_i &> -\beta E(q) + \tau d(x, y) + p \\ &= -\beta + \tau d(x, y) + p\end{aligned}$$

which holds for a mass

$$\begin{aligned}D_1 &= z(1 - (-\beta + \tau E(d(x, y)) + p)) \\ &= z(1 + \beta - z\tau - p)\end{aligned}$$

of generation 1 consumers, independent of y . With the purchasing consumer from location x who receives the minimum utility out of all purchasing consumers in location x having utility of:

$$u_x|_{\alpha_i = -\beta + d(x, y)\tau + p} = \beta(q - 1)$$

the purchasing consumer at location x who receives the maximum utility out of all consumers in location x having utility of

$$u_x|_{\alpha_i = 1} = 1 + \beta q - d(x, y)\tau - p$$

mean utility of purchasing consumers at location x being

$$\frac{1 - d(x, y)\tau - p + \beta(2q - 1)}{2}$$

the fraction of consumers in location x who purchased being:

$$1 + \beta - d(x, y)\tau - p$$

and mean utility of purchasing consumers in this group being:

$$\begin{aligned}\mu_1(z) &= \frac{\int_0^{2z} \frac{1 - r\tau - p + \beta(2q - 1)}{2} (1 + \beta - r\tau - p) dr}{\int_0^{2z} (1 + \beta - r\tau - p) dr} \\ &= \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(1 + \beta - z\tau - p)}\end{aligned}$$

and

$$\frac{\partial \mu_1}{\partial z} = -\frac{1}{6} \frac{\tau}{(p - \beta + z\tau - 1)^2} (3p^2 + 8pz\tau - 6p\beta - 6p + 4z^2\tau^2 - 8z\beta\tau - 8z\tau + 3\beta^2 + 6\beta + 3) < 0.$$

To see why $\frac{\partial \mu_1}{\partial z} < 0$ note that it has the opposite sign of

$$3p^2 + 8pz\tau - 6p\beta - 6p + 4z^2\tau^2 - 8z\beta\tau - 8z\tau + 3\beta^2 + 6\beta + 3 > 0$$

where the inequality holds because $3p^2 + 8pz\tau - 6p\beta - 6p + 4z^2\tau^2 - 8z\beta\tau - 8z\tau + 3\beta^2 + 6\beta + 3$ is decreasing in z and $z \leq 1$ and

$$3p^2 + 8p\tau - 6p\beta - 6p + 4\tau^2 - 8\beta\tau - 8\tau + 3\beta^2 + 6\beta + 3 > 0$$

where this latter is true because $3p^2 + 8p\tau - 6p\beta - 6p + 4\tau^2 - 8\beta\tau - 8\tau + 3\beta^2 + 6\beta + 3$ is increasing in β and $\beta \geq 0$ and

$$3p^2 + 8p\tau - 6p + 4\tau^2 - 8\tau + 3 > 0$$

which holds because $3p^2 + 8p\tau - 6p + 4\tau^2 - 8\tau + 3$ is decreasing in p and $p < 1 - 2\tau$ and $4\tau^2 > 0$.

■

Lemma 2 *In the no information benchmark, the demand of generation 2 is*

$$D_2^0 = 1 + \beta - \tau - p$$

with the mean utility (i.e. average rating) of purchasing consumers being

$$\mu_2^0(z) = \frac{1 - p - \tau + (2q - 1)\beta}{2}.$$

Proof. Consumer i adopts iff

$$\alpha_i > -\beta + \tau + p$$

which holds for a mass

$$\begin{aligned} D_2^0 &= 1 - (-\beta + \tau + p) \\ &= 1 + \beta - \tau - p \end{aligned}$$

of generation 2 consumers.

The mean utility of a 2nd generation purchasing consumers is then

$$\begin{aligned} \mu_2^0 &= u_x|_{\alpha_i + \beta - \tau - p > 0} = \frac{1 + \beta q - \tau - p + (-\beta + \tau + p + \beta q - \tau - p)}{2} \\ &= \frac{1 - p - \tau + (2q - 1)\beta}{2}. \end{aligned}$$

■

Lemma 3 *In the AR case, the demand of generation 2 is*

$$D_2^{AR} = 1 + \beta q^e - \tau - p$$

where

$$q^e = -\frac{-6p - 6\mu_1 + 4(z^e)^2\tau^2 + 6p\mu_1 - 6z^e\tau - 3\beta^2 - 6\beta\mu_1 + 3p^2 + 6pz^e\tau + 6z^e\tau\mu_1 + 3}{6\beta - 6p\beta + 6\beta^2 - 6z^e\beta\tau}.$$

In equilibrium, $z^e = z$ and $q^e = q$, and therefore the demand of generation 2 consumers become

$$D_2^{AR} = 1 + \beta q - \tau - p.$$

If $q \gtrless 1$ then $D_2^{AR} \gtrless D^0$. The mean utility of a generation 2 consumer is

$$\mu_2^{AR} = u_x|_{\alpha_i + \beta q - \tau - p > 0} = \frac{1 + \beta q - \tau - p}{2}.$$

Proof. Consumers have expectations z^e with respect to the interval the firm advertised on. Then

consumers observe μ_1 and use z^e to infer q^e that solves the following equation

$$\mu_1 = \frac{4(z^e)^2 \tau^2 - 6p + 6q^e \beta - 6z^e \tau - 3\beta^2 + 6q^e \beta^2 + 3p^2 - 6pq^e \beta + 6pz^e \tau - 6q^e z^e \beta \tau + 3}{6(\beta - p - z^e \tau + 1)}$$

or

$$q^e = -\frac{-6p - 6\mu_1 + 4(z^e)^2 \tau^2 + 6p\mu_1 - 6z^e \tau - 3\beta^2 - 6\beta\mu_1 + 3p^2 + 6pz^e \tau + 6z^e \tau \mu_1 + 3}{6\beta - 6p\beta + 6\beta^2 - 6z^e \beta \tau}$$

and

$$\frac{\partial q^e}{\partial \mu_1} = \frac{1}{\beta} > 0.$$

Then, consumer i with inference q^e adopts iff

$$\alpha_i > -\beta q^e + \tau + p$$

which holds for a mass

$$1 + \beta q^e - \tau - p$$

of generation 2 consumers. ■

Lemma 4 *In the Reviews case, the demand of generation 2 is*

$$D_2^R = 1 + \beta q - \tau - p$$

That is, $D_2^R(p) = D_2^{AR}(p)$. If $q \gtrless 1$ then $D_2^R(p) \gtrless D^0(p)$.¹¹ The mean utility of a generation 2 consumer is

$$\mu_2^R = \frac{3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 4\tau^2 - 6\tau + 3}{6(1 + \beta q - p - \tau)}$$

Proof. Consumers infer z , q , and y accurately. Then, consumer i in location x purchases the product iff

$$\alpha_i > -\beta q + d(x, y) \tau + p$$

¹¹Note that D_2^R might be different from D_2^{AR} because p^R might be different from p^{AR} .

which holds for a fraction

$$1 + \beta q - d(x, y) \tau - p$$

of consumers in location x . And for a mass of

$$D_2^R = 1 + \beta q - \tau - p$$

consumers overall.

The mean utility of a consumer in location x is

$$\frac{1 + \beta q - d(x, y) \tau - p}{2}$$

and therefore the mean utility across all consumers is

$$\begin{aligned} \mu_2^R &= \frac{\int_{x=0}^4 \frac{1}{4} \frac{(1 + \beta q - d(x, y) \tau - p)^2}{2} dx}{1 + \beta q - \tau - p} \\ &= \frac{\int_{r=0}^2 \frac{1}{2} \frac{(1 + \beta q - r \tau - p)^2}{2} dr}{1 + \beta q - \tau - p} \\ &= \frac{1}{4} \frac{\int_{r=0}^2 (1 + \beta q - r \tau - p)^2 dr}{1 + \beta q - \tau - p} \\ &= \frac{3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 4\tau^2 - 6\tau + 3}{6(1 + \beta q - p - \tau)}. \end{aligned}$$

■

Lemma 5 *In the no information benchmark, the monopoly price and marketing strategies solve the following equations*

$$\begin{aligned} p &= \frac{1}{2z + 2} (\beta - \tau + z(\beta - z\tau + 1) + 1) \\ p(1 + \beta - 2z\tau - p) &= A'_y(z). \end{aligned}$$

Proof. The monopoly solves

$$\max_{z, p} \Pi = p(D_1(z, p) + E_q(D_2^0(p))) - A_y(z)$$

In the case of no information this becomes

$$\max_{z,p} \Pi = pz(1 + \beta - z\tau - p) + p(1 - \tau - p + \beta) - A_y(z)$$

Prices FOC

$$z(1 + \beta - z\tau - p) - pz + (1 - \tau - p + \beta) - p = 0$$

$$p = \frac{1}{2z + 2} (\beta - \tau + z(\beta - z\tau + 1) + 1)$$

Marketing FOC

$$p(1 + \beta - 2z\tau - p) - A'_y(z) = 0$$

■

Lemma 6 *In the AR case, the monopoly price and marketing strategies solve the following equations*

$$p = \frac{1}{2z + 2} (z + \beta - \tau + z\beta - z^2\tau + 1)$$

$$p(1 + \beta - 2z\tau - p) + p\beta \frac{\partial E(q^e)}{\partial z} = A'_y(z).$$

The advertising interval in the AR case is smaller than the advertising interval in the benchmark, and the price in the AR case can be either higher or lower than the price in the benchmark. We can say more:

- *There exist functions \bar{f} and \underline{f} such that for any $z \in R^+$, $\bar{f}(z) \geq \underline{f}(z)$, and such that if for all $x \in R^+$, $A_y(z) \geq \bar{f}(z)$ then the price in the AR case is lower than the price in the benchmark, and if for all $x \in R^+$, $A_y(z) \leq \underline{f}(z)$ then the price in the AR case is higher than the price in the benchmark. That is, if advertising is sufficiently costly, then the price in the AR case is lower than the price in the benchmark, whereas if advertising is sufficiently cheap, then the price in the AR case is higher than the price in the benchmark.*

Proof. The monopoly solves

$$\max_{z,p} \Pi = p(D_1(z) + E_q(1 + \beta q^e - \tau - p)) - A_y(z)$$

or

$$\max_{z,p} \Pi = pz(1 + \beta - z\tau - p) + p(1 + \beta E(q^e) - \tau - p) - A_y(z)$$

Prices FOC

$$z(1 + \beta - z\tau - p) - pz + 1 - \tau - 2p + \beta E(q^e) + p\beta \frac{dE(q^e)}{dp} = 0$$

Next, recall that

$$\mu_1 = \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)}$$

and

$$q^e = -\frac{-6p - 6\mu_1 + 4(z^e)^2\tau^2 + 6p\mu_1 - 6z^e\tau - 3\beta^2 - 6\beta\mu_1 + 3p^2 + 6pz^e\tau + 6z^e\tau\mu_1 + 3}{6\beta - 6p\beta + 6\beta^2 - 6z^e\beta\tau}$$

therefore

$$E(\mu_1) = \frac{4z^2\tau^2 - 6p + 6\beta - 6z\tau + 3\beta^2 + 3p^2 - 6p\beta + 6pz\tau - 6z\beta\tau + 3}{6(\beta - p - z\tau + 1)}$$

and

$$\begin{aligned} E(q^e) &= -\frac{-6 + 6p - 6\beta + 6z^e\tau}{6\beta - 6p\beta + 6\beta^2 - 6z^e\beta\tau} E(\mu_1) \\ &\quad - \frac{-6p + 4(z^e)^2\tau^2 - 6z^e\tau - 3\beta^2 + 3p^2 + 6pz^e\tau + 3}{6\beta - 6p\beta + 6\beta^2 - 6z^e\beta\tau} \end{aligned}$$

and

$$\frac{\partial E(q^e)}{\partial p} = -\frac{\tau^2(z^e - z)}{6\beta} \frac{(\beta - p + 1)(z^e + z + z^e\beta + z\beta - pz^e - pz - 2z^ez\tau)}{(p - \beta + z^e\tau - 1)^2(p - \beta + z\tau - 1)^2}.$$

Next note that in equilibrium marketing level, it is always the case that $z^e = z = z^*$ and therefore

$$E(q^e)|_{\text{equilibrium } z} = 1$$

and

$$\frac{\partial E(q^e)}{\partial p} \Big|_{\text{equilibrium } z=0}$$

and therefore the FOC for price becomes

$$z(1 + \beta - z\tau - p) - pz + 1 - \tau - 2p + \beta = 0$$

or

$$p = \frac{1}{2z+2} (\beta - \tau + z(\beta - z\tau + 1) + 1)$$

which is an identical condition to the benchmark case. Notably, the price will be different from the benchmark case because z will be different.

Marketing FOC

$$p(1 + \beta - 2z\tau - p) + p\beta \frac{\partial E(q^e)}{\partial z} = A'_y(z)$$

where

$$\frac{\partial E(q^e)}{\partial z} = -\frac{1}{6\beta} \frac{\tau}{(p - \beta + z\tau - 1)^2} (3p^2 + 8pz\tau - 6p\beta - 6p + 4z^2\tau^2 - 8z\beta\tau - 8z\tau + 3\beta^2 + 6\beta + 3) < 0.$$

To see why the inequality holds note that $\frac{\partial E(q^e)}{\partial z}$ has the opposite sign of

$$3p^2 + 8pz\tau - 6p\beta - 6p + 4z^2\tau^2 - 8z\beta\tau - 8z\tau + 3\beta^2 + 6\beta + 3 > 0$$

which is positive if the following is positive (because the above expression is increasing in β)

$$3p^2 + 8pz\tau - 6p + 4z^2\tau^2 - 8z\tau + 3$$

which is positive if the following is positive (because the above expression is decreasing in p and $p < 1$)

$$4z^2\tau^2$$

which always holds.

Now note that $\frac{\partial E(q^e)}{\partial z} < 0$, $p(1 + \beta - 2z\tau - p)$ is decreasing in z and $A'_y(z)$ is increasing in

z . This implies that holding p constant, the marketing interval is larger in the benchmark than in the AR case. Recalling that for both the benchmark and the AR cases the price is determined by

$$p = \frac{1}{2z+2} (z + \beta - \tau + z\beta - z^2\tau + 1)$$

we can say that in equilibrium the marketing interval is larger in the benchmark than in the AR case.

We now turn to study whether the price in the AR is higher or lower than in the benchmark. Note that

$$\frac{\partial \left(\frac{1}{2z+2} (z + \beta - \tau + z\beta - z^2\tau + 1) \right)}{\partial z} = -\frac{1}{2} \frac{\tau}{(z+1)^2} (z^2 + 2z - 1)$$

That is, price first increases and then decrease in z . As a result, if z is small (for example, if $A'(z)$ is high) the price in the AR case is lower than in the benchmark, whereas if z is large the price in the AR case is higher than in the benchmark. ■

Lemma 7 *In the Reviews case, the monopoly price and marketing strategies solve the following equations*

$$p = \frac{1}{2z+2} (z + \beta - \tau + z\beta - z^2\tau + 1)$$

$$p(1 + \beta - 2z\tau - p) = A'_y(z).$$

That is, the price and advertising interval are the same in the Reviews case and the benchmark. Subsequently, the advertising interval in the Reviews case is larger than in the AR case, and the price can be either lower or higher than the AR case.

Proof. The monopoly solves

$$\begin{aligned} \max_{z,p} \Pi &= p(z(1 + \beta - z\tau - p) + E_q(1 + \beta q - \tau - p)) - A_y(z) \\ &= p(z(1 + \beta - z\tau - p) + (1 + \beta - \tau - p)) - A_y(z) \end{aligned}$$

Price FOC:

$$z(1 + \beta - z\tau - p) + (1 + \beta - \tau - p) + p(-z - 1) = 0$$

or

$$p = \frac{1}{2z+2} (z + \beta - \tau + z\beta - z^2\tau + 1)$$

Advertising FOC:

$$p(1 + \beta - 2z\tau - p) = A'_y(z)$$

or

$$4p(1 + \beta - 2z\tau - p) = A'_y(z).$$

■

Lemma 8 *In the benchmark case, average rating of generation 2 consumers is lower than the average rating of generation 1 consumers.*

Proof. We show that for all $z \in [0, 1]$, $\mu_2^0 < \mu_1(z)$. That is,

$$\frac{1 - p - \tau + (2q - 1)\beta}{2} < \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)}$$

or

$$0 < 3\beta - 3z - 3p - 3z\beta - 3z\tau + 4z^2\tau + 3pz + 3$$

which holds for the entire range of parameters that we consider. To see why, note that $3\beta - 3z - 3p - 3z\beta - 3z\tau + 4z^2\tau + 3pz + 3$ is decreasing in z when $1 - 2\tau > p > 2\beta$, and thus it is sufficient to show that the inequality holds when $z = 1$, which always holds. Note that this ratings dynamics holds regardless of the firm's choice of z (although the magnitude of the difference is affected). ■

Lemma 9 *In the AR case, there exists $\bar{q}^{AR} \leq 1$ such that average rating of generation 2 consumers is lower than the average rating of generation 1 consumers if and only if $q > \bar{q}^{AR}$.*

Proof. We first note that because $z^0 > z^{AR}$ we know that, holding the price fixed, $\mu_1^0 < \mu_1^{AR}$. That is, holding the price fixed, relative to generation 1 in the no information setup, the ratings of generation 1 in the AR case are higher because the firm advertises only to lower distance consumers.

The mean utility of a 2nd generation purchasing consumers is

$$\mu_2^{AR} = u_x|_{\alpha_i + \beta q - \tau - p > 0} = \frac{1 + \beta q - \tau - p}{2}$$

Notably, if $q \geq 1$ then $\mu_2^{AR} \leq \mu_1(z)$ for any z and therefore also for equilibrium z . To see why, note that for any $q > 1$ and all z

$$\begin{aligned}\mu_2^{AR} &= \frac{1 + \beta q - \tau - p}{2} < \frac{1 - p - \tau + (2q - 1)\beta}{2} \\ &< \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)} = \mu_1(z)\end{aligned}$$

where the second inequality follows the same analysis as in the rating dynamics analysis of the benchmark case. To complete the lemma we next show that $\mu_2^{AR} - \mu_1(z)$ is monotonically decreasing in q for any z :

$$\frac{\partial (\mu_2^{AR} - \mu_1(z))}{\partial q} = \frac{\beta}{2} - \frac{6\beta + 6\beta^2 - 6p\beta - 6z\beta\tau}{6(\beta - p - z\tau + 1)} = -\frac{1}{2}\beta < 0.$$

■

Lemma 10 *In the Reviews case, there exists \bar{q}^R such that average rating of generation 2 consumers is lower than the average rating of generation 1 consumers if and only if $q > \bar{q}^R$. Moreover, $\bar{q}^R > \bar{q}^{AR}$. That is, In the Reviews case, ratings increase for a larger range of qualities than in the AR case.*

Proof. We first note that $z^R = z^0$ and therefore $\mu_1^R = \mu_1^0$, with the implied comparison to μ_1^{AR} .

The mean utility of a 2nd generation purchasing consumers is

$$\mu_2^R = \frac{3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 4\tau^2 - 6\tau + 3}{6(1 + \beta q - p - \tau)}.$$

Note that

$$\mu_2^R = \frac{3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 4\tau^2 - 6\tau + 3}{6(1 + \beta q - p - \tau)} > \frac{1 + \beta q - \tau - p}{2} = \mu_2^{AR}.$$

To see why note that with some algebra the above inequality becomes

$$\frac{1}{3}\tau^2 > 0$$

which always holds.

In addition, because $z^{AR} < z^R$, and because $\mu_1(z)$ is decreasing in z , we know that $\mu_1^{AR} > \mu_1^R$. Subsequently, for any q such that $\mu_1^{AR} < \mu_2^{AR}$ it is also true that $\mu_1^R < \mu_2^R$, and for any q for which $\mu_1^R > \mu_2^R$, it is also true that $\mu_1^{AR} > \mu_2^{AR}$. That is, if there exists \bar{q}^R such that in the Reviews case average rating of generation 2 consumers is lower than the average rating of generation 1 consumers if and only if $q > \bar{q}^R$ then it must be the case that $\bar{q}^R > \bar{q}^{AR}$. We next show that there exists such \bar{q}^R . To show that exists such \bar{q}^R we show that $\mu_2^R - \mu_1(z)$ is monotonically decreasing in q .

$$\begin{aligned} \frac{\partial (\mu_2^R - \mu_1(z))}{\partial q} &= \frac{\beta (3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 2\tau^2 - 6\tau + 3)}{6(p + \tau - q\beta - 1)^2} \\ &\quad - \frac{(p + \tau - q\beta - 1)^2}{6(p + \tau - q\beta - 1)^2} \end{aligned}$$

To see that $\frac{\partial (\mu_2^R - \mu_1(z))}{\partial q}$ is negative, note that $\frac{\partial (\mu_2^R - \mu_1(z))}{\partial q}$ has the same sign as

$$\beta (3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 2\tau^2 - 6\tau + 3) - (p + \tau - q\beta - 1)^2$$

which is decreasing in q . Therefore, it is sufficient to show that

$$\beta (3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 2\tau^2 - 6\tau + 3) - (p + \tau - q\beta - 1)^2 < 0 \text{ for } q = 0 \text{ or that}$$

$$\beta (3p^2 + 6p\tau - 6p + 2\tau^2 - 6\tau + 3) - (p + \tau - 1)^2 < 0.$$

Which holds because $\beta (3p^2 + 6p\tau - 6p + 2\tau^2 - 6\tau + 3) - (p + \tau - 1)^2$ is decreasing in p , and when substituting in $p = 0$ we get

$$\begin{aligned} &\beta (2\tau^2 - 6\tau + 3) - (\tau - 1)^2 \\ &= \beta (1 - 2\tau) + (2\beta - 1) (1 - 2\tau + \tau^2) \\ &< \beta (1 - 2\tau + \tau^2) + (2\beta - 1) (1 - 2\tau + \tau^2) \\ &= (3\beta - 1) (1 - 2\tau + \tau^2) < 0 \end{aligned}$$

where the last inequality holds because $1 > 3\beta$. ■

Proof of Proposition 3. Recall that $p^0 = p^R$ and $z^0 = z^R$. In the benchmark case,

$$E(\Pi^0) = p^0 z^0 (1 + \beta - z^0 \tau - p^0) + p^0 (1 - \tau - p^0 + \beta) - A_y(z^0).$$

Similarly, in the Reviews case

$$\begin{aligned} E(\Pi^R) &= p^R (z^R (1 + \beta - z^R \tau - p^R) + E_q(1 + \beta q - \tau - p^R)) - A_y(z^R) \\ &= p^R (z^R (1 + \beta - z^R \tau - p^R) + (1 + \beta - \tau - p^R)) - A_y(z^R) \end{aligned}$$

which is identical to the benchmark and therefore expected profits in the Reviews case are the same as in the benchmark.

Conversely, in the AR case

$$E(\Pi^{AR}) = p^{AR} z^{AR} (1 + \beta - z^{AR} \tau - p^{AR}) + p^{AR} (1 + \beta E(q^e) - \tau - p^{AR}) - A_y(z^{AR})$$

and in equilibrium

$$E(\Pi^{AR}) = p^{AR} z^{AR} (1 + \beta - z^{AR} \tau - p^{AR}) + p^{AR} (1 + \beta E(q) - \tau - p^{AR}) - A_y(z^{AR})$$

or

$$E(\Pi^{AR}) = p^{AR} z^{AR} (1 + \beta - z^{AR} \tau - p^{AR}) + p^{AR} (1 + \beta - \tau - p^{AR}) - A_y(z^{AR}).$$

This profit function is the same as in the benchmark case. However, in the AR case the firm maximizes this function as if it can affect q^e , which introduces a distortion.

Proposition 9 *Period 2 demand under the various information structures is*

$$D_2^{N-0}(z) = z(1 - p + \beta - \tau + \tau\eta - z^e\tau\eta), \quad D_2^{N-AR}(z) = z(1 - p + q^e\beta - \tau + \tau\eta - z^e\tau\eta),$$

and $D_2^{N-R}(z) = z(1 - p + q\beta - \tau + \tau\eta - z\tau\eta).$

Proof. We begin with the analysis of the benchmark condition.

Let $\zeta = [1 - (-\beta + \tau E[d(x, y) | \text{positive sample}] + p)]$. In the Benchmark, generation 2 consumers do not get any direct information on q or y . Therefore, if $d(x, y) > 2z$, generation 2

demand for the product in location x is

$$(1 - \eta) z \zeta,$$

and otherwise it is

$$\eta \zeta + (1 - \eta) z \zeta = (\eta + (1 - \eta) z) \zeta$$

where

$$\begin{aligned} E[d(x, y) | \text{positive sample}] &= \frac{\eta z^e}{z^e} z^e + \frac{(1 - \eta) z^e}{z^e} 1 \\ &= \eta z^e + (1 - \eta) \end{aligned}$$

The actual total generation 2 demand is then

$$D_2^{N-0}(z) = z(1 - p + \beta - \tau + \tau\eta - z^e\tau\eta).$$

The analysis of the AR condition is similar, with the exception that generation 2 consumers observe the average ratings of generation 1 and establish expectations about the quality based on μ_1 and z^e . Therefore,

$$D_2^{N-AR}(z) = z(1 - p + q^e\beta - \tau + \tau\eta - z^e\tau\eta).$$

Finally, in the Reviews case, generation 2 consumers learn q and y directly, and do not need to infer q^e or z^e . Therefore, if $d(x, y) > 2z$, the generation 2 demand for the product in location x is

$$(1 - \eta) z [1 - (-\beta q + \tau d(x, y) + p)]$$

and otherwise it is

$$\begin{aligned} &\eta [1 - (-\beta q + \tau d(x, y) + p)] + (1 - \eta) z [1 - (-\beta q + \tau d(x, y) + p)] \\ &= (\eta + (1 - \eta) z) [1 - (-\beta q + \tau d(x, y) + p)]. \end{aligned}$$

The actual total generation 2 demand is then

$$D_2^{N-R}(z) = z(-p - \tau + q\beta + \tau\eta - z\tau\eta + 1).$$

■

Lemma 11 *In the benchmark condition, the monopoly advertising interval solves the following equation.*

$$p(2 + 2\beta - 2z\tau - 2p - \tau + \tau\eta - z\tau\eta) = A'_y(z).$$

Proof. In the benchmark condition, the firm maximizes the following in choosing the advertising interval

$$\max_{z,p} \Pi = p(D_1(z, p) + E_q(D_2^0(p))) - A_y(z)$$

or

$$\begin{aligned} \max_{z,p} \Pi &= p(z(1 + \beta - z\tau - p) + z(1 - p + \beta - \tau + \tau\eta - z^e\tau\eta)) - A_y(z) \\ &= pz(2 + 2\beta - z\tau - 2p - \tau + \tau\eta - z^e\tau\eta) - A_y(z) \end{aligned}$$

The advertising first-order condition is then

$$p(2 + 2\beta - 2z\tau - 2p - \tau + \tau\eta - z^e\tau\eta) = A'_y(z)$$

and in equilibrium

$$p(2 + 2\beta - 2z\tau - 2p - \tau + \tau\eta - z\tau\eta) = A'_y(z).$$

■

Lemma 12 *In the AR condition, the monopoly advertising interval solves the following equation.*

$$p\left(2 - 2p + 2\beta - \tau - 2z\tau + \tau\eta(1 - z) + z\beta\frac{dE_q(q^e)}{dz}\right) = A'_y(z).$$

Proof. Recall that

$$q^e = -\frac{-6p - 6\mu_1 + 4(z^e)^2\tau^2 + 6p\mu_1 - 6z^e\tau - 3\beta^2 - 6\beta\mu_1 + 3p^2 + 6pz^e\tau + 6z^e\tau\mu_1 + 3}{6\beta - 6p\beta + 6\beta^2 - 6z^e\beta\tau}.$$

The firm then maximizes the following in choosing price and interval

$$\max_{z,p} \Pi = p \left(D_1(z, p) + E_q(D_2^{AR}(p)) \right) - A_y(z)$$

or

$$\begin{aligned} \max_{z,p} \Pi &= p(z(1 + \beta - z\tau - p) + E_q(z(1 - p - \tau + q^e\beta + \tau\eta - z^e\tau\eta))) - A_y(z) \\ &= p(z(1 + \beta - z\tau - p) + z(1 - p - \tau + \beta E_q(q^e) + \tau\eta - z^e\tau\eta)) - A_y(z). \end{aligned}$$

Advertising FOC

$$p \left((1 + \beta - z\tau - p) - z\tau + (1 - p - \tau + \beta E_q(q^e) + \tau\eta - z^e\tau\eta) + z \left(\beta \frac{dE_q(q^e)}{dz} \right) \right) = A'_y(z)$$

Substituting that $E(q^e)|_{\text{equilibrium } z} = 1$ and $z^e = z$ and simplifying we get

$$p \left(2 - 2p + 2\beta - \tau - 2z\tau + \tau\eta(1 - z) + z\beta \frac{dE_q(q^e)}{dz} \right) = A'_y(z).$$

■

Lemma 13 *In the Reviews condition, the monopoly advertising interval solves the following equation.*

$$p(2 - 2p + 2\beta - \tau - 2z\tau + \tau\eta(1 - 2z)) = A'_y(z).$$

Proof. The firm maximizes the following in choosing the advertising interval

$$\max_{z,p} \Pi = p \left(D_1(z, p) + E_q(D_2^R(p)) \right) - A_y(z)$$

or

$$\max_{z,p} \Pi = pz(1 + \beta - z\tau - p) + pz(-p + \beta - \tau + \tau\eta - z\tau\eta + 1) - A_y(z)$$

Marketing interval FOC

$$p(2 - 2p + 2\beta - \tau - 2z\tau + \tau\eta(1 - 2z)) = A'_y(z).$$

■

Proof of Proposition 4. In the Benchmark, AR, and Reviews conditions, that the advertising interval is larger in the network case than the respective no-network case, follow from a direct comparison of the first order conditions in the network and no-network case, and by recalling that $A_y(z)$ is increasing and convex. Similarly, that the advertising interval is increasing in homophily in the benchmark and AR conditions, and that the advertising interval is increasing in the level of homophily if and only if $z < 1/2$, follow from an inspection of the same first-order conditions.

Proof of Proposition 5. We first analyze the relationship between the advertising intervals in the Reviews and AR cases. We note that the advertising interval is larger in the Reviews condition if and only if

$$\begin{aligned} & p(2 - 2p + 2\beta - \tau - 2z^R\tau + \tau\eta(1 - 2z^R)) \\ > & p\left(2 - 2p + 2\beta - \tau - 2z^{AR}\tau + \tau\eta(1 - z^{AR}) + z^{AR}\beta\frac{dE_q(q^e)}{dz}\right) \end{aligned}$$

which in the exogenous prices case becomes

$$0 > 2\tau z^R(1 + \eta) - \tau z^{AR}(2 + \eta) + z^{AR}\beta\frac{dE_q(q^e)}{dz}.$$

A sufficient condition for the above inequality to hold is that η is sufficiently small. To see why note that when $\eta \rightarrow 0$ the condition becomes

$$0 > 2\tau(z^R - z^{AR}) + z^{AR}\beta\frac{dE_q(q^e)}{dz}.$$

Then assume by contradiction that $z^R < z^{AR}$, and note that given that $\frac{dE_q(q^e)}{dz} < 0$ the condition holds. Thus contradicting the assumption that $z^R < z^{AR}$.

Next, the condition for the advertising interval in the benchmark being larger than in the AR

is

$$2 + 2\beta - 2z\tau - 2p - \tau + \tau\eta - z\tau\eta > 2 - 2p + 2\beta - \tau - 2z\tau + \tau\eta(1 - z) + z\beta \frac{dE_q(q^e)}{dz}$$

or

$$0 > z\beta \frac{dE_q(q^e)}{dz}$$

which always holds. The intuition is similar to the one in the non network case.

The condition for the advertising interval in the benchmark being larger than in the Reviews is

$$2 + 2\beta - 2z\tau - 2p - \tau + \tau\eta - z\tau\eta > 2 - 2p + 2\beta - \tau - 2z\tau + \tau\eta(1 - 2z)$$

or

$$(1 - z) > (1 - 2z)$$

which always holds.

Proof of Proposition 6. We first consider the Benchmark case. Let $\varrho = \tau E[d(x, y) | \text{positive sample}]$. The mean utility of a consumer who purchases, in equilibrium, in period 2 is

$$\begin{aligned} \mu_2^0(z) &= u_x|_{\alpha_i + \beta - \tau - p > 0} \\ &= \frac{1 + \beta q - \varrho - p + (-\beta + \varrho + p + \beta q - \varrho - p)}{2} \\ &= \frac{1 - p - \tau(\eta z + (1 - \eta)) + (2q - 1)\beta}{2} \end{aligned}$$

which is larger than the equivalent in the no-network case as long as $z < 1$. We next compare it to generation 1 ratings. For the ratings of generation 2 to be lower than the ratings of generation 1 the following must hold

$$\begin{aligned} &\frac{1 - p - \tau(\eta z + (1 - \eta)) + (2q - 1)\beta}{2} \\ &< \frac{4z^2\tau^2 - 6p + 6q\beta - 6z\tau - 3\beta^2 + 6q\beta^2 + 3p^2 - 6pq\beta + 6pz\tau - 6qz\beta\tau + 3}{6(\beta - p - z\tau + 1)} \end{aligned}$$

which simplifies to

$$3(-\eta z - 1 + \eta) < -3z + \frac{z^2\tau}{(1-p-z\tau+\beta)}$$

which always holds because $3(-\eta z - 1 + \eta) < -3z$ and $\frac{z^2\tau}{(1-p-z\tau+\beta)} > 0$.

Next, we consider the AR case. The mean utility of a consumer who purchases, in equilibrium, in period 2 is

$$\begin{aligned}\mu_2^{AR} &= \frac{1 + \beta q - \tau E[d(x, y) | \text{positive sample}] - p}{2} \\ &= \frac{1 + \beta q - \tau(\eta z + (1 - \eta)) - p}{2}\end{aligned}$$

which is larger than the equivalent in the non network case as long as $z < 1$.

Finally, in the Reviews case, the mean utility of a purchasing consumer in location x is

$$\frac{1 + \beta q - d(x, y)\tau - p}{2}$$

and therefore the mean utility across all consumers is

$$\begin{aligned}\mu_2^R &= \frac{\int_0^{2z} \frac{1}{2} \frac{1+\beta q-r\tau-p}{2} (\eta + (1-\eta)z) [1 - (-\beta q + \tau r + p)] dr}{z(-p - \tau + q\beta + \tau\eta - z\tau\eta + 1)} \\ &\quad + \frac{\int_{2z}^2 \frac{1}{2} \frac{1+\beta q-r\tau-p}{2} (1-\eta)z [1 - (-\beta q + \tau r + p)] dr}{z(-p - \tau + q\beta + \tau\eta - z\tau\eta + 1)} \\ &= \frac{1}{4} \frac{(\eta + (1-\eta)z) \int_0^{2z} (1 + \beta q - r\tau - p)^2 dr + (1-\eta)z \int_{2z}^2 (1 + \beta q - r\tau - p)^2 dr}{z(-p - \tau + q\beta + \tau\eta - z\tau\eta + 1)} \\ &= \frac{1}{4} \left(\eta \frac{\frac{1}{z} \int_0^{2z} (1 + \beta q - r\tau - p)^2 dr}{(-p - \tau + q\beta + \tau\eta - z\tau\eta + 1)} + (1-\eta) \frac{\int_0^2 (1 + \beta q - r\tau - p)^2 dr}{(-p - \tau + q\beta + \tau\eta - z\tau\eta + 1)} \right) \\ &= -\frac{3q^2\beta^2 - 6\tau - 6p + 6p\tau + 6q\beta + 4\tau^2 + 6\tau\eta - 4\tau^2\eta + 3p^2 - 6pq\beta}{6p + 6\tau - 6q\beta - 6\tau\eta + 6z\tau\eta - 6} \\ &\quad - \frac{4z^2\tau^2\eta - 6q\beta\tau - 6p\tau\eta - 6z\tau\eta + 6pz\tau\eta + 6q\beta\tau\eta - 6qz\beta\tau\eta + 3}{6p + 6\tau - 6q\beta - 6\tau\eta + 6z\tau\eta - 6}\end{aligned}$$

In the case of no homophily ($\eta = 0$) this becomes

$$\mu_2^R|_{\eta=0} = \frac{3p^2 - 6pq\beta + 6p\tau - 6p + 3q^2\beta^2 - 6q\beta\tau + 6q\beta + 4\tau^2 - 6\tau + 3}{6(1 + q\beta - p - \tau)}$$

which equals the second period ratings in the non network Reviews case. Then note that

$$\frac{\partial \mu_2^R}{\partial \eta} = \tau \frac{1 - z}{6} \frac{3q^2\beta^2 - 4\tau - 6p + 4p\tau + 6q\beta - 4z\tau + 4z\tau^2 + 3p^2 - 6pq\beta + 4pz\tau - 4q\beta\tau - 4qz\beta\tau + 3}{(p + \tau - q\beta - \tau\eta + z\tau\eta - 1)^2}$$

which has the same sign as

$$3q^2\beta^2 - 4\tau - 6p + 4p\tau + 6q\beta - 4z\tau + 4z\tau^2 + 3p^2 - 6pq\beta + 4pz\tau - 4q\beta\tau - 4qz\beta\tau + 3 > 0$$

where the inequality holds because $3q^2\beta^2 - 4\tau - 6p + 4p\tau + 6q\beta - 4z\tau + 4z\tau^2 + 3p^2 - 6pq\beta + 4pz\tau - 4q\beta\tau - 4qz\beta\tau + 3$ is decreasing in τ and z and increasing in β and if we substitute $z = 1, \beta = 0, 1 - p = 2\tau$ equals zero.

To complete the proof we recall that according to Proposition 4 in all conditions, the advertising interval is larger in the network case than in the no-network case and thus generation 1 ratings are lower in the network case than in the no-network case.

Proof of Proposition 7. We first show that in the Reviews condition, the profits in the network case are smaller than the profits in the no-network case. The proof follows a simple revealed preference argument. First, we note that conditional on being informed, a generation 2 consumer has the same information on the product location in the network and non network case. Next we note that for any advertising interval $z < 1$, there are more informed generation 2 consumers in the non network case than in the network case. Subsequently, if we take z^{R-Net} to be the interval in the network case, then we know that the profit in the no-network case is larger even if we constrain the advertising interval to equal z^{R-Net} . Next, a simple revealed preference argument suggests that the profit in the no-network case when the advertising interval is z^R must be larger even if we constrain the advertising interval to equal z^{R-Net} . This arguments extends immediately to the case where prices are endogenous.

Next note that in equilibrium in the AR condition with network case

$$E(\Pi^{N-AR}) = p(z^{N-AR}(1 + \beta - z^{N-AR}\tau - p) + z^{N-AR}(1 - p - \tau + \beta + \tau\eta - z^{N-AR}\tau\eta)) - A_y(z)$$

which is identical to the Reviews case. However, in the AR z^{N-AR} is not chosen to maximize this function but rather a function that takes z^e as given. Now assume by contradiction that z^{N-AR}

generates profits that are higher than the profits generated by z^{N-R} . This contradicts that z^{N-R} maximizes the same function. An almost identical argument can be used to prove that profits are lower in the Benchmark case than in the Reviews case and is omitted.

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