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Paddy PADMANABHAN
Ilia TSETLIN
Timothy VAN ZANDT
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Setting Price or Quantity:
Depends on the Seller is Uncertain About

V. (Paddy) Padmanabhan*

Ilia Tsetlin**

And

Timothy Van Zandt***

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* The INSEAD Chaired Professor of Marketing at INSEAD, 1 Ayer Rajah Avenue, Singapore 138676, phone: +65.6799.5335; fax: +65.6799.5446 paddy.padmanabhan@insead.edu

** Assistant Professor of Decision Sciences at INSEAD, 1 Ayer Rajah Avenue, Singapore 138676, phone: +65.6799.5339; fax: +65.6799.5365; ilia.tsetlin@insead.edu

*** Professor of Economics at INSEAD, Boulevard de Constance, 77305 Fontainebleau Cedex; Phone: +33.6072.4981; fax: +33.1.6074.6192; timothy.van-zandt@insead.edu

Abstract

We consider a seller with uncertain demand for its product. If the demand curve were certain, then setting price and setting quantity would be equivalent ways to frame the seller's problem of choosing a profit-maximizing point on its demand curve. With uncertain demand, these become distinct sales mechanisms. We distinguish between uncertainty about the market size and uncertainty about the consumers' valuations. Our main results are that (i) for a given marginal cost, an increase in uncertainty about valuation favors setting quantity whereas an increase in uncertainty about market size favos setting price; (ii) keeping demand uncertainty fixed, there is a nonmonotonic relationship between marginal costs and the optimal selling mechanism (setting price or quantity); and (iii) in a bilateral monopoly channel setting, channel coordination occurs except for a conflict zone in which the retailer's choice of a selling mechanism deviates from the coordinated channel selling mechanism.

Keywords: Demand Uncertainty, Setting Price, Setting Quantity, Auctions, Posted Prices

1. Introduction

For an individual firm with market power that has no uncertainty about the demand curve it faces, it is irrelevant whether the firm frames its decision problem as a choice of price or as a choice of quantity—one variable uniquely determines the other according to the demand curve. However, as soon as the firm is uncertain about its demand curve, such framing is relevant and captures the relative flexibility the firm retains for price or quantity. A price-setting firm commits to a price before the resolution of uncertainty and allows output to adjust to the realization of demand. A quantity-setting firm commits to its output level before the resolution of uncertainty and allows some market-clearing mechanism (such as auctions or promotions) to adjust the price so that realized demand equals the firm's supply.

The purpose of this paper is to relate the advantages of setting price versus setting quantity to the nature of the demand uncertainty. We distinguish between two kinds of demand uncertainty: about the distribution of consumer valuations and about the size of the market. (This is similar to a distinction made by Marvel and Peck (1995), who derive the implications of the nature of uncertainty on the design of the returns policy for a product.) For example, in the case of linear demand, uncertainty about the distribution of valuations is represented by uncertainty about the vertical (price) intercept; uncertainty about the size of the market is represented by uncertainty about the horizontal (quantity) intercept. Firms face uncertainty about both aspects of the market, but one type of uncertainty may be more significant.

- A publishing house set to launch a book from a new author faces uncertainty about both the market size for the book as well as consumers' valuation. However, events such as the book's being featured on the Oprah Winfrey show or garnering great reviews have an enormous impact on awareness about the book (and hence on the market size for the book) but less effect on the price elasticity.
- A gasoline company set to launch a new additive that improves fuel economy also faces uncertainty about market size and valuation. However, it faces lesser uncertainty about size of the market because the number of cars in a market does not change appreciably over a time period. It has relatively greater uncertainty about valuation.

The relative advantages of the two sales mechanisms depend in part on practical concerns such as the difficulty of flexible production (when setting price) and the difficulty of implementing an auction or another flexible price adjustment mechanism (when setting quantity). Our interest is in showing that, apart from these concerns, relative uncertainty about valuation versus market size has an important role to play in the choice of a sales mechanism. We obtain the following results.

1. In a benchmark with zero marginal cost and uncertainty about only one aspect of the market, we show (Section 4) that setting quantity is optimal when the uncertainty is about valuations and that setting price is optimal when the uncertainty is about market size.

2. In Sections 5 and 6, we consider the case in which there is uncertainty about both aspects of the market. We show that an increase in uncertainty about valuations favors setting quantity while an increase in uncertainty about market size favors setting price.
3. For linear demand, we find (Section 7) that there is a nonmonotonic relationship between marginal cost and the choice of mechanism. In a typical case, setting price is optimal for low marginal costs and for high marginal costs, whereas setting quantity is optimal for intermediate marginal costs.
4. In Section 8, we further explore the implications of the interaction between marginal costs and relative uncertainty in the context of a bilateral channel monopoly model. We show that double marginalization can create a "conflict zone" wherein the retailer's choice of a selling mechanism deviates from the coordinated channel selling mechanism.

We begin, in Section 2, with a review of related research. Section 3 introduces the model with uncertainty about market size and valuations. We conclude in Section 9 with a summary of our central results and directions for future research.

2. Related research

We consider two selling mechanisms—one where the firm sets the price and market determines the quantity, and the other where the firm sets the quantity and the market determines the price. One mechanism that allows for market determination of price is the auction mechanism. In that sense, setting quantity instead of setting price is analogous to a firm choosing an auction instead of posted prices. Our work is thus related to the auctions literature that compares two different types of selling methods: auctions and posted prices. For example, Wang (1993) compares auctions and posted price for the case in which the seller has a single unit of the good to sell. This is very different from our model, in which the seller can supply any quantity of the good at constant marginal cost; furthermore, the action he considers includes a reserve price, whereas we consider only quantity mechanisms without a reserve price. However, his main result is loosely related to one of ours. He shows that a steeper marginal revenue curve, which corresponds to greater uncertainty about the valuations of the buyers, increases the advantage of the auction; this paper shows that an increase in uncertainty about valuations increases the relative advantage of the quantity mechanism. Furthermore, the reserve price in his auctions avoids a possibility that we consider, whereby—with enough uncertainty about valuations—setting price becomes optimal because it avoids the risk of low selling prices.

The auctions-versus-prices literature has grown with the introduction on eBay of the "buy-it-now" option. Thus eBay, which began solely as an auctions website, now allows selling by posted prices and by a mixture in which the seller adds a reserve price and a "buy-it-now" option to a regular auction. The papers in that literature differ from ours in that they focus on the case of a seller with a single unit to sell and consider different factors that drive the choice between auctions and posted prices. For instance, Matthews (2004) suggests that buy-it-now appeals to bidders who are impatient (in the sense of discounting future utility). A similar idea drives the results of Caldentey and Vulcano (2007) and Etzion, Pinker, and Seidmann (2006), but the latter two papers also incorporate the stochastic distribution of arriving bidders. Budish and Takeyama (2001) argue for the optimality of buy-it-now in the context of risk-averse consumers. Hidvegi, Wang, and Whinston (2006) suggest that the optimality of buy-it-now stems from risk aversion on the part of sellers. Wang, Montgomery, and Srinivasan (2004) suggest that, by setting a buy-it-now price, the seller can reduce the transaction costs borne by the bidders. From the seller's perspective, the drawbacks of the buy-it-now option are that (i) it sets an upper bound on the price at which an item can be sold; and (ii) it limits the seller's ability to price discriminate. On the other hand, by reducing the transactions costs for bidders it improves the liquidity of the market and hence increases the probability that the item will be sold.

Klemperer and Meyer (1986) is the closest to our paper in that the authors also consider how a firm's choice between setting price and setting quantity depends on the nature of the demand uncertainty. They take up two themes that we do not consider: (a) How does that choice depend on the curvature of the marginal cost curve? (We assume that marginal cost is constant throughout.) (b) How do these factors affect the equilibrium of a duopoly in which each duopolist chooses its sales mechanism? In contrast, we assume constant marginal costs and a monopoly setting in order to highlight the impact of the nature of demand uncertainty (where they have much more limited results) on the choice between the selling mechanisms.

Specifically, Sections 3 and 4 of their paper take up linear demand, assuming that the uncertainty is small enough that the non-negativity condition is not binding (an assumption we sometimes relax in our treatment of linear demand). Section 3 uses the additive shock parameterization of uncertainty that we discuss in our Section 6.4; we note that this case is knife edge in that setting price and setting quantity are equivalent if marginal cost is constant. Klemperer and Meyer consider instead non-constant marginal cost; they show that setting quantity is better if marginal cost is increasing and setting price is better if marginal cost is decreasing. Since these results concern the curvature rather than magnitude of the cost curve, they differ from our analysis of how the choice of sales mechanisms depends on the magnitude of a constant marginal cost.

Their Section 4 then considers the case of linear demand with known valuations and uncertain market size. In our Section 4 we note that, for such uncertainty and whether demand is linear or nonlinear, setting price dominates when marginal cost is constant. Klemperer and Meyer make the same observation for linear demand. They show, furthermore, that setting price dominates as long as marginal cost does not increase too quickly but that otherwise setting quantity can dominate. Their Section 5 generalizes the linear additive shock model to $\tilde{P}(q) = f(q) + \tilde{\theta}$, where f is a known nonlinear function and $\tilde{\theta}$ is uncertain. However, as we point out in the linear case, this one-dimensional shock $\tilde{\theta}$ conflates uncertainty about valuations and uncertainty about market size and hence is not useful for the questions we study in this paper. In summary, for the most part, the two papers examine complementary but distinct cases and questions.

There is also a loose link between our paper and the research on whether to regulate by prices or quantities, as in Weitzman (1974), in the face of uncertainty.

For example, is it better to regulate carbon emissions by setting a carbon tax and allowing quantities to adjust or by setting carbon quotas and allowing prices to adjust? The answer depends on the uncertainty about the production costs of reducing carbon emissions, on the uncertainty about the social environmental costs of carbon emissions, and on the curvature of these cost curves. We ask analogous questions, but we will not look for closer links because the objective functions in the environmental regulation problem and the monopoly profit-maximization problem are so different.

Finally, related to our paper is a large literature in the production and operations management area on the topic of postponement. The focus of this work has been highlighting the benefits of delaying the point of commitment of work-in-process inventory in the context of a dynamic and uncertain market environment. Zinn and Bowersox (1988) and Lee (1993) were among the first to describe the operational benefits of postponement in assembly, manufacturing, packaging, and labeling. Swaminathan and Tayur (1998) show how moving the point of differentiation from the beginning of the assembly process to later stage helps reduce order fulfillment times for the consumer. Whang and Lee (1998) demonstrate how postponement by delaying product customization can improve forecasting ability and also lower inventory. More recently, Van Mieghem and Dada (1999) and Chod and Rudi (2005) have highlighted the benefits of postponing pricing decisions in the context of demand uncertainty (see Venkatesh and Swaminathan (2003) for a survey of this literature). In short, it has been shown that postponement strategies are beneficial for firms operating in an environment characterized by high uncertainty in demand and short product life cycles. Our paper provides additional perspective on this issue by considering the trade-offs faced by a firm choosing between postponing the decision of production (quantity) and the decision on price. We show that the relative uncertainty about valuation and market size influences which strategic variable is most worth keeping flexible.

3. Model of demand uncertainty

3.1. Setting quantity or setting price

We use a stylized model to study the implications of demand uncertainty for the sales mechanism of a single risk-neutral seller of a good or a service with constant marginal cost c. If the demand curve were known, the seller would pick the point (p, q) on the demand curve that maximizes its profit $(p - c)q$; "choosing price" and "choosing quantity" would merely be two equivalent ways to frame the decision problem. However, with uncertain demand, these sales mechanisms are not equivalent to each other.

Setting quantity. The firm can set its output or capacity to q and let the market price adjust to the uncertain value $\tilde{P}(q)$ (a random variable whose distribution depends on q).

Setting price. Alternatively, the firm can set a price p and let its output adjust to meet the uncertain demand $\tilde{Q}(p)$.

The realized profits for the quantity and price mechanisms are, respectively,

$$\tilde{\Pi}_q(q) = \tilde{P}(q)q - cq \qquad \text{and} \qquad \tilde{\Pi}_p(p) = (p-c)\tilde{Q}(p),$$

leading to expected profits of

$$\Pi_q(q) = E\left[\tilde{P}(q)\right] q - cq \qquad \text{and} \qquad \Pi_p(p) = (p-c)E\left[\tilde{Q}(p)\right].$$

Then the firm's maximum expected profits for the two mechanism are

$$\Pi_q^* = \max_q \Pi_q(q) \qquad \text{and} \qquad \Pi_p^* = \max_p \Pi_p(p).$$

Our goal is to understand how the choice of the optimal mechanism (i.e., the comparison between Π_q^* and Π_p^*) depends on the nature of the demand uncertainty.

3.2. Parameterization of demand uncertainty

We distinguish between the effects of two types of demand uncertainty.

Uncertainty about market size. On the one hand, the firm may know the distribution of the consumer's characteristics in the market but not the number of consumers (market size). This leads to the following specification. Denote the known per capita demand by $g(p)$ and let $\tilde{n}$ be the number of consumers in the market. Then demand as a function of price is $\tilde{Q}(p) = \tilde{n}g(p)$. If we let f be the inverse of g then the inverse demand is $\tilde{P}(q) = f(q/\tilde{n})$.

Uncertainty about valuations. Alternatively, the firm may know the exact size of the market but not the valuations (in the case of unit demand) or the marginal valuations (in the case of multi-unit demand) of the consumers. Unlike uncertainty about market size, such uncertainty is not inherently one-dimensional. We restrict attention to a one-dimensional parameterization in which a single random variable $\tilde{a}$ scales each consumer's valuation or marginal valuations linearly. If we let $f(q)$ be the inverse demand curve (hence marginal valuation curve) of the market when $\tilde{a} = 1$, then the inverse demand for other realizations of $\tilde{a}$ is $\tilde{P}(q) = \tilde{a}f(q)$. If g is the inverse of f then the demand curve is $\tilde{Q}(p) = g(p/\tilde{a})$.

Combining these two kinds of uncertainty, the inverse demand curve and the demand curve are given, respectively, by

$$\tilde{P}(q) = \tilde{a}\, f(q/\tilde{n}) \qquad \text{and} \qquad \tilde{Q}(p) = \tilde{n}\, g(p/\tilde{a}),$$

where g is the inverse of f. We thereby have a general specification of uncertain demand that explicitly distinguishes between uncertainty about market size ($\tilde{n}$) and uncertainty about valuations ($\tilde{a}$). For example, in the case of unit demand with a finite number of consumers, any demand curve has both a quantity and price intercept. A change in $\tilde{n}$ is then a movement of the quantity intercept alone; a change in $\tilde{a}$ is a movement in the price intercept alone.

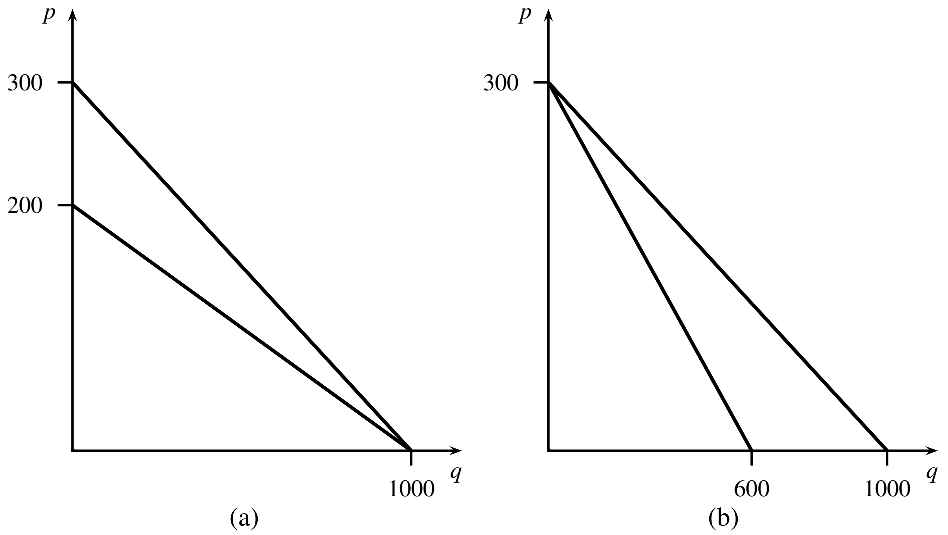

FIGURE 1. Possible linear demand curves if $g(p) = 1 - p$. In panel (a), n is known to be 1000 and $\tilde{a}$ is either 200 or 300. In panel (b), a is known to be 300 and $\tilde{n}$ is either 600 or 1000.

3.3. Linear case

As a special case we consider linear demand, where all realizations of the demand and inverse demand curves are linear. Then we have a two-dimensional family of demand curves that we parameterize by the quantity intercept $\tilde{n}$ and the price intercept $\tilde{a}$. This matches our general specification when $f(q) = 1-q$ and $g(p) = 1 - p$. (Other linear forms of f and q can be normalized to these canonical forms by absorbing the coefficients into the distributions of $\tilde{n}$ and $\tilde{a}$.) Then the inverse demand and demand curves are

$$\tilde{P}(q) = \tilde{a}(1 - q/\tilde{n}) \qquad \text{and} \qquad \tilde{Q}(p) = \tilde{n}(1 - p/\tilde{a}). \tag{1}$$

This specification corresponds, for example, to the case of unit demand with $\tilde{n}$ consumers whose valuations are uniformly distributed on $[0, \tilde{a}]$. (The standard representation $\tilde{Q}(p) = \tilde{\alpha} - \beta p$, where β is known, is not adequate for our purposes because it does not distinguish between these two types of uncertainty.)

To illustrate the two types of demand uncertainty, we consider the following cases.

1. Perhaps, as with the new fuel additive example from Section 1, the number of consumers is known to be 1000 and $\tilde{a}$ is 200 if a competing product is introduced and 300 otherwise. The competing product reduces each consumer's willingness to pay by one third. Figure 1(a) shows the two possible demand curves.
2. Alternatively, as in the new book example in Section 1, a is known to be 300 but the publisher is unsure whether a newspaper review will make more customers aware of the product. The number $\tilde{n}$ of customers equals 1000 if there is a positive review and 600 otherwise. Figure 1(b) shows the two possible demand curves.

3. If, on the other hand, the introduction of a competing fuel additive would not only reduce $\tilde{a}$ but also make more consumers aware of the product, then both $\tilde{a}$ and $\tilde{n}$ are uncertain and are negatively correlated.

The formulas for linear demand in equation (1) do not take into account that price and quantity cannot be negative. The fully specified formulas are

$$\tilde{P}(q) = \max\{\tilde{a}(1 - q/\tilde{n}), 0\} \qquad \text{and} \qquad \tilde{Q}(p) = \max\{\tilde{n}(1 - p/\tilde{a}), 0\}. \tag{2}$$

Such formulas are unnecessarily pedantic when there is no demand uncertainty; we understand that the firm will limit its price and quantity decisions to the region where the simpler formulas in equation (1) are correct. However, with uncertainty, a firm's optimal price (or quantity) could be such that, for some realizations of demand, the non-negativity constraint in (2) is binding and the quantity (or price) is zero. The linear case then shifts from being simple to complicated. To avoid this concern, we will sometimes assume (albeit informally) that demand uncertainty is small enough and costs are low enough that these boundaries are not reached in the range of potential profit-maximizing prices or quantities.

4. Uncertainty about only one aspect of the market

4.1. Statewise dominance

We first consider two special cases as follows.

1. Uncertainty is only about market size.
2. Uncertainty is only about valuation and $c = 0$.

The analysis is based on comparative statics of the optimal price and quantity in the full-information case as a function of known parameters n and a. We can thereby see how the firm would wish to adjust its price and quantity if it could learn demand in advance. This, in turn, gives us some intuition as to whether and why the firm would prefer to preserve flexibility with respect to quantity (by setting price) or price (by setting quantity).

For the two special cases, this intuition translates into propositions about the comparison between the mechanisms.

1. In the full-information case, the optimal price is insensitive to market size whereas the optimal quantity depends on this parameter. Therefore, if the uncertainty is only about market size, then setting price achieves the ex post maximum profit and therefore dominates, state by state, setting quantity.
2. In the full-information case with zero marginal cost, the optimal quantity is insensitive to consumer valuations whereas the optimal price depends on this parameter. Therefore, if the uncertainty is only about valuations, then setting quantity achieves the ex post maximum profit and thus dominates, state by state, setting price.

Observe that these results show statewise dominance of one mechanism over the other and hence do not require that the firm be risk neutral.

4.2. Uncertainty only about market size

Consider first the full-information comparative statics with respect to n. Varying n has no effect on the elasticity of demand at any price. Therefore, with constant marginal cost c, the firm's optimal price does not depend on n. This is easily seen by writing the firm's price-setting problem as maximizing markup times volume:

$$\max_p \; (p-c)\,(n\,g(p/a))\,.$$

Then parameter n merely scales up the objective function (a monotonic transformation) and does not affect the optimal price p^*. In contrast, the optimal quantity varies linearly with n: it is equal to $ng(p^*/a)$.

It follows that if a is known but $\tilde{n}$ is uncertain, then the firm can achieve its full-information profit by setting its price to maximize $(p-c)g(p/a)$. In contrast, it can achieve its full-information profit when setting quantity in at most one state; in all other states it earns a lower profit than if it sets price.

PROPOSITION 1. *Suppose that a is known but $\tilde{n}$ is uncertain. Then setting price leads to the ex post maximum profit. Setting quantity leads to a lower profit except in at most one state.*

4.3. Uncertainty only about valuations

There is an analogous result with respect to quantity and the parameter a if $c = 0$, because we can then write the firm's full-information problem as one of maximizing revenue:

$$\max_q \; a\,f(q/n)\,q.$$

The parameter a merely scales the objective function and does not change the optimal quantity q^*. The optimal price does vary linearly with a: it is equal to $a\,f(q^*/n)$.

It follows that if n is known but $\tilde{a}$ is uncertain and if the firm has zero marginal cost, then the firm can achieve its full-information profit by setting quantity to maximize $f(q/n)\,q$; it earns a lower profit if it sets price.

PROPOSITION 2. *Suppose that $c = 0$ and that n is known but $\tilde{a}$ is uncertain. Then setting quantity leads to the ex post maximum profit. Setting price leads to a lower profit except in at most one state.*

5. Increasing uncertainty and the relative advantage of price versus quantity

The results of Section 4 suggest that greater uncertainty about market size would favor setting price and greater uncertainty about valuations would favor setting quantity. We show in Sections 5 and 6 that such intuition remains valid when uncertainty is about both market size and valuations. Unlike in Section 4, we cannot show statewise dominance of one mechanism over another. Instead, we compare the sales mechanisms' *expected* profits.

5.1. Comparative statics with respect to risk

Rather than rank Π_q^* and Π_p^*, we characterize how their difference, $\Pi_p^* - \Pi_q^*$ (which measures the value, perhaps negative, of switching from flexible prices to flexible quantities), changes when either valuations become more uncertain or market size becomes more uncertain. "More uncertain" means "more risky"—roughly in the sense of Rothschild and Stiglitz (1970), except that we must take into account that $\tilde{n}$ and $\tilde{a}$ may not be independent.

Definition 1. Given random variables $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{y}$, we say that $\tilde{x}_2$ *is riskier than* $\tilde{x}_1$ *given* $\tilde{y}$ if the following holds: for almost every realization y of $\tilde{y}$, the distribution of $\tilde{x}_2$ conditional on $\tilde{y} = y$ is riskier than the distribution of $\tilde{x}_1$ conditional on $\tilde{y} = y$.

For conciseness, we often let the "given $\tilde{n}$" or "given $\tilde{a}$" be implicit and say, for example, "$\tilde{n}_2$ is riskier than $\tilde{n}_1$" to mean "$\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$".

Our main results are the following.

1. An increase in uncertainty about market size favors a price mechanism ($\Pi_p^* - \Pi_q^*$ rises) because it reduces the expected profit of the quantity mechanism without affecting the expected profit of the price mechanism.
2. An increase in uncertainty about valuations favors a quantity mechanism ($\Pi_p^* - \Pi_q^*$ falls) because it reduces the expected profit of the price mechanism without affecting the expected profit of the quantity mechanism.

We explain these results in terms of shifts in the expected demand curves and then show that the changes in expected profit occur not merely around the optimum but for any value of the decision variable.

5.2. Certainty-equivalent demand curves

Recall that expected profit as function of q and as a function of p are given by

$$\Pi_q(q) = E\left[\tilde{P}(q)\right] q - cq \qquad \text{and} \qquad \Pi_p(p) = (p-c)E\left[\tilde{Q}(p)\right].$$

We can convert each decision problem (setting quantity and setting price) into a familiar problem without uncertainty by defining

$$P(q) = E\left[\tilde{P}(q)\right] \qquad \text{and} \qquad Q(p) = E\left[\tilde{Q}(p)\right],$$

so that the objective functions become

$$\Pi_q(q) = P(q)\, q - cq \qquad \text{and} \qquad \Pi_p(p) = (p-c)\, Q(q)\,.$$

That is, a quantity-setting (resp., price-setting) firm faces the same objective—thus the same solution and maximum profit—as if it had the deterministic inverse demand curve $P(q)$ (resp., demand curve $Q(p)$). For this reason, we call $P(q)$ and $Q(p)$ the *certainty-equivalent* (CE) inverse demand and demand curves. Whereas without uncertainty a firm's inverse demand and demand curves would

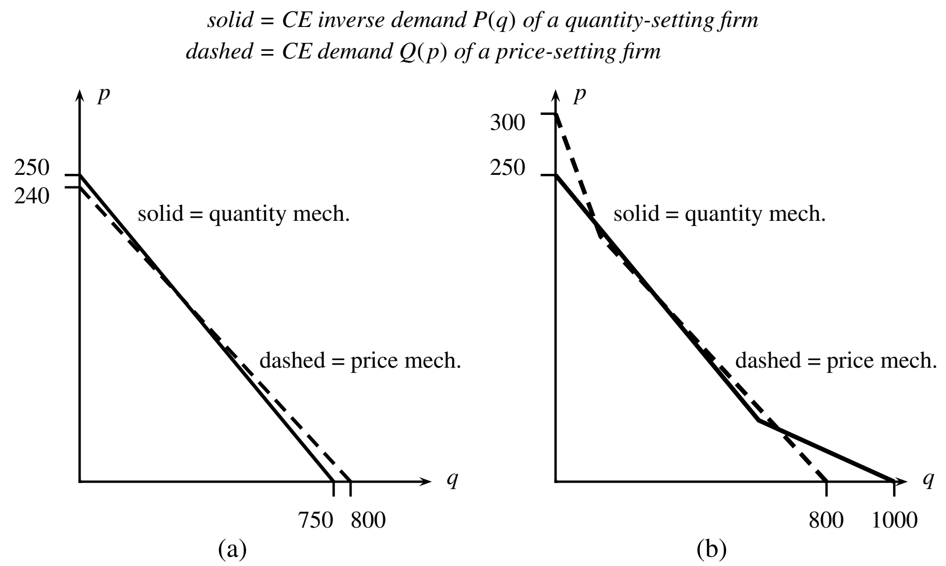

FIGURE 2. Certainty-equivalent demand curves for linear demand. Demand is linear, $\tilde{n}$ and $\tilde{a}$ are independent, $\tilde{n}$ equals 600 and 1000 with equal probability, and $\tilde{a}$ equals 200 and 300 with equal probability. Panel (a) ignores the non-negativity condition; panel (b) takes it into account.

be inverses of each other and hence equivalent, $P(q)$ and $Q(p)$ are, in general, different.

The choice of sales mechanism can thus be viewed as the choice between two demand curves by a firm with deterministic demand. The distribution of $(\tilde{n}, \tilde{a})$ affects this choice entirely through its impact on the CE demand curves. We exploit this viewpoint whenever possible in our analysis.

When we need to emphasize the dependency of these functions and values on the distribution of $(\tilde{n}, \tilde{a})$, we will write $P(q; \tilde{n}, \tilde{a})$, $Q(p; \tilde{n}, \tilde{a})$, $\Pi_q(q; \tilde{n}, \tilde{a})$, $\Pi_p(p; \tilde{n}, \tilde{a})$, $\Pi^*_q(\tilde{n}, \tilde{a})$, and $\Pi^*_p(\tilde{n}, \tilde{a})$. The reader should keep in mind that, for example, $\Pi_q(q; \tilde{n}, \tilde{a})$ is the expected profit given the joint distribution of $\tilde{n}$ and $\tilde{a}$, not the realized profit as a function of the random variables $\tilde{n}$ and $\tilde{a}$.

To illustrate the CE demand curves for the case of linear demand, we assume that $f(q) = 1 - q$ and $g(p) = 1 - p$. Over the range of quantities and prices for which the boundary of the linear demand curves are not reached with positive probability, we have the following CE demand curves:

$$P(q) = E[\tilde{a}] - qE[\tilde{a}/\tilde{n}] \qquad \text{and} \qquad Q(p) = E[\tilde{n}] - pE[\tilde{n}/\tilde{a}]. \tag{3}$$

For example, suppose that $\tilde{n}$ equals 600 and 1000 with equal probability, that $\tilde{a}$ equals 200 and 300 with equal probability, and that $\tilde{n}$ and $\tilde{a}$ are independent. Then the CE demand curves based on the formulas in (3) are shown in Figure 2(a). When we take into account the boundary conditions, applying the formulas in equation (2), we obtain the curves in Figure 2(b).

5.3. Assumption of decreasing marginal revenue

Some of the upcoming results assume that the revenue curves (as a function of quantity and as a function of price) are strictly concave, even though we do not rely on second-order conditions of the maximization problem. Below we define these revenue curves and explain how such assumptions will be treated.

Denote revenue as a function of quantity by $\tilde{R}(q) \equiv \tilde{P}(q)q$ and revenue as a function of price by $\tilde{S}(p) \equiv p\tilde{Q}(p)$. Observe that these curves, which equal

$$\tilde{R}(q) = \tilde{a}\, f(q/\tilde{n})\, q \qquad \text{and} \qquad \tilde{S}(p) = p\tilde{n}\, g(p/\tilde{a})\,,$$

are strictly concave for all realizations of $\tilde{a}$ and $\tilde{n}$ if and only if

$$\hat{R}(q) = f(q)\, q \qquad \text{and} \qquad \hat{S}(p) = p\, g(p)$$

(respectively) are strictly concave.

However, because $\hat{R}(q)$ and $\hat{S}(p)$ are positive, they could be strictly concave on all of $[0, \infty)$ only if they were strictly increasing. Therefore, we assume instead that $\tilde{R}(q)$ is strictly concave on an interval $[0, \bar{q}]$ for any realization of the demand curve. This means that $\hat{R}(q)$ is strictly concave on $[0, \bar{q}/n^{\min}]$, where $n^{\min}$ is the greatest lower bound of the support of $\tilde{n}$. Likewise, we assume that $\tilde{S}(p)$ is strictly concave on an interval $[0, \bar{p}]$ for any realization of demand. This means that $\hat{S}(p)$ is strictly concave on $[0, \bar{p}/a^{\min}]$, where $a^{\min}$ is the greatest lower bound of the support of $\tilde{a}$.

We explain the significance of these assumptions for the case of linear demand in Section 6 and for general demand curves in Appendix B.

5.4. How demand uncertainty affects certainty-equivalent demand curves

Proposition 3 states that an increase in uncertainty about $\tilde{n}$ has no effect on the CE demand of a price-setting firm (under no extra assumptions) but dampens the CE demand of a quantity-setting firm (assuming that revenue is a strictly concave function of quantity). Proposition 4 states that an increase in uncertainty about $\tilde{a}$ has no effect on the CE demand of a quantity-setting firm (under no extra assumptions) but dampens the CE demand of a price-setting firm (assuming that revenue is a strictly concave function of price).

PROPOSITION 3. *Suppose $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ are such that $\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$. Then:*

1. *$Q(p; \tilde{n}_2, \tilde{a}) = Q(p; \tilde{n}_1, \tilde{a})$ for all p;*
2. *if $\hat{R}(q)$ is strictly concave on $[0, \bar{q}/n_2^{\min}]$, then $P(q; \tilde{n}_2, \tilde{a}) < P(q; \tilde{n}_1, \tilde{a})$ for all $q \in [0, \bar{q}]$.*

PROPOSITION 4. *Suppose $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ are such that $\tilde{a}_2$ is riskier than $\tilde{a}_1$ given $\tilde{n}$. Then:*

1. *$P(q; \tilde{n}, \tilde{a}_2) = P(q; \tilde{n}, \tilde{a}_2)$ for all q;*
2. *if $\hat{S}(p)$ is strictly concave on $[0, \bar{p}/a_2^{\min}]$, then $Q(p; \tilde{n}, \tilde{a}_2) < Q(p; \tilde{n}, \tilde{a}_2)$ for all $p \in [0, \bar{p}]$.*

The proofs of these two propositions are identical except for a substitution of the roles of $\tilde{a}$ and $\tilde{n}$, of f and g, et cetera. Therefore, we include only the proof of Proposition 3, which can be found in Appendix A.

5.5. How demand uncertainty affects expected profits

We have shown that an increase in the uncertainty about market size has no effect on a price-setting firm but hurts a quantity-setting firm, and that an increase in the uncertainty about valuations has no effect on the quantity-setting firm but hurts a price-setting firm. As corollaries to Proposition 3 and 4, we have the following main results of this section.

THEOREM 1. *Suppose $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ are such that $\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$.*

1. *Then $\Pi_p(p; \tilde{n}_2, \tilde{a}) = \Pi_p(p; \tilde{n}_1, \tilde{a})$ for all p; therefore, $\Pi_p^*(\tilde{n}_2, \tilde{a}) = \Pi_p^*(\tilde{n}_1, \tilde{a})$.*
2. *Suppose $\hat{R}(q)$ is strictly concave on $[0, \bar{q}/n_2^{\min}]$. Then $\Pi_q(q; \tilde{n}_2, \tilde{a}) < \Pi_q(q; \tilde{n}_1, \tilde{a})$ for all $q \in [0, \bar{q}]$; therefore, if a solution to $\max_q \Pi_q(q; \tilde{n}_2, \tilde{a})$ lies in $[0, \bar{q}]$, then $\Pi_q^*(\tilde{n}_2, \tilde{a}) < \Pi_q^*(\tilde{n}_1, \tilde{a})$.*

THEOREM 2. *Suppose $\{\tilde{n}, \tilde{a}_1, \tilde{a}_2\}$ are such that $\tilde{a}_2$ is riskier than $\tilde{a}_1$ given $\tilde{n}$.*

1. *Then $\Pi_q(q; \tilde{n}, \tilde{a}_2) = \Pi_q(q; \tilde{n}, \tilde{a}_1)$ for all q; therefore, $\Pi_q^*(\tilde{n}, \tilde{a}_2) = \Pi_q^*(\tilde{n}, \tilde{a}_1)$.*
2. *Suppose $\hat{S}(p)$ is strictly concave on $[0, \bar{p}/a_2^{\min}]$. Then $\Pi_p(p; \tilde{n}, \tilde{a}_2) < \Pi_p(p; \tilde{n}, \tilde{a}_1)$ for all $p \in [0, \bar{p}]$; therefore, if a solution to $\max_p \Pi_p(p; \tilde{n}, \tilde{a}_2)$ lies in $[0, \bar{p}]$, then $\Pi_p^*(\tilde{n}, \tilde{a}_2) < \Pi_p^*(\tilde{n}, \tilde{a}_1)$.*

Theorems 1 and 2 imply that (under the strict concavity assumptions) greater uncertainty about market size causes $\Pi_p^* - \Pi_q^*$ to rise whereas greater uncertainty about valuation causes $\Pi_p^* - \Pi_q^*$ to fall. This is consistent with our analysis in the previous section, where uncertainty was only about market size or only about valuations.

6. Application to linear demand

To understand the implications of the results in Section 5, we need to explore our assumptions about the strict concavity of the revenue curves. In this section, we do this for the case of linear demand. The main ideas extend to nonlinear demand, as discussed in Appendix B.

6.1. Shifts in the CE demand curves

Assume that $f(q) = 1 - q$ and $g(p) = 1 - p$. The full-information revenue curves are shown in Figure 3 for any realizations n and a of $\tilde{n}$ and $\tilde{a}$. They are quadratic (and hence strictly concave) functions up until the intercepts n and a. Thus, over the domains $[0, n^{\min}]$ for quantities or $[0, a^{\min}]$ for prices, the CE demand curves will shift as predicted by the second parts of Propositions 3 and 4. This observation is summarized in the following proposition.

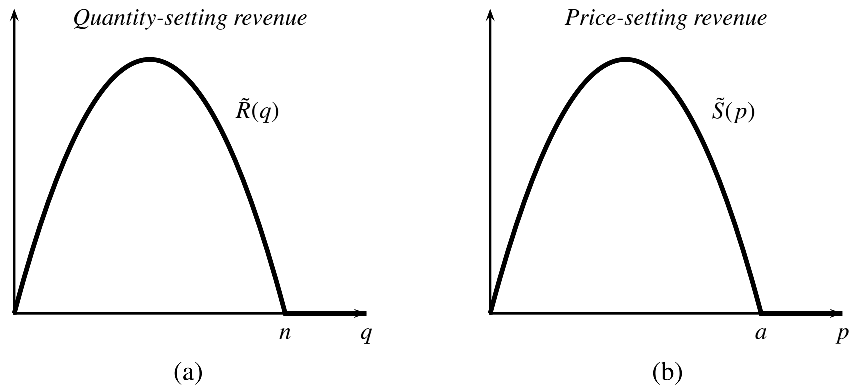

FIGURE 3. The full-information revenue curves for any realizations n and a of $\tilde{n}$ and $\tilde{a}$, assuming linear demand $f(q) = 1 - q$ and $g(p) = 1 - p$. If $n = a = 1$, then these are the canonical revenue curves $\hat{R} = q - q^2$ and $\hat{S}(p) = p - p^2$.

PROPOSITION 5. *Assume that demand is linear.*

1. *Suppose that* $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ *are such that* $\tilde{n}_2$ *is riskier than* $\tilde{n}_1$ *given* $\tilde{a}$. *Then* $P(q; \tilde{n}_2, \tilde{a}) < P(q; \tilde{n}_1, \tilde{a})$ *for* $q \in [0, n_2^{\min}]$.
2. *Suppose that* $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ *are such that* $\tilde{a}_2$ *is riskier than* $\tilde{a}_1$ *given* $\tilde{n}$. *Then* $Q(p; \tilde{n}, \tilde{a}_2) < Q(p; \tilde{n}, \tilde{a}_2)$ *for* $p \in [0, a_2^{\min}]$.

Proof. In Propositions 3 and 4, the "strict concavity of revenue" conditions are stated in terms of the canonical revenue curves: $\hat{R}(q) = \max\{q - q^2, 0\}$ and $\hat{R}(q) = \max\{q - q^2, 0\}$. Observe that each is quadratic and hence strictly concave on $[0, 1]$. The assumptions in Propositions 3 and 4 are that they be quadratic and strictly concave on $[0, \bar{q}/n_2^{\min}]$ and $[0, \bar{p}/a_2^{\min}]$, respectively; thus they hold for $\bar{q} = n_2^{\min}$ and $\bar{p} = a_2^{\min}$, respectively. □

For $q > n_2^{\min}$ or $p > a_2^{\min}$, the CE demand curves might not shift as predicted by Proposition 5 because of the nonconcavity in the revenue curves. Figure 4(a) illustrates the shift in the CE inverse demand curve when $\tilde{n}$ becomes more uncertain. In the example, $\tilde{n}$ and $\tilde{a}$ are independent. The dashed line assumes that $\tilde{n}$ equals 700 or 900 with equal probability; the solid line assumes that $\tilde{n}$ equals 600 or 1000 with equal probability. As predicted by the first part of Proposition 5, the inverse demand curve shifts downward for $q \in [0, 600]$. For high enough values of q, the shift is in the opposite direction.

Figure 4(b) illustrates the shift in the CE demand curve when $\tilde{a}$ becomes more uncertain. In the example, $\tilde{n}$ and $\tilde{a}$ are independent. The dashed line assumes that $\tilde{a}$ equals 240 or 260 with equal probability; the solid line assumes that $\tilde{a}$ equals 200 or 300 with equal probability. As predicted by the second part of Proposition 5, the demand curve shifts to the left on $p \in [0, 200]$. For high enough values of p, the shift is in the opposite direction.

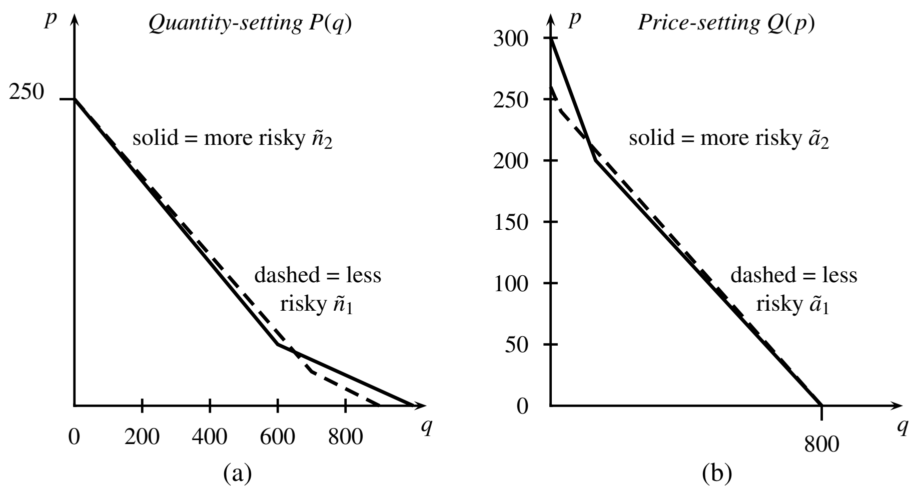

FIGURE 4. CE demand curves. In all cases, $\tilde{n}$ and $\tilde{a}$ are independent. Panel (a): Quantity-setting $P(q)$. Dashed curve is for less risky $\tilde{n}_1$, equal to 700 or 900 with same probability. Solid curve is for more risky $\tilde{n}_2$, equal to 600 or 1000 with same probability. Panel (b): Price-setting $Q(p)$. Dashed curve is for less risky $\tilde{a}_1$, equal to 240 or 260 with same probability. Solid curve is for more risky $\tilde{a}_2$, equal to 200 or 300 with same probability.

6.2. Shifts in expected profit

The shifts in the CE demand curves translate directly into shifts in expected profit on the same range of quantity or price.

COROLLARY 1. *Assume that demand is linear.*

1. *Suppose that* $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ *are such that* $\tilde{n}_2$ *is riskier than* $\tilde{n}_1$ *given* $\tilde{a}$. *Then* $\Pi_q(q; \tilde{n}_2, \tilde{a}) < \Pi_q(q; \tilde{n}_1, \tilde{a})$ *for* $q \in [0, n_2^{\min}]$.
2. *Suppose that* $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ *are such that* $\tilde{a}_2$ *is riskier than* $\tilde{a}_1$ *given* $\tilde{n}$. *Then* $\Pi_p(p; \tilde{n}, \tilde{a}_2) < \Pi_p(p; \tilde{n}, \tilde{a}_2)$ *for* $p \in [0, a_2^{\min}]$.

This is illustrated in Figure 5(a) for a quantity-setting firm, using the example from Figure 4(a) and assuming zero marginal cost (so that expected profit equals expected revenue). In the region before the CE inverse demand curves cross (i.e., where the expected price is lower when $\tilde{n}$ is more uncertain), the expected profit is also lower when $\tilde{n}$ is more uncertain. The opposite happens at higher quantities.

Likewise, Figure 5(b) illustrates the affect on expected profit of a price-setting firm when valuations become more uncertain. The profit curves are based on the example in Figure 4(b) and assume zero marginal cost (so that expected profit equals expected revenue). For prices lower than where the CE demand curves cross (i.e., such that the expected quantity is lower when $\tilde{a}$ is more uncertain), the expected profit is also lower when $\tilde{a}$ is more uncertain. The opposite happens at higher prices.

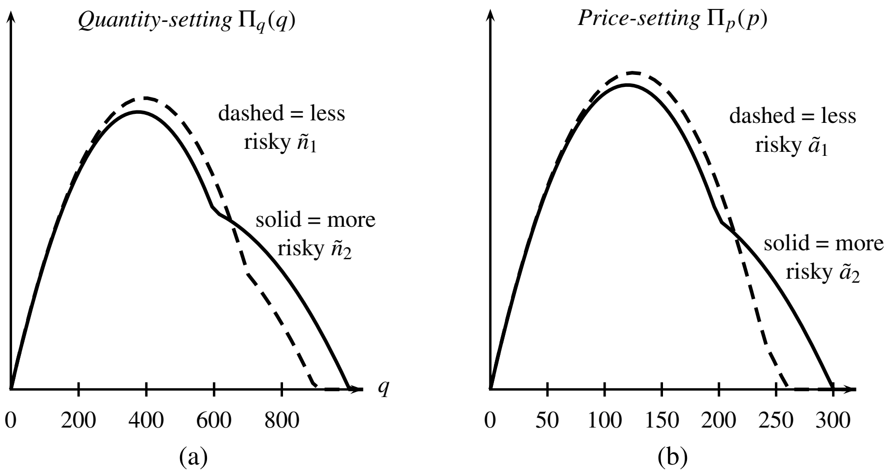

FIGURE 5. Profit and uncertainty. Panel (a): Profit function for a quantity-setting firm, comparing less risky $\tilde{n}_1$ (dashed) with more risky $\tilde{n}_2$ (solid) as in Figure 4(a). Panel (b): Profit function for a price-setting firm, comparing less risky $\tilde{a}_1$ (dashed) with more risky $\tilde{a}_2$ (solid) as in Figure 2(b).

6.3. Shifts in maximal expected profits

We now examine the net effects on the optimal profit of (a) an increase in uncertainty about $\tilde{n}$ when the seller sets quantity (Π_q^*) and (b) an increase in uncertainty about $\tilde{a}$ when the seller sets price (Π_p^*).

Consider our graphical examples in Figure 5. In each case, the increase in uncertainty causes the maximum expected profit to fall. This is because the optimal quantity (resp., price) is in the region where the CE inverse demand curve (resp., CE demand curve) shifts downward (resp., to the left) and hence the expected profit curve shifts downward. Proposition 6 formalizes simple and precise restrictions on the joint distribution of $(\tilde{n}, \tilde{a})$ needed for such consequences to hold.

Now consider the optimal quantity in the full-information benchmark with linear demand. Like the revenue curve, the profit curve (as a function of quantity or of price) is single-peaked. Therefore, if $q^{\max}$ is the greatest full-information profit-maximizing quantity across possible realizations of the demand curve, then

$$\Pi_q(q^{\max}; \tilde{n}, \tilde{a}) \geq \Pi_q(q; \tilde{n}, \tilde{a}) \quad \forall q \geq q^{\max}$$

for all possible realizations of $\tilde{n}$ and $\tilde{a}$. It follows that $q^{\max}$ bounds the quantity that maximizes *expected* profit with incomplete information. The full-information profit-maximizing quantity is $(1/2)n(1-c/a)$. Therefore, $q^{\max} = (1/2)n^{\max}(1-c/a^{\max})$, which does not exceed $n^{\min}$ if $n^{\max} \leq 2(a^{\max}/(a^{\max}-c))n^{\min}$. A sufficient condition is that $n^{\max} \leq 2n^{\min}$. This yields the first part of Proposition 6.

PROPOSITION 6. *Assume that demand is linear.*

1. *Suppose that $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ are such that $\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$ and that $n_2^{\max} \leq 2(a^{\max}/(a^{\max}-c))n^{\min}$ (a sufficient condition is $n^{\max} \leq 2n^{\min}$). Then $\Pi_q^*(\tilde{n}_2, \tilde{a}) < \Pi_q^*(\tilde{n}_1, \tilde{a})$.*

2. *Suppose that* $\{\tilde{n}, \tilde{a}_1, \tilde{a}_2\}$ *are such that* $\tilde{a}_2$ *is riskier than* $\tilde{a}_1$ *given* $\tilde{n}$ *and that* $a_2^{\max} \leq 2a_2^{\min} - c$. *Then* $\Pi_p^*(\tilde{n}, \tilde{a}_2) < \Pi_p^*(\tilde{n}, \tilde{a}_1)$.

Next consider the price decision in the full-information benchmark. Again, the profit curve is single-peaked. Therefore, if $p^{\max}$ is the greatest full-information profit-maximizing price across possible realizations of the demand curve, then

$$\Pi_p(p^{\max}; \tilde{n}, \tilde{a}) \geq \Pi_p(p; \tilde{n}, \tilde{a}) \quad \forall p \geq p^{\max}$$

for all possible realizations of $\tilde{n}$ and $\tilde{a}$. It follows that $p^{\max}$ bounds the price that maximizes *expected* profit with incomplete information. The full-information profit-maximizing price is $(a+c)/2$. Therefore, $p^{\max} = (a^{\max}+c)/2$, which does not exceed $a^{\min}$ if $a^{\max} \leq 2a^{\min} - c$. This yields the second part of Proposition 6.

We could summarize Proposition 6 by saying that we obtain the reductions in expected profit predicted by Theorems 1 and 2 when "the uncertainty is not too large". However, Proposition 6.1, concerning the effect of more uncertain market size on the profit of a quantity-setting firm, is more robust than Proposition 6.2. Even with zero marginal cost, the firm chooses quantity and price in the upper part of the demand curves, whereas the possible inversion of the CE inverse demand curves occurs in the lower part of the demand curves. Higher marginal cost pushes the firm toward lower quantity and higher price and thus away from the region where the revenue curve may not be concave. This is why we could state a sufficient condition, $n_2^{\max} \leq 2n_2^{\min}$, independent of the marginal cost.

Consider instead Proposition 6.2 regarding the effect of more uncertain valuations on the profit of a price-setting firm. If we fix the marginal cost below the mean of $\tilde{a}$, then for small enough uncertainty about valuations the condition in this proposition holds. On the other hand, if we fix $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$, then for high-enough marginal cost the "action" (the optimal price) ends up where the CE demand curve for $\tilde{a}_2$ is to the right of the one for $\tilde{a}_1$, and the firm is better off with the more uncertain valuations. The intuition is as follows. When the marginal cost is high, there is a risk when setting quantity of selling below marginal cost. If the firm sets a high price, it can do no worse than to have zero sales, but some times it will be lucky and sell at a high price.

6.4. Remark on the standard additive-shock model

A standard representation of uncertain linear demand is $\tilde{Q}(p) = \tilde{\alpha} - \beta p$, where β is known. The single parameter α scales both the horizontal and vertical intercepts; in our model, this means that $\tilde{n} = \tilde{\alpha}$ and $\tilde{a} = \tilde{\alpha}/\beta$. As a consequence, the CE inverse demand and demand curves are equivalent (assuming that p and q are such that formulas (1) do not result in negative values.) Therefore, this representation has the knife-edge property property that the seller is indifferent between setting price and setting quantity no matter what the distribution of $\tilde{\alpha}$.

This observation is interesting for two reasons. First, it means that this widely used additive demand model (see e.g. Petruzzi and Dada 1999; Bhardwaj 2001; Chod and Rudi 2005) is very special. It also highlights that explicit modeling of the nature of demand uncertainty, which is the subject of our paper, is important.

It is also a benchmark that is useful for ranking the sales mechanisms. If we deviate from the representation by increasing the uncertainty about valuations (resp., by increasing the uncertainty about market size), then by Proposition 6 we know that setting quantity is better than setting price (resp., that setting price is better than setting quantity). We summarize this conclusion in Proposition 7.

PROPOSITION 7. *Suppose that demand is linear.*

1. *If* $\tilde{n} = \beta\tilde{a} + \tilde{x}$*, where* $\beta > 0$ *and* $E[\tilde{x} \mid \tilde{a}] = 0$ *almost surely, then setting price is better than setting quantity.*
2. *If* $\tilde{a} = \tilde{n}/\beta + \tilde{x}$*, where* $\beta > 0$ *and* $E[\tilde{x} \mid \tilde{n}] = 0$ *almost surely, then setting quantity is better than setting price.*

7. How the price-versus-quantity decision depends on the marginal cost

We now examine, for the case of linear demand, how the choice of sales mechanism depends on the magnitude of the marginal cost. We first ignore the boundary constraint (i.e., we assume that the uncertainty about market size and valuations is not too large) and then show how the answer changes when we take it into account.

7.1. Analysis ignoring non-negativity constraints

Consider the example of the CE demand curves in Figure 2(a), which do not take into account the non-negativity constraints on quantity and price. Recall that (i) the choice of sales mechanism for a firm that faces uncertain demand is equivalent to (ii) the choice of CE demand curves for a firm with certain demand but two demand curves from which to choose. A firm with decision problem (ii) would always choose a point on the outer (i.e., upper) envelope of the two demand curves. We can thus frame that the problem of a firm with decision problem (i) as:

1. choose a profit maximizing point on the outer envelope of the CE demand curves $P(q)$ and $Q(p)$; then
2. see whether that solution lies in a region of the outer envelope that comes from the $P(q)$ curve or the $Q(p)$ curve in order to identify the best sales mechanisms.

Figure 6(a) shows the outer envelope of the CE demand curves from Figure 2(a). The kink, where the outer envelope switches between $P(q)$ and $Q(p)$, is marked with a dot. Because of the kink, the optimal price/quantity on this outer envelope does not vary continuously with marginal cost; in fact, it will jump over the kink as the marginal cost rises. Nevertheless, as a simple application of monotone comparative statics, we know that the optimal quantity is decreasing and the optimal price is increasing as the cost rises because the objective function $\pi(q; c) = pq - cq$ has strictly decreasing differences in q and c. Therefore, as the marginal cost rises, the optimal price/quantity may switch from the dashed section (meaning that setting price is optimal) to the solid section (meaning that

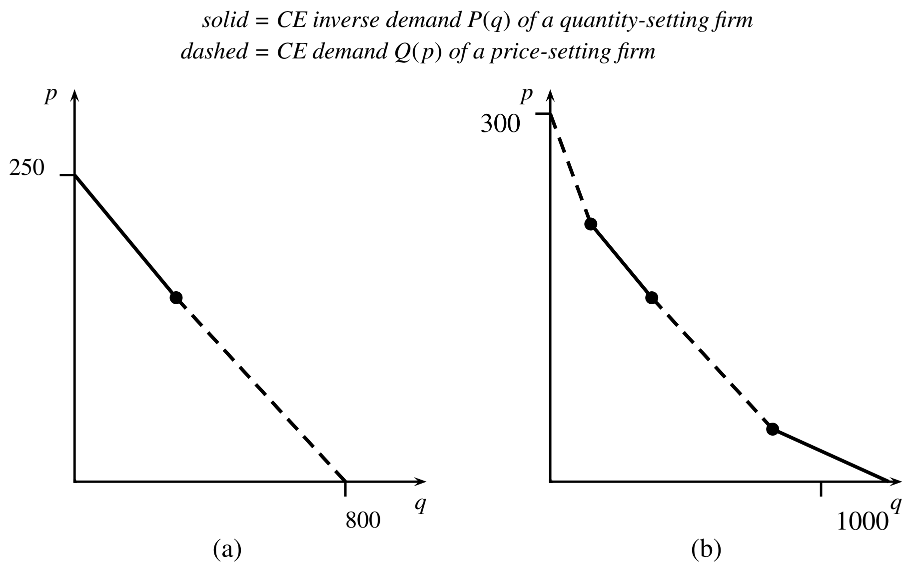

FIGURE 6. Outer envelope of the CE demand curves for the example of linear demand in Figure 2. Panel (a) ignores the non-negativity condition, as in Figure 2(a); panel (b) takes into account the condition, as in Figure 2(b).

Intercept	$P(q)$	$Q(p)$
Quantity	$E[\tilde{a}]/E[\tilde{a}/\tilde{n}]$	$E[\tilde{n}]$
Price	$E[\tilde{a}]$	$E[\tilde{n}]/E[\tilde{n}/\tilde{a}]$

TABLE 1. Intercepts of the CE inverse demand and demand curves for the linear case, ignoring non-negativity constraints.

setting quantity is optimal) but not vice versa.

To make sure that this conclusion is robust (as long as we ignore the non-negativity conditions) we just have to check that the CE curves can cross only as shown in Figure 2(a).

The formulas for the intercepts of the CE demand curves are shown in Table 1 for easy reference.

PROPOSITION 8. *Consider linear demand and ignore the non-negativity constraints. Then either the CE demand curves do not cross or $P(q)$ crosses $Q(p)$ from above, as illustrated in Figure 2(a).*

Proof. We are ruling out the case in which the CE curves cross the other way, which would mean that the quantity intercept of $Q(p)$ is lower than that of $P(q)$ whereas the price intercept of $P(q)$ is lower than that of $Q(p)$. From Table 1, this translates into the following inequalities:

$$E[\tilde{n}] < \frac{E[\tilde{a}]}{E[\tilde{a}/\tilde{n}]} \quad \text{and} \quad E[\tilde{a}] < \frac{E[\tilde{n}]}{E[\tilde{n}/\tilde{a}]}. \tag{4}$$

Intercept	$P(q)$	$Q(p)$
Quantity	$n^{\max}$	$E[\tilde{n}]$
Price	$E[\tilde{a}]$	$a^{\max}$

TABLE 2. Intercepts of the CE inverse demand and demand curves for the linear case, taking into account the non-negativity constraints.

By multiplying the two inequalities together (LHS×LHS and RHS×RHS), then canceling $E[\tilde{n}]E[\tilde{a}]$ from the resulting inequality, and then rearranging, we obtain $E[\tilde{a}/\tilde{n}]E[\tilde{n}/\tilde{a}] < 1$. This is impossible: for any nonnegative random variable $\tilde{z}$, $E[\tilde{z}]E[1/\tilde{z}] \geq 1$. □

Proposition 8 leads to our main result on how the price-versus-quantity decision depends on the magnitude of marginal cost.

COROLLARY 2. *Suppose that demand is linear and consider the range of cost c such that the non-negativity constraints are not reached. If for some marginal cost c^* setting quantity is better for the seller, then it is better for any $c > c^*$. If for some marginal cost c^* setting price is better for the seller, then it is better for any $c < c^*$.*

7.2. Full analysis with non-negativity constraints

We now consider how the answer changes when we take into account the non-negativity constraints.

Consider the example of the CE demand curves in Figure 2(b). Figure 6(b) shows their outer envelope. The kinks, where the outer envelope switches between $P(q)$ and $Q(p)$, are marked with a dot. As can be seen from Figure 6(b), the story is more complicated than in Figure 6(a). If the CE curves cross three times, as in this picture, then the optimal sales mechanism could switch from quantity to price to quantity and then again to price as the marginal cost rises.

We can slightly simplify the possibilities by assuming that the bottom right kink (crossing point) occurs at quantities beyond those that could maximize profit. This is true in the example because the quantity is greater than $n^{\max}/2$, which bounds the optimal quantity. There are then three possible zones:

1. at low enough costs, the firm sets price;
2. at an intermediate range of costs, the firm sets quantity;
3. at an upper range of costs, the firm sets price.

To ensure that these conclusions are robust, we must verify that the CE demand curves cross as in Figure 2(b). In fact, this is true. Table 2 shows the intercepts of the CE curves. Along the vertical axis (low quantity, high price), the price-setting curve $Q(p)$ must dominate. As long as the firm sets price below $a^{\max}$, it gets some sales; in contrast, when setting quantity it can never get an expected price that exceeds $E[\tilde{a}]$. By a similar argument, along the horizontal axis (high quantity, low price) the quantity-setting curve $P(q)$ dominates. Thus, the curves must cross at least once. In fact, there are three possibilities.

1. *The CE curves ignoring boundaries never cross because setting price dominates everywhere.* Then the true CE curves cross only once, at low price and high quantity, in a region that is never optimal even for zero marginal cost. Therefore, setting price is always optimal.
2. *The CE curves ignoring boundaries never cross because setting quantity dominates everywhere. Then the true CE curves cross only once, at high price and low quantity.* If $c = 0$ then setting quantity is optimal; if $c \geq E[\tilde{a}]$ then setting price is optimal; in between, there is a single cost at which the optimal mechanism switches from quantity to price.
3. *The CE curves ignoring boundaries cross as illustrated in Figure 2(a).* Then the true CE curves cross at this point and at two other points, one above and one below. If the optimal price/quantity at zero marginal cost is above the middle crossing point, then this case ends up the same as case 1. Otherwise, At $c = 0$ and for $c \geq E[\tilde{a}]$, setting price is optimal. There may be a single interval of costs between 0 and $E[\tilde{a}]$ for which setting quantity is optimal.

Results of this section (in particular, Corollary 2), where we have shown how the price-versus-quantity decision depends on the marginal cost, are relevant for manufacturer–retailer interactions. The retailer faces the problem we have so far studied in this paper, but with its marginal cost equal to the wholesale price set by the manufacturer. Because of double marginalization, this wholesale price is greater than the marginal cost of the manufacturer. As a consequence, the price-versus-quantity preferences of the retailer and the manufacturer might not be aligned. We study this issue in the following section.

8. Manufacturer–retailer interaction

We now consider a supply chain with an upstream manufacturer and a downstream retailer, both risk neutral. The manufacturer has constant marginal cost c. We restrict the manufacturer to a linear posted-price mechanism and denote its price—which becomes the marginal cost of the retailer—by w. The retailer, on the other hand, can either set quantity or set price. We consider three cases.

1. The manufacturer can stipulate the mechanism used by the retailer (this is meant to be a benchmark).
2. The retailer must commit to one of the two mechanisms before the manufacturer sets w.
3. The retailer selects its sales mechanism after the manufacturer sets w.

We assume that demand is linear and that the uncertainty is low enough relative to the cost that the boundaries of the linear demand curves are not relevant. (The higher is the cost, the less uncertainty there can be about demand so that the boundaries are avoided.) Thus, we use the formulas for linear demand in equation (1), which do not take into account the non-negativity conditions.

We use subscript m (resp., r) to denote variables for the manufacturer (resp., retailer). For comparison, we will refer to the single firm that we studied in the previous sections as a “coordinated channel”.

8.1. Mechanism is selected before the wholesale price is set

Our analysis begins with the observation that the “certainty equivalent” approach extends to the supply chain if the mechanism is selected (by the manufacturer or retailer) in advance of the wholesale price w. That is, the quantity, price, and profit (or expectations thereof) for the parties are the same as for a supply chain whose certain demand curve equals the mechanism’s certainty equivalent demand curve.

For example, suppose the retailer is committed to setting quantity. Let $P(q)$ be the CE inverse demand curve. Given w, the retailer chooses q to maximize $(P(q) - w)q$, just as it would if its inverse demand curve were known to be $P(q)$. Let $Q_r(w)$ be the solution as a function of w. Then, whether demand is certain or uncertain, the manufacturer chooses w to maximize $(w - c)Q_r(w)$.

Suppose instead the retailer is committed to setting price. Let $Q(p)$ be the CE demand curve. Then the retailer chooses p to maximize $(p - w)Q(p)$, just as it would if its demand curve were known to be $Q(p)$. Let $Q_r(w)$ be the resulting *expected* sales as a function of w, which would equal its actual sales if its demand curve were certain. Either way, the manufacturer chooses w to maximize $(w - c)Q_r(w)$.

Therefore, the choice of sales mechanism by the manufacturer or by the retailer is equivalent to the choice of CE demand curve $P(q)$ or $Q(p)$ in a supply chain with two possible demand curves. Our next step is to review the equilibrium profits for a supply chain with known demand.

The preceding observation does not depend on the assumption that demand is linear, but we now restrict attention to this case. Suppose, then, that the inverse demand and demand curves are known to be

$$p = a(1 - q/n) \qquad \text{and} \qquad q = n(1 - p/a).$$

One can show that the manufacturer sets $w = (1/2)(a + c)$ (the midpoint between the constant marginal cost c and the intercept a) and the retailer then sets $p = (3/4)a + (1/4)$ (the midpoint between its constant marginal cost w and the intercept a). The manufacturer’s price w is the same as the retail price of the coordinated channel. However, because the retailer’s price p is higher than w, sales are half of what they would be in the coordinate channel. Therefore, the manufacturer’s profit is one half of the profit of the coordinated channel. In the supply chain, the manufacturer and retailer sell the same amount, but the manufacturer’s markup is twice that of the retailers: $(1/2)(a-c)$ versus $(1/4)(a-c)$. Therefore, the manufacturer’s profit is twice that of the retailers.

With uncertainty and ignoring the boundaries, the CE inverse demand and demand curves are linear. Applying the summary of the previous paragraph, we see that, for either sales mechanism, the retailer’s profit is half that of the manufacturer, which in turn is half that of the coordinated channel:

$$\Pi^*_q = 2\Pi^*_{qm} = 4\Pi^*_{qr} \qquad \text{and} \qquad \Pi^*_p = 2\Pi^*_{pm} = 4\Pi^*_{pr}.$$

It follows that whichever CE demand curve (sales mechanism) gives the highest profit to the coordinated channel also gives the highest profit to the manufacturer and to the retailer. It does not matter which party chooses the sales mechanism—the decision would be the same. Furthermore, all our results from Sections 4–7 about how the nature of the uncertainty and the marginal cost c affect the choice of sales mechanism for the coordinated channel apply to the choice of sales mechanism for the supply chain.

8.2. Retailer selects the mechanism after the wholesale price is set

Let M' be the mechanism the manufacturer would select if it controlled the choice, let w' be the price the manufacturer would select given that the mechanism is set to M', and let Π'_m be the resulting profit for the manufacturer.

Here we suppose that the retailer selects the mechanism after w is set. Consider whether the outcome for the manufacturer is still $\{M', w', \Pi'_m\}$ or whether it earns a lower profit than Π'_m because the retailer can choose the sales mechanism after observing the wholesale price. We refer to parameters where the latter holds as a "conflict zone" because the manufacturer would prefer to be able to force the mechanism M' on the retailer and not allow for the retailer's discretion.

Such a conflict occurs if, given w', the retailer would not choose sales mechanism M'. The manufacturer cannot achieve profit Π'_m because either (a) it modifies w away from w', so that the retailer chooses m', or (b) the retailer uses a mechanism that yields a profit lower than Π'_m for the manufacturer.

Such a conflict is possible because the choice of M' is based on the marginal cost c, whereas the retailer's choice of sales mechanism is qualitatively like that of the coordinated channel but with marginal cost w' rather than c. We saw in Section 7 that the marginal cost influences the choice of sales mechanism.

Consider the CE curves shown in Figure 2(a) and suppose that $c = 0$. For the coordinated channel, the optimal price for either demand curve is one half the vertical intercept; we can see that the price-setting curve $Q(p)$ dominates in this region. Hence this is the mechanism that the manufacturer would like to specify and that the retailer would also choose if it had to commit to a mechanism in advance. Let w^*_p be the manufacturer's optimal price if the retailer had to set price; w^*_p is one half the intercept of the dashed curve. Given w^*_p, the retailer would set a price halfway between w^*_p and the vertical intercept—in the region where the quantity-setting curve $P(q)$ dominates. Therefore, the retailer prefers the quantity-setting mechanism.

Given our assumption that the boundaries of the demand curves are not relevant, we know that an increase in marginal cost can cause a switch from setting price to setting quantity but not vice versa. Therefore, the only type of conflict that can exist is as in the previous example: the manufacturer would prefer to commit to a price-setting mechanism but, given the manufacturer's wholesale price, the retailer would end up setting quantity.

This happens for an intermediate level of uncertainty about the valuations of the consumers. With very little uncertainty about $\tilde{a}$, the CE demand curves intersect near the vertical axis, and setting price will be optimal for both the manufacturer and for the retailer given w. With a lot of uncertainty about $\tilde{a}$, setting quantity dominates for the manufacturer and hence also for the retailer.

What does the manufacturer do in this conflict zone? It would not naively set

w_p^* when it knows that the retailer would choose to set quantity. There are two possibilities: (a) the manufacturer may lower w to below w_p^* in order to induce the retailer to set price; or (b) the manufacturer may accept that the retailer will set quantity and so chooses the price $w_q^* > w_p^*$ that is optimal given that mechanism.

Consider the following example. Let $\tilde{a}$ and $\tilde{n}$ be independent, with $\tilde{n}$ equal to $1 - \delta_n$ and $1 + \delta_n$ with equal probability and $\tilde{a}$ equal to $1 - \delta_a$ and $1 + \delta_a$ with equal probability. Parameters δ_n and δ_a capture uncertainty about market size and valuation, respectively.

Fix $c = 0$ and $\delta_n = 0.1$. Figure 7 plots wholesale price, manufacturer's profit, and total channel profit as functions of δ_a under three possible scenarios:

1. the retailer sets price;
2. the retailer sets quantity;
3. the retailer chooses to set price or quantity.

In the range $\delta_a \leq 0.05$, the uncertainty about valuation is relatively low and thus the manufacturer is better off when the retailer sets price, which is also in the retailer's interests. In the range $\delta_a > 0.1$, the uncertainty about valuation is large (compared to uncertainty about market size, $\delta_n = 0.1$) and thus the manufacturer is better off when the retailer sets quantity; this is also in the retailer's interests. Hence $\delta_a \leq 0.05$ and $\delta_a \geq 0.1$ correspond to "non-conflict" zones. In the range $0.05 < \delta_a < 0.1$, we observe the conflict zone: the manufacturer is better off if the retailer sets price, but the retailer prefers to set quantity when faced with the wholesale price w_p^*. For $\delta_a = 0.06$ the manufacturer is better off by lowering the wholesale price to 0.47, so that the retailer then prefers to set price. For higher values of δ_a the manufacturer is better off letting the retailer to set quantity and thus setting the wholesale price at w_q^*. In other words, in the conflict zone the manufacturer must pay careful attention to how its choice of wholesale price determines whether the retailer sets price or quantity as its decision variable. Finally, observe that the total channel profit has a peak at $\delta_a = 0.06$. The intuition is as follows: The lower is the wholesale price, the better is channel coordination; since at that value of δ_a the manufacturer is better off by lowering the wholesale price, total channel profit goes up.

9. Conclusions

We examine how uncertainty about the demand curve influences a seller's choice to set price or quantity. In particular, the seller could be uncertain about the market size (e.g., the number of buyers) for its product or about how much consumers are willing to pay for the product. We show that greater uncertainty about market size favors setting price and greater uncertainty about valuations favors setting quantity. We also find a nonmonotonic relationship between marginal cost and the choice of sales mechanism.

Framing the issues differently, we show that the seller needs to consider which of these two decision variables it should be more flexible on. Clearly, there are different costs to the seller of retaining flexibility of prices versus flexibility of quantity. Our analysis allows the seller to be informed on how the nature of a)

b)

c)

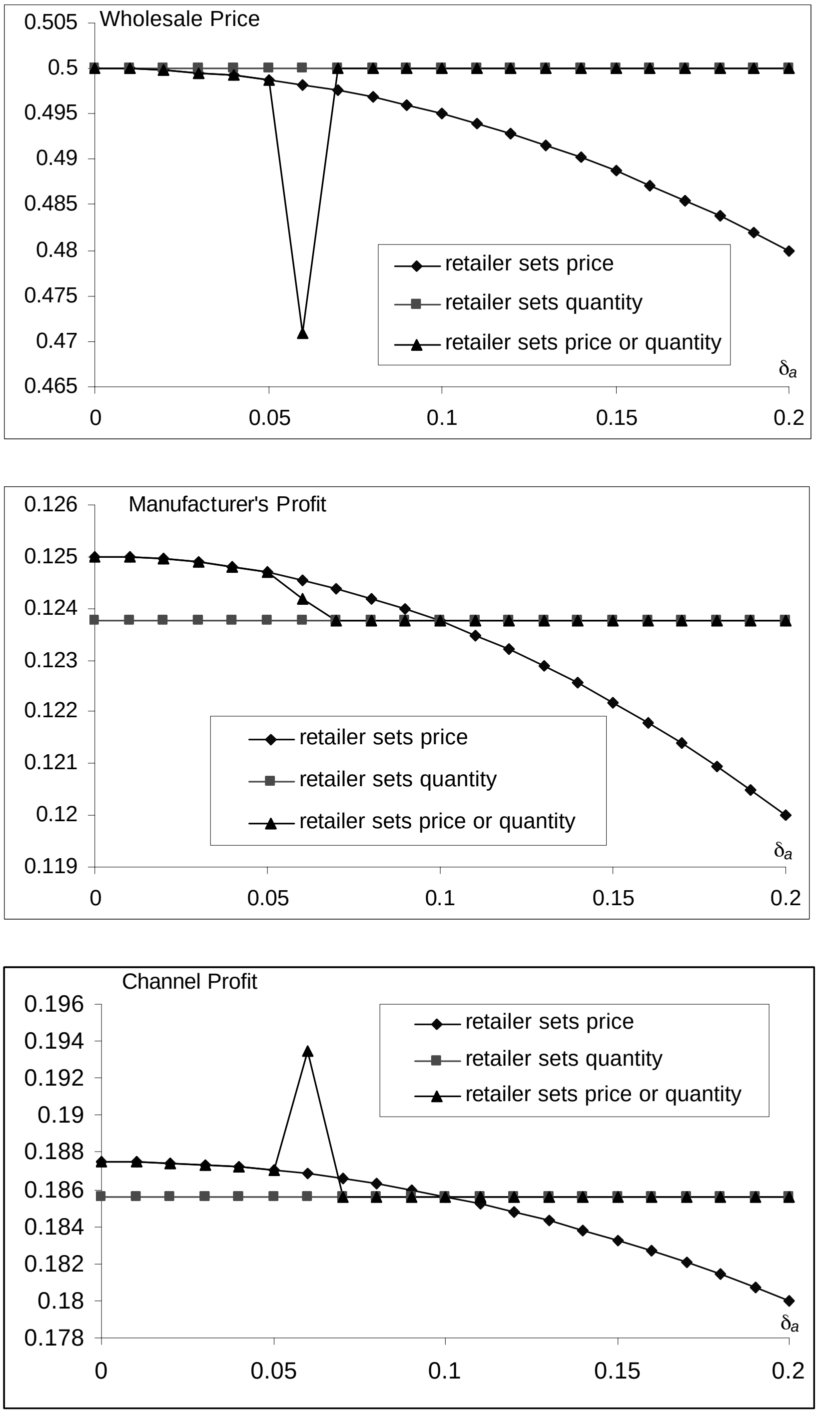

FIGURE 7. Wholesale price (a), manufacturer's profit (b), and total channel profit (c) as a function of δ_a for cases where the retailer sets price, the retailer sets quantity, and the retailer chooses between setting price or quantity.

uncertainty and the magnitude of marginal cost affect this trade-off.

We also consider the channel setting with a single manufacturer and a single retailer. The interests of the manufacturer and retailer are aligned if there is prevailing uncertainty about either market size or about valuation. If both uncertainties are comparable, then there exists a "conflict zone" in which the manufacturer prefers the retailer to set price but the retailer prefers to set quantity. In this situation the manufacturer must either (a) lower the wholesale price to the level at which the retailer prefers to set price or (b) let the retailer set quantity and adjust the wholesale price accordingly. The manufacturer would then be better off if it could force the retailer to set the price.

The focus of our paper is on the effect of the nature of demand uncertainty on the basic operation mode (to set price or to set quantity) preferred by a seller. We also show how the results extend to the channel setting. An interesting direction for future research could be the impact of the nature of demand uncertainty on other types of decisions, either for a single seller or for the channel.

Appendix A: Proof of Proposition 3

Observe that

$$E\left[\tilde{Q}(p)\right] = E[\tilde{n}\, g(p/\tilde{a})] = E\left[E[\tilde{n}\, g(p/\tilde{a}) \,|\, \tilde{a}]\right] = E\left[E[\tilde{n} \,|\, \tilde{a}]\, E[g(p/\tilde{a})]\right], \quad \text{(A.1)}$$

$$E\left[\tilde{P}(q)\right] = E[\tilde{a}\, f(q/\tilde{n})] = E\left[E[\tilde{a}\, f(q/\tilde{n}) \,|\, \tilde{a}]\right] = E\left[\tilde{a}\, E[f(q/\tilde{n}) \,|\, \tilde{a}]\right]. \qquad \text{(A.2)}$$

In each equation, the first equality is by substitution of the formulas for $\tilde{Q}(p)$ and $\tilde{P}(q)$. The second equality is by iterative expectations. The third equality follows because we can treat $\tilde{a}$ as a constant term when conditioning on it and hence move it out of the expectation.

Suppose that $\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$. By definition, this means that the conditional distribution of $\tilde{n}_2$ is riskier than the conditional distribution of $\tilde{n}_1$, conditioning on $\tilde{a}$.

One implication is that $E[\tilde{n}_1 \,|\, \tilde{a}] = E[\tilde{n}_2 \,|\, \tilde{a}]$ almost surely. Therefore, by equation (A.1), $E[\tilde{Q}(p; \tilde{n}_1, \tilde{a}] = E[\tilde{Q}(p; \tilde{n}_2, \tilde{a}]$.

A second implication is that $E[u(\tilde{n}_2) \,|\, \tilde{a}] < E[u(\tilde{n}_1) \,|\, \tilde{a}]$ almost surely for any strictly concave u. Therefore, for any $q \in [0, \bar{q}]$, we have $E[f(q/\tilde{n}_2) \,|\, \tilde{a}] < E[f(q/\tilde{n}_1) \,|\, \tilde{a}]$ almost surely, and hence by equation (A.2) we have $E\left[\tilde{P}(q; \tilde{n}_2, \tilde{a})\right] < E\left[\tilde{P}(q; \tilde{n}_1, \tilde{a})\right]$ if

(a) $n \mapsto f(q/n)$ is a strictly concave function of n on an interval that contains the supports of $\tilde{n}_2$ and $\tilde{n}_1$.

To conclude the proof,we need to show that (a) is a consequence of

(b) $\hat{R}(q)$ is strictly concave on $[0, \bar{q}/n_2^{\min}]$.

Since $\tilde{n}_2$ is riskier than $\tilde{n}_1$ conditional on $\tilde{a}$, $[n_2^{\min}, \infty)$ is an interval that contains the supports of $\tilde{n}_2$ and $\tilde{n}_1$. (Recall that $n_2^{\min}$ is the greatest lower bound of the support of $\tilde{n}_2$.)

For any functional $u(x)$ on $\mathbb{R}$ and any $\lambda > 0$, u is strictly concave on $[z', z'']$ if and only if $z \mapsto u(\lambda z)$ is strictly concave on $[z'/\lambda, z''/\lambda]$. Therefore, $n \mapsto f(q/n)$

is strictly concave on $[n_2^{\min}, \infty)$ if and only if $n \mapsto f(1/n)$ is strictly concave on $[n_2^{\min}/q, \infty)$. The latter holds for all $q \in (0, \bar{q}]$ if and only if $n \mapsto f(1/n)$ is strictly concave on $[n_2^{\min}/\bar{q}, \infty)$.

The proof of Proposition 3 is then completed by using Lemma A.1. It says that $n \mapsto f(1/n)$ is strictly concave in n on $[n_2^{\min}/\bar{q}, \infty)$ if and only if $\hat{R}(q) = f(q)\, q$ is strictly concave in q on $[0, \bar{q}/n_2^{\min}]$.

LEMMA A.1. *Let $F\colon \mathbb{R}_+ \to \mathbb{R}_+$ be differentiable. Define $G(x) \equiv xF(x)$ and $H(y) \equiv F(1/y)$. Let $\bar{y} \in \mathbb{R}_{++}$. Then $H(y)$ is strictly concave on $[\bar{y}, \infty)$ if and only if G is strictly concave on $[0, 1/\bar{y}]$.*

Proof of Lemma A.1. We provide a simple proof for differentiable F by showing that $H''(y) < 0$ for all $y \in [\bar{y}, \infty)$ if and only if $G''(x) < 0$ for all $x \in [0, 1/\bar{y}]$. The lemma can also be proved algebraically without differentiability; we omit the details.[1] Observe that

$$G'(x) = xF'(x) + F(x), \qquad H'(y) = -\frac{1}{y^2}F'(1/y),$$

$$G''(x) = xF''(x) + 2F'(x), \qquad H''(y) = \frac{1}{y^4}F''(1/y) + \frac{2}{y^3}F'(1/y).$$

Observe, after canceling $1/y^3$ in the expression for $H''(y)$, that $H''(y) < 0$ is equivalent to

$$\hat{H}(y) \equiv \frac{1}{y}F''(1/y) + 2F'(1/y) < 0.$$

By substituting $x = 1/y$, we see that $\hat{H}(y) < 0$ for all $y \geq \bar{y}$ if and only if $G''(x) < 0$ for all $x \leq 1/\bar{y}$. □

Appendix B: Extension of Section 6 to nonlinear demand

Section 5 derives conditions under which an increase in uncertainty about valuation (resp., market size) favors setting quantity (resp., setting price). Section 6 characterizes those conditions for the case of linear demand and shows that they often hold. In this appendix, we extend that analysis to nonlinear demand.

For the nonlinear case, we will be less precise but will argue that the main ideas coming from the linear example are robust. Specifically, we build on the following ideas.

1. An empirical regularity is that demand becomes weakly more elastic at higher prices.
2. This implies that the "strict concavity of $\hat{R}$" and "strict concavity of $\hat{S}$" conditions hold on a range $q \in [0, \bar{q}]$ and $p \in [0, \bar{p}]$, respectively, that includes the optimal quantity and price as long as there is not too much uncertainty and the marginal cost is low enough.

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1. We thank Oleg Chuprinin and Kaifu Zhang for providing us with the algebraic proof.

3. For the effect of market-size uncertainty on the expected profit of a quantity-setting firm, higher marginal cost merely strengthens the result (relaxes the condition on the amount of uncertainty).
4. In contrast, for the effect of valuation uncertainty on the expected profit of a price-setting firm, higher marginal cost weakens the result (strengthens the condition on the amount of uncertainty).

As noted, an empirical regularity is that demand becomes weakly more elastic at higher prices. Given our parameterization of demand uncertainty, this holds for a realization of the demand curve if and only if it is a property of the canonical demand curves f and g. Assume that this property holds. Denote by e the elasticity of demand at some point on the curve. We are thus assuming that e, which is negative, becomes greater in magnitude when moving up the demand curve to higher p and lower q.

Consider first the canonical revenue curve $\hat{R}(q)$. Marginal revenue can be written as

$$\hat{R}'(q) = p\left(1 + \frac{1}{e}\right),$$

where p is the price $f(q)$ when quantity is q and e is the elasticity at the point (q, p) on the demand curves. One can use this formula to show the following.

R1. $\hat{R}$ is maximized at the point $\hat{q}^r$ at which $e = -1$.
R2. $\hat{R}$ is strictly concave on an interval $[0, \hat{q}]$, where $\hat{q} > \hat{q}^r$.
R3. $\hat{R}$ is single peaked, meaning that it is decreasing for $q > \hat{q}^r$.

Likewise, consider the canonical revenue curve $\hat{S}(p)$. Marginal revenue can be written

$$\hat{S}'(p) = q\,(1 + e)\,.$$

One can use this formula to show the following.

S1. $\hat{S}$ is maximized at the point $\hat{p}^r$ at which $e = -1$.
S2. $\hat{S}$ is strictly concave on an interval $[0, \hat{p}]$, where $\hat{p} > \hat{p}^r$.
S3. $\hat{S}$ is single peaked, meaning that it is decreasing for $p > \hat{p}^r$.

From R2 and S2, we have the following corollary to Proposition 5 and Corollary 1.

Proposition B.1.

1. *Suppose that* $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ *are such that* $\tilde{n}_2$ *is riskier than* $\tilde{n}_1$ *given* $\tilde{a}$. *Then* $P(q; \tilde{n}_2, \tilde{a}) < P(q; \tilde{n}_1, \tilde{a})$ *and* $\Pi_q(q; \tilde{n}_2, \tilde{a}) < \Pi_q(q; \tilde{n}_1, \tilde{a})$ *for* $q \in [0, \hat{q}n_2^{\min}]$.
2. *Suppose that* $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ *are such that* $\tilde{a}_2$ *is riskier than* $\tilde{a}_1$ *given* $\tilde{n}$. *Then* $Q(p; \tilde{n}, \tilde{a}_2) < Q(p; \tilde{n}, \tilde{a}_1)$ *and* $\Pi_p(p; \tilde{n}, \tilde{a}_2) < \Pi_p(p; \tilde{n}, \tilde{a}_1)$ *for* $p \in [0, \hat{p}a_2^{\min}]$.

For any realization of the inverse demand curve f, revenue—and hence profit—is a decreasing function of q on $[\hat{q}^r n, \infty)$. Therefore, $\hat{q}^r n^{\max}$ is an upper bound on quantity that can maximize expected profit. As long as $\hat{q}^r n_2^{\max} < \hat{q} n_2^{\min}$ (i.e., $n_2^{\max}/n_2^{\min} < \hat{q}/\hat{q}^r$), we can conclude that the shift in the CE inverse demand curve causes the maximum expected profit to fall.

For the effect of an increase in the uncertainty about valuation on a price-setting firm, we must be more restrictive. The problem is that the profit-maximizing price necessarily gets pushed up to the region where the revenue curve is not concave as the marginal cost rises. Otherwise, however, the argument is similar. The condition $a_2^{\max}/a_2^{\min} < \hat{p}/\hat{p}^r$ guarantees that the price that maximizes expected revenue is in the strictly concave region of the revenue curve. As long as the marginal cost is small enough, the profit-maximizing price is also in this region.

We summarize these two observations in Proposition B.2.

PROPOSITION B.2.

1. *Suppose that* $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ *are such that* $\tilde{n}_2$ *is riskier than* $\tilde{n}_1$ *given* $\tilde{a}$. *If* $n_2^{\max}/n_2^{\min} < \hat{q}/\hat{q}^r$ *then* $\Pi_q^*(\tilde{n}_2, \tilde{a}) < \Pi^*(\tilde{n}_1, \tilde{a})$.
2. *Suppose that* $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ *are such that* $\tilde{a}_2$ *is riskier than* $\tilde{a}_1$ *given* $\tilde{n}$. *If* $a_2^{\max}/a_2^{\min} < \hat{p}/\hat{p}^r$ *then there is a* $\bar{c}$ *such that* $\Pi_p^*(\tilde{n}, \tilde{a}_2) < \Pi_p^*(\tilde{n}, \tilde{a}_1)$ *for* $c \in [0, \bar{c}]$.

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Europe Campus
Boulevard de Constance,
77305 Fontainebleau Cedex, France
Tel: +33 (0)1 6072 40 00
Fax: +33 (0)1 60 74 00/01

Asia Campus
1 Ayer Rajah Avenue, Singapore 138676
Tel: +65 67 99 53 88
Fax: +65 67 99 53 99

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