# Dynamic Coordination and Bankruptcy Regulations

Hongda Zhong, Zhen Zhou*

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**Abstract**

The "automatic stay" and "avoidable preference" (clawback of some pre-bankruptcy repayments), two key provisions in many countries' bankruptcy codes, seek to avoid creditor runs on insolvent firms. However, by making it harder to exit distressed firms in or near bankruptcy, these provisions could motivate creditors to run ex-ante. We develop a theoretical framework based on "clock game" and derive the optimal design of these regulations. We show that inside creditors should face a longer clawback window. Furthermore, firms can survive longer by committing to filing for bankruptcy earlier because the extra payoff in bankruptcy mitigates creditors' incentive to run.

**Keywords**: Avoidable Preference, Bankruptcy Protection, Automatic Stay, Clock Game, Runs

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# 1 Introduction

When firms borrow from multiple sources, coordination failure among creditors can make the bankruptcy process chaotic, because individual creditors find it optimal to run on firms' assets. Accordingly, bankruptcy laws around the world usually impose regulations to promote orderly resolution of bankruptcy by eliminating the first-come-first-serve nature of the process. For example, when a debtor seeks "bankruptcy protection," creditors must stop collecting their debt individually and instead join others in bankruptcy court to negotiate a more efficient outcome, a feature known as the "automatic stay." Another widely cited regulation with similar aims is the legal treatment of "avoidable preference."[1] This regulation prevents a distressed firm from treating some creditors more favorably than others and is implemented by clawing back repayments made shortly before bankruptcy. The proceeds are then shared among all creditors in bankruptcy court.[2] However, restricting creditors' ability to collect debt around bankruptcy also affects their incentives to stay invested while the firm is still relatively healthy. Concerned about the possibility of being stuck in a failing firm, creditors may exit even sooner, at the first sign of trouble, so that the firm is pushed into bankruptcy earlier.[3] Hence, there is a fundamental tension between coordinating creditors in bankruptcy ex-post and keeping them ex-ante.

In this paper, we build a dynamic model to analyze how bankruptcy regulations affect creditors' willingness to stay invested ex-ante, and explore the optimal design of these regulations as well as the optimal timing for a firm to seek bankruptcy protection. To focus on events around bankruptcy, we model a failing firm with a continuum of creditors. Specifically, at some random time, an unobservable bad shock hits the firm's asset growth rate. As a result, liabilities grow more rapidly than assets, and the firm will eventually go bankrupt. In an asynchronous manner, creditors gradually and privately become informed that the bad shock has arrived, but are not informed about the exact time of its arrival.

[1]Between 2017 and 2019, among the 595 bankruptcy cases collected by Westlaw legal research service, 290 cases (or 48.74%) cited avoidable preference in the United States. In the past, many well-known bankruptcies, such as WorldCom, General Motors, and Lehman Brothers, resorted to avoidable preference legislation to settle disputes among creditors.

[2]The typical clawback window is between 90 days and one year in the United States (see Chapter 11, Sections 547 and 550), between six months and one year in China (see Enterprise Bankruptcy Law, Articles 31 and 32) and between six months and two years in the United Kingdom (see Insolvency Act 1986, Sections 239 and 240).

[3]For instance, under the avoidable preference regulation, creditors who intend to exit a troubled firm need to leave sooner in order to make it more likely that this payment is outside of the clawback window — a concern that legal professionals also share (see McCoid (1981) and Countryman (1985)).

Therefore, they do not know how many other creditors are informed about the bad shock. For simplicity, we assume that creditors continuously roll over the existing debt, and upon being informed, each creditor can then decide when to refuse debt rollover and exit the firm. One can alternatively interpret such an exit as long-term creditors not waving covenant violations upon a technical default. The firm is forced into bankruptcy when a sufficient fraction of creditors choose to exit, a threshold that the firm later can choose in order to maximize equity value. Bankruptcy triggers the clawback window under the avoidable preference regulation. The length of the clawback window is chosen by a regulator in order to maximize welfare, which is the total payoff to all creditors.[4] In deciding when to exit, each creditor faces the following trade-off: Staying invested for longer earns the creditor additional interests if the firm survives during this time but simultaneously increases the risk that the firm may fail before the creditor exits. In a symmetric equilibrium, each creditor waits for the same amount of time after privately learning about the bad shock and then exits the firm. The equilibrium variable – creditors' waiting time – reflects their willingness to stay invested.

The timing of creditors' exit and the clawback window in the avoidable preference clause are only meaningful when time is explicitly modeled. We build on the clock-game literature (e.g., Abreu and Brunnermeier (2003) and Brunnermeier and Morgan (2010)) to capture these dynamics. One of our key contributions is to endogenize creditors' payoff in bankruptcy through various policies and creditors' exit strategies. This novel feature introduces a fixed point problem: Creditors' willingness to stay invested (waiting time) depends on the bankruptcy payoff they can expect, which in turn depends on their equilibrium waiting time. Despite this technical complication, the model remains tractable, and we provide analytical solutions for most equilibrium variables. Endogenizing the bankruptcy payoff offers new insights into how regulations such as avoidable preference should be designed, and when firms should seek bankruptcy protection.

We begin with the avoidable preference. Because the clawback of repayments per se is purely redistributional among creditors, it affects welfare only through creditors' willingness to stay invested – their equilibrium waiting time. A longer waiting time is more efficient as any capital taken away from a productive firm hurts welfare. Hence, the avoidable preference creates the following trade-off. On the one hand, a longer clawback window improves the payoff in bankruptcy by seizing more repayments made prior to bankruptcy. This channel incentivizes creditors to stay invested. On the other hand, a

[4]As will be clear, the total payoff to creditors is a welfare measure because the firm will ultimately go bankrupt in this baseline model, and equity holders do not receive anything in bankruptcy.

longer clawback window makes it less likely for creditors to exit successfully, which in turn motivates them to exit sooner. Based on this trade-off, we find a closed-form solution for the optimal clawback window and offer several implications on policy design.

First, the optimal clawback window depends on features of debt contracts but not on certain firm characteristics, such as leverage and the post-shock growth rate. This explains why the avoidable preference provision in practice often varies by different types of creditors but is largely universal for all firms. Specifically, inside creditors are often subject to longer clawback windows than those for outsiders in many legal regimes. In the context of our model, bad news travels among insiders more quickly. As a result, insiders are more eager to exit their debt positions, and a longer clawback window provides them stronger incentives to stay. To see why the optimal clawback window is independent of certain firm characteristics, we revisit the aforementioned trade-off. On the one hand, the total repayments subject to clawback only depend on the terms and conditions of the debt contract, such as the interest rate. On the other hand, the incentive to outrun the clawback window only depends on the probabilistic structure of the game, such as the likelihood of the bad shock and how quickly creditors become informed. Hence, the trade-off associated with the avoidable preference does not depend on the firm's characteristics, and the independence result follows. Second, we show that if the bad shock is more likely to occur, creditors are less willing to stay invested, and a longer clawback window alleviates the problem. Third, a higher interest rate implies that more repayments can be seized, thereby reducing the optimal length of the clawback window. Finally, in an extension, when avoidable preference is only imperfectly enforced, we show that the optimal clawback window must expand in order to compensate for the weaker execution of the policy.

We then turn our attention to the effect of the automatic stay provision on creditors' ex-ante incentive to stay invested and the firm's optimal timing to seek bankruptcy protection. A bankruptcy process without automatic stay has a first-come-first-serve feature in that early creditors receive full repayments whereas latecomers receive nothing. We show that in this case, creditors exit immediately upon learning the bad shock in order to avoid the disastrous consequence of receiving nothing should they arrive late. Creditors' frantic demand for repayments may in turn push the firm into bankruptcy more quickly.

A related and counterintuitive result is that firms can survive longer by committing to filing for bankruptcy protection early even when they still have assets to honor additional repayment. One might think that in order to survive longer, the firm should never declare bankruptcy early but instead meet creditors' demand for repayments until all assets are depleted. After all, it takes longer for enough creditors to exhaust all assets, thereby delaying bankruptcy. The counteracting force again comes through our novel channel — an endogenous payoff in bankruptcy. If the firm does not commit to a cessation of full repayments and seeking bankruptcy protection while some assets are still available, then creditors receive little payoff in bankruptcy, similar to the aforementioned case without bankruptcy protection. Creditors' decision to exit immediately upon being informed about the bad shock can force the firm into bankruptcy even sooner.

A firm can ex-ante commit to a bankruptcy policy to some extent by holding some illiquid assets: Once liquid assets are depleted by the repayments to exiting creditors, the firm is bankrupt and remaining creditors share the illiquid assets. Our finding therefore offers new insights on the role of illiquid assets and recovery rates in bankruptcy. Maintaining some illiquid assets on the balance sheet can help mitigate runs, as they ensure some payoffs to creditors in bankruptcy. In addition, comparative static analysis suggests that firms should file for bankruptcy earlier, resulting in a higher recovery rate, when the bad shock is more likely to happen or its magnitude is smaller. The opposite implication for the intensity and magnitude of the bad shock also highlights the importance of having a dynamic model to separate the two aspects.

Finally, we extend our model to allow for imperfect enforcement of avoidable preference and show an ultra-long clawback window that prevents any creditor from exiting the firm is suboptimal. Our model can also accommodate a recovery from the bad shock and negative growth rates. The key insights remain robust.

## Literature Review

The coordination problem at the center of our model is related to a large literature on creditor (depositor) coordination following the seminal work by Bryant (1980) and Diamond and Dybvig (1983).[5] This literature studies bank runs and how policies such as suspension of convertibility and deposit insurance can alleviate such runs. Recently, more policies have been analyzed, including bank stress tests (see, for example, Goldstein and Huang (2016); Inostroza and Pavan (2020); Basak and Zhou (2020*b*)) and direct payoff intervention policies (see, for example, Sakovics and Steiner (2012); Cong, Grenadier and Hu (2020)). Among this literature, Schilling (2020) is closely related. In a global game setting, Schilling (2020) studies how many full withdrawals the regulator should allow

[5] See, for example, Gorton (1985); Chari and Jagannathan (1988); Jacklin and Bhattacharya (1988); Green and Lin (2003); Peck and Shell (2003).

(forbearance level) before imposing a costly resolution, after which depositors equally share the remaining assets. There is a similar trade-off: Allowing for more withdrawals makes it more likely for a depositor to receive full repayment, but simultaneously lowers the payoff it receives in a resolution. Depositors therefore preempt the regulator's forbearance level by withdrawing more often. Most of this literature is static in that depositors only choose whether to run but do not choose the precise timing of their exits.[6] In contrast, the continuous-time dynamic framework in this paper allows us to study the time aspect of the coordination problem more carefully, such as when creditors exit and when firms should seek bankruptcy protection. Furthermore, we can also investigate regulations that are intrinsically dynamic, such as the clawback window in the avoidable preference clause.

Our paper also contributes to a large literature on corporate bankruptcy. For example, Bebchuk (2002) finds that the ex-post violation of absolute priority in bankruptcy can aggravate the moral hazard problem and thereby lower ex-ante efficiency. Bolton and Oehmke (2015) investigate the ex-ante impact of exempting derivatives from automatic stay in bankruptcy. Donaldson, Gromb and Piacentino (2020*a*,*b*) study the role of collateral and covenants in regulating creditors' payoffs in bankruptcy. Donaldson et al. (2020) compare out-of-court restructuring and a formal bankruptcy procedure. Different from the existing studies, we focus on two novel aspects of bankruptcy regulation – avoidable preference and automatic stay in bankruptcy protection – and focus on how these ex-post regulations affect creditors' ex-ante incentive to exit the firm. Our study also generates implications on the timing of bankruptcy, the recovery rate, and social welfare.

The theoretical foundation of our model – asynchronous clock – was first introduced in the computer science literature (see Halpern and Moses (1990)). The key idea is that not everyone learns about a piece of news at the same time, but instead some individuals become aware of the news sooner than others. Morris (1995) applies this information structure to a dynamic coordination game in labor economics. Abreu and Brunnermeier (2003), Brunnermeier and Morgan (2010), and Doblas-Madrid (2012) use the clock game setup to understand the formation of bubbles and their subsequent crashes. In the context of banking, He and Manela (2016) allow investors to actively acquire information and show that such information acquisition may accelerate runs. We contribute to this literature by constructing a tractable framework to endogenize the bankruptcy payoff of creditors

[6]Among the exceptions, He and Xiong (2012) study runs on staggered corporate debt and show that a creditor's decision not to roll over maturing debt poses an externality on other creditors whose claims have not yet matured. In contrast, our paper focuses more on various policies regulating this market.

and use this framework to study the optimal response of firms and creditors and efficient policy design.

From a theoretical perspective, clock game is a methodology to relax common knowledge and eliminate equilibrium multiplicity in coordinate games. A popular alternative is the global game approach introduced by Carlsson and Van Damme (1993), which has been extensively applied to modeling currency attacks and bank runs (see Morris and Shin (1998, 2004); Goldstein and Pauzner (2005) for details). The dynamic global game models (see, for example, Dasgupta (2007); Angeletos, Hellwig and Pavan (2007); Basak and Zhou (2020*a*)) focus on how learning from the past history of play can affect future coordination.[7] While the global game models can offer a tractable way of modeling learning, the clock game setup we adopt in this paper offers better tractability for investigating the timing of creditors' exits, firms' choice of bankruptcy, and regulations that are intrinsically dynamic.

We structure the paper as follows. Section 2 introduces the baseline model featuring endogenous bankruptcy payoff shaped by regulations and creditors' strategic decision to exit the firm. In Section 3, we analytically solve for the optimal clawback window in the baseline model. Section 4 focuses on bankruptcy protection and automatic stay. We show how bankruptcy protection affects creditors' willingness to stay invested ex-ante and when firms should commit to seek bankruptcy protection. We extend the model in Section 5 to incorporate imperfect policy enforcement, recovery from the bad shock, and negative growth rates. Finally, Section 6 concludes.

# 2 Baseline Model

We start by building the baseline model in Subsection 2.1 and then discuss several key model assumptions in Subsection 2.2.

## 2.1 Model Setup

Time $t \in [0, \infty)$ is continuous, and the discount rate is normalized to zero. To begin, consider a mature and stable firm with total initial assets $A$ and one unit of liability at $t = 0$, both growing at the rate of $g$. Specifically, the liability is financed by a continuum of identical risk-neutral creditors indexed by $i \in [0, 1]$, each holding a unit face value at $t = 0$. For simplicity, we assume that the debt contracts are continuously refinanced at

[7] It is important to note that, in our dynamic model, creditors perfectly understand the nature of the fundamental shock but do not have perfect information about when the bad shock arrives. They are not waiting for more information but rather are waiting for a higher payoff before the firm goes bankrupt.

the interest rate $g$.[8] As such, without additional shocks, the leverage ratio at any time $t$ is a constant $\frac{e^{gt}}{Ae^{gt}} = \frac{1}{A} \leq 1$.

We are interested in the events around bankruptcy in this paper and therefore focus on a failing firm heading for an eventual bankruptcy. To capture the essence of a failing firm, we assume that at some random time $t_0$, a bad shock hits, permanently reducing the firm's asset growth rate to $g' \in (0, g)$. The arrival intensity of the bad shock is $\lambda$, and $t_0$ therefore follows an exponential distribution with density function $\phi(t_0) = \lambda e^{-\lambda t_0}$ for any $t_0 > 0$. Neither the firm nor the creditors directly observe the arrival of the bad shock, and therefore no renegotiation of the debt contract, in particular the interest rate $g$, is feasible.[9] Instead, each creditor $i$ independently learns about the bad shock at some random time $t_i$ following its arrival, and $t_i$ is uniformly distributed over $[t_0, t_0 + \eta]$ with conditional density $f(t_i|t_0) = \frac{1}{\eta}\mathbf{1}_{t_i \in [t_0, t_0+\eta]}$. In what follows, we sometimes refer to this event as the creditor becoming "informed" at time $t_i$. Upon becoming informed, creditors only learn that the bad shock has arrived but are not informed about its exact time $t_0$, which in turn means that creditors do not exactly know how many other creditors have been informed. As a result, creditor $i$'s posterior belief about $t_0$ admits the following conditional density:

$$\psi(t_0|t_i) = \frac{f(t_i|t_0)\phi(t_0)}{\int_{t_i-\eta}^{t_i} f(t_i|s)\phi(s)ds} = \frac{\frac{1}{\eta}\lambda e^{-\lambda t_0}}{\frac{1}{\eta}(e^{-\lambda(t_i-\eta)} - e^{-\lambda t_i})} = \frac{\lambda e^{-\lambda(t_0-t_i)}}{e^{\lambda\eta} - 1},$$

for $t_0 \in [t_i - \eta, t_i]$, and $\psi(t_0|t_i) = 0$ otherwise. We denote by $\Psi(t_0|t_i)$ the corresponding cumulative distribution function.

After learning about the bad shock at $t_i$, each creditor can then choose to stop rolling over the debt at $\beta_i(t_i) \geq t_i$ privately and demand the promised repayment $e^{g\beta_i(t_i)}$ from the firm. We normalize creditors' outside return to 0, and hence, once having left the firm, creditors' payoff stops growing – an assumption we discuss in Subsection 2.2. One can alternatively interpret the bad shock as a violation of certain performance-based covenants and creditors' exit from the firm as long-term creditors' refusal to waive such a violation, resulting in an acceleration of the repayment. The density (rate) of those creditors who

[8] We discuss in Subsection 2.2 that endogenizing the interest rate makes the coordination problem at the center of the model more severe.

[9] As will be clear, in the baseline model, the firm has no strategic action. Consequently, it is irrelevant whether the firm learns about the shock as it arrives. We provide more discussions of this assumption and the possibility of renegotiation in Subsection 2.2.

decide to exit at time $t \geq t_0$ (i.e., $\beta_i(t_i) = t$) is given by

$$w_t(t_0, \{\beta_i\}) \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\beta_i(t_i) \in [t, t+\Delta t]} f(t_i | t_0) dt_i. \tag{1}$$

The total assets remaining in the firm, denoted by $Y_t$, evolve as follows:

$$dY_t = \begin{cases} gY_t dt & \text{if } t \leq t_0 \\ (g' Y_t - w_t e^{gt})\, dt & \text{if } t > t_0 \end{cases}. \tag{2}$$

The total assets grow at a rate of $g$ prior to the bad shock's arrival at $t_0$, and $g'$ afterward. At any instant $t$ following $t_0$, a density of $w_t$ creditors receive a total repayment of $w_t e^{gt}$ from the firm, which is captured by the second case in (2).

Since each creditor is small, we assume that both individual and total repayments are unobservable to other creditors. When a $k \in (0, 1)$ fraction of creditors leave the firm, the firm files for bankruptcy protection.[10] One can interpret $k$ as the portion of the firm's liquid assets, including those that can easily be sold without affecting its operation. When the firm depletes its liquid assets, any additional demand for repayment from creditors fundamentally disrupts the firm's operation, and the firm goes bankrupt. Alternatively, one can imagine that when sufficiently many creditors exit (i.e., a fraction $k$), the bad shock becomes public information, and all remaining creditors immediately demand repayment, forcing the firm into bankruptcy. The time of bankruptcy $\hat{t}$ can therefore be defined as

$$\hat{t}\,(k, t_0, \{\beta_i\}) = \inf \left\{ u \middle| \int_{t_0}^{u} w_t\,(t_0, \{\beta_i\})\, dt \geq k \right\}. \tag{3}$$

For now, $k$ is an exogenous parameter. We will later allow (the manager of) the firm to choose $k$ in Subsection 4.1. In Subsection 2.2, we discuss why raising new financing may be difficult at this stage and show that whether the firm continues or is terminated at $\hat{t}$ is inconsequential for our analysis.

To rule out infinite values, we make a purely technical assumption that the bad shock at $t_0$ leads to an exogenous termination of the firm at $t_0 + T$ regardless of creditors' actions. The remaining creditors in this case share the final assets $Y_{t_0+T}$ equally. Throughout the paper, we focus on the case where $T$ is large enough and therefore nonbinding such that

[10] As will soon be clear, risk-neutral creditors do not have an incentive to keep partial investments. Also, since the firm is doomed to fail, creditors also do not have an incentive to reinvest the capital back into the firm. We adopt this model specification to focus on the key events around bankruptcy and the associated coordination problems.

the firm is always endogenously bankrupt (i.e., when $k$ creditors withdraw their capital) before reaching $t_0 + T$ in equilibrium.

We now introduce the key aspect of our paper: avoidable preference in the bankruptcy process. Under the avoidable preference legislation in practice, creditors who receive repayments shortly before bankruptcy, resulting in a more favorable treatment than other remaining creditors, need to return those payments. These returned repayments, together with other assets left in the firm, are shared among all creditors in the bankruptcy process.

In the United States, for example, Chapter 11, Section 547 b) of the Bankruptcy Code states that "the trustee may, ... avoid any transfer of an interest of the debtor in property ... made (A) on or within 90 days before the date of the filing of the petition; or (B) between ninety days and one year before the date of the filing of the petition, if such creditor at the time of such transfer was an insider." This clause is commonly cited in bankruptcy litigations. Among the 595 bankruptcy cases collected by Westlaw legal research service between 2017 and 2019, 290 cases (or 48.74%) cite avoidable preference in the United States. For instance, when General Motors filed for bankruptcy in 2009, the bankruptcy trustee sued creditor JPMorgan Chase Bank to recover approximately \$28 million in interest and \$1.4 billion in principal repayment, citing the avoidable preference clause.

We formally model this feature as a "clawback" window $m$ chosen by a regulator with the objective of maximizing social welfare. Specifically, suppose the pivotal creditor, that is, the $k$th creditor whose exit triggers bankruptcy, leaves the firm at time $\hat{t}$. If the firm is unable to honor the full repayment of $e^{g\hat{t}}$ to all remaining creditors (i.e., $(1-k)\, e^{g\hat{t}} > Y_{\hat{t}}$), then equity receives nothing and the avoidable preference clause becomes effective.[11] Specifically, only creditors who withdraw at least $m$ dates before bankruptcy (i.e., withdraw at time $\beta_i(t_i) < \hat{t} - m$), can keep the full repayment $e^{g\beta_i(t_i)}$, which we later refer to as the "exit payoff." Any creditor who withdraws during the final $m$ dates prior to bankruptcy at $\beta_i(t_i) \in \left[\hat{t} - m, \hat{t}\right]$ initially receives $e^{g\beta_i(t_i)}$ but will later be required to return the money (clawback) when the firm goes bankrupt at $\hat{t}$. Denote by $n_c$ and $RC_{\hat{t}}$ the number of creditors and the total repayments that are subject to clawback, respectively. Mathematically:

$$n_c = \int_{\beta_i(t_i) \in \left[\hat{t}-m, \hat{t}\right]} f(t_i|t_0) dt_i,$$

[11] In the baseline model, an ultimate bankruptcy is always the equilibrium outcome. In Subsection 4.1, we extend the model so that equity may sometimes receive a positive payoff in bankruptcy. All intuitions and trade-offs remain robust.

and

$$RC_{\hat{t}} = \int_{\beta_i(t_i)\in\left[\hat{t}-m,\hat{t}\right]} e^{g\beta_i(t_i)} f(t_i|t_0)dt_i. \tag{4}$$

For reasons that will become clear in Subsections 5.1 and 5.2, we introduce imperfect enforcement of avoidable preference; that is, the clawback of each individual repayment made at $\beta_i(t_i) \in \left[\hat{t}-m, \hat{t}\right]$ is only successful with probability $p$ independently, and with probability $1-p$, this creditor can keep the full repayment of $e^{g\beta_i(t_i)}$. This assumption is motivated by the fact that the outcome of a lawsuit in the bankruptcy court can be uncertain, affected by many factors such as the preference of the judge, the skill of the lawyers involved in the case, the specific purpose of each repayment, and so on. Since the intuitions for most results are robust to such an enforcement imperfection, we focus on the case of perfect enforcement (i.e., $p = 1$) for the main discussion in the paper. This case highlights the key trade-off and offers a simple closed-form solution. We discuss a theoretical subtlety associated with perfect enforcement in Subsection 5.1 and then explicitly study the effect of imperfect enforcement in Subsection 5.2.

There are two groups of creditors in bankruptcy, including the $pn_c$ exiting creditors whose repayments are clawed back and the $1-k$ creditors who never have the chance to exit prior to bankruptcy at $\hat{t}$. These creditors will share the remaining assets $Y_{\hat{t}}$ and the expected clawback proceeds $pRC_{\hat{t}}$ of the payments made during $t \in [\hat{t}-m, \hat{t}]$. Each creditor therefore receives $\frac{Y_{\hat{t}}+pRC_{\hat{t}}}{1-k+pn_c}$.

In what follows, we specify the payoff functions of creditors and the regulator and define the equilibrium concept. The equilibrium includes a set of creditors' withdrawal strategy $\beta^* = \{\beta_i^*(t_i)\,|i \in [0,1]\}$ and the regulator's optimal design of the avoidable preference window $m^*$. Given other creditors' equilibrium strategy $\beta_{-i}^*$ and avoidable preference window $m^*$ specified by the regulator, creditor $i$'s withdrawal strategy $\beta_i^*\left(t_i|\beta_{-i}^*, m^*\right)$ maximizes her expected payoff, denoted by $\Pi_i$:

$$\begin{aligned}
\beta_i^*(t_i) &= \arg\max_{\beta_i} \Pi_i(\beta_i|t_i, \beta_{-i}^*, m^*) \\
&\equiv \underbrace{\int_{\beta_i\le\hat{t}-m} e^{g\beta_i}\psi(t_0|t_i)dt_0}_{\text{successful withdrawal: exit payoff}} + \underbrace{\int_{\hat{t}-m<\beta_i\le\hat{t}} \left((1-p)\,e^{g\beta_i} + p\frac{Y_{\hat{t}}+pRC_{\hat{t}}}{1-k+pn_c}\right)\psi(t_0|t_i)dt_0}_{\text{exit payoff or bankruptcy payoff within clawback window}} \\
&+ \underbrace{\int_{\beta_i>\hat{t}} \frac{Y_{\hat{t}}+pRC_{\hat{t}}}{1-k+pn_c}\psi\left(t_0|t_i\right)dt_0}_{\text{bankruptcy payoff}}.
\end{aligned} \tag{5}$$

It is worth noting, from (3), that the time of bankruptcy $\hat{t}\,(k, t_0, \{\beta_i\})$ depends on other creditors' equilibrium withdrawal strategies $\{\beta^*_{-i}\}$ but not creditor $i$'s own strategy $\beta_i$, because any individual creditor is infinitesimally small. Since the creditor does not know the exact value of $t_0$, she must update her belief based on the time of being informed $t_i$, reflected by the posterior belief $\psi(t_0|t_i)$. One of the following three outcomes occurs. If the creditor withdraws at least $m$ periods before the firm goes bankrupt (i.e., $\beta_i \leq \hat{t}-m$), then the creditor receives the full repayment $e^{g\beta_i}$, reflected by the first integral in equation (5). Alternatively, if the creditor withdraws within the clawback window (i.e., $\hat{t}-m < \beta_i \leq \hat{t}$,) then she gets full repayment with probability $1-p$ (when the clawback policy is not enforced) or the bankruptcy payoff with probability $p$, captured by the second integral in equation (5). Finally, if the firm goes bankrupt before the creditor withdraws (i.e., $\beta_i > \hat{t}$), then she receives the bankruptcy payoff, as in the last integral in equation (5).

Given creditors' equilibrium strategy $\beta^* = \{\beta_i^*|i \in [0,1]\}$, the regulator chooses an optimal clawback window $m^*$ to maximize ex-ante welfare:

$$m^* = \arg\max_m W\,(m) \equiv \int_{t_0}^{t_0+\eta} \Pi_i\,(\beta_i^*|t_i)\, f(t_i|t_0)dt_i. \tag{6}$$

Equation (6) is the total payoff to all creditors indexed by the time when they become informed. Rigorously speaking, one should also include the payoff to equity holders in the welfare calculation. As will soon be clear, with the exception of Subsection 4.1, equity is eventually wiped out because not all creditors are paid in full in equilibrium. To simplify the exposition, we ignore the zero payoff to equity.[12]

Finally, for technical consideration, we restrict our attention to relatively short clawback windows in our baseline model,

$$m < k\eta, \tag{7}$$

which is only needed under perfect enforcement ($p = 1$). The essence of condition (7) is to allow some creditors to exit successfully in a symmetric equilibrium. Otherwise, no creditors can exit with full repayment and arbitrary outcomes can be supported as an equilibrium under perfect enforcement. We discuss the details of this assumption in Subsection 5.1 and relax it in Subsection 5.2. In the baseline model, we present our main results with $p = 1$ for maximum clarity and tractability.

[12]Note that the welfare defined in (6) is conditional on a specific realization of $t_0$. One could alternatively define ex-ante welfare by integrating $W$ with respect to $t_0$. As will soon be clear, this alternative definition does not change any of our results because the welfare of different outcomes is uniformly ranked for all $t_0$.

## 2.2 Discussion of Modeling Assumptions

**Liquidation vs. Reorganization** Even though our formal model ends when the firm goes bankrupt at $\hat{t}$, this bankruptcy should not be narrowly interpreted as a liquidation of the firm. The setting can be equivalently interpreted as the outcome of a costly reorganization, and each creditor in bankruptcy receives the continuation value of the firm until the project naturally matures at $t_0 + T$. More specifically, the total value of assets at bankruptcy $\hat{t}$ is $Y_{\hat{t}} + pRC_{\hat{t}}$. Assume the bankruptcy cost per unit asset is $(1-\gamma)$, and the continuation value of the firm is a multiple $\omega$ of its current assets in place.[13] Hence, each creditor's payoff is

$$(1-\gamma)\,\omega\frac{Y_{\hat{t}} + pRC_{\hat{t}}}{1-k+pn_c}.$$

One can immediately see that if $(1-\gamma)\,\omega = 1$, the bankruptcy payoff above coincides with that in (5). Even when $(1-\gamma)\,\omega \neq 1$, the model can still be similarly solved and all economic intuitions remain robust.

**Renegotiation and New Financing** In the model, we assume that the firm and its creditors cannot engage in a renegotiation of debt contracts following the bad shock. Indeed, if all creditors can accept a deal to lower the interest rate from $g$ to $g'$, then the coordination problem in our model disappears. However, such a spontaneously coordinated reduction of interest is unlikely for several reasons, which arguably is why formal reorganization in bankruptcy exists in the first place.

First, it is widely accepted that renegotiation with dispersed creditors, such as bond holders, is notoriously difficult if not impossible in practice. In our model, there is a continuum of creditors, each having a dominant strategy to refuse renegotiation if all other creditors accept a reduction of interest. This free-rider problem may prevent creditors from accepting any renegotiation in a decentralized fashion. In fact, empirical evidence suggests that when firms receive bad news, such as a violation of covenants, the renegotiated interest is often higher instead of lower than the original interest (see Roberts and Sufi (2009*a*,*b*) and Roberts (2015)). For the same reason, no new debt financing is possible as no creditor has incentives to accept an interest rate lower than $g'$. Therefore, we argue that endogenizing the interest rate exacerbates the coordination problem and the insights of the model remain robust. Equity financing is also infeasible because it will ultimately be wiped out by debt holders in the inevitable bankruptcy. Second, it is well documented

[13]The valuation multiple $\omega$ can be further microfounded by having the firm operating to its natural maturity $t_0 + T$, where $T$ is subject to an exponential distribution rather than a constant.

that CEOs are prone to overconfidence. They might not recognize the bad shock or perhaps may be unwilling to admit that (the growth rate of) the firm is in trouble. In our model, (the manager of) the firm does not need to know when the bad shock happens as they do not have any strategic action. Finally, since creditors only become aware of the bad shock gradually in the model, it is not in the firm's best interest to broadcast this negative information voluntarily by renegotiating with all creditors. Keeping the negative information private might slow the rate of withdrawal from creditors, providing the firm a longer opportunity to recover.

**Asynchronous Learning** The asynchronous learning specification has a couple of interpretations. First, it is possible that different creditors exert different monitoring effort on the firm and therefore learn about the bad shock at different times. The parameter $\eta$ represents the time it takes for the news to disseminate among all creditors, and conversely $\frac{1}{\eta}$ captures creditors' speed of learning. Alternatively, one can interpret $\eta$ as a proxy for maturity when the debt structure is staggered, as is often the case in practice (see Almeida et al. (2012) and Choi, Hackbarth and Zechner (2018)). Each debt contract has a maturity of $\eta$ dates, and at every unit of time, a $\frac{1}{\eta}$ fraction of the total debt contracts mature. Without the bad shock, creditors automatically renew their debt contracts with the same interest rate and maturity. Under this interpretation, all creditors learn about the bad shock simultaneously upon its arrival but can take action only when their respective debt contracts mature. The creditor maturing at $t_i$ can decide whether to renew a debt contract and, if so, with what maturity. The creditor's exit strategy $\beta_i\left(t_i\right)$ can be interpreted as offering a new and final round of debt financing with $\beta_i\left(t_i\right) - t_i$ maturity. The extreme case $\beta_i\left(t_i\right) = t_i$ means that the creditor does not roll over the debt contract and demand the full repayment immediately.

**Creditor's Outside Option** For simplicity, we normalize the return of creditors' outside option to 0; that is, any capital taken away from the firm stops growing. The essence of this mild assumption is that the promised return of the current investment ($g$) is higher than that of an average project in the economy (normalized to 0). For instance, through the investment relationship, creditors gradually learn about the quality of the firms in their portfolios and only keep those with higher risk-adjusted returns. Also, due to search friction, it may take creditors considerable amount of time to find the next good investment opportunity. Hence, we argue it is likely that the currently invested firms generate higher returns than an average new investment that a creditor can find in a short period of time (i.e., before the creditor exits at $\beta_i(t_i)$).

# 3 Avoidable Preference

As preluded in the model setup, in this section, we focus on the case where avoidable preference is perfect enforced (i.e., $p = 1$) and a clawback window that is not too long as in (7). Following the literature, we study symmetric linear equilibria throughout the paper: Every creditor, upon learning about the bad shock at $t_i$, waits for the same amount of time $\tau \geq 0$ and divests at

$$\beta(t_i) \equiv \beta_i(t_i) = t_i + \tau. \tag{8}$$

This type of equilibria is natural since all creditors are ex-ante identical. We first compute the key variable – bankruptcy payoff – in Subsection 3.1, followed by the characterization of creditors' optimal waiting strategy $\tau^*$ in Subsection 3.2 and the regulator's choice of the optimal clawback window $m^*$ in Subsection 3.3.

## 3.1 Bankruptcy Payoff

Under the conjectured strategy in (8), we can explicitly rewrite the withdrawal rate defined in (1) as

$$w_t(t_0, \beta) = \begin{cases} 0 & t \leq t_0 + \tau \\ \frac{1}{\eta} & t_0 + \tau < t \leq t_0 + \eta + \tau \end{cases}. \tag{9}$$

Before the first creditor becomes informed at $t_0$ and then exits at $t_0 + \tau$, there is no capital outflow. At every unit of time after $t_0 + \tau$, a $\frac{1}{\eta}$ fraction of creditors, who learned about the bad shock $\tau$ dates ago, exit the firm. Therefore, the pivotal ($k$th) creditor becomes informed at $t_0 + k\eta$ and subsequently exits at

$$\hat{t}\,(k, t_0, \beta) = t_0 + k\eta + \tau, \tag{10}$$

driving the firm into bankruptcy. Mathematically, expression (10) is a simplification of (3) using (9). A total of $\frac{m}{\eta}$ mass of creditors receive repayments during the final $m$ dates $[\hat{t} - m, \hat{t}]$ and are therefore subject to clawback:

$$n_c = \frac{m}{\eta}. \tag{11}$$

The affected $n_c$ creditors need to return the full repayment $e^{gt}$ that they have received, and the total proceeds defined in (4) can be simplified to

$$RC_{\hat{t}} = \int_{\hat{t}-m}^{\hat{t}} e^{gt} \frac{1}{\eta} dt = \frac{e^{g(t_0+\tau)}}{g\eta} \left( e^{gk\eta} - e^{g(k\eta-m)} \right). \tag{12}$$

Together with the assets in the firm at bankruptcy $Y_{\hat{t}}$, the total resources are shared among $1-k+n_c$ creditors, including those who did not exit $(1-k)$ and those who are subject to clawback $(n_c)$. Lemma 1 characterizes the bankruptcy payoff under avoidable preference.

**Lemma 1** *Suppose all creditors use an identical exit strategy: $\beta(t) = t + \tau$, then the total assets left in the firm at time $t$ is*

$$Y_t = \begin{cases} Ae^{gt} & 0 \le t \le t_0 \\ Ae^{gt_0+g'(t-t_0)} & t_0 < t \le t_0+\tau \\ Ae^{gt_0+g'(t-t_0)} - \frac{e^{gt}-e^{g(t_0+\tau)+g'(t-t_0-\tau)}}{(g-g')\eta} & t_0+\tau < t \le t_0+\tau+k\eta \end{cases} \tag{13}$$

*At the time of bankruptcy $\hat{t}$, each of the remaining $1-k+n_c$ creditors receives*

$$\alpha(\tau,k,m)e^{gt_0} \equiv \frac{Y_{\hat{t}} + RC_{\hat{t}}}{1-k+n_c} = \frac{Ae^{g'(k\eta+\tau)} - \frac{e^{g\tau}}{(g-g')\eta}\left(e^{gk\eta} - e^{g'k\eta}\right) + \frac{e^{g\tau}}{g\eta}\left(e^{gk\eta} - e^{g(k\eta-m)}\right)}{1-k+\frac{m}{\eta}} e^{gt_0} \tag{14}$$

The first two cases in (13) reflect the high growth rate $g$ prior to $t_0$, the low growth rate $g'$ between $(t_0, t_0+\tau)$, and the fact that no creditor exits before $t_0+\tau$. After $t_0+\tau$, creditors start to take assets out of the firm at the rate of $\frac{1}{\eta}$ according to (9). Its effect on $Y_t$ is reflected by the second term in the third case of (13).

Figure 1 summarizes the evolution of the game and the payoff to creditors. The bad shock hits at $t_0$ reducing the asset growth rate from $g$ to $g'$ (the dashed curve). Creditors gradually learn about the bad shock between $[t_0, t_0+\eta]$ (the red region). The pivotal creditor becomes informed at $t_0+k\eta$ and exits at $\hat{t} = t_0+k\eta+\tau$, triggering bankruptcy. Creditors, who exit the firm at least $m$ dates prior to bankruptcy between $[t_0+\tau, \hat{t}-m]$, receive the promised payment $e^{gt}$ in full (the dark black curve). Repayments made within the final $m$ dates are subject to clawback (the green region), increasing total assets available in bankruptcy. The affected creditors together with the remaining ones in bankruptcy receive the bankruptcy payoff $\alpha(\tau,k,m)e^{gt_0}$ (the blue line).

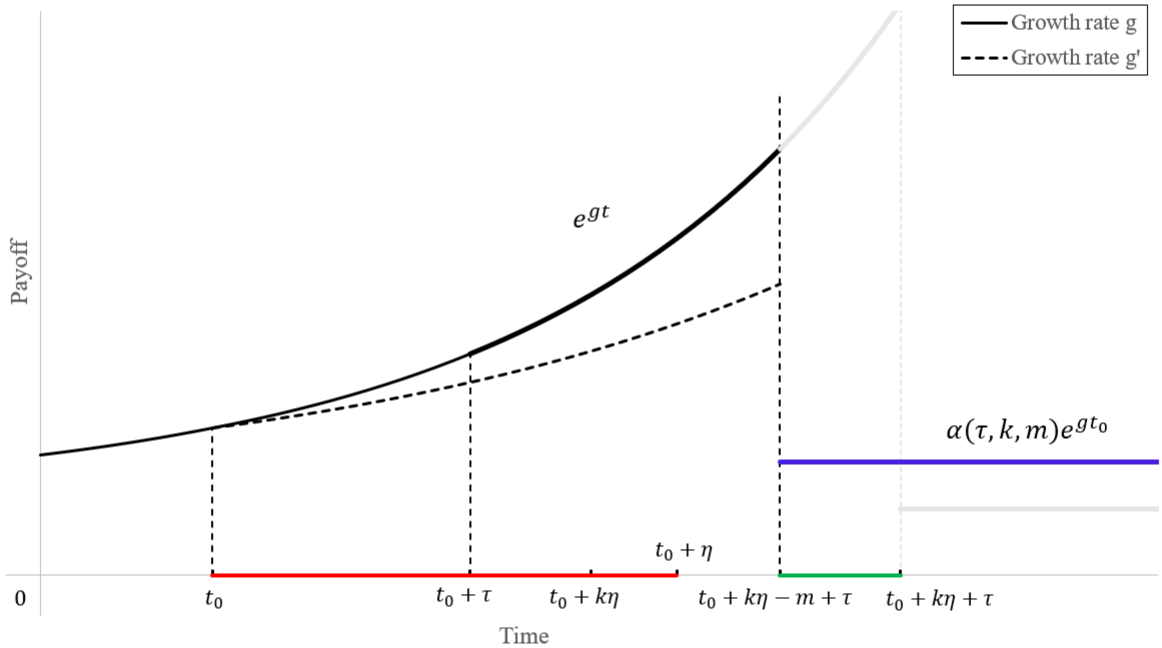



Figure 1: Evolution of the game and payoff to creditors

## 3.2 Equilibrium Waiting Time

In this subsection, we solve for creditors' optimal exit strategy $\beta^*(t_i)$, or equivalently, the common waiting time $\tau^*$. To conserve notations, we denote by $\beta_i = t_i + \tau_i$ the exit time of a generic individual creditor $i$ who becomes informed at time $t_i$ and by $\tau^*$ the equilibrium strategy of other creditors. Throughout the paper, we impose the following parameter restriction to ensure that the equilibrium waiting time $\tau^*$ is well defined. We postpone its intuition until after the first-order condition in (19).

**Assumption 1** *The parameters $\lambda$, $\eta$, and $g$ satisfy* $0 < \eta - \frac{1}{g-\lambda} < \frac{1}{g}$.

Given $\tau^*$, creditor $i$ can receive the full repayment if and only if it is outside of the clawback window: $t_i + \tau_i \leq \hat{t} - m$. Using (10), this condition can be explicitly expressed as the bad shock occurring sufficiently late relative to $t_i$:

$$t_0 \geq t_i + \tau_i - k\eta - \tau^* + m. \tag{15}$$

From Lemma 1 and expression (5), the expected payoff to creditor $i$ becomes

$$\Pi_i(\tau_i|t_i,\tau^*) = \int_{t_i+\tau_i\leq\hat{t}-m} e^{g(t_i+\tau_i)}\psi(t_0|t_i)dt_0 + \int_{t_i+\tau_i>\hat{t}-m} \alpha(\tau^*,k,m)e^{gt_0}\psi(t_0|t_i)dt_0. \quad (16)$$

Expression (16) is the simplified version of (5) under perfect enforcement of the clawback policy ($p=1$). Creditor $i$ receives $e^{g(t_i+\tau_i)}$ if she exits outside of the clawback window as in (15), or otherwise, the bankruptcy payoff $\alpha(\tau^*,k,m)e^{gt_0}$. The optimal choice of $\tau_i$ satisfies the first-order condition from (16):

$$[1-\Psi(t_i+\tau_i-\tau^*-k\eta+m|t_i)]ge^{g(t_i+\tau_i)} = \psi(t_i+\tau_i-\tau^*-k\eta+m|t_i)\left(1-\alpha e^{-g(\tau^*+k\eta-m)}\right)e^{g(t_i+\tau_i)} \quad (17)$$

Condition (17) highlights the trade-off associated with waiting for an additional moment $\Delta t$. On the left-hand side, the marginal benefit is that the exit payoff increases by $g\Delta t e^{g(t_i+\tau_i)}$ (recall the promised repayment grows at the rate of $g$) before the clawback window becomes effective. This case, captured by (15), happens with probability $1-\Psi(t_i+\tau_i-\tau^*-k\eta+m|t_i)$. The right-hand side of condition (17) reflects the marginal costs. It is possible that the clawback window becomes effective during the next $\Delta t$ instant, that is,

$$\hat{t}-m \in (t_i+\tau_i, t_i+\tau_i+\Delta t), \quad (18)$$

which occurs with probability $\psi(t_i+\tau_i-\tau^*-k\eta+m|t_i)\Delta t$. In this case, creditor $i$ loses the exit payoff $e^{g(t_i+\tau_i)}$ and instead receives the bankruptcy payoff $\alpha(\tau^*,k,m)e^{gt_0}$. As $\Delta t\to 0$, condition (18) boils down to an equality,

$$t_i+\tau_i = t_0+k\eta+\tau^*-m,$$

and the bankruptcy payoff $\alpha(\tau^*,k,m)e^{gt_0}$ equals $\alpha(\tau^*,k,m)e^{g(t_i+\tau_i-\tau^*+m-k\eta)}$, establishing the right-hand side of condition (17).

In a symmetric equilibrium, creditor $i$ adopts the same strategy as other creditors (i.e., $\tau_i=\tau^*$), and the first-order condition (17) simplifies to

$$ge^{g(t_i+\tau^*)}[1-\Psi(t_i-k\eta+m|t_i)] = [e^{g(t_i+\tau^*)}-\alpha e^{g(t_i-k\eta+m)}]\psi(t_i-k\eta+m|t_i). \quad (19)$$

Now is a good time to take a short detour to discuss the essence of Assumption 1, which requires that the interest rate $g$ is neither too big nor too small. On the one hand, if the interest rate is too high, then the marginal benefit of waiting becomes excessively attractive, preventing an interior solution. On the other hand, if the interest rate is too low, early repayments post few negative externalities on future creditors, leading to a diminishing marginal cost of waiting and again preventing an interior solution.

For convenience, we define the following hazard rate:

$$h(k,m) \equiv \frac{\psi(t_0 = t_i - k\eta + m|t_i)}{1 - \Psi(t_0 = t_i - k\eta + m|t_i)} = \frac{\lambda e^{\lambda(k\eta - m)}}{e^{\lambda(k\eta - m)} - 1}. \tag{20}$$

Reorganizing (19), we arrive at an intuitive characterization for the equilibrium waiting time $\tau^*$, as summarized by the following proposition.

**Proposition 1** *Any symmetric equilibrium with $\beta(t_i) = t_i + \tau^*$, where the waiting time $\tau^*$ is strictly positive, satisfies*

$$\tau^* = \frac{1}{g}\ln\left[\alpha(\tau^*, k, m)e^{-g(k\eta - m)}\right] + \frac{1}{g}\ln\left(\frac{h(k,m)}{h(k,m) - g}\right). \tag{21}$$

*In equilibrium, the equity holders receive nothing at $\hat{t} = t_0 + k\eta + \tau^*$, that is,*

$$\alpha(\tau^*, k, m)e^{gt_0} < e^{g\hat{t}}. \tag{22}$$

Even though (21) is an implicit function of $\tau^*$, this decomposition intuitively reflects the two factors affecting creditors' incentive to run: the gap between the exit and bankruptcy payoffs and the probability of a successful exit. First, a higher repayment from a successful exit motivates creditors to leave early. If a creditor exits immediately before the clawback window becomes effective, she receives the full repayment $e^{g(t_0 + k\eta + \tau^* - m)}$. Otherwise, she receives the bankruptcy payoff $\alpha(\tau^*, k, m)e^{gt_0}$. The ratio between the bankruptcy payoff and the full repayment is proportional to $\alpha(\tau^*, k, m)e^{-g(k\eta - m)}$ for a given $\tau^*$. When this ratio is higher, the gap between these two payoffs is narrower, and, thus, not being able to exit successfully becomes less costly for creditors. Consequently, creditors are more willing to wait, reflected by the first term in (21). Second, if creditors are more likely to face bankruptcy (reflected by a higher hazard rate $h(k,m)$), they are more eager to run on the troubled firm, thereby reducing the equilibrium waiting time $\tau^*$. This feature is captured by the fact that a higher $h(k,m)$ reduces the second term ($\frac{h(k,m)}{h(k,m)-g}$) in (21).

A simple rearrangement of (21) yields

$$\frac{h(k,m) - g}{h(k,m)} = \alpha e^{-g(k\eta - m + \tau^*)} \geq \frac{\alpha(\tau^*, k, m)e^{gt_0}}{e^{g(t_0 + k\eta + \tau^*)}},$$

where the inequality uses (10) and $m \geq 0$. Since $\frac{h(k,m)-g}{h(k,m)} < 1$, the above condition implies (22). Intuitively, the bankruptcy payoff must be dominated by the highest payoff from a successful exit, or otherwise, all creditors would wait for the superior bankruptcy payoff. This fact in turn means that the remaining $1-k$ creditors in bankruptcy are not paid in full, thereby triggering bankruptcy and the clawback provision.

Condition (21) highlights our contribution to the existing literature. In many dynamic coordination models, the bankruptcy (termination) payoff $\alpha(\tau^*, k, m)$ is typically assumed to be exogenous, independent of both the creditor's strategy and the termination rule. For example, in Brunnermeier and Morgan (2010), among other differences, the coefficient of termination payoff $\alpha \equiv 1$. We develop a setting in which the bankruptcy payoff is endogenously determined, allowing us to explicitly model how different bankruptcy regulations affect creditors' payoffs, which in turn affect their decision to stay invested and the efficiency of the economy. Such an interdependence leads to a fixed-point problem as in the implicit characterization of $\tau^*$ in (21). Despite this complication, the model remains tractable. As will soon be clear, the endogenous bankruptcy payoff is the key channel that qualitatively shapes the optimal clawback window in Subsection 3.3 and the firm's optimal timing of bankruptcy filing in Section 4. Using (14) to replace $\alpha(\tau^*, k, m)$, we can further characterize the closed-form expression of $\tau^*$ from (21).

**Proposition 2** *The equilibrium waiting time is*

$$\tau^*(k,m) = \max\{0, \frac{1}{g-g'}(\ln A - \ln v(k,m))\}, \tag{23}$$

*where*

$$\begin{aligned} v(k,m) \equiv\ & \frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - \frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta} \\ & + (1-k+\frac{m}{\eta})\left[\frac{g}{\lambda}e^{(g-\lambda)(k\eta-m)-g'k\eta} - (\frac{g}{\lambda}-1)e^{(g-g')k\eta-gm}\right]. \end{aligned} \tag{24}$$

*Such a symmetric equilibrium exists if $v(k,m) > 0$.*

As we show in Appendix A, it is possible that $v(k,m) \geq A$. In this case, given any conjectured equilibrium waiting time $\tau^*$ adopted by other creditors, creditor $i$'s best response is to undercut: $\tau_i < \tau^*$. That leads to a unique symmetric equilibrium, in which all creditors exit immediately after hearing the bad news (i.e., $\tau^* = 0$ and $\beta^*(t_i) = t_i$).

## 3.3 Optimal Clawback Window

In this subsection, we solve for the regulator's optimal clawback policy $m^*$. We first show that the regulator's objective to maximize welfare is equivalent to delaying bankruptcy of the firm, or more precisely, maximizing the equilibrium waiting time $\tau^*$. Next, we characterize the analytical solution of $m^*$, perform comparative static analysis, and discuss related economic and policy implications.

### An Equivalent Welfare Measure

**Proposition 3** *The total welfare defined in (6) depends on the clawback window $m$ only through $\tau^*$ and can be rewritten as*

$$W(\tau^*) = \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*} \frac{1}{\eta} e^{gt} dt + Y_{t_0+k\eta+\tau^*}. \tag{25}$$

*In addition, the total welfare $W(\tau^*)$ is increasing in $\tau^*$.*

Since avoidable preference is purely redistributional among creditors in bankruptcy, the choice of clawback window $m$ per se does not directly affect welfare. However, the clawback policy does affect creditors' withdrawal decisions $\tau^*$, which in turn has a welfare impact. Hence, total welfare can be equivalently calculated as the aggregate payoff to creditors as if there were no clawback: The first $k$ creditors withdrawing at $t \in [t_0 + \tau^*, t_0 + k\eta + \tau^*]$ receive $e^{gt}$, and the remaining creditors share the firm's assets at bankruptcy $Y_{t_0+k\eta+\tau^*}$. This establishes equation (25). A longer waiting time $\tau^*$ improves welfare in two ways. First, any repayments reduce the productive assets in the firm, which appreciate at a rate of $g' > 0$. By delaying the initial repayment at $t_0 + \tau^*$ and consequently all subsequent ones, the firm can keep more productive assets growing, thereby increasing welfare. Second, bankruptcy occurs later at $t_0 + k\eta + \tau^*$, which also keeps productive assets growing for a longer time, improving welfare. In summary, the total welfare $W(\tau^*)$ is increasing in $\tau^*$.

An immediate implication of Proposition 3 is that the social planner's objective to maximize total welfare is equivalent to maximizing creditors' equilibrium waiting time $\tau^*$:

$$m^* = \arg\max_{m \leq k\eta} W(\tau^*) = \arg\max_{m \leq k\eta} \tau^*(k, m). \tag{26}$$

#### The Optimal Clawback Window

To visualize the welfare implications (or equivalently $\tau^*$ due to (26)) associated with the clawback window $m$, we numerically decompose $\tau^*$ into two components according to (21) in Figure 2. On the one hand, a bigger $m$ subjects more creditors $(\frac{m}{\eta})$ to the clawback regulation and thereby makes more resources available for creditors in bankruptcy. As a result, the gap between the bankruptcy payoff (i.e., $\alpha(\tau, k, m) e^{gt_0}$) and the highest full repayment immediately prior to the clawback window (i.e., $e^{g(t_0+k\eta+\tau-m)}$) becomes smaller. The ratio between these two payoffs, which is proportional to $\alpha(\tau, k, m) e^{-g(k\eta-m)}$ for a given $\tau$ (i.e., the first term in the decomposition (21)), is therefore increasing in $m$. This payoff channel increases the equilibrium waiting time $\tau^*$, reflected by the red curve in Figure 2.

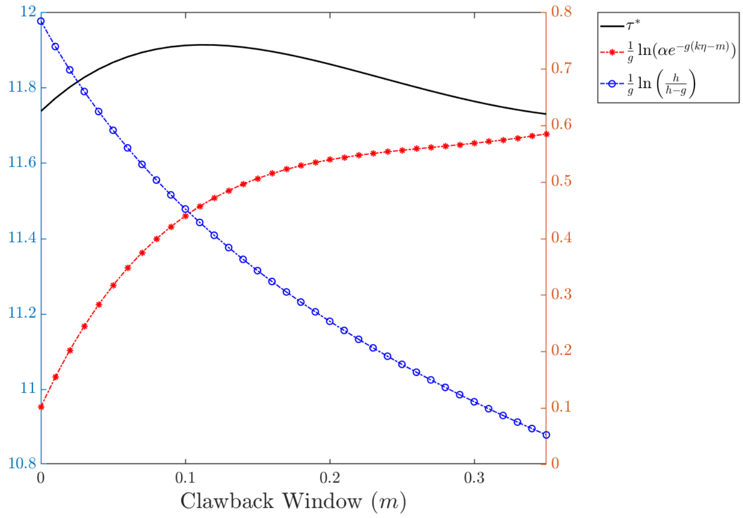



Figure 2: Decomposition of the equilibrium waiting time $\tau^*$.

We decompose the equilibrium waiting time $\tau^*$ into the payoff channel (i.e., $\frac{1}{g}\ln\left(\alpha(\tau, k, m) e^{-g(k\eta-m)}\right)$ in red) and the hazard rate channel (i.e., $\frac{1}{g}\ln\left(\frac{h(k,m)}{h(k,m)-g}\right)$ in blue) according to the two terms in (21). The parameters we use to generate this figure are: $g = 2, g' = 1.9, \lambda = 0.05, \eta = 0.8, A = 2, T = 40$, and $k = 0.5$.

On the other hand, a longer clawback window makes it more difficult for creditors to exit successfully, making them more eager to run ex-ante. Intuitively, without avoidable preference, a successful exit only requires a creditor to outrun the pivotal creditor (i.e., the $k$th creditor to become informed), whereas under avoidable preference, this creditor needs to leave at least $m$ dates before the pivotal creditor in order to receive a full repayment. A longer clawback window allows fewer creditors to exit successfully and therefore makes waiting riskier, resulting in a higher hazard rate (i.e., $h(k, m)$). This higher hazard rate in turn reduces creditors' incentives to wait, reflected by a reduction of the second term in the decomposition (21) (i.e., $\frac{h(k,m)}{h(k,m)-g}$). The blue curve in Figure 2 depicts this hazard rate channel. The optimal clawback window $m^*$ that maximizes $\tau^*$ (black curve in Figure 2) trades off these two channels. Proposition 4 analytically characterizes $m^*$.

**Proposition 4** *The optimal $m^* \in [0, k\eta)$ that maximizes welfare (or equivalently, the equilibrium waiting time $\tau^*(k, m)$) is given by*

$$m^* = \max\left\{0, \frac{1}{g-\lambda} - (1-k)\eta\right\}. \tag{27}$$

Thanks to the closed-form solution, we can perform clean comparative static analysis of $m^*$ on all relevant parameters and draw clear policy implications. The first striking feature is that the optimal avoidable preference regulation $m^*$ is independent of certain firm performance characteristics, such as the initial leverage $A$ and the post-shock growth rate $g'$.[14] This feature implies that a universal regulation can be universally applied to a wide class of firms. Although striking at first glance, this result is rather robust and intuitive. Consider the trade-off associated with a longer clawback window. On the one hand, the marginal benefit is the extra resources made available in bankruptcy, which comes from the full repayment made to $\frac{m}{\eta}$ creditors prior to default. The total proceeds only depend on the contractual terms of debt, such as the interest rate $g$, but not on the firm's performance. On the other hand, the marginal cost is the extra probability of a creditor being trapped in bankruptcy, which only depends on the stochastic structure of events (shock intensity $\lambda$, bankruptcy threshold $k$, and dissemination speed of the news $\eta$), but again not on the firm's performance. Taken together, the firm's performance characteristics should not affect the optimal clawback window.

Second, the result naturally implies that an insider who receives repayment prior to bankruptcy should be subject to a longer clawback window, a common feature across many legal regimes. For instance, for outside creditors the typical clawback window is 90 days in the United States and six months in the United Kingdom, whereas for insiders,

[14] One might notice that the optimal $m^*$ in (27) does depend on the pre-shock growth rate $g$, which is a firm performance parameter. We argue that this is because $g$ also represents the interest rate. Based on the intuition that immediately follows, we conjecture that in a more complicated model where the pre-shock growth rate is different from the interest rate, the former parameter does not affect $m^*$.

this window extends to one year and two years, respectively. In the context of our model, one can argue that bad news travels among insiders more quickly, hence a lower $\eta$. As a result, insiders are more eager to exit their debt positions, and a longer clawback window provides them stronger incentives to stay. Similarly, if the shock intensity $\lambda$ is higher, creditors are more likely to end up in bankruptcy, motivating them to exit sooner. A longer clawback window in this case helps mitigate this effect.

Third, if the debt contracts feature a higher interest rate $g$, then more proceeds are subject to clawback for any given $m$, reducing the need to have a long clawback window. Finally, any increase in the bankruptcy threshold (a larger $k$) should be offset by a longer clawback window $m$. Intuitively, when avoidable preference is in place, the pivotal creditor who can successfully exit the firm is the $k - \frac{m}{\eta}$th to be informed, and anyone receiving repayments afterward needs to return the money. We can rewrite (27) as

$$k - \frac{m^*}{\eta} = 1 - \frac{1}{(g - \lambda)\,\eta},$$

which shows that any inefficient delay of bankruptcy caused by a larger $k$ should be compensated by a longer clawback window $m^*$.[15]

We conclude the baseline model with a short note that the optimal design of avoidable preference is time consistent: The regulator ex-ante specifies $m^*$ and does not have any incentive to deviate ex-post when the firm goes bankrupt at $\hat{t}$. This is because ex-post clawback is purely redistributional, and the welfare of different equilibrium outcomes is uniformly ranked by $\tau^*$ regardless of the realization of $t_0$.

# 4 Bankruptcy Protection

Thus far, we have focused on the optimal design of a bankruptcy procedure: avoidable preference. In this section, we consider another key feature of bankruptcy protection – automatic stay – which prevents creditors from individually seizing assets in a bankrupt firm. While most literature on corporate debt assumes automatic stay if the firm goes bankrupt, its ex-ante impact on creditors' willingness to stay invested remains largely unexplored. Unlike the baseline model in Section 2, we now need to consider the firm's

[15] Clearly, because the clawback window $m$ must be weakly positive, the regulator can only move the effective pivotal creditor earlier than $k$. If the bankruptcy threshold $k$ is already inefficiently small (i.e., $k \leq 1 - \frac{1}{(g-\lambda)\eta}$), then delaying bankruptcy through a larger $k$ is efficient. In this case, the optimal clawback window admits a corner solution, $m^* = 0$.

strategic decisions. As such, we start by introducing positive payoff to equity and its decision to trigger bankruptcy in Subsection 4.1. Next, in Subsection 4.2, we show that automatic stay not only promotes coordination in bankruptcy by definition but can also incentivize creditors to stay invested early on. In absence of bankruptcy protection and automatic stay, creditors would frantically run on a relatively healthy firm. Finally, we analyze the firms' optimal timing to seek bankruptcy protection in Subsection 4.3. The result is perhaps surprising. In order to survive longer, firms need to commit to declaring bankruptcy early when they still have assets to make additional repayments.

## 4.1 Equity Stake and Endogenous Bankruptcy Threshold $k$

To accommodate equity, we modify the baseline model by assuming that the bad shock reduces the growth rate to $g'$ only with probability $q \in (0, 1)$ in this section. With complementary probability $1 - q$, the growth rate remains at $g$ even after the bad shock, although the project still terminates exogenously at $t_0 + T$, as in the baseline model. In addition, we allow the firm to choose and commit at time 0 to a threshold $k^*$ that triggers bankruptcy – a parameter we exogenously fix in the baseline model. In making this choice, the firm maximizes its expected equity value: the total assets net of all payouts to creditors. Since the firm's choice of bankruptcy threshold $k^*$ is our focus in this section, we assume away the avoidable preference clause (i.e., $m = 0$) in order to simplify exposition.[16] All other ingredients are the same as in the baseline model. Similar to the baseline model, no one directly observes the bad shock nor its realized effect on the growth rate, and each creditor decides when to withdraw investment after becoming informed at $t_i$. We again focus on a symmetric equilibrium where all creditors choose a common waiting time; that is, $\beta_i = t_i + \tau_i$ and in equilibrium $\tau_i = \tau^*$.

Our analysis of the ex-ante optimal bankruptcy threshold is economically relevant. Indeed, firms can choose when to stop servicing their debt and file for bankruptcy. We argue that firms can also commit up front to such a bankruptcy policy to some extent through holding some illiquid assets. Specifically, suppose distressed firms primarily rely on selling liquid assets as a last resort to service debt repayments shortly before bankruptcy; then the fraction of liquid assets on the balance sheet can be broadly interpreted as the

[16] One can also study the optimal threshold $k^*$ given any clawback window $m$. The results and intuitions are similar with more complicated mathematics. The problem becomes more challenging if one studies the ex-ante design of the optimal clawback window $m^*$, taking into consideration that firms' decision $k^*(m^*)$ depends on $m^*$. In this case, total welfare can no longer be equivalently measured by creditors' equilibrium waiting time $\tau^*$. Given the length of this article, we leave this problem for future research.

bankruptcy threshold $k$ in the model. Once firms deplete the liquid assets, they are forced into bankruptcy, and the remaining creditors share the illiquid portion of the firm's assets. To the extent that firms cannot quickly ex-post adjust the ratio of their liquid assets, we view their bankruptcy policy $k$ as being committed ex-ante. Granted that firm's ex-post choice of bankruptcy filing is an equally interesting question, we only provide a short discussion at the end of this section, considering the length of this article.

To begin, we specify the payoff to a generic creditor $i$:

$$\Pi_i(\tau_i|t_i,\tau^*) = \left(1-q+q\int_{t_i+\tau_i\le\hat{t}}\psi(t_0|t_i)dt_0\right)e^{g(t_i+\tau_i)} + q\int_{t_i+\tau_i>\hat{t}}\alpha(\tau^*,k)e^{gt_0}\psi(t_0|t_i)dt_0, \tag{28}$$

where the time of bankruptcy $\hat{t}$ is given by (10). The first term in (28) summarizes two possibilities to receive a full repayment. With probability $1-q$, the bad shock does not affect the asset growth rate, and the creditor receives the full repayment of $e^{g(t_i+\tau_i)}$ regardless of the exit time. In addition, when the bad shock reduces the asset growth rate, the creditor may still be fully repaid if it exits before bankruptcy (i.e., $t_i+\tau_i\le\hat{t}$). The second term in (28) captures the bankruptcy payoff when the asset growth rate is reduced to $g'$, and the creditor fails to exit prior to bankruptcy.

The following first-order condition is conceptually similar to (19), with one more marginal benefit of waiting for an additional $\Delta t$ moment: If the asset growth rate remains at $g$ (with probability $1-q$), then the extra repayment of $g\Delta t e^{g(t_i+\tau_i)}$ is guaranteed:

$$\begin{aligned} &ge^{g(t_i+\tau_i)}[1-q+q(1-\Psi(t_i+\tau_i-\tau^*-k\eta|t_i)] \\ &= q\psi(t_i+\tau_i-\tau^*-k\eta|t_i)\left(e^{g(t_i+\tau_i)}-\alpha(\tau^*,k)e^{g(t_i+\tau_i-\tau^*-k\eta)}\right). \end{aligned} \tag{29}$$

We can then replicate Propositions 1 and 2 in the baseline model.

**Proposition 5** *When the bad shock reduces the growth rate with probability $q$, the equilibrium waiting time $\tau^*>0$ can be decomposed into*

$$\tau^*(k,q) = \frac{1}{g}\ln\left[\alpha(\tau^*,k)e^{-gk\eta}\right] + \frac{1}{g}\ln\left(\frac{h(k,m=0)}{h(k,m=0)-g-g\frac{1-q}{q}\frac{1}{1-\Psi(t_0=t_i-k\eta|t_i)}}\right). \tag{30}$$

*The analytical solution of $\tau^*$ is given by*

$$\tau^*(k,q) = \max\left\{0, \frac{1}{g-g'}\left(\ln A - \ln v(k,q)\right)\right\},$$

*where*

$$v(k,q) \equiv \frac{e^{(g-g')k\eta}-1}{(g-g')\eta} + (1-k)\left(e^{(g-g'-\lambda)k\eta}\frac{g}{\lambda}\left[e^{\lambda\eta} - \frac{1}{q}(e^{\lambda\eta}-1)\right] - (\frac{g}{\lambda}-1)e^{(g-g')k\eta}\right). \tag{31}$$

Comparing the decompositions in (30) and (21) reveals that when $q = 1$, the current setup admits the baseline model (with $m = 0$) as a special case. As the bad shock becomes more benign (i.e., as $q$ decreases), creditors are more willing to wait (i.e., the equilibrium $\tau^*$ increases), driven by the hazard rate channel (i.e., the second term in (30)).

## 4.2 Bankruptcy Protection and Ex-Ante Runs

Our model offers new insights into the role of bankruptcy protection. The received wisdom is that automatic stay prevents creditors from running after the firm declares bankruptcy, thereby promoting coordination among them. We show that it also has an ex-ante effect when the firm is not yet bankrupt. A world without automatic stay is similar to allowing the maximum number of creditors ($k = k_{\max}$, which will be defined later) to exit with full repayments in our model. Put differently, creditors can freely run on the firm until no assets are left, and their payoff has a first-come-first-serve feature.

Formally, we define $k_{\max}$ to be the natural upper bound of the bankruptcy threshold such that exactly zero assets are left at the time of bankruptcy when the bad shock reduces the asset growth rate to $g'$. Mathematically, $k_{\max}$ is the solution to

$$Y_{t_0+k_{\max}\eta}|_{\text{asset growth rate}=g' \text{ after } t_0} = 0 \tag{32}$$

under a conjectured equilibrium strategy $\tau^* = 0$, where the evolution of $Y_t$ in this case is given by Lemma 1, with $\tau = 0$. Since we do not consider clawback window $m$ here, the bankruptcy payoff $\alpha(\tau^* = 0, k_{\max}, m = 0)$ is also 0. The next result characterizes $k_{\max}$ and proves that the conjectured "no-waiting" strategy $\tau^* = 0$ is indeed an equilibrium under some conditions.

**Proposition 6** *Define* $\overline{A} \equiv \frac{e^{(g-g')\eta}-1}{(g-g')\eta} > 0$ *and* $q_0 \equiv \frac{e^{\lambda\eta}-1}{e^{\lambda\eta}-(1-\frac{\lambda}{g})(1+A(g-g')\eta)^{\frac{\lambda}{g-g'}}}$. *For any* $A \in [1, \overline{A})$ *and* $q \in [q_0, 1]$, *the maximum bankruptcy threshold given by (32),*

$$k_{\max} \equiv \frac{1}{(g-g')\eta}\ln\left(A(g-g')\eta + 1\right),$$

*is well defined. In addition, when $k = k_{\max}$, the equilibrium waiting time is $\tau^* = 0$.*

Intuitively, without bankruptcy protection, creditors who are late in collecting repayments receive nothing. As a result, creditors have such a strong incentive to exit ex-ante that there is a corner solution: Every creditor leaves immediately upon learning about the bad shock ($\tau^* = 0$).[17] The extreme outcome of zero waiting time may accelerate the firm's bankruptcy compared with a firm who commits to seeking bankruptcy protection when there are still some assets left. In the latter case, the remaining assets preserved in bankruptcy offer creditors more incentives to stay invested up front. The natural question that follows is when a firm should seek bankruptcy protection (i.e., the optimal threshold $k$). We address this question in the next subsection.

## 4.3 Optimal Timing to Seek Bankruptcy Protection

Similar to the baseline model, if the bad shock lowers the asset growth rate to $g'$, then equity receives nothing in bankruptcy. Hence, equity is only valuable when the asset growth rate stays at $g$ following the bad shock at $t_0$. A convenient feature of this setup is that the leverage is a constant $\frac{1}{A}$. When the firm is terminated at $\hat{t} = t_0 + k\eta + \tau^*(k)$, the assets and liabilities are $(A-k)e^{g\hat{t}}$ and $(1-k)e^{g\hat{t}}$, respectively. Therefore, the expected value of equity can be succinctly expressed as follows:

$$\mathbb{E}\,(1-q)\,(A-1)\,e^{g(t_0+k\eta+\tau^*)}, \tag{33}$$

where the expectation is taken over all possible realization of $t_0$. Clearly, the firm's objective of maximizing the expected value of equity in (33) is equivalent to maximizing its life span:

$$k^* = \arg\max_k k\eta + \tau^*(k,m). \tag{34}$$

Such an objective can be alternatively motivated by a manager's desire to stay in the job and the fact that a bankruptcy filing is typically associated with a shakeup of the management. The manager would therefore like to choose an optimal timing to declare

[17] Technically, the existence of such an equilibrium relies on $A$ being not too big. Otherwise, even in the $g'$ case, after all creditors withdraw their capital immediately at $t_i$, the firm still has some assets left, which would encourage creditors to deviate and wait for some time. In addition, we need $q$ to not be too small or otherwise the likelihood of the bad shock becomes too negligible to motivate any creditors to exit.

bankruptcy (implemented by holding some illiquid assets as previously discussed) in order to maximize the expected life span of the firm.

How can a firm maximize its expected life span? One might naturally think that the optimal $k^*$ to prolong firm life is simply $k_{\max}$: Repay creditors until no assets are left. After all, why would the firm commit to an early bankruptcy if the goal is to maximize its life? Surprisingly, this intuition is incomplete. As we show in Proposition 7, the optimal threshold to trigger bankruptcy $k^*$ can be strictly smaller than $k_{\max}$. The key driver of this result is in fact the intuition of Proposition 6: As $k$ approaches $k_{\max}$, creditors are eager to exit up front (i.e., $\tau^* = 0$), driving the firm into bankruptcy more quickly. The next proposition, which is the main finding of this subsection, characterizes the optimal threshold $k^*$ and shows that $k^*$ is typically an interior solution strictly smaller than $k_{\max}$.

**Proposition 7** *The optimal threshold $k^*$ to seek bankruptcy protection satisfies*

$$e^{(g-g')k^*\eta}\left[-\frac{g}{\lambda}+1+\frac{g}{q}\left(\eta(1-k^*)+\frac{1}{\lambda}\right)\left(-(1-q)e^{\lambda\eta}+1\right)e^{-\lambda k^*\eta}\right]=1.$$

*Under the parameter condition $\left(\frac{\lambda\eta+1}{1-\lambda/g}\right)^{\frac{g-g'}{\lambda}} < A(g-g')\eta+1$, the optimal threshold is strictly interior, $0 < k^* < k_{\max}$.*

To demonstrate how the threshold to seek bankruptcy protection ($k$) affects the firm's life span, we plot $k\eta + \tau^*$ in Figure 3 and decompose it into three components using (30). First, a larger $k$ mechanically makes the firm more robust to creditors' exit as it allows more creditors to exit before bankruptcy. This effect is captured by the $k\eta$ term in the firm's objective function (34) and the green line in Figure 3. Second, as $k$ increases, it becomes relatively easier for creditors to exit successfully, which alleviates their incentives to run. This hazard rate channel (i.e., the second term in the decomposition (30)) is reflected by the increasing blue curve in Figure 3. Finally, the novel payoff channel in our model works in the opposite direction. A higher bankruptcy threshold $k$ allows more creditors to exit with full repayments, leaving fewer assets for the remaining creditors to share in bankruptcy. This effect widens the gap between the full exit payoff and the bankruptcy payoff, which motivates creditors to run more frantically, reflected by the red curve in Figure 3. Mathematically, the first term in (30) (i.e., $\alpha(k,\tau^*)e^{-gk\eta}$), capturing the payoff gap, is decreasing in $k$ when $k$ is sufficiently large. The optimal bankruptcy threshold $k^*$ balances all three effects and is in general interior.

Our model offers a novel insight into the role of illiquid assets in the firm: Holding some illiquid assets can mitigate potential creditor runs. Recall that the firm can commit

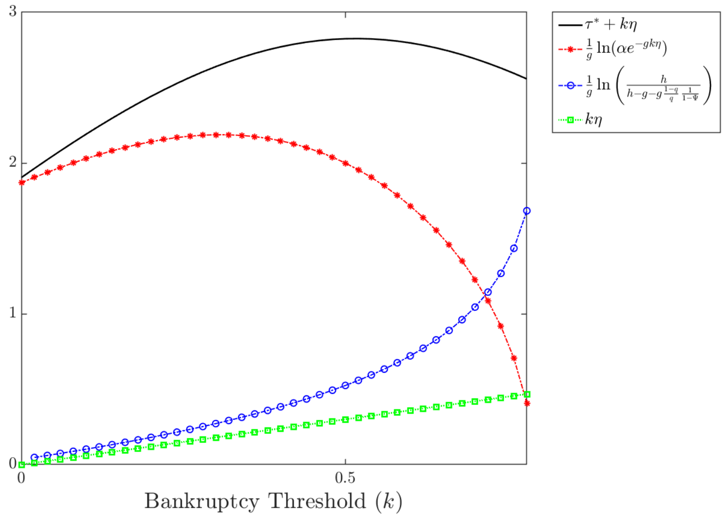



Figure 3: Decomposition of the firm's life span $k\eta + \tau^*$

We decompose the firm's objective into three components: the number of creditors allowed to exit (i.e., $k\eta$ in green), the payoff gap channels (the first term in (30), in red), and the hazard rate channel (the second term in (30), in blue). The parameters we use to generate this figure are: $g = 2, g' = 1.6, \lambda = 0.15, \eta = 0.6, T = 40, A = 2, k = 0.5$.

to filing for bankruptcy earlier (i.e., a smaller $k^*$) by holding more illiquid assets. Our result therefore suggests that firm optimal liquidity level $k^*$ is less than what the firm can maximally support $k_{\max}$. Intuitively, illiquid assets guarantee some positive payoffs for creditors in bankruptcy and provide them some assurance to stay invested early on.

Another surprising feature of the model is that a higher intensity ($\lambda$) and a bigger magnitude ($g - g'$) of the bad shock lead to opposite implications on the timing of bankruptcy filing $k^*$ and creditors' recovery rate, even though both parameter changes make the bad shock more severe. One can define the recovery rate as the ratio between the actual payoff in bankruptcy and the promised full repayment:

$$\frac{\alpha e^{gt_0}}{e^{g(t_0+k^*\eta+\tau^*)}} = \alpha e^{-g(k^*\eta+\tau^*)}.$$

On the one hand, when the bad shock is more likely to occur (a higher $\lambda$), creditors are less likely to exit successfully. Firms should therefore seek bankruptcy protection early (a lower $k^*$ as in Panel A of Figure 4) in order to preserve more capital for creditors in bankruptcy, resulting in a higher recovery rate (Panel C of Figure 4). On the other hand, when the magnitude of the bad shock is larger (a lower $g'$ and therefore a bigger

$g - g'$), the gap between the full exit payoff and bankruptcy payoff is wider. Unlike the previous case for $\lambda$, firms now should file for bankruptcy later (a higher $k^*$ as in Panel B) in order to allow more creditors to exit, thereby reducing the chance for them to receive the bankruptcy payoff, resulting in a lower recovery rate (Panel D). Two similar aspects of the bad shock – intensity and magnitude – generate distinctively different effects on the optimal bankruptcy threshold $k^*$ and the equilibrium recovery rate. This contrast demonstrates the importance of using a dynamic model to separate the time aspect (intensity) and the static aspect (magnitude) of the problem.

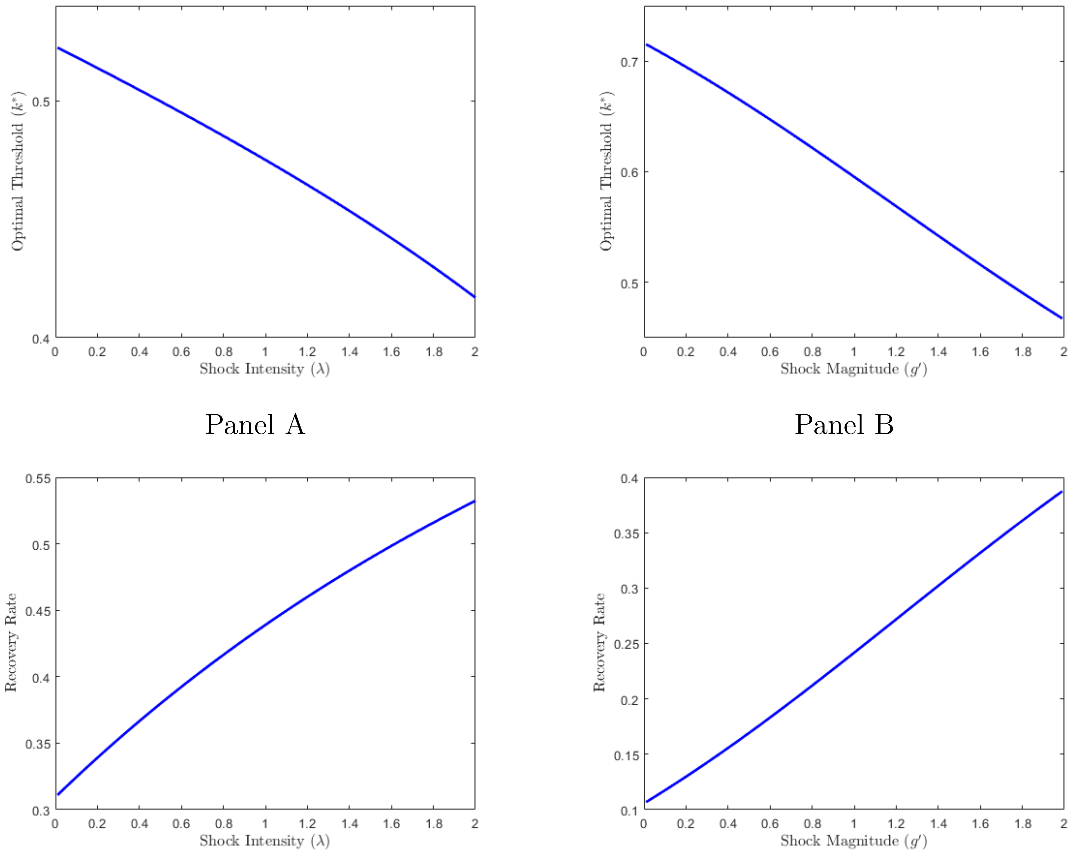
The model's prediction on recovery rate is consistent with a well-known empirical fact that when an industry is in distress, creditors' recovery rate is typically lower (see, for example, Acharya, Bharath and Srinivasan (2007); Mora (2012); Jankowitsch, Nagler and Subrahmanyam (2014)). In the context of our model, the growth rate (following the bad shock) tends to be lower in a distressed industry, and firms tend to file for bankruptcy later, leaving fewer assets for creditors in bankruptcy.

We conclude this subsection with a brief discussion on the time-inconsistency problem. Unlike the optimal design of the clawback window $m^*$ in the baseline model, the firm's choice of the bankruptcy threshold is time-inconsistent and therefore requires commitment. Although it is ex-ante optimal for the firm to commit to filing for bankruptcy early by choosing a relatively lower threshold $k^* < k_{\max}$, once creditors start to exit, the firm always prefers to make additional repayments in order to survive longer ex-post. As such, the firm has incentives to deviate from $k^*$. While we have argued that the ex-ante optimal choice of $k^*$ is an interesting problem in its own right, we acknowledge that firms in practice can choose a bankruptcy threshold ex-post based on their performance and the action of their creditors. A time-consistent bankruptcy policy is interesting to study, and we look forward to future research on this matter.

# 5 Extensions

## 5.1 A Fragile First Best with Ultra-Long Clawback Window

In this subsection, we consider the first-best outcome that maximizes social welfare and show that the policy that may achieve such an outcome is fragile in that it suffers from the multiple equilibria problem. Even after the bad shock, the firm still generates a positive growth rate $g' > 0$ that is superior to creditors' outside option (normalized to 0). Consequently, any repayments made prior to the exogenous termination at $t_0 + T$




Panel A

Panel B

Panel C

Panel D

Figure 4: Optimal Timing to File for Bankruptcy and Recovery Rate

We plot the firm's optimal timing to seek bankruptcy protection $k^*$ and the equilibrium recovery rate $\alpha e^{-g(k^*\eta+\tau^*)}$ as functions of shock intensity $\lambda$ (in Panels A and C) and the post-shock growth rate $g'$ (in Panels B and D). A higher $\lambda$ indicates a more likely shock whereas a lower $g'$ indicates a greater magnitude $(g - g')$. The common parameters are: $A = 2, g = 2, \eta = 0.6$ and $q = 0.95$ across all four panels. In Panels A and C, we fix $g' = 1.6$ and vary $\lambda$. In Panels B and D, we fix $\lambda = 0.15$ and vary $g'$.

cannibalize productive assets in the firm and are therefore inefficient. The first-best outcome is to have all creditors wait for at least $T$ dates,

$$\tau^*_{FB} \geq T, \tag{35}$$

and let the project terminate naturally at $t_0 + T$, delivering each creditor $Ae^{gt_0+g'T}$.

Interestingly, when the clawback window can be enforced perfectly ($p = 1$), an ultra-long clawback window (for example, $m^* \geq T$) offers a chance, albeit fragile as discussed below, to restore the first-best outcome. In this case, even when the first creditor who becomes informed at $t_0$ exits immediately, she is still subject to clawback because $t_0 \in [t_0 + T - m^*, t_0 + T]$. As a result, no creditor receives the exit payoff, and everyone instead receives the bankruptcy payoff. Consequently, individual creditors are indifferent about the timing to exit because the bankruptcy payoff is determined by the collective action of all creditors rather than individual ones. Hence, any strategy profile (not even necessarily symmetric ones as in (8)) that leads to bankruptcy can be supported as an equilibrium. The most efficient outcome is given by (35), and the most inefficient outcome is $\tau^* \equiv 0$.[18] Because of the indeterminacy, we argue that these outcomes are fragile, which is also why we assume (7) in the baseline model to rule out such pathological (indeterministic) cases.

We would like to emphasize that imposing (7) is purely for technical consideration under perfect enforcement ($p = 1$) and does not eliminate any interesting outcomes. As shown in Subsection 5.2, when the clawback enforcement is imperfect ($p < 1$), such equilibrium indeterminacy disappears. The unique symmetric equilibrium converges to the one we solved in Section 3, as clawback enforcement approaches perfection $p \to 1$. In this case, the optimal clawback window $m^*$ indeed satisfies (7) even when ultra-long clawback window $m \geq k\eta$ is possible. In other words, condition (7) can be established as a result rather than an assumption, and the equilibrium we study in the baseline model is robust, whereas others do not survive even a slight perturbation in enforcement.

## 5.2 Imperfect Enforcement

In this subsection, we solve the general model specified in (5) with imperfect enforcement (i.e., $p < 1$). As discussed, such an imperfection can be interpreted as the uncertainty associated with lawsuits in the bankruptcy court, for instance, the preference of the judge, the skill of the lawyers involved, and so on. We show that the unique symmetric equilibrium converges to the one in the baseline model as $p \to 1$. In addition, we find that the optimal clawback window is decreasing in enforcement.

[18] Note that this discussion relies on the clawback clause being triggered upon termination, namely, at least some creditors do not receive the full repayment. Otherwise, creditors simply receive their full repayment and the clawback window becomes irrelevant. For exogenous termination at $t_0 + T$ to trigger clawback, we require $T$ to be large enough such that the final assets are less than final liabilities: $Ae^{gt_0 + g'T} < e^{g(t_0+T)}$. For the outcome with $\tau^* \equiv 0$ to trigger clawback, we require $A \geq 1$ to be small enough such that the firm goes bankrupt when all creditors leave immediately upon being informed during $[t_0, t_0 + \eta]$: $A < \bar{A} \equiv \frac{1}{(g-g')\eta}[e^{(g-g')\eta} - 1]$. The derivation for this condition is provided in the proof of Proposition 6 in Appendix A.

We begin by calculating the terms in (5). The time of bankruptcy $\hat{t}$ is still given by (10). The number of creditors $n_c$ and the total proceeds $RC_{\hat{t}}$, given by (10) and (12) in the baseline model, should be modified as follows:

$$n_c = \min\left\{\frac{m}{\eta}, k\right\},$$

and

$$RC_{\hat{t}} = \int_{\hat{t}-\min\{m,k\eta\}}^{\hat{t}} e^{gt}\frac{1}{\eta}dt = \frac{e^{g(t_0+\tau)}}{g\eta}\left(e^{gk\eta} - e^{g\max\{k\eta-m,0\}}\right).$$

Using these expressions, the bankruptcy payoff in (5) can be explicitly calculated:

$$\alpha_p(\tau, m)\, e^{gt_0} = \frac{Ae^{g'(k\eta+\tau)} - \frac{e^{g\tau}}{(g-g')\eta}\left(e^{gk\eta} - e^{g'k\eta}\right) + p\frac{e^{g\tau}}{g\eta}\left(e^{gk\eta} - e^{g\max\{k\eta-m,0\}}\right)}{1-k+p\min\{\frac{m}{\eta},k\}}e^{gt_0}. \quad (36)$$

Compared with (14), the bankruptcy payoff in (36) reflects the fact that clawback is only successful with probability $p$.

Similar to the baseline model, we focus on a symmetric equilibrium with $\beta_i = t_i + \tau_i$ and $\tau_i^* = \tau^*$ for any creditor $i$. The first-order approach is slightly more complicated in this extension since there is a kink in the payoff function (5) at $m = k\eta$ and $\tau_i = \tau^*$. We discuss three cases $m < k\eta$, $m > k\eta$, and $m = k\eta$ in Appendix B, and the respective equilibrium waiting time is summarized in the following proposition.

**Proposition 8** *For any parameter $0 \le p < 1$, a symmetric equilibrium $\beta(t_i) = t_i + \tau^*(m,p)$ can be characterized as follows:*

1. *When $m \in [0, k\eta)$, the equilibrium is unique and*

$$\begin{aligned}\tau^*(m,p) =& \frac{1}{g-g'}\left\{\ln A - \ln\left[\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - p\frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta}\right.\right.\\ &\left.\left. + \left(1-k+\frac{p}{\eta}m\right)\frac{-(g-\lambda)[pe^{\lambda(k\eta-m)}+(1-p)e^{\lambda k\eta}]+g}{p\lambda e^{-(g-g'-\lambda)k\eta+(g-\lambda)m}+(1-p)\lambda e^{-(g-g'-\lambda)k\eta}}\right]\right\}.\end{aligned}$$

2. *When $m \in (k\eta, +\infty)$, if a symmetric equilibrium $\tau^*$ exists, then it is unique and*

$$\begin{aligned}\tau^*(m,p) =& \frac{1}{g-g'}\left\{\ln A - \ln\left[\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - p\frac{e^{-g'k\eta}[e^{gk\eta}-1]}{g\eta}\right.\right.\\ &\left.\left. + (1-(1-p)k)\left[-\left(\frac{g}{\lambda}-1\right)e^{(g-g')k\eta} + \frac{g}{\lambda}e^{(g-g'-\lambda)k\eta}\right]\right]\right\}.\end{aligned} \quad (37)$$

*3. When $m = k\eta$, no symmetric equilibrium exists.*

When the clawback window $m$ is relatively short as in (7), the outcome and intuition are similar to the baseline model. The additional effect is that clawback is only effective with probability $p$, affecting both the likelihood of a successful exit and the bankruptcy payoff. When the clawback window is long $m > k\eta$, the equilibrium waiting time $\tau^*$ in (37) is insensitive to $m$. Intuitively, all withdrawing creditors are subject to clawback, albeit only successful with probability $p$, and the policy is already at its maximum power for any $m > k\eta$. Unlike in the perfect enforcement case, symmetric equilibrium can be unique because the creditors choose the optimal exiting time considering the scenario where the clawback is not enforced. Finally, the kink in the payoff function (5) at $m = k\eta$ and $\tau_i = \tau^*$ prevents any symmetric equilibrium.

We further show that, when $m > k\eta$, the symmetric equilibrium, if it exists, admits a waiting time $\tau^*(m, p)$ that is strictly decreasing in $p$ (see Lemma 3 in Appendix B). Since zero enforcement $(p = 0, m > k\eta)$ is obviously equivalent to having no clawback window $(m = 0)$ to begin with, we have $\tau^*(m, p) \leq \tau^*(m, p = 0) = \tau^*(m = 0, p)$ for any $m > k\eta$. Therefore, without loss of generality, for any $p \in (0, 1)$, to find the optimal length of the clawback window that maximizes $\tau^*(m, p)$, we only need to consider $m \in [0, k\eta)$. We solve the optimal clawback window $m_p^*$ and show that $m_p^*$ converges to $m^*$ in (27), when policy enforcement becomes perfect $(p \to 1)$ as in the baseline model (see Lemma 4 in Appendix B). This observation provides us with reassurance that the equilibrium we focus on in the baseline model is the only one robust to arbitrary small imperfections in policy enforcement. Finally, we study how policy enforcement affects the optimal design of the clawback window.

**Proposition 9** *For any $p > \underline{p}(k) \equiv \frac{e^{-(g-\lambda)k\eta} - (1-k)(g-\lambda)\eta}{e^{-(g-\lambda)k\eta} + (g-\lambda)k\eta - 1}$, $m_p^* \in [m^*, k\eta]$ is decreasing in $p$.*

Weaker enforcement makes it easier for creditors to leave and claw back fewer assets into the bankruptcy state. Hence, in order to restore the power of avoidable preference, the optimal clawback window must expand to compensate for the smaller probability of a successful clawback.

## 5.3 Recovery of Growth and Negative Growth $g'$

Throughout the paper, we have assumed that the bad shock is permanent in order to model a firm heading for bankruptcy in a succinct manner. However, in practice, many bad shocks are temporary. One can modify the baseline model to incorporate the possibility that the growth rate $g'$ might revert back to $g$ after $t_0$ with intensity $\hat{\lambda}$. Denote by $t_1 \geq t_0$ the random time when this event happens. Assume for simplicity that creditors stop learning about the bad shock at $t_1$ as it no longer exists, and those who have exited the firm cannot reinvest. All remaining creditors automatically stay invested until the project naturally terminates at $t_0 + T$. All other model ingredients are the same as in Section 2.

Instead of formally deriving this extension, we provide the key intuitions behind it to conserve space. When the growth rate recovers sufficiently quickly (i.e., $t_1$ is sufficiently close to $t_0$), the firm can avoid bankruptcy in equilibrium. Consequently, when creditors decide their exit time, waiting involves an additional marginal benefit: The growth rate might improve, thereby eliminating the risk of bankruptcy. Although orthogonal to the trade-off in the baseline model, this channel motivates creditors to stay invested for a longer period of time. On the other hand, the regulator's preference to delay exit is not affected by this new ingredient. A longer waiting time $\tau$ provides the firm more chance to recover its original growth rate and is therefore more efficient. As such, we argue that the results in our baseline model are robust to the extension of growth recovery. A surprising insight related to Proposition 7 is that by committing to filing for bankruptcy, the firm could avoid bankruptcy altogether because creditors are more willing to stay invested, thereby offering the firm a better chance to recover.

When the post-shock growth rate $g'$ is negative, the equilibrium waiting time $\tau^*$ in Proposition 1 does not change mathematically. However, a negative growth rate does alter the regulator's welfare measure in Proposition 3. Specifically, since keeping money inside the firm is no longer productive, welfare is decreasing in $\tau^*$. The numerical simulation in Figure 5 suggests that the hump-shaped relation between $\tau^*$ and the clawback window $m$ remains a robust feature. Therefore, the welfare-maximizing $m^*$ should be either 0 or (the left limit of) $k\eta$, depending on which boundary generates a smaller $\tau^*$. This observation suggests that for firms without a profitable business model, the bankruptcy court should take extreme actions: to either not enforce avoidable preference at all (equivalent to no clawback window $m^* = 0$) or impose a long clawback window.

# 6 Conclusion

In this paper, we build a dynamic coordination model in which atomistic creditors learn about a hidden bad shock in an asynchronous manner and then decide when to withdraw capital. We study the impact of two bankruptcy regulations – avoidable preference and

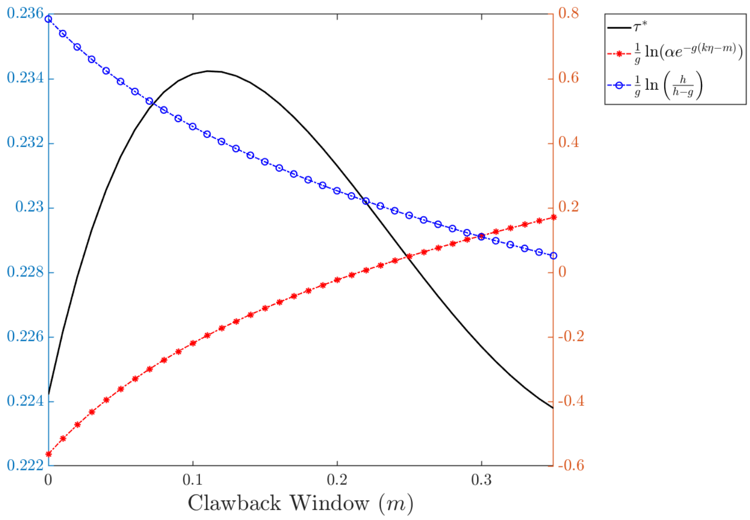



Figure 5: Decomposition of the equilibrium waiting time $\tau^*$ $(g' < 0)$

Similar to Figure 2, we decompose the equilibrium waiting time $\tau^*$ into the payoff channel (i.e., $\frac{1}{g}\ln\left(\alpha\left(\tau,k,m\right)e^{-g(k\eta-m)}\right)$ in red) and the hazard rate channel (i.e., $\frac{1}{g}\ln\left(\frac{h(k,m)}{h(k,m)-g}\right)$ in blue) according to the two terms in (21). The key difference is that the post-shock growth rate $g'$ is negative. The parameters we use to generate this figure are: $g = 2, g' = -0.5, \lambda = 0.05, \eta = 0.8, T = 40$ and $k = 0.5$.

automatic stay – on creditors' ex-ante decision to stay invested. The optimal clawback window features a closed-form solution and should be longer when creditors learn more quickly about the bad shock (such as insiders), when the interest rate is lower, when the bad shock is more likely to occur, or when the bankruptcy is triggered later.

We then show that bankruptcy protection such as automatic stay helps mitigate creditors' incentives to exit ex-ante. Surprisingly, firms can survive longer if they commit to filing for bankruptcy when they still have some assets available to honor more repayments because the resulting higher payoff to creditors in bankruptcy alleviates creditors' ex-ante incentives to run. Finally, the analytical framework provided in this paper is general enough to allow for more work on this topic. We look forward to future research on how other policies regulating creditors' ex-post payoffs affect their ex-ante investment decisions – for example, a redemption fee in the mutual fund industry, a suspension of redemption in the banking industry, and so on.

# A Appendix A. Omitted Proofs

**Proof of Lemma 1.** Under the symmetric exiting strategy $\beta(t) = t + \tau$, $w_t = 0$ for $t < t_0 + \tau$. Based on the dynamic evolution of the total asset value (2) and $Y(0) = A$, we have

$$Y_t = \begin{cases} Ae^{gt} & 0 \le t \le t_0 \\ Ae^{gt_0 + g'(t-t_0)} & t_0 < t \le t_0 + \tau \end{cases}.$$

When $t > t_0 + \tau$, $w_t = \frac{1}{\eta}$ and $dY_t = \left(g'Y_t - \frac{1}{\eta}e^{gt}\right) dt$. Thus, we have $d\left(Y_t e^{-g't}\right) = e^{-g't}\left(dY_t - g'Y_t dt\right) = -\frac{e^{(g-g')t}}{\eta} dt$. Solving the above differential equation with the following initial condition $Y_{t_0+\tau} e^{-g'(t_0+\tau)} = Ae^{gt_0 + g'\tau} e^{-g'(t_0+\tau)} = Ae^{(g-g')t_0}$, we have

$$\int_{u=t_0+\tau}^{t} d\left(Y_u e^{-g'u}\right) = Y_t e^{-g't} - Ae^{(g-g')t_0} = \int_{u=t_0+\tau}^{t} -\frac{e^{(g-g')u}}{\eta} du.$$

Hence,

$$Y_t = Ae^{gt_0 + g'(t-t_0)} - \frac{1}{(g-g')\,\eta}\left[e^{gt} - e^{g(t_0+\tau) + g'(t-t_0-\tau)}\right].$$

Given this dynamic process of $Y_t$, at time $\hat{t} = t_0 + \tau + k\eta$,

$$Y_{\hat{t}} = Ae^{gt_0 + g'(\tau + k\eta)} - \frac{1}{(g-g')\,\eta}\left[e^{g(t_0+\tau+k\eta)} - e^{g(t_0+\tau) + g'k\eta}\right].$$

According to (12), the total proceeds that will be clawed back is $RC_{\hat{t}} = \frac{e^{g(t_0+\tau)}}{g\eta}\left(e^{gk\eta} - e^{g(k\eta - m)}\right)$. Hence, each remaining creditor receives $\alpha(\tau, k, m)e^{gt_0}$; that is,

$$\alpha(\tau, k, m) = \frac{Y_{\hat{t}} + RC_{\hat{t}}}{1 - k + n_c} e^{-gt_0} = \frac{Ae^{g'(k\eta+\tau)} - \frac{e^{g\tau}}{(g-g')\eta}\left(e^{gk\eta} - e^{g'k\eta}\right) + \frac{e^{g\tau}}{g\eta}\left(e^{gk\eta} - e^{g(k\eta-m)}\right)}{1 - k + \frac{m}{\eta}}.$$

■

**Proof of Proposition 1.** We can rewrite the equilibrium condition (19) as

$$ge^{g(t_i + \tau^*)} = h(k, m)[e^{g(t_i+\tau^*)} - \alpha e^{g(t_i - k\eta + m)}]. \tag{38}$$

Eliminating $e^{gt_i}$ on both sides of (38), we have $e^{g\tau^*} = \alpha \frac{h(k,m)}{h(k,m)-g} e^{-g(k\eta-m)}$. Hence, for any $\tau^* > 0$ that makes the first-order condition satisfied, we have

$$\tau^* = \frac{1}{g} \ln \left[\alpha(\tau^*, k, m) e^{-g(k\eta-m)}\right] + \frac{1}{g} \ln \left( \frac{h(k,m)}{h(k,m) - g} \right),$$

thereby completing the proof. ■

**Proof of Proposition 2.** According to (20), the hazard rate $h(k,m) = \frac{\lambda e^{\lambda(k\eta-m)}}{e^{\lambda(k\eta-m)}-1}$, and, thus,

$$\frac{h(k,m)}{h(k,m)-g} = \frac{\lambda e^{\lambda(k\eta-m)}}{g-(g-\lambda)e^{\lambda(k\eta-m)}}.$$

When $m \in (0, k\eta)$, we can plug in $\alpha(\tau^*, k, m)$ into equation (21) and rearrange it to find a unique $\tau^*$ that satisfies the equilibrium condition. If the equilibrium waiting time $\tau^*$ is well defined, then

$$\tau^*(k,m) = \frac{1}{g-g'} \left(\ln A - \ln v(k,m)\right).$$

Please refer to (24) to see the expression of $v(k,m)$. Obviously, when $v(k,m) < 0$ or $v(k,m) > A$, then the above expression cannot give a well defined equilibrium waiting time. Next, we are concerned with the conditions under which $\tau^*(k,m)$ is well defined and is non-negative. We first discuss three mutually exclusive and collectively exhaustive cases regarding $v(k,m)$.

**Case 1:** $v(k,m) > A$. Given all the other creditors choose to wait $\tau^* \geq 0$, the first-order derivative for creditor $i$ is

$$\frac{\partial \mathbb{E}(\pi_i|t_i)}{\partial \tau_i} = \frac{e^{g(t_i+\tau_i)}}{e^{\lambda\eta}-1} \left( (g - [1-\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}]\lambda) e^{\lambda(\tau^*-\tau_i+k\eta-m)} - g \right).$$

Thus, the best response for creditor $i$, given that others choose $\tau^*$, is

$$\tau_i = \tau^* + k\eta - m - \frac{1}{\lambda} \ln \left( \frac{1}{1-[1-\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}]\frac{\lambda}{g}} \right).$$

The condition $v(k,m) > A$ is equivalent to

$$\frac{\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - \frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta}}{1-k+\frac{m}{\eta}} > \frac{A}{1-k+\frac{m}{\eta}} - \left[\frac{g}{\lambda}e^{(g-\lambda)(k\eta-m)-g'k\eta} - (\frac{g}{\lambda}-1)e^{(g-g')k\eta-gm}\right].$$

Multiplying both sides by $e^{-(g-g')k\eta+gm}$, we have

$$\frac{A}{1-k+\frac{m}{\eta}}e^{-(g-g')k\eta+gm}-\Big[\frac{g}{\lambda}e^{-\lambda(k\eta-m)}-(\frac{g}{\lambda}-1)\Big]<\frac{\frac{e^{gm}(1-e^{-(g-g')k\eta})}{(g-g')\eta}-\frac{e^{gm}-1}{g\eta}}{1-k+\frac{m}{\eta}}. \tag{39}$$

Based on the inequality in (39), we have

$$\begin{aligned}
&\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}=\frac{Ae^{-(g-g')(\tau^*+k\eta)+gm}-\frac{e^{gm}(1-e^{-(g-g')k\eta})}{(g-g')\eta}+\frac{e^{gm}-1}{g\eta}}{1-k+\frac{m}{\eta}}\\
<&\frac{Ae^{-(g-g')(\tau^*+k\eta)+gm}}{1-k+\frac{m}{\eta}}-\frac{A}{1-k+\frac{m}{\eta}}e^{-(g-g')k\eta+gm}+\Big[\frac{g}{\lambda}e^{-\lambda(k\eta-m)}-(\frac{g}{\lambda}-1)\Big]\\
=&\frac{Ae^{-(g-g')k\eta+gm}(e^{-(g-g')\tau^*}-1)}{1-k+\frac{m}{\eta}}+\frac{g}{\lambda}e^{-\lambda(k\eta-m)}-(\frac{g}{\lambda}-1).
\end{aligned}$$

That, in turn, implies

$$1-[1-\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}]\frac{\lambda}{g}<\frac{\lambda}{g}\frac{Ae^{-(g-g')k\eta+gm}(e^{-(g-g')\tau^*}-1)}{1-k+\frac{m}{\eta}}+e^{-\lambda(k\eta-m)}.$$

Therefore, for any $\tau^*$ that is well defined (i.e., $\tau^*\geq 0$), $e^{-(g-g')\tau^*}-1\leq 0$, and, thus,

$$1-[1-\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}]\frac{\lambda}{g}<e^{-\lambda(k\eta-m)}.$$

Based on the above inequality, the optimal $\tau_i$ as a response to others' waiting strategy $\tau^*$ satisfies that $\tau_i<\tau^*+k\eta-m-\frac{1}{\lambda}\ln\frac{1}{e^{-\lambda(k\eta-m)}}=\tau^*$. In sum, if $v(k,m)>A$, the best response $\tau_i$, which makes the first order condition satisfied, has to be strictly smaller than $\tau^*$. Next, we consider a "corner solution." Given all the other creditors choose $\tau^*=0$, the marginal benefit from waiting for any $\tau_i$ is

$$\begin{aligned}
\frac{\partial\mathbb{E}(\pi_i|t_i)}{\partial\tau_i}&=\frac{e^{g(t_i+\tau_i)}}{e^{\lambda\eta}-1}\left((g-[1-\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}]\lambda)e^{\lambda(\tau^*-\tau_i+k\eta-m)}-g\right)\\
&<\frac{ge^{g(t_i+\tau_i)}}{e^{\lambda\eta}-1}\left(e^{-\lambda(k\eta-m)}e^{\lambda(-\tau_i+k\eta-m)}-1\right)=\frac{ge^{g(t_i+\tau_i)}}{e^{\lambda\eta}-1}\left(e^{-\lambda\tau_i}-1\right)<0
\end{aligned}$$

The best response for creditor $i$ is $\tau_i=0$. Therefore, if $v(k,m)>A$, in the unique symmetric equilibrium, $\tau^*=0$.

**Case 2:** $v(k,m) \leq 0$**.** If the condition $v(k,m) \leq 0$ holds, then we have

$$\tfrac{e^{(g-g')k\eta}-1}{(g-g')\eta} - \tfrac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta} + (1-k+\tfrac{m}{\eta})\left[\tfrac{g}{\lambda}e^{(g-\lambda)(k\eta-m)-g'k\eta} - (\tfrac{g}{\lambda}-1)e^{(g-g')k\eta-gm}\right] \leq 0.$$

That, in turn, implies

$$\frac{\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - \frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta}}{1-k+\frac{m}{\eta}} \leq \left(\frac{g}{\lambda}-1\right)e^{(g-g')k\eta-gm} - \frac{g}{\lambda}e^{(g-\lambda)(k\eta-m)-g'k\eta}.$$

Multiplying both sides by $e^{-(g-g')k\eta+gm}$, we have

$$\frac{\frac{e^{gm}(1-e^{-(g-g')k\eta})}{(g-g')\eta} - \frac{e^{gm}-1}{g\eta}}{1-k+\frac{m}{\eta}} \leq \frac{g}{\lambda} - 1 - \frac{g}{\lambda}e^{-\lambda(k\eta-m)}.$$

Based on the above inequality and the fact that $\tau^*$ is well defined (i.e., $\tau^* \geq 0$), following the same procedures as in the case where $v(k,m) > A$, we have

$$1 - [1-\alpha(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)}]\frac{\lambda}{g} \geq \frac{\lambda}{g}\frac{Ae^{-(g-g')(\tau^*+k\eta)+gm}}{1-k+\frac{m}{\eta}} + e^{-\lambda(k\eta-m)} > e^{-\lambda(k\eta-m)}.$$

Therefore, the best response $\tau_i$ to others choosing $\tau^*$ is

$$\begin{aligned}\tau_i =& \tau^* + k\eta - m - \frac{1}{\lambda}\ln\frac{1}{1-[1-\alpha(\tau^*,m)e^{-g(\tau^*+k\eta-m)}]\frac{\lambda}{g}} \\ >& \tau^* + k\eta - m - \frac{1}{\lambda}\ln\frac{1}{e^{-\lambda(k\eta-m)}} = \tau^*.\end{aligned}$$

That implies, if $v(k,m) \leq 0$, $\tau_i > \tau^*$ for any $\tau^*$, thereby proving the non-existence of a symmetric equilibrium.

**Case 3:** $0 < v(k,m) \leq A$**.** In the case where $v(k,m) \in (0,A]$, based on the above discussions, the equilibrium waiting time $\tau^*(k,m) \geq 0$ is unique and can be solved explicitly as $\tau^*(k,m) = \frac{1}{g-g'}\left(\ln A - \ln v(k,m)\right)$.

To summarize, if $v(k,m) \in (0,A]$, there is a unique equilibrium waiting time $\tau^* \geq 0$, which satisfies the first-order condition as an interior solution. If $v(k,m) > A$, $\tau^*(k,m) = 0$ constitutes the unique symmetric equilibrium; otherwise, if $v(k,m) \leq 0$, a symmetric equilibrium with a linear waiting strategy does not exist. Next, we show that under

Assumption 1, $v(k,m) > 0$ for any $m \in (0, k\eta)$. Based on that, we can exclude the possibility that a symmetric equilibrium does not exist.

First, consider $\tau^*(k,m) = \frac{1}{g-g'}(\ln A - \ln v(k,m))$, we have

$$\frac{\partial \tau^*(k,m)}{\partial m} = -\frac{1}{g-g'}\frac{v_m(k,m)}{v(k,m)},$$

in which, $v_m(k,m) = \frac{g}{\lambda\eta}e^{(g-g')k\eta - gm}(1-e^{-\lambda(k\eta-m)})[-1+(\eta(1-k)+m)(g-\lambda)]$. Therefore, $v_m(k,m^*) = 0$ if

$$m^* = k\eta - (\eta - \frac{1}{g-\lambda}). \tag{40}$$

Further, since $v_{mm}(k,m)|_{m=m^*} = \frac{g}{\lambda\eta}e^{(g-g')k\eta-gm^*}(1-e^{-\lambda(k\eta-m^*)})(g-\lambda) > 0$, we know that $v(k,m)$ decreases with $m$ for $m \in [0, m^*]$ and increases with $m$, $m \in (m^*, k\eta]$. As such, $v(k,m)$ attains its minimum, and, accordingly, $\tau^*(m)$ attains its maximum at $m^*$. To prove that $v(k,m) > 0$ for all $m \in (0, k\eta)$, it suffices to show that $v(k,m^*) > 0$. Plug $m^*$ (solved in (40)) into $v(k, m = m^*)$, we have

$$v(k,m^*) = \frac{g'e^{(g-g')k\eta}}{g\eta(g-g')} - \frac{(g-\lambda)e^{-g'k\eta+g(\eta-\frac{1}{g-\lambda})}}{g\lambda\eta} + \frac{ge^{\eta(g-\lambda)-1-g'k\eta}}{\lambda\eta(g-\lambda)} - \frac{1}{(g-g')\eta}.$$

So, to prove that $v(k,m^*) > 0$, it suffices to show that

$$\frac{1}{(g-g')g}(g'e^{gk\eta} - ge^{g'k\eta}) > \frac{1}{\lambda}e^{g(\eta-\frac{1}{g-\lambda})}\left(\frac{g-\lambda}{g} - \frac{g}{g-\lambda}e^{-\lambda(\eta-\frac{1}{g-\lambda})}\right). \tag{41}$$

Since $\frac{\partial(g'e^{gk\eta}-ge^{g'k\eta})}{\partial k} = g'g\eta(e^{gk\eta} - e^{g'k\eta}) > 0$, the LHS of (41) is increasing in $k$. For the inequality in (41) to hold true, we need

$$\frac{1}{(g-g')g}(g'e^{gk\eta} - ge^{g'k\eta})\Big|_{k=1-\frac{1}{\eta(g-\lambda)}} > \frac{1}{\lambda}e^{g(\eta-\frac{1}{g-\lambda})}\left(\frac{g-\lambda}{g} - \frac{g}{g-\lambda}e^{-\lambda(\eta-\frac{1}{g-\lambda})}\right).$$

This is equivalent to

$$\frac{g}{g-\lambda}e^{-\lambda(\eta-\frac{1}{g-\lambda})} - \frac{\lambda}{g-g'}e^{-(g-g')(\eta-\frac{1}{(g-\lambda)})} > \frac{g-\lambda}{g} - \frac{\lambda g'}{(g-g')g} = \frac{g-g'-\lambda}{g-g'}. \tag{42}$$

According to Assumption 1, $\eta - \frac{1}{g-\lambda} > 0$, and, thus, the LHS of (42) satisfies

$$\frac{g}{g-\lambda}e^{-\lambda(\eta-\frac{1}{g-\lambda})} - \frac{\lambda}{g-g'}e^{-(g-g')(\eta-\frac{1}{g-\lambda})} > \frac{g}{g-\lambda}(1-\lambda(\eta-\frac{1}{g-\lambda})) - \frac{\lambda}{g-g'}.$$

Moreover, since $\eta - \frac{1}{g-\lambda} < \frac{1}{g}$ (see Assumption 1), the RHS of (42) satisfies

$$\frac{g-g'-\lambda}{g-g'} = \frac{g}{g-\lambda}(1-\frac{\lambda}{g}) - \frac{\lambda}{g-g'} < \frac{g}{g-\lambda}(1-\lambda(\eta - \frac{1}{g-\lambda})) - \frac{\lambda}{g-g'}.$$

Therefore, the inequality in (42) holds and $v(k, m^*) > 0$ for any $m \in (0, k\eta)$.

Furthermore, we check the second-order condition to make sure that $\tau_i = \tau^*(k, m)$ maximizes the creditor $i$'s expected payoff when other creditors take the same strategy $\tau^*$. Given $\tau = \tau^*(m, k)$, the second-order derivative at $\tau_i = \tau^*(m, k)$ is

$$\left.\frac{\partial^2 \mathbb{E}(\pi_i|t_i)}{\partial \tau_i^2}\right|_{\tau_i=\tau^*} = -g(1+\lambda)\frac{e^{g(t_i+\tau^*)}}{e^{\lambda\eta}-1} < 0.$$

Therefore, $\tau_i = \tau^*(k, m)$ is indeed the local maximizer, thereby completing the proof. ■

**Proof of Proposition 3.** Based on the definition of total welfare $W$ in (6) and the symmetric equilibrium captured by $\tau^*$, we can rewrite the total welfare as

$$\begin{aligned} W(\tau^*) &= \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*-m} \frac{1}{\eta} e^{gt} dt + \int_{t_0+k\eta+\tau^*-m}^{t_0+\eta+\tau^*} \frac{1}{\eta}\alpha(\tau^*, k, m) e^{gt_0} dt \\ &= \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*-m} \frac{1}{\eta} e^{gt} dt + \frac{Y_{t_0+k\eta+\tau^*} + \int_{t_0+k\eta+\tau^*-m}^{t_0+\tau^*+k\eta} \frac{1}{\eta} e^{gt} dt}{1-k+\frac{m}{\eta}} \int_{t_0+k\eta-m}^{t_0+\eta} \frac{1}{\eta} dt. \end{aligned}$$

Based on the definition of $\alpha$ and the fact that $\int_{t_0+k\eta-m}^{t_0+\eta} \frac{1}{\eta} = 1-k+\frac{m}{\eta}$, we have

$$W(\tau^*) = \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*-m} \frac{1}{\eta} e^{gt} dt + \int_{t_0+k\eta+\tau^*-m}^{t_0+\tau^*+k\eta} \frac{1}{\eta} e^{gt} dt + Y_{t_0+k\eta+\tau^*} = \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*} \frac{1}{\eta} e^{gt} dt + Y_{\hat{t}}.$$

Next, we leverage on the following Lemma to prove the monotonicity of $W(\tau^*)$ on $\tau^*$. For convenience, we introduce $Y_{t,\tau}$ and $w_{t,s}$ to denote the firm's asset value and the fraction of exiting creditors at time $t$ respectively, if the waiting time is $\tau$ for each creditor.

**Lemma 2** *For any $\tau_2 > \tau_1$,*

$$Y_{t,\tau_2} \geq Y_{t,\tau_1},$$

*for any $t \leq t_0 + k\eta + \tau_1$ with equality holding if and only if $t \leq t_0 + \tau_1$.*

**Proof.** Given the process of asset value $Y_t$ (see (13)), if $t \le t_0 + \tau_1$, clearly, $Y_{t,\tau_2} = Y_{t,\tau_1}$. Moreover, when $t_0 + \tau_1 < t \le t_0 + \tau_2$, we have

$$Y_{t,\tau_2} = Ae^{gt_0 + g'(t-t_0)} > Ae^{gt_0 + g'(t-t_0)} - \frac{1}{(g-g')\,\eta}\left[e^{gt} - e^{g(t_0+\tau_1)+g'(t-t_0-\tau_1)}\right] = Y_{t,\tau_1}.$$

Finally, if $t > t_0 + \tau_2$, $Y_{t,\tau_2} > Y_{t,\tau_1}$ because the term $e^{gt} - e^{g(t_0+\tau)+g'(t-t_0-\tau)}$ in the third case scenario in (13) is strictly decreasing in $\tau$. ■

We can rewrite $Y_{t,\tau}$ in its integral form: $Y_{t,\tau} = Y_{t_0} + \int_{t_0}^{t} g'Y_{s,\tau} - w_{s,\tau}e^{gs}ds$, where

$$w_{s,\tau} = \begin{cases} \frac{1}{\eta} & \text{if } s \in [t_0+\tau, t_0+k\eta+\tau] \\ 0 & \text{otherwise} \end{cases}.$$

Consider any $\tau_2 > \tau_1 > 0$. The difference between welfare associated with waiting times $\tau_1$ and $\tau_2$ is given by:

$$\begin{aligned} W(\tau_2) - W(\tau_1) = &\int_{t_0+\tau_2}^{t_0+\tau_2+k\eta} \frac{1}{\eta}e^{gs}ds - \int_{t_0+\tau_1}^{t_0+\tau_1+k\eta} \frac{1}{\eta}e^{gs}ds \\ &+ \int_{t_0}^{t_0+k\eta+\tau_2} \left(g'Y_{s,\tau_2} - w_{s,\tau_2}e^{gs}\right)ds - \int_{t_0}^{t_0+k\eta+\tau_1} \left(g'Y_{s,\tau_1} - w_{s,\tau_1}e^{gs}\right)ds. \end{aligned}$$

For any $s \le t_0 + \tau_1 < t_0 + \tau_2$, $Y_{s,\tau_2} = Y_{s,\tau_1}$ (Lemma 2), and $w_{s,\tau_1} = w_{s,\tau_2} = 0$. Therefore, we have

$$\begin{aligned} W(\tau_2) - W(\tau_1) = &\int_{t_0+\tau_2}^{t_0+\tau_2+k\eta} \frac{1}{\eta}e^{gs}ds - \int_{t_0+\tau_1}^{t_0+\tau_1+k\eta} \frac{1}{\eta}e^{gs}ds \\ &+ \int_{t_0+\tau_1}^{t_0+k\eta+\tau_2} \left(g'Y_{s,\tau_2} - w_{s,\tau_2}e^{gs}\right)ds - \int_{t_0+\tau_1}^{t_0+k\eta+\tau_1} \left(g'Y_{s,\tau_1} - w_{s,\tau_1}e^{gs}\right)ds \\ = &\int_{t_0+\tau_1}^{t_0+k\eta+\tau_2} g'Y_{s,\tau_2}ds - \int_{t_0+\tau_1}^{t_0+k\eta+\tau_1} g'Y_{s,\tau_1}ds \\ = &\int_{t_0+k\eta+\tau_1}^{t_0+k\eta+\tau_2} g'Y_{s,\tau_2}ds + \int_{t_0+\tau_1}^{t_0+k\eta+\tau_1} g'\left(Y_{s,\tau_2} - Y_{s,\tau_1}\right)ds. \end{aligned}$$

Since $Y_{t,\tau} > 0$ and $Y_{s,\tau_2} - Y_{s,\tau_1} > 0$ (Lemma 2), the above expression is strictly positive, thereby completing the proof. ■

**Proof of Proposition 4.** Please refer to the proof of Proposition 2 to see that, when the symmetric equilibrium exists, the equilibrium waiting time $\tau^*$ increases with $m$ when $m \in [0, k\eta - (\eta - \frac{1}{g-\lambda})]$ and decreases with $m$ when $m \in (k\eta - (\eta - \frac{1}{g-\lambda}), k\eta]$. Therefore, $m^* = \max\{0, k\eta - (\eta - \frac{1}{g-\lambda})\}$. ■

**Proof of Proposition 5.** In any symmetric equilibrium, when other creditors choose $\tau^*$, the optimal choice $\tau_i = \tau^*$. Plug this into the first-order condition (29), we have

$$\left(1 - \alpha(\tau^*, k)e^{-g(\tau^*+k\eta)}\right) q\psi(t_i - k\eta|t_i) = g\left[1 - q + q(1 - \Psi(t_i - k\eta|t_i))\right], \tag{43}$$

in which, $\alpha(\tau^*, k) = \alpha(\tau = \tau^*, k, m = 0)$ (see (14)), or, equivalently,

$$\alpha(\tau^*, k) = \frac{Ae^{g'(\tau+k\eta)} - \frac{e^{g\tau}(e^{gk\eta} - e^{g'k\eta})}{(g-g')\eta}}{1-k}.$$

Rearranging (43), we have

$$\tau^*(k, q) = \frac{1}{g}\ln\alpha(\tau^*, k)e^{-gk\eta} + \frac{1}{g}\ln\frac{h(k, m = 0)}{h(k, m = 0) - g - g\frac{1-q}{q}\frac{1}{1-\Psi(t_i - k\eta|t_i)}},$$

in which, $h(k, m = 0) = \frac{\lambda e^{\lambda k\eta}}{e^{\lambda k\eta}-1}$ (see (20)) and $\Psi(t_i - k\eta|t_i) = \frac{e^{\lambda\eta}-e^{\lambda k\eta}}{e^{\lambda\eta}-1}$. Then, plug $\alpha$, $h$ and $\Psi$ into the above decomposition of $\tau^*$ and follow the same procedure as we did in the proof of Proposition 1, we can show that $\tau^*(k, q) = \max\{0, \frac{1}{g-g'}\ln A - \ln v(k, q)\}$. The definition of $v(k, q)$ can be found in (31). ■

**Proof of Proposition 6.**

**Condition for $A$** Recall that

$$\alpha(\tau = 0, k, m = 0) = \frac{Ae^{g'k\eta} - \frac{e^{gk\eta}-e^{g'k\eta}}{(g-g')\eta}}{1-k} = e^{g'k\eta}\frac{A - \frac{e^{(g-g')k\eta}-1}{(g-g')\eta}}{1-k}$$

By definition, $k_{\max}$ makes $\alpha(\tau = 0, k = k_{\max}, m = 0) = 0$ satisfied, and, thus, $k_{\max} = \frac{1}{(g-g')\eta}\ln\left[1 + A(g - g')\eta\right]$. For the existence of such $k_{\max} \in (0, 1)$, we need $A < \bar{A} \equiv \frac{e^{(g-g')\eta}-1}{(g-g')\eta}$.

**Condition for $q$** When $\tau^* = 0$ and $m = 0$, the first order derivative with respect to $\tau_i$ is

$$e^{g(t_i+\tau_i)} \left( g[1 - q + q(1 - \Psi(t_0 = t_i - k\eta | t_i))] - q\psi(t_0 = t_i - k\eta | t_i) \right).$$

To guarantee that $\tau^* = 0$ is an equilibrium when $k = k_{\max}$, we need

$$\left(\frac{1-q}{q}\right) g \leq \psi(t_0 = t_i - k_{\max}\eta | t_i) - g\left(1 - \Psi(t_0 = t_i - k_{\max}\eta | t_i)\right) = \frac{g - (g-\lambda)e^{\lambda k_{\max}\eta}}{e^{\lambda\eta} - 1}.$$

That is equivalent to

$$q \geq \frac{1}{\frac{g-(g-\lambda)e^{\lambda k_{\max}\eta}}{g(e^{\lambda\eta}-1)} + 1} = \frac{ge^{\lambda\eta} - g}{ge^{\lambda\eta} - (g-\lambda)e^{\lambda k_{\max}\eta}}.$$

Given that $k_{\max} = \frac{1}{(g-g')\eta} \ln\left[1 + A(g-g')\eta\right]$, the above condition can be re-written as

$$q \geq q_0 = \frac{e^{\lambda\eta} - 1}{e^{\lambda\eta} - (1 - \frac{\lambda}{g})\left(1 + A(g-g')\eta\right)^{\frac{\lambda}{g-g'}}}.$$

■

**Proof of Proposition 7.** The first-order derivative of $k\eta + \tau^*$ with respect to $k$ is

$$\frac{\partial(k\eta + \tau^*(k,q))}{\partial k} = \eta + \frac{\partial \tau^*(k,q)}{\partial k} = \frac{1}{(g-g')v(k,q)}[(g-g')\eta v(k,q) - v_k(k,q)]$$

where

$$v(k,q) = \frac{e^{(g-g')k\eta} - 1}{(g-g')\eta} + (1-k)\left(-(\frac{g}{\lambda} - 1)e^{(g-g')k\eta} - (\frac{1}{p} - 1)\frac{g}{\lambda}e^{\lambda\eta + (g-g'-\lambda)k\eta} + \frac{g}{p\lambda}e^{(g-\lambda)k\eta - g'k\eta}\right)$$

and

$$v_k(k,q) = e^{(g-g')k\eta} + \left[\frac{g}{\lambda} - 1 - (1-k)\frac{(g-\lambda)(g-g')\eta}{\lambda}\right]e^{(g-g')k\eta} + \left[(\frac{1}{q} - 1)\frac{g}{\lambda} - (1-k)(\frac{1}{q}\right.$$
$$\left. - 1)\frac{g(g-g'-\lambda)\eta}{\lambda}\right]e^{\lambda\eta + (g-g'-\lambda)k\eta} + \left[-\frac{g}{q\lambda} + (1-k)\frac{g(g-\lambda-g')\eta}{q\lambda}\right]e^{(g-\lambda)k\eta - g'k\eta}.$$

Given that, the optimal $k^*$, which satisfies $(g - g')\eta v(k, q) - v_k(k, q) = 0$, solves

$$e^{(g-g')k^*\eta}\left[-\frac{g}{\lambda}+1+\frac{g}{q}\left(\eta(1-k^*)+\frac{1}{\lambda}\right)\left(-(1-q)e^{\lambda\eta}+1\right)e^{-\lambda k^*\eta}\right]=1.$$

Next, we want to show that, under the assumption $A(g-g')\eta+1 > \left(\frac{g(\lambda\eta+1)}{g-\lambda}\right)^{\frac{g-g'}{\lambda}}$, $k^* \in (0, k)$. First of all, under this assumption, we have,

$$\begin{aligned}
&\frac{g}{q}\left(\eta(1-k_{max})+\frac{1}{\lambda}\right)(1-(1-q)e^{\lambda\eta})e^{-\lambda k_{max}\eta} < \frac{g}{q}\left(\eta+\frac{1}{\lambda}\right)(1-(1-q)e^{\lambda\eta})e^{-\lambda k_{max}\eta}\\
=&\frac{g}{q}\left(\eta+\frac{1}{\lambda}\right)(1-(1-q)e^{\lambda\eta})[A(g-g')\eta+1]^{-\frac{\lambda}{g-g'}} < \frac{g}{q}\left(\eta+\frac{1}{\lambda}\right)(1-(1-q)e^{\lambda\eta})\frac{g-\lambda}{g(\lambda\eta+1)}.
\end{aligned}$$

Then, since $\frac{1-(1-q)e^{\lambda\eta}}{q}$ is increasing in $q \in (0, 1)$, we know that $\frac{1-(1-q)e^{\lambda\eta}}{q} < 1$. Following the above inequality, we have

$$\begin{aligned}
&\frac{g}{q}\left(\eta(1-k_{max})+\frac{1}{\lambda}\right)(1-(1-q)e^{\lambda\eta})e^{-\lambda k_{max}\eta}\\
<&\frac{g}{q}\left(\eta+\frac{1}{\lambda}\right)(1-(1-q)e^{\lambda\eta})\frac{g-\lambda}{g(\lambda\eta+1)}\frac{q}{1-(1-q)e^{\lambda\eta}}=\frac{g}{\lambda}-1.
\end{aligned}$$

Therefore, we have $\left.\frac{\partial(k\eta+\tau^*(k,q))}{\partial k}\right|_{k=k_{max}} < 0$, which demonstrates that $k^* \in (0, k_{max})$. ■

# B Appendix B. Imperfect Enforcement

**Proof of Proposition 8.** Creditor $i$'s expected payoff from choosing $\tau_i$ is

$$\mathbb{E}(\pi_i|t_i) = \int_{t_i+\tau_i-(\tau^*+k\eta-m)}^{\infty} e^{g(t_i+\tau_i)}\psi(t_0|t_i)dt_0 + \int_0^{t_i+\tau_i-(\tau^*+k\eta)} \alpha_p(\tau^*,k,m)e^{gt_0}\psi(t_0|t_i)dt_0$$
$$+ \int_{t_i+\tau_i-(\tau^*+k\eta)}^{t_i+\tau_i-(\tau^*+k\eta-m)} [(1-p)e^{g(t_i+\tau_i)} + p\alpha_p(\tau^*,k,m)e^{gt_0}]\psi(t_0|t_i)dt_0.$$

Accordingly, the first-order derivative with respect to $\tau_i$ is

$$\frac{\partial\mathbb{E}(\pi_i|t_i)}{\partial\tau_i} = e^{g(t_i+\tau_i)}[g(1-\Psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i))$$
$$-(1-\alpha_p(\tau^*,k,m)e^{-g(\tau^*+k\eta-m)})\psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i)]$$
$$+(1-p)e^{g(t_i+\tau_i)}\{g(\Psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i)-\Psi(t_i+\tau_i-(\tau^*+k\eta)|t_i))$$
$$+\psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i)-\psi(t_i+\tau_i-(\tau^*+k\eta)|t_i)$$
$$-\alpha_p(\tau^*,k,m)e^{-g(\tau^*+k\eta)}[e^{gm}\psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i)-\psi(t_i+\tau_i-(\tau^*+k\eta)|t_i)]\}.$$

**Case I.** $m < k\eta$ When $m < k\eta$, in any possible symmetric equilibrium, $\tau_i = \tau^*$, which is in the range of $[\tau^*+k\eta-\eta, \tau^*+k\eta-m)$. We consider $\frac{\partial\mathbb{E}(\pi_i|t_i)}{\partial\tau_i}$ in this range, and find the first-order condition as follows.

$$\begin{aligned}&pe^{g(t_i+\tau_i)}[g(1-\Psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i))-(1-\alpha_p(\tau^*,m)e^{-g(\tau^*+k\eta-m)})\\&\times\psi(t_i+\tau_i-(\tau^*+k\eta-m)|t_i)]+(1-p)e^{g(t_i+\tau_i)}[g(1-\Psi(t_i+\tau_i-(\tau^*+k\eta)|t_i))\\&-(1-\alpha_p(\tau^*,m)e^{-g(\tau^*+k\eta)})\psi(t_i+\tau_i-(\tau^*+k\eta)|t_i)]=0.\end{aligned}\tag{44}$$

Based on this first-order condition, if a symmetric equilibrium exists, we can solve for the unique $\tau^*(m,p)$, that is,

$$\tau^*(m,p) = \frac{1}{g-g'}\left\{\ln A - \ln\left[\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - p\frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta}\right.\right.$$
$$\left.\left.+\left(1-k+\frac{p}{\eta}m\right)\frac{-(g-\lambda)[pe^{\lambda(k\eta-m)}+(1-p)e^{\lambda k\eta}]+g}{p\lambda e^{-(g-g'-\lambda)k\eta+(g-\lambda)m}+(1-p)\lambda e^{-(g-g'-\lambda)k\eta}}\right]\right\}.$$

Next, as we have the following the second-order condition

$$\left.\frac{\partial^2\mathbb{E}(\pi_i|t_i)}{\partial\tau_i^2}\right|_{\tau_i=\tau^*} = -e^{g(t_i+\tau^*)}\frac{g\lambda}{e^{\lambda\eta}-1} < 0,$$

we know that $\tau_i = \tau^*(m,p)$ obtains the maximal expected payoff when others take the same strategy.

**Case II.** $m > k\eta$ When $m > k\eta$, in any possible symmetric equilibrium with $\tau^*(m,p)$, we only need to consider the range that $\tau_i = \tau^*(m,p) \in [\tau^* + k\eta - \eta, \tau^* + k\eta)$. Considering the first-order condition in this range, we have

$$(1-p)e^{g(t_i+\tau_i)}\{g[1-\Psi(t_i+\tau_i-(\tau^*+k\eta)|t_i)] - [1-\alpha_p(\tau^*,m)e^{-g(\tau^*+k\eta)}]\psi(t_i+\tau_i-(\tau^*+k\eta)|t_i)\} = 0. \quad (45)$$

Then, based on the above first order condition, if such a symmetric equilibrium exists, we can solve for the unique $\tau^*(m,p)$, that is,

$$\begin{aligned}\tau^*(m,p) =& \frac{1}{g-g'}\left\{\ln A - \ln\left[\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - p\frac{e^{-g'k\eta}[e^{gk\eta}-1]}{g\eta}\right.\right.\\ &\left.\left. + (1-(1-p)k)\left[-\left(\frac{g}{\lambda}-1\right)e^{(g-g')k\eta} + \frac{g}{\lambda}e^{(g-g'-\lambda)k\eta}\right]\right]\right\}.\end{aligned}$$

Further, we check the the second-order condition and find that the sign of second-order derivative is negative; that is,

$$\left.\frac{\partial^2\mathbb{E}(\pi_i|t_i)}{\partial\tau_i^2}\right|_{\tau_i=\tau^*} = -(1-p)e^{g(t_i+\tau^*)}\frac{g\lambda}{e^{\lambda\eta}-1} < 0.$$

Therefore, $\tau_i = \tau^*(m,p)$ is a profit maximizing choice.

**Case III.** $m = k\eta$ To show that symmetric equilibrium does not exist when $m = k\eta$, observe that when creditor $i$ chooses to wait longer than the other creditors' symmetric choice $\tau^*$ (i.e., $\tau_i > \tau^*$), the first-order condition is in the form of (45). However, if creditor $i$ chooses any $\tau_i < \tau^*$, then the first-order condition of $\tau_i$ is in the form of (44). As this holds true for any $\tau^*$ when $m = k\eta$, no symmetric equilibrium exists given this discontinuity in the marginal condition at $\tau_i = \tau^*$. ■

The following Lemma demonstrates how the equilibrium waiting time $\tau^*(m,p)$ depends on the enforcement parameter $p$ (if a symmetric equilibrium exists when $m > k\eta$).

**Lemma 3** *For any $k$ and $m > k\eta$, $\tau^*(m,p)$ is strictly decreasing in $p$.*

**Proof of Lemma 3.** With $m > k\eta$, the equilibrium waiting time $\tau^*(m,p)$ can be written as $\frac{1}{g-g'}\{\ln A - \ln v(k,m,p)\}$ (see (37)), where

$$v(k,m,p) = \frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - p\frac{e^{-g'k\eta}[e^{gk\eta}-1]}{g\eta} + (1-(1-p)k)\left[-\left(\frac{g}{\lambda}-1\right)e^{(g-g')k\eta} + \frac{g}{\lambda}e^{(g-g'-\lambda)k\eta}\right].$$

Next, we show that $v(k,m,p)$ is increasing in $p$. First, observe that

$$\begin{aligned}\frac{\partial v(k,m,p)}{\partial p} &= -\frac{e^{-g'k\eta}[e^{gk\eta}-1]}{g\eta} + k\left[-\left(\frac{g}{\lambda}-1\right)e^{(g-g')k\eta} + \frac{g}{\lambda}e^{(g-g'-\lambda)k\eta}\right] \\ &= e^{(g-g')k\eta}\left[\left(k - \frac{1-e^{-gk\eta}}{g\eta}\right) + k\frac{g}{\lambda}(e^{-\lambda k\eta}-1)\right].\end{aligned}$$

Define a new function $z(k) := \left(k - \frac{1-e^{-gk\eta}}{g\eta}\right) + k\frac{g}{\lambda}(e^{-\lambda k\eta}-1)$. It is easy to check that $z(k=0) = 0$, and $z'(k) > 0$ for any $k \in (0,1)$ given the parameter restriction in Assumption 1. Therefore, for any $k \in (0,1)$, $\frac{\partial v(k,m,p)}{\partial p} = e^{(g-g')k\eta}z(k) > 0$. Therefore, $v(k,m,p)$ in increasing in $p$, and, accordingly, $\tau^*(m,p)$ is decreasing in $p$. ■

The next Lemma characterizes the optimal clawback window $m_p^*$ in our extension. One can easily check that the condition (46) converges to $m_p^*(g-\lambda) - 1 + (1-k)\eta(g-\lambda) = 0$ when $p$ converges to 1, which echoes the optimal length of clawback window when there is perfect enforcement (i.e., $m^*$ in (27)).

**Lemma 4** *For any given $p \in (0,1)$, the optimal choice $m_p^* \equiv \arg\max_{m\geq 0}\tau^*(m,p) \in [0,k\eta]$ and $m_p^*$ satisfies*

$$m_p^*(g-\lambda)p - (1-p)e^{-(g-\lambda)m_p^*} - p + (1-k)\eta(g-\lambda) = 0. \tag{46}$$

**Proof of Lemma 4.** When $m \in [0,k\eta)$, taking the first-order derivative for $\tau^*(m,p)$ with respect to $m$, we have $\frac{\partial \tau^*(m,p)}{\partial m} = -\frac{1}{g-g'}\frac{\frac{\partial v(k,m,p)}{\partial m}}{v(k,m,p)}$, in which,

$$\begin{aligned}\frac{\partial v(k,m,p)}{\partial m} =& \frac{gpe^{\lambda(k\eta-m)} + (g-\lambda)(1-p)e^{\lambda k\eta} - g + (1-p)\lambda e^{\lambda k\eta - gm}}{\lambda pe^{-(g-g'-\lambda)k\eta+(g-\lambda)m} + (1-p)\lambda e^{-(g-g'-\lambda)k\eta}} \\ &\times\frac{-(1-p)e^{m(g-\lambda)p-(g-\lambda)m} - p + (1-k)\eta(g-\lambda)}{\frac{\eta}{p}[p+(1-p)e^{-(g-\lambda)m}]}.\end{aligned}$$

Since $g-(g-\lambda)e^{\lambda k\eta}-\lambda e^{-(g-\lambda)k\eta}\leq 0$, the following inequality, that is,

$$gpe^{\lambda(k\eta-m)}+(g-\lambda)(1-p)e^{\lambda k\eta}-g+(1-p)\lambda e^{\lambda k\eta-gm}\geq 0,$$

holds true for all $m\in[0,k\eta)$ and $p\in(0,1)$. Therefore, to have $\frac{\partial\tau^*(m,p)}{\partial m}|_{m=m_p^*}=0$, $m_p^*$ must satisfy

$$m_p^*(g-\lambda)p-(1-p)e^{-(g-\lambda)m_p^*}-p+(1-k)\eta(g-\lambda)=0. \tag{47}$$

Moreover, the second-order derivative $\frac{\partial^2 v(k,m,p)}{\partial m^2}$ for $m=m_p^*$ is

$$\frac{gpe^{\lambda(k\eta-m_p^*)}+(g-\lambda)(1-p)e^{\lambda k\eta}-g+(1-p)\lambda e^{\lambda k\eta-gm_p^*}}{\lambda pe^{-(g-g'-\lambda)k\eta+(g-\lambda)m_p^*}+(1-p)\lambda e^{-(g-g'-\lambda)k\eta}}\,\frac{(g-\lambda)(1-p)e^{-(g-\lambda)m_p^*}+(g-\lambda)p}{\frac{\eta}{p}[p+(1-p)e^{-(g-\lambda)m_p^*}]}>0.$$

Therefore, $m_p^*$ that satisfies (47) minimizes $v(k,m,p)$ and maximizes $\tau^*(m,p)$. ∎

**Proof of Proposition 9.** Define $h(m,p):=m(g-\lambda)p-(1-p)e^{-(g-\lambda)m}-p+(1-k)\eta(g-\lambda)$. First, it is easy to see that $h(m,p)$ is increasing in $m$ because, for any $m\geq 0$ and $p\in(0,1)$,

$$\frac{\partial h(m,p)}{\partial m}=(g-\lambda)p+(1-p)(g-\lambda)e^{-(g-\lambda)m}>0.$$

Next, it is easy to check that the condition $p>\underline{p}(k)$ holds, $h(m=k\eta,p)>0$; and, under the condition that $(1-k)\eta(g-\lambda)<1$, $h(m=0,p)<0$.[19] Further, since $e^{-(g-\lambda)m}\geq-(g-\lambda)m+1$, we have

$$\frac{\partial h(m,p)}{\partial p}=m(g-\lambda)+e^{-(g-\lambda)m}-1>0.$$

Therefore, $h(m,p)$ is increasing in $p$. Given that $h(m,p)$ is increasing in $m$ and $m_p^*$ uniquely solves $h(m_p^*,p)=0$, we know that $m_p^*$ is decreasing in $p$. ∎

[19] Notice that, under Assumption 1, $\eta(g-\lambda)>1$, and, thus, $\underline{p}(k)<1$ for any $k\in(0,1)$. Moreover, $(1-k)\eta(g-\lambda)<1$ is a condition that is needed for the existence of a positive $m_p^*$ (same for $m^*$ defined in (27)).

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