Cost Allocation in Manufacturing-Remanufacturing Operations
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2005/08/TOM

Working Paper Series
COST ALLOCATION IN MANUFACTURING - REMANUFACTURING OPERATIONS

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Abstract
In many firms, manufacturing and remanufacturing operations are carried out in different divisions whose managers’ performances are evaluated separately. In this research, we expose the decision-making inefficiencies in such a structure and propose a mechanism to align incentives. To this end, we consider a manufacturer who also undertakes remanufacturing operations. In a two-period model, we characterize the global profit-maximizing prices of the two products. If the same decisions are delegated to managers whose goal is to maximize divisional profits, we show their decisions would be sub-optimal. We then find a mechanism that achieves optimality in a decentralized system with two self-interested decision-makers who are responsible for the two processes respectively. This mechanism is a cost allocation mechanism that allocates a portion of the initial production cost of the product to each of the two stages of the product life-cycle. Our results provide several recommendations that can assist in coordinating the manufacturing and remanufacturing operations in a firm.

Keywords: cost allocation, remanufacturing, closed-loop supply chain, transfer prices.

January 2005
1 Introduction

Remanufacturing is a process by which value is recovered from used products. In this process, used products are disassembled, parts are tested and refurbished if necessary, and these parts are used in producing remanufactured products. Examples of remanufactured products include copiers, cell-phones, computers, automobile parts, printers, toner cartridges, etc. In this article, we focus on a manufacturer who sells a new product, and subsequently remanufactures it. In the context of the WEEE Directive (Directive 2002/96/EC) that promotes producer responsibility, we expect this trend to grow.

Consumers typically value the remanufactured product less than the corresponding new product. In many countries, remanufactured products must be labelled as such, so the consumers are informed about their purchases. Therefore, a manufacturer/remanufacturer needs to make two distinct pricing decisions. At the same time, current sales of new products will influence the potential for future sales of remanufactured ones, linking the two pricing decisions. Previous research finds that optimal pricing may involve a small, or even negative, margin for the new product (Debo et al. 2004); this happens when there is a high market potential for the remanufactured unit. If the new and remanufactured product divisions are evaluated separately for profitability, the new product manager will prefer a higher margin than is optimal for the firm. This will improve his divisional profit, at the expense of the overall profitability of the firm.

Existing literature makes the implicit assumption that optimal prices for the two products will be reached by the firm (Groenevelt and Majumder 2001a,b, Ferrer and Swaminathan 2002, Savaşkan et al. 2004, Debo et al. 2004). In practice, different divisions are often responsible for manufacturing and remanufacturing operations. As a result, the two pricing decisions are made by different managers. Examples include Hewlett Packard (Guide and Van Wassenhove 2002), Bosch (Valenta 2004), DaimlerChrysler (Driesch and Flapper 2005) and Océ (Zuidwijk et al. 2005).

Whether division managers will reach the optimal decisions for the firm – versus for themselves – depends on the way their performances are measured. Large firms usually evaluate their division managers based on divisional profits. This is because more aggregate
measures such as firm profits may introduce free-riding, while not linking managers’ compensation to their performance may lead to shirking. (For a discussion on decentralization and the incentive problems it engenders in firms, see Chapter 7 of Kaplan and Atkinson, 1998). The calculation of divisional profits then becomes crucial. The divisional profit is particularly ambiguous when one division’s activities have an impact on those of the others, for instance, when there are internal transactions between different divisions. In our setting, the output of the new product division is an input of the remanufacturing division. The prevalent mechanism to account for such transactions is transfer pricing.

Transfer prices are the prices firms charge internally when goods or services are transferred from one part of the firm to another. This practice is put in place to make different segments of the firm aware of the value of the goods transferred, in attempt to alleviate the incentive problems. To the selling division, the transfer price is considered a revenue when divisional profits are computed. Likewise, it is considered a cost to the buying division. The revenues cancel out the costs when total firm profits are computed, but they have an impact on the decision-making of the individual division managers.

From both interviews and existing literature, we identified a variety of transfer pricing practices amongst the divisions responsible for manufacturing and remanufacturing, ranging from essentially charging nothing to full standard cost. In some cases, given the transfer pricing policy, a division decides how many products to return for remanufacturing at the said price, and how many units to scrap (Zuidwijk et al. 2005). While there is an extensive accounting literature on transfer pricing (Edlin and Reichelstein 1995, Vaysman 1998), the extant research offers no guidance specific to closed-loop supply chains. It is not clear how division managers’ incentives can be aligned to those of the firm when a firm undertakes both manufacturing and remanufacturing.

In the operations literature, the only papers that raise a related issue - that of inventory valuation for remanufacturable products - are Teunter et al. (2000), Teunter (2001), and Teunter and van der Laan (2002, 2004). These papers focus on setting the holding costs of returned products in decision support systems based on average cost models. They investigate whether the approximate average cost approach is appropriate to use, compared to the discounted cash flow approach; and if so, what the “correct” holding costs for returned
products should be. Unlike our paper, their focus is not coordination of incentives within the firm, but inventory valuation for centralized decision making.

Given the discord among existing transfer pricing practices, and the lack of academic research, there is a clear need for transfer pricing guidelines in closed-loop supply chains. To this end, we consider a manufacturer who also undertakes remanufacturing operations. We characterize the firm as having a distinct allocation of decision rights to individual managers. This is in contrast to treating the firm as one black box that maximizes profits centrally, which is implicit in previous literature. We determine the optimal decisions for the firm, and show that under certain performance measures, the division managers will arrive at sub-optimal decisions because of misaligned incentives. We then propose a mechanism that achieves the optimal centralized profit in the decentralized system. The mechanism allocates a portion of the initial production cost to each of the two stages in the product life-cycle, a method not previously suggested in the literature.

In Section 2, we describe the model. Section 3.1 characterizes the first-best decision that maximizes profits for the entire firm. Section 3.2 considers the decisions made by individual division managers when divisional profits are computed without cost allocation. We show the resulting decisions to be sub-optimal. We then consider a cost allocation mechanism in Section 3.3, and determine the proportion of costs that need to be allocated across divisions in order to provide respective managers proper incentives to make the first-best decisions. Finally, we discuss the robustness of our results in Section 3.4. We conclude with the managerial implications of our findings in Section 4.

2 The Model

In this section, we present the model that we develop to answer the research questions posed above.

Firm Structure. Manufacturing and remanufacturing operations are undertaken by two separate divisions. As discussed in the introduction, this structure is prevalent in firms that undertake both operations. We call the manufacturing division D1, and the remanufacturing division D2. Their respective managers will be referred to as M1 and M2. M1 makes decisions
regarding manufacturing, and M2 remanufacturing. We assume their compensation (e.g., annual bonus) is strictly increasing in their respective divisional profits. In the contracting literature, the delegation of decision rights can be a result of limited communication, or private information, i.e., the divisional managers are delegated certain decisions because they have more accurate or timely information than the headquarters, and therefore can make better decisions (Wei and Zeng 2004). We do not explicitly model limited communication or asymmetric information; we believe the decentralized structure is frequently adopted by firms and we investigate delegated decision-making in such contexts.

**Market Characteristics.** We assume that the new and remanufactured products are sold to different market segments that do not overlap. This is appropriate when the remanufactured product is sold in a different geographic region, or when the customers in these segments have distinct preferences. Specifically, firms can limit cannibalization by selling through different channels (Guide and Van Wassenhove 2002), or by remanufacturing only a subset of their product line where they believe cannibalization effects are limited (Valenta 2004).

**Product life-cycle.** We assume that the new product can be used for only one period. Subsequently, products are returned by customers. A fraction $q \in [0, 1]$ of returned products can be remanufactured; the rest cannot be used and will be scrapped. Remanufactured products also have a one-period lifetime, after which they are disposed of. Similar assumptions are adopted in other papers, e.g., Groenevelt and Majumder (2001a,b) and Debo et al. (2004). With the above assumptions, it is sufficient to analyze a two-period model. Only new products are sold in the first period by D1, and only remanufactured products are sold in the second period by D2.

**Consumer Characteristics.** We assume that consumer willingness-to-pay is heterogeneous and uniformly distributed in the interval $[0, 1]$. The market size is normalized to 1. Consumers typically value the remanufactured product less than the new product. We model this by letting a consumer of type $\phi \in [0, 1]$ have a willingness-to-pay of $\phi$ for a new product and $(1 - \delta)\phi$ for a remanufactured product. The parameter $\delta$ is “perceived depreciation” (Debo et al. 2004). The utility each consumer derives from purchasing a product is equal to the difference between his willingness-to-pay and the product price.
We let $p_n$ and $d_n$ denote the (first-period) new product price and demand, and $p_r$ and $d_r$ the (second-period) remanufactured product price and demand. With the consumer utility function defined as above, the following linear inverse demand functions are obtained:

$$p_n = 1 - d_n \quad (1)$$

$$p_r = (1 - \delta)(1 - d_r). \quad (2)$$

The Cost Structure. We denote the unit manufacturing cost by $c$; and the unit remanufacturing cost by $c_r$. Second-period profits are discounted by $\beta < 1$. Returned remanufacturable products have salvage value $s$ per unit. The disposal cost for nonremanufacturable units is assumed to be zero.

We make the following assumptions: (i) $0 \leq c - \beta sq \leq 1$, (ii) $0 \leq c_r \leq (1 - \delta) - s$, and (iii) $s < 1 - \delta$. These assumptions ensure that the optimal centralized solution satisfies the following properties: $0 < d^*_n < \infty$ and $d^*_r > 0$. All proofs can be found in the Appendix.

3 Analysis

3.1 Centralized Decision-making

We begin by establishing firm-wide profit-maximizing decisions, which will serve as benchmarks for our later results. Because of the one-to-one correspondence between price and demand, the optimization problem can be formulated either in terms of prices or demands. We proceed with the latter for ease of interpretation.

The first-best demand levels are those that maximize the present value of the profits from both periods subject to an availability constraint,

$$\max_{d_n, d_r} d_np_n(d_n) - cd_n + \beta (d_rp_r(d_r) - c_rd_r + s(qd_n - d_r)) \quad \text{s.t. } qd_n - d_r \geq 0. \quad (3)$$

Lemma 1 The first-best demand levels for new and remanufactured products are

$$(d^*_n, d^*_r) = \begin{cases} 
\left( \frac{1-c+\beta sq}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)} \right) & \text{if } c_r + s \geq (1 - q(1 - c + \beta sq))(1 - \delta) \\
\left( \frac{1-c+\beta q(1-\delta-c_r)}{2+2sq(1-\delta)}, qd^*_n \right) & \text{otherwise.}
\end{cases} \quad (4)$$
In the first case, the quantity of products remanufactured is unconstrained by the availability of remanufacturable products from past sales; some remanufacturables are salvaged. In the second case, the firm prefers to remanufacture more units were there any, but is constrained by the amount available. By examining the inequality in (4), we identify conditions driving each case. In particular, remanufacturing may become constrained by availability when the remanufacturing cost, the salvage value or the perceived depreciation is low. All these conditions increase the appeal of remanufactured products, making the constraint binding. Similarly, when manufacturing cost is high, obtaining revenues through remanufacturing becomes more attractive.

3.2 Decentralized Decision-making

Recall that decisions about the new and remanufactured products are made by different managers M1 and M2. Their compensation strictly increases in their respective divisional profits, which they strive to maximize. In particular, M1 maximizes \((p_n - c)d_n\) and M2 maximizes \((p_r - c_r)d_r + s(qd_n - d_r)\), s.t. \(d_r \leq qd_n\), where \(d_n\) has been chosen by M1.

**Lemma 2** Under decentralized decision-making, the demand levels are

\[
(\hat{d}_n, \hat{d}_r) = \begin{cases} 
(\frac{1-c}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)}) & c_r + s \geq (1 - q(1-c))(1-\delta) \\
(\frac{1-c}{2}, \frac{q(1-c)}{2}) & \text{otherwise.} 
\end{cases}
\]  

(5)

There are several observations from comparing the decisions reached in (4) and (5). First, consider the case where the second period’s production is *unconstrained* in both scenarios. It is easy to see that \(\hat{d}_n\) is less than the first-best \(d^*_n\); the difference is \(\frac{\beta sq}{2}\). Division 1’s profit only improves when a new product is made and sold; M1 does not benefit from the salvage value that can be obtained at the end of life. In contrast, both revenues are valuable to the firm. M1, therefore, under-produces. One can see that in the second period, the costs and revenues accruing to the firm are the same as those accruing to M2. Consequently, M2 applies the same decision rule as the firm, so that \(\hat{d}_r = d^*_r\). Note this equality only holds if \(\hat{d}_r \leq q\hat{d}_n\). The threshold for this constraint to be binding is higher under the decentralized setting: \((1-q(1-c))(1-\delta) \geq (1-q(1-c+\beta sq))(1-\delta)\). In other words, the remanufactured
product quantity is constrained for a larger set of parameters. This is a second-order effect of the under-production of the new product.

When \( c_r + s < (1 - q(1 - c))(1 - \delta) \), the demand levels in both periods are suboptimal. In this case, any additional new product produced will be remanufactured by M2. Similarly as in the unconstrained case, M1 ignores the effect of his decision on future operations. In particular, he ignores the benefit of future revenues from remanufacturing, since he does not reap any benefit from this activity. Consequently, he under-produces, leading M2 to under-produce as well. These observations are formalized in the following proposition.

**Proposition 1** Assume \( s > 0 \). With no cost allocation, M1 always under-produces in the decentralized setting and M2 under-produces when \( c_r + s < (1 - q(1 - c))(1 - \delta) \).

\[
\hat{d}_n < d_n^*, \quad and \quad \hat{d}_r = d_r^* \quad if \quad c_r + s \geq (1 - q(1 - c))(1 - \delta)
\]

\[
\hat{d}_n < d_n^*, \quad and \quad \hat{d}_r < d_r^* \quad if \quad c_r + s < (1 - q(1 - c))(1 - \delta).
\]

The only situation when there is no under-production of new products is for those scenarios with negligible salvage value \( (s = 0) \), and \( d_r \) unconstrained by \( d_n \). This captures the fact that subsequent to incurring the initial cost \( c \) during production, this cost becomes entirely sunk for the remanufacturing operation. In other words, given zero salvage value, one cannot recoup any economic value from the returns. In conjunction with the fact that M2 reaches the optimal decision in this case, we conclude that incentive problems do not exist. Consequently, for processes where products have negligible salvage value, and remanufacturing is not constrained by availability, the firm does not have to coordinate incentives between managers even in decentralized settings.

In most cases, however, the asynchronous nature of costs and benefits related to the production of the new product drives M1’s suboptimal decisions. In the next subsection, we examine cost allocation mechanisms that can resolve this conflict by aligning M1’s incentives to those of the firm as a whole.

### 3.3 Decentralized Decision-making with Cost Allocation

We consider a simple cost allocation scheme where a fraction \( \alpha \) of the unit manufacturing cost is charged to D1, and the rest, \( 1 - \alpha \), to D2. Thus, M1 maximizes \( (p_n - \alpha c_n) d_n \) and M2
maximizes \(-(1 - \alpha)cd_n + d_r p_r(d_r) - c_r d_r + s(qd_n - d_r)\). s.t. \(d_r \leq qd_n\), where \(d_n\) has already been chosen by M1.

**Proposition 2** In the presence of cost allocation scheme \(\{\alpha, 1 - \alpha\}\), first-best decisions can be reached when

\[
\alpha^* = \begin{cases} 
1 - \frac{\beta sq}{c} & \text{if } c_r + s \geq (1 - q(1 - c + \beta sq))(1 - \delta) \\
\frac{c + \beta q(1 - \delta)(q - 1)}{c + \beta q^2(1 - \delta)} & \text{otherwise.}
\end{cases}
\]

With this allocation scheme,

\[
(d_n, d_r) = \begin{cases} 
\left(\frac{1 - c + \beta sq}{2}, \frac{1 - \delta - c_r - s}{2(1 - \delta)} \right) & \text{if } c_r + s \geq (1 - q(1 - c + \beta sq))(1 - \delta) \\
\left(\frac{1 - c + \beta q(1 - \delta - c_r)}{2 + 2\beta q^2(1 - \delta)}, qd_n \right) & \text{otherwise.}
\end{cases}
\]

Note that under the cost allocation scheme \(\alpha^*\), we obtain decentralized demand levels that are exactly the same as in the centralized case. In other words, we have shown that a proportional cost allocation scheme with \(\alpha\) not necessarily equal to 1 is optimal. If \(s = 0\), then \(\alpha^* = 1\) in the unconstrained case: With no salvage value and sufficient supply, the decentralized solution with no cost allocation is optimal. This confirms the observation from Section 3.2 – that with negligible salvage value, under-production of the new product is not of concern when operating under the unconstrained scenario.

The logic behind the expression for \(\alpha^*\) is not immediately apparent. We interpret this result next. Let \(C_1\) denote the cost to D1 when the full production cost is allocated to this division. Let \(C\) denote the cost to the firm.

**Proposition 3** \(\alpha^* = \frac{\partial C}{\partial d_n} \bigg|_{d_n = d_n^*}\).

What this proposition says is that in both cases of Proposition 2, the expression obtained for \(\alpha^*\) equals the marginal cost to the firm divided by the marginal cost to the first division, evaluated at \(d_n = d_n^*\).

Let us interpret \(\alpha^* = 1 - \frac{\beta sq}{c}\); the other case follows the same logic. Rewriting \(\alpha^*\) as \(\frac{c - \beta sq}{c}\), we see that the numerator \(ca^*\) is the marginal cost of producing a new product to the entire firm when the next unit will be salvaged; this is the case since we are in the unconstrained region. This cost is less than the marginal cost to D1 because of the salvage value. The
denominator is the marginal cost of making a new product to D1. When we allocate only \( \alpha^* \) portion of the costs to M1 in the first period, he makes the following calculation:

\[
\max_{d_n} d_n(1 - d_n) - \alpha^* c d_n.
\]

Differentiating with respect to \( d_n \), we obtain:

\[
1 - 2d_n - \frac{c - \beta sq}{c} = 0,
\]

which yields \( d_n^* = \frac{1 - \frac{1}{2} \beta sq}{\beta} \), the optimal demand for new products. M1 always chooses the quantity that makes his marginal revenue equal to his marginal cost. His marginal revenue is already equal to that of the firm’s. By changing his marginal cost from \( c \) to \( \alpha^* c \), we are able to incorporate the firm-wide incentives into his performance measure. In particular, we substitute his cost function with that of the firm’s. Consequently, when equating his marginal revenue to his marginal cost, he essentially solves the firm’s problem. In other words, his incentives are aligned and his decisions become optimal.

For the other case, \( \alpha \alpha^* c \) can again be interpreted as the marginal cost of a new product at \( d_n^* \), but this time when the next unit would be remanufactured. Again, by charging \( \alpha^* c d_n \), we are substituting the firm’s incentives into M1’s performance measure, and thereby incorporating the firm’s tradeoffs into M1’s decision making.

### 3.4 Robustness of Results

The goals of this paper are (i) to underline the importance of coordinating the manufacturing and remanufacturing operations in firms undertaking both activities; and (ii) to explore internal cost allocation as a means of achieving coordination. To this end, we develop a simple model of a firm that undertakes both operations. Here, we comment on the robustness of our results.

First, we assume the disposal cost to be 0. Suppose that the unit disposal cost of unusable products is \( v > 0 \). In the first-best new product demand, the term \( \beta sq \) is replaced with \( \beta (sq - v(1 - q)) \) in the unconstrained case and in the constraint; and a term \( \beta v(1 - q) \) is subtracted from the numerator in the constrained case. In other words, the original production volume
decreases. This is intuitive because the total margin from new production is reduced. The demand for remanufactured products does not change under the unconstrained case, but it changes due to the change in the new product volume in the constrained case. Thus, a disposal cost only has a secondary effect on the remanufacturing volume. In the decentralized case, the new product quantity depends on which party is charged the cost of disposal. If D1 is charged this cost, his production level decreases by $\beta \nu (1-q)$ relative to the 0 cost case. The difference from the centralized solution is the same as before, $\frac{\beta \nu q}{2}$. Underproduction occurs due to the omission of the future income from salvaging unused remanufacturable products. If D2 is charged this cost, $d_n^* = \frac{1-c}{2}$ as in the 0 disposal cost case. Only now the difference from the centralized solution is $\beta (q - \nu (1-q))$. Thus, whether underproduction or overproduction occurs depends on the relative disposal cost and volume, and salvage value and volume. In either case, $\alpha^*$ has the same structure as in Proposition 3 and the logic behind the cost allocation is the same as before: To set $\alpha^*$ in a way that makes D1 internalize the firm’s cost of producing one more new product.

Second, we assume the salvage value of usable products to be positive, since there may be a market for these products even without remanufacturing. However, this quantity could be negative if there is a cost to dispose of these products. In this case, the under-production results relating to new products will change to over-production, but the structure of the results and the insights pertaining to the optimal cost allocation mechanism will remain unchanged.

Finally, our analysis assumes that costs are accurate and certain. In practice, cost figures often contain estimation errors and are subject to much debate and uncertainty. While uncertainty can be added to the analysis under similar assumptions without changing the nature of our results, estimation errors can be minimized through appropriate application of activity-based costing (commonly known as ABC). ABC is a process by which firms arrive at their cost parameters. Since we take cost parameters as given, our results are robust to the firm’s choice of a costing system.

Models incorporating more features such as fixed costs, cannibalization, and unobservable costs can be considered to refine the basic insights developed in this paper. We hope that our research provides a starting point for such analysis.
4 Conclusions

In many firms, manufacturing and remanufacturing operations are performed in different divisions whose performances are evaluated separately. Division managers may have goals incongruent to those of the firm, giving rise to suboptimal decisions. In this research, we expose one instance of the inefficiencies that can be created with such a structure. Incentive conflicts between manufacturing and remanufacturing operations extend to areas beyond those addressed here, such as sales effort and product development, and are crucial in determining the success of remanufacturing. The literature on remanufacturing has so far implicitly assumed that firms reach optimal decisions for manufacturing and remanufacturing operations in some manner. However, the mechanisms firms adopt to arrive at these decisions are not specified. We hope that this paper provides the impetus for future research in this area.

For our analysis, we develop a simple model that captures the essential characteristics of manufacturing/remanufacturing operations. We examine the causes of inefficiencies due to decentralization, and develop a cost allocation scheme that achieves the first-best solution. Our results provide several recommendations that can assist in coordinating the manufacturing and remanufacturing operations in practice. We also identify features of closed-loop supply chains that render direct application of results from forward supply chains unsuitable. These findings are discussed below.

First, cost allocation should be such that manufacturing one more new unit imposes the same marginal cost on the manufacturing division as on the firm. In other words, in a closed-loop supply chain, the production cost should not be borne only by the manufacturing division; a portion of the unit production cost should be allocated to each division that benefits from it. This is our central finding. It is not currently standard practice to allocate the production cost to any other division than manufacturing. However, this practice is driven by characteristics of forward supply chains, and is not appropriate for closed-loop supply chains where value accrues to the firm at different points in the life-cycle of the product and via operations of different divisions.

While novel in the literature, this result does not run counter to existing results. In
particular, it is consistent with the intuition from the literature on sharing the cost of capital investments such as new facilities, machines or technologies. The entire firm may benefit from such an investment, while one division incurs the investment cost. Or, while the investment is incurred in the current period, its benefits accrue over several years. The problem in these cases is that the division making the investment tends to underinvest, either because it does not reap the entire benefit of the investment, or because the gains are accrued over time and the current period profits suffer. In the literature, the issue is how to align incentives for the division making the investment in a way that benefits the entire firm. This is done by allocating part of the cost of the investment to other divisions (Wei 2004) or spreading it over time to reflect its pattern of use. These allocation schemes are developed for capital investments; this concept has not been previously proposed on a production cost basis. In a closed-loop supply chain, the production cost can be viewed as an investment whose benefit will be obtained later and/or by another party, so the same principle applies.

Second, we identify cases where the logic derived from forward supply chains is sufficient: If the salvage value of a product is negligible and there is sufficient supply of used products, decentralization is not an issue. This is because the marginal cost to the firm equals the marginal cost to the manufacturing division.

Finally, if part of the initial production cost is allocated to other divisions, it must be allocated so it is a fixed cost, not a variable cost. In other words, the allocated cost must not be unitized as in the case described in Zuidwijk et al. (2005). Notice this is how M2’s optimization problem in §3.3 was formulated. Doing otherwise would introduce double marginalization in the remanufacturing division, leading to a lower level of remanufacturing than is optimal for the firm.
References


Appendix

**Proof of Lemma 1.** The Hessian of the objective function in (3) is

\[
\begin{pmatrix}
-2 & 0 \\
0 & -2(1 - \delta)
\end{pmatrix},
\]

whose leading coefficient is negative and whose determinant \(4(1 - \delta)\) is positive since we assumed \(\delta < 1\). Thus, the Hessian is negative definite and the profit function is strictly concave in \((d_n, d_r)\). The associated Lagrangean is

\[
L(d_n, d_r, \lambda) = d_n p_n(d_n) - c d_n + \beta(d_r p_r(d_r) - c_r d_r + s(qd_n - d_r)) + \lambda(qd_n - d_r).
\]

The triplet \((d^*_n, d^*_r, \lambda^*)\) satisfy the Kuhn-Tucker conditions

\[
\frac{\partial L}{\partial d_n} = p_n + d_n p'_n - c + \beta s q + q \lambda = 1 - 2d_n - c + \beta s q + q \lambda = 0 \tag{6}
\]

\[
\frac{\partial L}{\partial d_r} = \beta(p_r + d_r p'_r - c_r - s) - \lambda = \beta((1 - \delta)(1 - 2d_r) - c_r - s) - \lambda = 0 \tag{7}
\]

\[
\lambda(qd_n - d_r) \geq 0, \lambda \geq 0.
\]

Here, we used \(p_n = 1 - d_n\), \(p_r = (1 - \delta)(1 - d_r)\), \(p'_n = -1\) and \(p'_r = -(1 - \delta)\).

**Case i.** Assume that \(qd^*_n - d^*_r > 0\). Then \(\lambda^* = 0\). From (6), we obtain \(d^*_n = \frac{1-c+\beta s q}{2}\); and from (7), we obtain \(d^*_r = \frac{1-\delta-c}{2(1-\delta)}\). We now need to check whether \(qd^*_n - d^*_r > 0\) holds. Substituting and simplifying, we find that this condition holds if \(c_r + s > (1 - \delta)(1 - q(1 - c + \beta s q))\). If \(c_r + s = (1 - \delta)(1 - q(1 - c + \beta s q))\), then \(qd^*_n - d^*_r = 0\) and \(\lambda^* = 0\).

**Case ii.** If \(c_r + s < (1 - \delta)(1 - q(1 - c + \beta s q))\), then \(\lambda^*\) cannot be 0; \(\lambda^* > 0\) must hold. In this case, \(qd^*_n = d^*_r\). To solve for \(d^*_n\), solve for \(\lambda^*\) in (7) and plug it into (6):

\[
1 - 2d^*_n - c + q \beta((1 - \delta)(1 - 2qd^*_n) - c_r) = 0.
\]

Then \(d^*_n = \frac{1+q \beta(1-\delta-c_r)-c}{2(1+q \beta(1-\delta))}\). To check that \(\lambda^* > 0\), we substitute \(d^*_n\) into (6), which yields the condition \(c_r + s < (1 - \delta)(1 - q(1 - c + \beta s q))\).

This is what we had assumed, so we are done. ■

**Proof of Lemma 2.** In period one, the profits are

\[
\Pi_1 = d_n p_n - c d_n = d_n (1 - d_n) - c d_n.
\]

The demand level that maximizes period 1’s profit, denoted by \(\hat{d}_n\), fulfills the following condition:

\[
\Pi_1'(\hat{d}_n) = 1 - 2 \hat{d}_n - c = 0
\]

\[
\hat{d}_n = \frac{1 - c}{2}.
\]
In period two, the profit has the following profit:

$$
\Pi_2 = d_rp_r - c_rd_r + s(q\hat{d}_n - d_r)
$$

$$
= d_r(1 - \delta)(1 - d_r) - c_rd_r + s(q\hat{d}_n - d_r) \quad \text{s.t. } d_r \leq q\hat{d}_n.
$$

$$
\Pi'_2 = (1 - \delta)(1 - 2d_r) - (c_r + s) \quad \text{and } \Pi''_2 = -2(1 - \delta). \quad \text{Therefore, } \Pi_2 \text{ is strictly concave in } d_r, \text{ and is maximized at } d_r = \frac{1 - c - c_r - s}{2(1 - \delta)}. \quad \text{Taking the constraint into account, we conclude that } \hat{d}_r = \min(q\hat{d}_n, \frac{1 - \delta - c_r - s}{2(1 - \delta)}). \quad \text{Using } \hat{d}_n = \frac{1 - c}{2}, \text{ the two cases can be rewritten as follows:}
$$

Case (i): If $c_r + s \geq (1 - q(1 - c))(1 - \delta)$, the second-period solution is unconstrained and $d_r = \hat{d}_r = \frac{1 - c}{2}.$ \[\square\]

**Proof of Proposition 1.**

Case (i): Let $c_r + s \geq (1 - q(1 - c))(1 - \delta)$. Comparing Lemmas 1 and 2, it is easy to see that $d^*_n = \frac{1 - c + \beta sq}{2} > \frac{1 - c}{2} = \hat{d}_n$ for $s > 0$, and that $d^*_r = \hat{d}_r$.

Case (ii): Let $(1 - q(1 - c + \beta sq))(1 - \delta) \leq c_r + s < (1 - q(1 - c))(1 - \delta)$. $d^*_n$ and $\hat{d}_n$ remain the same. We need to check whether $d^*_r > \hat{d}_r$, i.e., whether $\frac{1 - \delta - c_r - s}{2(1 - \delta)} > q\frac{1 - c}{2}$. This condition is equivalent to $c_r + s < (1 - q(1 - c))(1 - \delta)$, so we are done.

Case (iii): Let $c_r + s < (1 - q(1 - c + \beta sq))(1 - \delta)$. For $s > 0$, this inequality implies $c_r < (1 - q(1 - c))(1 - \delta)$, which can be rewritten as $(1 - \delta)(q - cq) < (1 - \delta)(1 - c_r).$ Also note that

$$
\hat{d}_n = \frac{1 - c}{2} = \frac{(1 - c)(1 + \beta q^2(1 - \delta))}{2(1 + \beta q^2(1 - \delta))} = \frac{1 - c + \beta q(1 - \delta)(q - cq)}{2 + 2\beta q^2(1 - \delta)} < \frac{1 - c + \beta q(1 - \delta - c_r)}{2 + 2\beta q^2(1 - \delta)} = d^*_n,
$$

where we use this inequality in the last step. We conclude that $qd^*_n > q\hat{d}_n$, which implies $d^*_r > \hat{d}_r$ for this case. \[\square\]

**Proof of Proposition 2.** The maximum profit in period 1 is

$$
\Pi_1(\alpha) = \max d_np_n(d_n) - \alpha cd_n = \max d_n(1 - d_n) - \alpha cd_n. \quad (8)
$$
The optimal solution is \( d_n^* = \frac{1-\alpha c}{2} \). The corresponding price is \( p_n^* = 1 - d_n^* = \frac{1+\alpha c}{2} \) and the corresponding profit is \( \Pi_1(\alpha) = \frac{(1-\alpha c)^2}{4} \).

The maximum profit in period 2 is

\[
\Pi_2(\alpha) = -\frac{1}{4} (1-\alpha) c d_n^* + \max_{d_r \leq q d_n^*} (1-\delta) d_r (1 - d_r) - c_r d_r + s (q d_n^* - d_r)
\]

The term \( (1-\delta) d_r (1 - d_r) - c_r d_r + s (q d_n^* - d_r) \) is concave unimodal and attains its maximum at \( \frac{1-\delta-c_r-s}{2(1-\delta)} \). Therefore, \( d_r^* = \min \left( \frac{1-\alpha c}{2}, \frac{1-\delta-c_r-s}{2(1-\delta)} \right) \). Note that \( \frac{1-\alpha c}{2} > \frac{1-\delta-c_r-s}{2(1-\delta)} \) if and only if \( c_r + s > (1-\delta) (1-q(1-\alpha c)) \), or, written in terms of \( \alpha \), \( \alpha < \bar{\alpha} \equiv \frac{(1-\delta)(q-1)+c_r+s}{(1-\delta)qc} \). That is,

\[
d_r^* = \begin{cases} 
\frac{1-\delta-c_r-s}{2(1-\delta)} & \text{if } c_r + s \geq (1-\delta)(1-q(1-\alpha c)) \ (\text{or } \alpha \leq \bar{\alpha}), \\
\frac{1-\alpha c}{2} & \text{otherwise.}
\end{cases}
\]

Let \( \Pi(\alpha) \) be the discounted total profit of the firm as a function of \( \alpha \). Then \( \Pi(\alpha) = d_n^* (1-d_n^*) - \frac{c d_n^* + \beta \max_{d_r \leq q d_n^*} (1-\delta) d_r (1 - d_r) - c_r d_r + s (q d_n^* - d_r) }{1} \).

Now let’s calculate \( d_n^* \) and \( \Pi(\alpha) \) under the two cases we identified.

Case (a): \( c_r + s \geq (1-\delta)(1-q(1-\alpha c)) \). In this case, \( d_r^* = \frac{1-\delta-c_r-s}{2(1-\delta)} \) and \( \Pi(\alpha) \) has the form \(-c^2\alpha^2/4 + (c^2 - \beta sqc)\alpha/2 + C \), call it \( \Pi_a(\alpha) \). This function is concave unimodal and the point that satisfies the first-order condition is \( \alpha_a^* = 1 - \frac{\beta sq}{c} \).

Case (b): \( c_r + s < (1-\delta)(1-q(1-\alpha c)) \). In this case, \( d_r^* = \frac{1-\alpha c}{2} \). Again, the profit function, denoted by \( \Pi_b(\alpha) \), is concave unimodal and the point that satisfies the first-order condition is \( \alpha_b^* = \frac{c+\beta q(c_r+(1-\delta)(q-1))}{c(1+\beta q^2(1-\delta))} \).

We can write \( \Pi(\alpha) = \Pi_a(\alpha) I_{\{\alpha \leq \bar{\alpha}\}} + \Pi_b(\alpha) I_{\{\alpha > \bar{\alpha}\}} \). We find that \( \Pi_a(\bar{\alpha}) = \Pi_b(\bar{\alpha}) \) and \( \Pi_a'(\bar{\alpha}) = \Pi_b'(\bar{\alpha}) = \frac{c((1-\delta)(1-q(1-c+\beta sq)) - c_r - s)}{2(1-\delta)qc} \). Therefore, \( \Pi(\alpha) = \Pi_a(\alpha) I_{\{\alpha \leq \bar{\alpha}\}} + \Pi_b(\alpha) I_{\{\alpha > \bar{\alpha}\}} \) is a continuous function and \( \alpha^* = \alpha_a^* \) if \( \Pi'(\bar{\alpha}) < 0 \), with \( \alpha^* = \alpha_b^* \) otherwise. Note that \( \Pi'(\bar{\alpha}) < 0 \Leftrightarrow (1-\delta)(1-q(1-c+\beta sq)) - c_r - s < 0 \Leftrightarrow c_r + s > (1-\delta)(1-q(1-c+\beta sq)) \).

This is the same condition as in the centralized case. We conclude that the threshold level where \( d_n \) becomes a constraint for \( d_r \) is the same as under the centralized case, which is the desired outcome.

If \( c_r + s \geq (1-\delta)(1-q(1-c+\beta sq)) \), we need to plug in \( \alpha_a^* \) in \( d_n^* = \frac{1-\alpha c}{2} \). We find \( d_n^*(\alpha_a^*) = \frac{1-c+\beta sq}{2} \).
If $c_r + s < (1 - \delta)(1 - q(1 - c + \beta sq))$, we need to plug in $\alpha^*_b$ in $d^*_n = \frac{1 - \alpha c}{2}$. We find $d^*_n(\alpha^*_b) = \frac{1 - c + \beta q(1 - c + \beta sq)}{2 + \beta q^2(1 - \delta)}$. 

**Proof of Proposition 3.** $\frac{\partial C_1}{\partial d_n} = c$ for any $d_n$. We also need to determine $\frac{\partial C}{\partial d_n} |_{d^*_n}$. At the centralized optimal solution, the marginal cost to the firm equals the marginal revenue of the firm: $\frac{\partial C}{\partial d_n} = \frac{\partial R}{\partial d_n} = 1 - 2d^*_n$. In case (i), $d^*_n = \frac{(1 - c + \beta sq)}{2}$ and $1 - 2d^*_n = c - \beta sq$, yielding $\alpha^* = 1 - \frac{\beta sq}{c}$. In case (ii), $d^*_n = \frac{1 - c + \beta q(1 - \delta - c_r)}{2(1 + \beta q^2(1 - \delta))}$ and $1 - 2d^*_n = \frac{c + \beta q(c_r + (1 - \delta)(q - 1))}{1 + \beta q^2(1 - \delta)}$, yielding $\alpha^* = \frac{c + \beta q(c_r + (1 - \delta)(q - 1))}{c(1 + \beta q^2(1 - \delta))}$. 

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