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ABSTRACT

We consider sequential games with incomplete information and analyze the impact of restrictions on exogenous beliefs when rational players reason according to a forward induction principle. A player **strongly believes** a proposition, or event  $E$  (about co-players) if she assigns probability 1 to  $E$  whenever possible, i.e., conditional on each history consistent with  $E$ . We say that players reason according to **forward induction** (FI) if they apply the “best rationalization principle”, that is, they always ascribe to co-players the highest degree of strategic sophistication consistent with co-players' observed behavior. In particular, this implies each player strongly believes that co-players are rational, i.e., that they are subjective expected utility maximizers. Strong belief in a proposition (e.g., “co-players are rational and hold uniform initial beliefs about the types of others”) does not imply strong belief in a weaker proposition (e.g., “co-players are rational”), because weaker propositions are consistent with and are believed at more histories. This lack of monotonicity of strong belief makes it hard to compare behavior consistent with rationality and FI reasoning under less or more restrictive assumptions about **exogenous beliefs**, i.e., about initial interactive beliefs concerning types. Yet, we can prove that *more restrictive assumptions about exogenous beliefs yield, for each state of nature, (weakly) more precise predictions about outcomes*. This allows us to also prove that, in any given social environment, the (virtual) *implementation of social choice functions with sequential mechanisms under FI reasoning is robust*, that is, if a social choice function can be implemented under FI reasoning with no assumptions about exogenous beliefs, it can also be implemented under FI reasoning with arbitrary assumptions about exogenous beliefs. This is important because Mueller (JET, 2016) proved that, under FI reasoning with no assumption about exogenous beliefs, using sequential mechanisms considerably expands the set of implementable social choice functions compared to static mechanisms.