PRODUCT WARRANTIES
AND DOUBLE ADVERSE SELECTION

by

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97/11/MKT

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Printed at INSEAD, Fontainebleau, France.
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Abstract

There is extensive literature analysing the use of warranties and their application as a marketing tool. Two of the most frequently cited roles for warranties are first, warranty policy can facilitate price discrimination when consumer-types are unobservable. This issue is analyzed by researchers including Kubo (1986), Matthews and Moore (1987), and Padmanabhan and Rao (1993). The use of warranties for price discrimination is analogous to the model for monopoly pricing of products of differing quality developed by Mussa and Rosen (1978). Second, warranties can signal product quality to consumers when quality is not observable. Nelson (1974) refers to products where buyers cannot evaluate the quality prior to buying as 'experience' goods. When there is variability in the quality of an 'experience' good, a long warranty can be used to signal better quality because sellers of high quality have a cost advantage over sellers of low quality in terms of offering warranty protection. This aspect of warranties is addressed by Spence (1977), Grossman (1981), Emons (1988), Lutz (1989) and Gal-Or (1989).

Frequently, a warranty does play one role or the other but there are markets in which there is a need for it to play both of these roles simultaneously. This need exists when buyers cannot observe product quality and sellers cannot identify buyers who have heterogeneous valuations for the product. Markets that meet these criteria include the used car market and the IBM-cloned personal computer market. The warranty literature has yet to consider the possibility of warranties acting as both a screen and signal simultaneously. Accordingly the objective of this study is to develop a model which examines optimal warranty policy under these conditions.

The model involves a seller who desires to sell a product and an optional extended warranty to heterogeneous consumers. Specifically, the seller chooses a base price and warranty for his product and the duration and pricing for optional extended coverage. The seller knows the quality of the product and the buyer does not. Using the terminology of Akerlof (1970), this situation of asymmetric information is described as a 'lemons market' and it occurs when there is variability in the quality of a good and buyers cannot evaluate it before buying. Sellers are assumed to know the quality of the product and this provides them with an informational advantage over the buyer. The objective is to understand how sellers set warranty menus to maximise profit when the only potential signal of quality to buyers are the prices and warranty lengths chosen by sellers.

The main results are first, a seller of high quality can use a warranty menu to
signal high quality and price discriminate simultaneously under most conditions. However, when the premium that buyers are willing to pay for quality is very high, the warranty menu cannot play both roles simultaneously. The menu continues to signal higher quality to buyers, but it loses its ability to screen. Second, when the premium for quality is above a certain level, a seller of high quality will distort the menu in order to simultaneously maximise his profit and deter sellers of low quality from mimicking. The optimal action for the seller of high quality invariably involves offering longer warranties to both types of buyers and generally charging higher prices. However, when the premium for high quality is relatively low, the menu chosen by a seller of high quality is unaffected by the existence of sellers of low quality. Finally, under conditions of two sided adverse selection, there are several equilibrium conditions that can occur in which sellers do not offer complete menus (a price/warranty bundle designed for each type of consumer). When the premium for quality is high, a seller of high quality may be forced to offer a 'collapsed' menu in which both types of buyers are offered the same bundle. In addition, there are regions in the feasible parameter space where a seller of low quality may choose to serve both types of buyers, only high valuation buyers or nobody.

Key Words: extended warranty, signalling, screening, unobservable quality, adverse selection, price discrimination, menu of contracts
1.0 Introduction

The following quote appeared in a brochure published by Compaq Computer Corp. in 1994 to assist rookies in buying a first personal computer:

"When comparing computer features, reliability is difficult to assess. But the length of the warranty is a clue to the dependability of its computers. Remember, it costs the company money to repair computers under warranty. A longer warranty period reflects the company's confidence that its products will last."

Even when consumers do not have the time or expertise to assess the quality of products, they can make useful inferences about a product's quality from the length of its warranty. Quite simply, when warranties act as signals, a longer warranty signals a better product.

On the other hand, a defining feature of buying products from major appliances to power tools is the persistent (if not aggressive) effort of a salesperson to sell some form of extended warranty protection. Clearly, retailers are making money by convincing some but not all consumers to buy extra warranty coverage. With heterogeneous consumers, a retailer can increase its profit by offering different price/warranty combinations and having buyers choose the combination that best suits them (in the economics literature, this process is called "screening").

But why are retailers so aggressive in the marketing of extended warranties? The answer lies in their tremendous profitability. According to a large Canadian retailer, margins on extended warranties range from 60 to 70 per cent. Business Week (January 14, 1991) reports that ½ of the operating profits for big consumer-electronic chains come from extended warranties and more recently, an article in the New York Times (July 23, 1995) indicates that retailers may earn as much as 75% of their gross income from the sales of extended warranty and service contracts.

These observations about the marketing of warranties underline two important facts. First, warranties (or commitments by the seller to service defective products for a specific period of time) are an increasingly important element in the marketing mix for durable products. Not only are manufacturers such as Samsung (a major electronics manufacturer) investing substantial funds ($125 million) to enhance their warranty programmes (Business Korea, Vol. 12, No. 1, July 1994), but consumers too are buying extended warranties on a wider array of goods than ever before.

Second, warranties can do a lot more than provide extra value to purchasers of

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1 Sears reports that its sales of extended warranties on durable goods exceeds $1 billion (San Francisco Chronicle, January 20, 1992).
durable products by insuring them against failure. The two roles cited above, signalling and screening, underline the capacity warranties have to play different roles in different situations.

A question that comes to mind (having cited two very different roles for warranties) is "can warranty policy play these two roles simultaneously?". Specifically, what happens in a market where the need to signal (i.e. potential consumers are uncertain about the quality of a product) and screen (i.e. consumers have different unobservable 'valuations' for a product) exist simultaneously.

Three issues are involved in providing a satisfactory answer to this general question:

1) Does the NEED for a warranty to play these roles simultaneously exist in observed markets?
2) If there are contexts where the need exists, CAN warranties screen and signal simultaneously?
3) If warranties can screen and signal simultaneously, HOW does it occur?

This paper will argue that warranties play these roles simultaneously when there is variability in the quality of products, sellers know more about the quality of products than buyers, all buyers want some degree of warranty protection and sellers cannot tell which buyers are most interested in buying warranty protection.

2.0 Background

Adverse selection in the economics literature refers to a class of problems where pre-contractual opportunism by parties possessing private information leads to inefficiency in the operation of a market. The term comes from the insurance industry where insurance companies face "adverse selection" in the sales of insurance policies. The problem is that the people most likely to purchase insurance policies are unfortunately those who are most likely to make claims. When sellers know more about the quality of products than buyers, we clearly have an adverse selection problem for buyers. However, in the conditions described in section 1.0, sellers also lack information that is important for contracting: namely, information on the preferences of buyers. Accordingly, I describe this as a condition of double adverse selection because both parties (the buyer and the seller) possess private information that has the potential to create inefficiencies.
Two markets characterized by double adverse selection are the used car market and the non-branded personal computer market.

In the used car market, extended warranties are popular because used cars frequently need repairs. In addition, there is high variability in quality. The Consumer Reports 1994 Buying Guide provides strong evidence of continued variability in automobile quality, across manufacturers, within manufacturers and even within make. Dealers usually have different qualities available because they purchase their stock through the wholesale market where quality is difficult to determine. Genesove (1993) points out the mechanics of the wholesale automobile auction provide very little time for buyers to examine the cars and reputation effects of the sellers are not strong.

In addition, dealers know more than buyers about the condition of cars they sell because they have computer systems to evaluate them. The problem for a potential purchaser is that the value of a car depends on its quality, an attribute of the product which she cannot observe. Conceivably, this problem could be quickly resolved if consumers could costlessly test cars to measure their quality (many dealers do allow potential buyers to take the cars to independent diagnostic centres for testing). The problem is that significant time and costs are associated with such testing and as a result, only a small minority of used car buyers actually pursue this option for getting better information about a car's quality.

With these conditions, there is strong motivation for a used car dealer to use warranty policy as both a screen and a signal. A recent survey of dealers in the Toronto area provides evidence that this is in fact, the case. By definition, used car dealers are "screening" because extended warranties are available on most cars but not all customers buy them (extended warranties are a useful option but they are not cheap!). In addition, substantial variance in the pricing of ostensibly similar cars and extended warranty protection is also observed (there are differences of up to 50% in the pricing of extended warranties for identical cars of the same year). This suggests that warranties and pricing are also being used to provide information to consumers about the hidden quality of the cars themselves. Of note, double adverse selection can be a problem in any secondhand market for used durable goods where different consumers desire different levels of warranty protection.

Another example of double adverse selection occurs in the market for 'IBM clone' computers. A number of manufacturers import components from the Far East and assemble personal computers in North America. When a consumer considers a purchase of a cloned computer, he/she cannot necessarily depend on a reliable brand name,
Consumer Reports or advertising. Consumers would like a credible signal that might provide them with useful information. Because it is more expensive for a firm with low quality computers to offer the same length of warranty as a firm with better quality computers, a firm with better quality computers will signal its higher quality by offering a longer warranty. At the same time, all firms (regardless of the quality of computer they are selling) have an incentive to offer an extended warranty option to extract additional profit from those purchasers who are more risk averse. Similar to the used car market, this is a situation where consumers cannot be identified a priori and consumers have the problem of not being able to observe the quality of the product in question.

These examples show that the need for simultaneous signalling and screening can exist in markets where warranties are important. The objective of this paper to provide insight about how and when warranty policy can play these roles using a game theoretic approach. A model is developed to analyze the optimal contracts offered by sellers to buyers where the contract is comprised of a price for a product with a warranty of finite duration. The seller provides the product and will repair any breakdowns that occur during the warranty period. In order to focus on the problem of simultaneous signalling and screening, I make three assumptions.

First, I assume that sellers are risk neutral. This assumption is justified because most sellers make numerous sales and this minimizes their aversion to the risk associated with an individual transaction. Second, I assume that producer moral hazard is insignificant. In general, for the markets discussed, the dealer has little ability to affect the performance of the product after it has been sold. Purchaser moral hazard is also assumed to be minimal since deductibles and maintenance programmes can be used to induce careful behaviour on the part of buyers.

Prior to presenting the model and its results, I will briefly review the relevant literature on warranties.

3.0 Background Literature

There is a rich literature in both economics and marketing that relates to the marketing of warranties on durable goods. The objective of this section is to review two subsets of this literature: first, the literature that analyses the use of warranties for price discrimination and second, the literature that considers the use of warranties to signal

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3 Producer moral hazard occurs when the performance of a product is affected by the care of the seller after purchase and that care is both costly to the seller and unobservable to the buyer. In contrast, purchaser moral hazard occurs when the performance of the product is affected by the care of the purchaser after purchase and that care is both costly to the purchaser and unobservable to the seller. A number of papers have looked at warranties within this context including Cooper and Ross (1985) and Dybvig and Lutz (1993).
quality to potential buyers.

Warranties to Price Discriminate or Screen

In situations where a firm has market power, it can extract additional surplus from the consumer by including a repair warranty with the product. This is the basic bundling result as discussed by Tirole (1990). An extension to this idea is that a warranty can be used to screen when consumers have heterogeneous valuations for the product. The warranty allows the seller to price discriminate based on these valuations; length of warranty is essentially a proxy for quantity in a typical second degree price discrimination model (consumers who have higher valuations for a product buy more of it but at a lower cost per unit). Since 1986, several papers have appeared which discuss this use of warranties. Kubo (1986), for example, shows how a monopolist can increase its profits through the use of an optional quality guarantee when consumers are heterogeneous (the basis for consumer heterogeneity in this paper is income).

A more complicated version of this problem is studied by Matthews and Moore (1987). The objective of the monopolist is similar to that in Kubo’s paper but the monopolist is assumed to have three decision variables: price, quality (which is fully observable), and warranty level, for each alternative they offer to consumers instead of two (price and warranty level). The main contribution of the paper is the characterization of an optimal screening menu where every bundle in the menu is a unique combination of price, quality and warranty coverage and showing how this menu relates to the risk preferences of consumers.

Padmanabhan and Rao (1993) show how customer heterogeneity can arise from risk tolerances which vary across consumers. Given this heterogeneity, sellers can increase their profits by offering a menu of price/warranty bundles. In addition, Padmanabhan and Rao provide a theoretical basis and empirical evidence for purchaser moral hazard in the context of extended service contracts.

Warranties as Signals

Signalling is important when one agent to a contract is unfamiliar with the quality of the other agent (or his product) and that agent’s quality cannot be observed prior to

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4 If a firm has market power in a homogeneous market, its ability to bundle products and warranties is unaffected by the presence of a competitive repair market. However, when markets are heterogeneous, the monopoly’s ability to price discriminate with warranties is affected. For more discussion of this issue see Hollis (1994).
contracting. In the context of durable goods, the principal (the buyer) selects an agent (the seller) to perform a task (provide him/her with a durable good) but cannot observe the characteristics of the good before purchase. As noted by Bergen, Dutta and Walker (1992), this problem (which is also known as a problem of ‘Hidden Information’) applies to many situations in marketing. Rothschild and Stiglitz (1976) analyze one of the most famous examples, the insurance market in which the principal (the insurance company) cannot distinguish between high and low risk customers.

With many durable products, quality cannot be evaluated prior to purchase and it becomes evident only after prolonged use. Nelson (1974) refers to these products as “experience goods”. Akerlof (1970) underlines how unobservable attributes can interfere with the operation of markets. He predicts that only poor quality cars (lemons) will be traded in the used car market for the following reason. Sellers have perfect information about quality, consumers have poor information about quality and both have similar valuations for quality. Simplifying Akerlof’s analysis somewhat, only poor quality products are traded because buyers are willing to pay for the expected quality of products being offered and sellers only offer products which have a value less than the market price.

Signalling can be used to mitigate the problem noted by Akerlof. When sellers of high quality have cost advantages over sellers of low quality in making signals then signalling can facilitate the exchange of higher quality products. The use of warranties as signals is analyzed by both Spence (1977) and Grossman (1981). Spence models the warranty as a source of insurance for the consumer and assumes that a competitive market will attempt to set price, quality, and the rate of liability in order to maximize the consumer’s utility (subject to a zero profit constraint). With risk averse consumers, Spence finds that the amount of coverage offered by the manufacturer is a perfect signal of quality. This result occurs as a by-product in Spence’s model due to profit maximising behaviour by firms and the requirement that, in equilibrium, consumers’ beliefs about quality be consistent with the bundles actually offered by firms.

Gal-Or (1989) examines the signalling roles of warranties and identifies the problem of multiple equilibria when two differentiated manufacturers make decisions about both the length of warranty and the quantity of product to produce. Multiple equilibria occur when the products are either close substitutes or far apart in their attributes and this limits the signalling capability of the warranty. Multiple equilibria do not arise in the model developed in this paper primarily because the products are less complex (given a warranty length, a high quality product is simply better than a low

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5 There is no desire by the firms in Spence’s model to use the warranties as a signal. It occurs endogenously as a result of variegated consumer preferences.
quality product).

4.0 Overview of the model

As noted previously, the objective of this analysis is to identify the optimal contracts that a seller of durable goods (with warranty coverage) will offer to a buyer under conditions of double adverse selection. To develop the mathematical formulation for the game, we begin by considering the key elements that make up the model: the buyers, the sellers, the informational assumptions and the order of play (or the extensive form of the game).

Buyers

The utility of the buyer is represented by the following quasi-linear utility function where \( x \) is the length of the warranty:

\[
U = \theta V(x) - P \quad \text{if the buyer purchases at price } P
\]

\[
U = 0 \quad \text{if buyer doesn't purchase}
\]

\( V(x) \) is a value function where \( V(0) = 0, V'(x) > 0, \) and \( V''(x) < 0. \) These are standard conditions which imply decreasing marginal utility for warranty length. The taste parameter \( \theta \) is assumed to vary across buyers. Heterogeneous preference for products can arise from differences in income\(^6\) [Shaked and Sutton (1982)] or differences in risk aversion as shown in Appendix A.

The total value \( \theta V(x) \) is obtained by a type \( i \) buyer from owning a product sold with a warranty of length \( x \). Thus, buyers who have a higher \( \theta \) place a higher value on a product sold with a warranty of length \( x \) than buyers with a lower \( \theta \).

The market is assumed to be comprised of two types of buyers who place different values on a product with warranty of length \( x \): high valuation buyers with \( \theta = \theta_h \) and low valuation buyers with \( \theta = \theta_i \) and \( \theta_h > \theta_i \). I assume that a fraction '\( \lambda \)' of the market is low valuation buyers and the fraction '1-\( \lambda \)' is high valuation buyers. There are two types of products on the market: a high quality product and a low quality product. The value of a higher quality product is captured by the premium '\( z \)' which is incorporated into the

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\(^6\) Identical preferences over goods and heterogeneous incomes imply taste parameters that are the inverse of the marginal utility of income when the amount of money spent on the good is relatively small (compared to total income).
utility function for both types of buyers as follows:

\[ U = (0 + z) V(x) - P \]  

(3)

This function implies that for any warranty length \( x \), a higher quality product is more valuable than a low quality product (to both types of buyers). In addition, warranty coverage of length \( x \) is more valuable for high quality products than for low quality products. The basis for this assumption is that the 'value' from owning a higher quality product flows from more than just superior reliability (i.e. higher quality implies superior performance, not just superior reliability or lower repair costs). The warranty is worth more to the owner of a high quality product because the warranty is a guarantee of worry free use of that product (a failed product is always repaired and is repaired more quickly)\(^7\).

I will assume that:

\[ V(x) = \frac{1 - (1-x)^2}{2} \]  

(4)

This utility function exhibits the required properties of \( V(0) = 0 \), \( V'(x) > 0 \), and \( V''(x) < 0 \) for \( x \in [0,1] \). This function exhibits satiation at \( x = 1 \) (i.e. values of \( x > 1 \) are not observed)\(^8\). Since virtually all durable goods have a finite life, a warranty must similarly have a limited life beyond which it is of little value to buyers\(^9\).

**Sellers**

The seller’s profit function is a function of the relative distribution of types in the market and profit that is made from sales to each type. A key assumption is that the proportion of high and low valuation buyers who patronize any seller are the same as their relative distribution in the population. For purposes of simplification, the marginal

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\(^7\) In some markets, warranties may be worth more to owners of low quality products when their principal value is as insurance against repair payments (which, for low quality products, are greater). In this situation, as long as the single crossing property holds for consumers and high quality products are worth more than low quality ones (normalizing for warranty coverage), simultaneous signalling and screening (similar to the model discussed in this paper) occurs. A model and results which incorporate this alternate assumption are available from the author on request.

\(^8\) A solution using the value function: \( V(x) = 1 - \exp(-x) \) (which does not exhibit satiation) has been derived and is available from the author on request.

\(^9\) Any appropriate satiation limit can be incorporated into the model by suitably scaling the parameters. The limit could be expected life of the product, the expected timing of innovations which will make the product obsolete, or the number of years left in the consumer's life.
cost of the product is assumed to be zero\textsuperscript{10}.

A seller’s expected profits will be:

$$\pi_q = (1-\lambda)(P_h - c_q x_h) + \lambda(P_l - c_q x_l)$$  \hfill (5)

where \((P_h, x_h)\) and \((P_l, x_l)\) are the ‘price/warranty’ bundles purchased by the high and low valuation buyers respectively and \(c_q\) is the cost per unit time of providing warranty coverage for a seller of quality ‘\(q\)’ (\(q=h, l\)). This formulation assumes that the seller’s of providing warranty protection are a linear function of warranty length\textsuperscript{11}. The warranty length \(x_i\) can be thought of as the base warranty and \((x_h - x_l)\) is the length of the extended warranty which can be purchased for \((P_h - P_l)\). To facilitate computations, it is easier to treat the two options available to consumers (the car without an extended warranty, the car with an extended warranty) as two points \((P_l, x_l)\) and \((P_h, x_h)\) in price/warranty length space.

It is assumed that \(c_h < c_l\) which means that a seller of a high quality product can provide warranty coverage more cheaply than the seller of a low quality product. This is reasonable given that higher quality products are assumed to fail less frequently. Basically, \(c_h\) applies to products which receive the premium ‘\(z\)’ in buyers’ valuations; \(c_l\) applies to products which do not receive the premium.

An additional assumption ensures that both high and low valuation buyers participate in the market under all three distributions of information:

\(c_h < (\theta_l - \theta_h + \lambda(\theta_h + z))/\lambda\) (this ensures that the seller of a high quality product has an incentive to serve both high and low valuation buyers\textsuperscript{12}). If this assumption is violated, the market is homogeneous (only high valuation consumers buy) and we are left with a simple problem of one dimensional signalling.

**Informational Assumptions**

The sellers’ cost structures, both types of buyers’ utility functions, and the value of the premium are assumed to be common knowledge. Double adverse selection implies that a) the buyer cannot tell a priori whether the seller is offering high or low quality and b) the seller cannot tell a priori whether the buyer has a low or high valuation for the

\textsuperscript{10}This assumption does not affect the results. Because quantity does not vary in this model, production cost does not affect the first order conditions.

\textsuperscript{11}A linear cost function can be endogenously generated by an exponential distribution of failure coupled with instantaneous repair (see Appendix A).

\textsuperscript{12}The basis for this assumption can be found in section 6. Proposition 4 as applied to a seller of high quality ensures that the market is heterogeneous.
product. In order for the buyer to figure out whether the seller is offering high or low quality, the buyer forms beliefs about a) what a high quality seller will do and b) what a low quality seller will do. The Cho-Kreps criterion (1987) is used to find a set of beliefs (for buyers) that are reasonable. Formally, it is buyers' beliefs about seller strategies (using their knowledge of the seller's cost functions) that allow the derivation of unique separating equilibria.

**Extensive Form of the Game**

Although the game is modelled as a simultaneous single shot game in which the players choose optimal strategies, there is an implied order of play:

**Stage 1.** The seller chooses a menu of bundles and will announce them to any buyer who has interest in the particular product that the seller is offering.

**Stage 2.** Buyers arrive randomly at the seller's place of business. The proportion of each type of buyer arriving at the seller's place of business is the same as in the population.

**Stage 3.** Buyers decide whether to purchase any of the bundles that the seller has available.

Allowing the seller to move first implies that he has price setting ability and will attract buyers as long as they receive positive utility. This is a reasonable assumption in a market where the cost of finding a second product that meets the buyer's criteria is significant.\(^{13}\)

**Solution Procedure**

I examine equilibrium contracts under three different informational assumptions. The first is the contracts that would be observed under full information (neither screening nor signalling is necessary). The second is the contracts that occur when sellers cannot observe buyer types but buyers can observe product quality (here screening is important but not signalling). Finally, I will derive the contracts that would be observed when sellers cannot observe buyer types and buyers cannot observe product quality (both

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\(^{13}\) Diamond (1971) notes that a market which has subsequent search costs greater than zero results in monopoly-like pricing.
screening and signalling are important). This procedure will allow us to examine how different levels of information asymmetry (first on the part of sellers and second on the part of both sellers and buyers) affect the contracts that are observed in the market.

5.0 Equilibrium Contracts when Buyer Type and Product Quality is Observable

The preceding overview provides a framework to analyze the equilibrium that results with full information. It will be a basis for comparison in terms of understanding how different informational assumptions affect market outcomes. Two observations are critical for describing the full information equilibrium:

1) Since sellers know a priori whether a buyer is high or low valuation, a seller can (if he wishes) quote a different price/warranty bundle to different types. The seller is assumed to have market power: as long as the price and warranty bundle offered by the seller yields positive surplus to the buyer, she will buy it.

2) The buyer’s valuation of a given price/warranty bundle depends on the quality of the product that she observes a priori. Thus, the seller knows (even before choosing a warranty length) the value that the buyer will place on his product with a given warranty length.

The problem for the low quality dealer under full information can be represented as follows:

Maximize \( \pi_{ip} = (1-\lambda)[P_h - c_r x_h] + \lambda[P_I - x_r] \)

subject to:

\[ \theta_h V(x_h) - P_h \geq 0 \]  
\[ \theta_I V(x_I) - P_I \geq 0 \]  
\[ 0 \leq x_h \leq 1 \]  
\[ 0 \leq x_I \leq 1 \]
If we assume the feasibility conditions are not binding, the solution for this problem is straightforward (and is omitted for brevity). In equilibrium, the low quality seller sets the warranty length for each type of buyer at the 'welfare maximising' warranty length. He uses price to extract all of the surplus from the buyer. When both types of buyers participate in the market, both equations 7 and 8 are binding.

The problem for the high quality seller under full information is similar to the problem for the low quality seller (with \( c_i \) replaced by \( c_h \), \( \theta_i \) replaced by \( \theta_h + z \), and \( \theta_h \) replaced by \( \theta_h + z \)). The explicit solutions for both types of sellers are shown in Table 1.

### Table 1
Equilibrium Warranty Lengths and Prices under Full Information

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<thead>
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<th>High Quality Seller (HQS)</th>
<th>Low Quality Seller (LQS)</th>
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<tr>
<td>( x_h = 1 - \frac{c_h}{\theta_h + z} )</td>
<td>( x_h = 1 - \frac{c_l}{\theta_h} )</td>
<td></td>
</tr>
<tr>
<td>( x_l = 1 - \frac{c_h}{(\theta_l + z)^2} )</td>
<td>( x_l = 1 - \frac{c_l}{\theta_l} )</td>
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<tr>
<td>( P_l = \frac{\theta_l + z}{2} \left[ 1 - \frac{c_l^2}{(\theta_l + z)^2} \right] )</td>
<td>( P_l = \frac{\theta_l}{2} \left[ 1 - \frac{c_l^2}{\theta_l^2} \right] )</td>
<td></td>
</tr>
<tr>
<td>( P_h = \frac{\theta_h + z}{2} \left[ 1 - \frac{c_h^2}{(\theta_h + z)^2} \right] )</td>
<td>( P_h = \frac{\theta_h}{2} \left[ 1 - \frac{c_i^2}{\theta_i^2} \right] )</td>
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The feasibility conditions are never binding for a high quality seller in this problem because of the assumption noted in section 4.0. However, the feasibility conditions may
be binding for a low quality seller if his cost of providing warranty coverage \( c_1 \) is high enough.

**PROPOSITION 1.** Under full information, the low quality seller does not participate in the market if \( c_1 > \theta_h \).

**PROOF:** The expressions given for both \( x_h \) and \( x_i \) in Table 1 for a seller of low quality yield negative lengths if \( c_1 > \theta_h \). Thus, when \( c_1 > \theta_h \) constraints 9 and 10 are binding and the seller of low quality effectively drops out of the market.

**PROPOSITION 2.** Under full information, the low quality seller serves only high valuation consumers if \( \theta_i < c_i < \theta_h \).

**PROOF:** In Table 1, the expression for \( x_i \) for a seller of low quality yields a negative length when \( c_i > \theta_i \). Thus, when \( c_i > \theta_i \) constraint 10 is binding and a seller of low quality serves only high valuation buyers.

Under full information two key results should be underlined. First, Propositions 1 and 2 imply that a seller of low quality products may not serve all types of buyers: his equilibrium menu is a function of his cost structure. The intuition behind this finding is that a seller of any quality must provide some minimal level of warranty coverage to participate in the market\(^{14}\); if the cost of providing even a minimal level of coverage is too high, the seller will cease operations. Second, the optimal menu for sellers of high or low quality products involves setting the warranty length for each type of consumer such that the marginal value of warranty length for each type of consumer equals the marginal cost of providing that coverage.

**6.0 Equilibrium Contracts when Quality is Observable and Buyer Type is Not**

We now proceed to examine the equilibrium that results when seller cannot observe buyer type but buyers can observe product quality.\(^{15}\) The problem is similar to the one presented in the previous section; however, when buyer types are unobservable, the bundles designed for each type of buyer must meet incentive compatibility constraints: the seller cannot quote a price designed for a specific buyer.

\(^{14}\) This is a reasonable assumption given that all merchants are bound by implied warranties under the Uniform Commercial Code (the Sale of Goods Act in Canada).

\(^{15}\) When quality is observable, a seller of low quality cannot pretend to be a seller of high quality.
type; he must design a ‘menu’ where each type of buyer chooses the bundle designed for her. When quality is observable but buyer types are not, the problem for a seller of low quality is analogous to the problem described in section 5.0 (equations 6-11) subject to the following two incentive compatibility constraints:

\[ \theta_h V(x_h) - P_h \geq \theta_i V(x_i) - P_i \]  
\[ \theta_i V(x_i) - P_i \geq \theta_h V(x_h) - P_h \]  

The solution for this optimization problem is contained in Appendix B.

The problem for the seller of high quality under observability is similar to the solution for the low quality seller (with \( c_i \) replaced by \( c_h \), \( \theta_i \) replaced by \( \theta_i + z \), and \( \theta_h \) replaced by \( \theta_h + z \)). The explicit solutions for both types of sellers are shown in Table 2.

**Table 2**

Equilibrium Warranty Lengths and Prices when Quality is Observable and Buyer Types are not

<table>
<thead>
<tr>
<th>High Quality Seller (HQS)</th>
<th>Low Quality Seller (LQS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ x_h = 1 - \frac{c_h}{\theta_h + z} ]</td>
<td>[ x_h = 1 - \frac{c_i}{\theta_h} ]</td>
</tr>
<tr>
<td>[ x_i = 1 - \frac{\lambda c_h}{\theta_i - \theta_h + \lambda (\theta_h + z)} ]</td>
<td>[ x_i = 1 - \frac{\lambda c_i}{\theta_i + (\lambda - 1)\theta_h} ]</td>
</tr>
<tr>
<td>[ P_i = \frac{\theta_i + z}{2} \left[ 1 - \frac{(\lambda c)^2}{(\theta_i - \theta_h + \lambda (\theta_h + z))^2} \right] ]</td>
<td>[ P_i = \frac{\theta_i}{2} \left[ 1 - \frac{(\lambda c_i)^2}{(\theta_i + (\lambda - 1)\theta_h)^2} \right] ]</td>
</tr>
<tr>
<td>[ P_h = \frac{\theta_i + z}{2} \left[ \frac{\lambda c_h^2}{(\theta_i - \theta_h + \lambda (\theta_h + z))^2} - \frac{c_h^2}{\theta_h^2} \right] ]</td>
<td>[ P_h = \frac{\theta_i}{2} \left[ \frac{(\lambda c_i)^2}{(\theta_i + (\lambda - 1)\theta_h)^2} - \frac{c_i^2}{\theta_h^2} \right] ]</td>
</tr>
</tbody>
</table>
Similar to the full information case, the feasibility conditions are never binding for a high quality seller because of the assumption noted in section 6.0. However, the feasibility conditions may be binding for a low quality seller if his cost of providing warranty coverage $'c_l'$ is high enough.

**PROPOSITION 3.** When buyer types are not observable, a seller of low quality seller does not participate in the market if $c_l > \theta_h$.

**PROPOSITION 4.** The seller of low quality will serve both types of buyers if and only if $\theta_h > c_l$ and $\theta_l > (1-\lambda)\theta_h + \lambda c_l$.

**PROOF OF PROPOSITIONS 3 AND 4:** see Appendix C.

When $\theta_h > c_l$ but $\theta_l < (1-\lambda)\theta_h + \lambda c_l$, the low quality seller only serves the high valuation buyers. The reason is that when $\theta_l < (1-\lambda)\theta_h + \lambda c_l$, the proportion of low valuation buyers is so low that the seller can make more profit by selling to high valuation buyers only. When the seller serves only high valuation buyers, he can extract all the surplus associated with their consumption of the product. However, when he serves both types of buyers, he is forced to leave high valuation buyers with positive surplus in order to meet the incentive compatibility constraint (the seller must price the bundle intended for the high valuation buyer low enough so that the high valuation buyer does not ‘defect’ and purchase the bundle intended for the low valuation buyer). Thus, if the proportion of low valuation buyers is sufficiently low, when both types of buyers are served, the profit that can be extracted from low valuation buyers does not compensate for the lost surplus due to the reduction in price for the high valuation buyers. In sum, Propositions 3 and 4 imply that the cost of providing warranty coverage determines whether a seller of low quality serves both types of buyers, only high valuation buyers or nobody.

In summary, when quality is observable and buyer types are not, the equilibrium obtained is similar to the full information case with two key differences:

1) When buyer types are unobservable and both types of buyers are served, a high valuation buyer always pays a price lower than the full information price. This is evident by comparing the expressions for $P_i$ and $P_h$ in Table 2 with those in Table 1. The reduction in price for the low valuation buyer occurs because the warranty coverage in the bundle designed for her in Table 2 is shorter than the coverage she receives under full information (compare the values of $x_l$ in Table 2 with those of Table 1). In contrast, the lower price paid by the high valuation buyer occurs because the seller must ensure that the bundle designed for him is incentive
compatible. That is the seller must reduce the price of the bundle for the high valuation buyer so that she does not defect to the bundle designed for the low valuation buyer.

2) When \( c_i \) lies in the interval \( (\theta_h, [\theta_l - (1-\lambda)\theta_h]/\lambda) \) and buyer types are unobservable, low valuation buyers are not served. This is in stark contrast with the equilibrium under full information. When the seller cannot identify buyer types, he reduces the price on the bundle designed for high valuation buyer (as noted in point 1). This reduction is not necessary if the seller chooses not to serve low valuation buyers. If \( c_i \) lies in the interval \( (\theta_l, [\theta_l - (1-\lambda)\theta_h]/\lambda) \), low valuation buyers do not offer ‘enough profit’ to justify a reduction in the price charged to high valuation consumers.

Consider a market in which sellers of both high and low quality have an incentive to sell to both types of buyers (ie. \( \theta_i > (1-\lambda)\theta_h + \lambda c_i \)). When buyer types are not observable but the quality of the product is, the seller of high quality offers strictly longer and more expensive bundles than the seller of low quality to each type of purchaser\(^{16}\). Figure 1 illustrates how the menu offered by the seller of high quality lies to the northeast of the menu offered by the seller of low quality. The dashed lines represent reservation utility indifference curves for low valuation buyers and the curved solid lines are indifference curves for high valuation buyers. The indifference curves for high quality lie above the indifference curves for low quality reflecting the premium ‘\( z \)’ which applies to high quality products. The high valuation buyer receives the surplus maximising bundle (ie. this is reflected by the tangency of the isoprofit lines and indifference curves at \( P_h \)) regardless of whether she finds herself dealing with a seller of high or low quality. On the other hand, the low valuation buyer receives a bundle which lies on his reservation utility indifference curve but which is \textit{strictly shorter} than his surplus maximising bundle. Its exact location is determined by the relative distribution of high and low quality buyers in the population (when the proportion of low valuation buyers is higher, the bundle is longer and less distorted).

In Figure 1, both of the bundles offered by the high quality seller lie above the isoprofit curves of the low quality seller.

(Figure 1)

When product quality is not observable, buyers may be unwilling to accept the menu

\(^{16}\) This can be shown easily by subtracting column 2 from column 1 in Table 2.
offered by the seller of high quality shown in Figure 1 because there is nothing to stop a seller of low quality from masquerading as a seller of high quality.

7.0 Equilibrium Contracts when Buyer Type and Product Quality are not Observable

In order to identify the equilibrium contracts that would be observed when product quality and buyer type are unobservable, I need to find menus that would be offered by each type of seller where neither seller has an incentive to deviate and pretend to be the other. In addition, when players lack information that they require to make decisions, they form beliefs about this information\(^{17}\). Following Harsanyi (1967), any reasonable equilibrium that is proposed in such a situation must involve players maximizing utility (or profit) and players' actions must be consistent with the beliefs that they have formed. In signalling games such as this, the Bayesian equilibrium concept does not generate unique solutions because it does not allow sufficient restriction on the buyer's belief set. However, the Cho-Kreps (1987) criterion places a restriction on the beliefs of buyers such that a seller of high quality will choose but one set of profit maximizing bundles. Formally, a Perfect Bayesian Equilibrium violates the intuitive criterion if there exists an action which will yield one player strictly greater payoffs given that the beliefs of players in the game do not ascribe positive probability to any player for whom the said action is equilibrium dominated\(^{18}\).

It is self evident that a seller of high quality will not mimic a seller of low quality (in fact, the optimal menu for a seller of low quality represents a lower limit in terms of profit for a seller of high quality). The seller of low quality, on the other hand, may have a strong motivation to mimic the seller of high quality because buyers are willing to pay more for the high quality seller's products. Assume that buyers assign a belief of probability \(\phi\) that a given menu will be offered by a seller of high quality and a belief of \(1-\phi\) that it is offered by a seller of low quality. The Cho-Kreps criterion allows us to fix \(\phi=1\) for any menu observed that would generate less profit for the seller of low quality than the menu that the seller of low quality would offer when quality is

\(^{17}\) These beliefs can be based on prior information or on prior information that has been updated based on any observations that a player has made since the beginning of the game. Bayesian equilibria, in which actions and beliefs (of players) are specified, are used to derive fixed points in games of incomplete information.

\(^{18}\) In any Bayesian equilibrium, the belief set of the players as well as their actions must be specified. When there are two types of players, invoking the Cho-Kreps criteria is equivalent to restricting attention to equilibria with reasonable beliefs. In this context, reasonable beliefs means that an equilibrium cannot be justified using beliefs that ascribe positive probability to an action by a type for whom the action would make him strictly worse off.
observable\textsuperscript{19}.

Therefore, without loss of generality, any feasible separating menu for the seller of high quality must be a strategy for which $\phi = 1$. This is reflected implicitly in the high quality seller’s optimization problem as a ‘no mimic’ constraint. This constraint obliges the seller of high quality to choose a contract which is unattractive to the seller of low quality\textsuperscript{20}. When the ‘no mimic’ constraint is binding, a seller of high quality chooses a menu to which the seller of low quality is just indifferent between mimicking and not mimicking\textsuperscript{21}.

To simplify the notation, $P_h$, $P_1$, $x_h$, and $x_1$ are the prices and warranty lengths that are chosen by the seller of high quality when quality is not observable. The problem for a seller is analogous to the problem described in section 6.0 subject to the previously discussed no-mimic constraint:

$$\pi_{\text{lp}} \max > (1-\lambda)(P_h - c_P h) + \lambda(P_1 - c_P 1)$$  \hspace{1cm} (30)

where the maximum $\pi_{\text{lp}}$ is the value shown in Appendix B: equation 33\textsuperscript{22}. With several simplifications (that are analogous to the simplifications used in Appendix B), this problem can be rewritten as a Kuhn Tucker problem\textsuperscript{23}:

\textsuperscript{19} In this context, it is reasonable to assume that consumers believe that quality is low ($\Phi = 0$) unless they observe a contract which could not feasibly be offered by a low quality dealer. In the strategy space defined by $\Phi = 0$, the optimal menu for the low quality dealer is the one he would offer when quality is observable.

\textsuperscript{20} The optimal profit that a seller of low quality obtains when quality is observable (but not buyer type) is an important benchmark: it represents the minimum profit necessary for a low quality seller to consider changing his action.

\textsuperscript{21} In the model as specified, I will show that the 'no mimic' constraint is not binding for sufficiently low premium levels.

\textsuperscript{22} Equation 33 applies when a seller of low quality finds it profitable to serve both high and low valuation buyers. The appropriate expression for $\pi_{\text{lp}}$ when only high valuation buyers are served is:

$$\pi_{\text{lp}} = \frac{(1-\lambda)\theta_h}{2} \left(1 - \frac{c^2_1}{\theta^2_h}\right)$$

When a seller of low quality serves neither type of buyer (i.e. $c_i > \theta_h$), the no-mimic constraint applies with $\pi_{\text{lp}} = 0$.

\textsuperscript{23} Once again, we ignore the feasibility conditions because in most cases they are not binding. When the upper limit on $x$ is binding ($x_h$ or $x_1$ equals 1), the relevant Lagrangean multiplier is negative unless the shadow price on the no mimic constraint is one (proof available from author). As will be shown later, this can only occur when $z$ exceeds a value $z_{\max}$ implying that the high quality dealer’s menu has collapsed.
This objective function is maximised with respect to \( x_l, x_i, P_h, P_l, \mu_1, \mu_2, \mu_3, \) and \( \mu_4 \) and the solution is contained in Appendix D.

The length of warranty coverage for the low valuation buyer \((x_i)\) is obtained by solving equation 19 in Appendix D. The equations for \( x_h, P_h, \) and \( P_l \) are determined by substituting back into the expressions that have been derived for each variable in terms of \( x_l \).

This solution has several interesting properties. First, when the added value of a high quality product, reflected by the premium \('z'\) lies in the range \((0, z^*)\), the menu chosen by a seller of high quality is unaffected by the presence of sellers of low quality.

PROPOSITION 5: When \( z < z^* \) and quality is not observable, the menu chosen by a seller of high quality is unaffected. He offers the same menu to the market that he would offer if product quality were observable.

PROOF: see Appendix E.

The intuition behind this result is that in the region of low premium levels \((0, z^*)\), the cost advantage of the seller of high quality products (i.e., the degree to which \( c_h \) is less than \( c_l \)) overshadows the premium that buyers are willing to pay for high quality. Basically, the optimal menu chosen by a seller of high quality would be expensive for the low quality seller to mimic (because \( c_l \) is significantly higher than \( c_h \)) but not sufficiently attractive (because \( z \) is not sufficiently high). Above \( z^* \), the premium that buyers are willing to pay for high quality overshadows the cost advantage possessed by sellers of high quality. The menu that would be chosen by the seller of high quality (in the absence of low quality) is sufficiently attractive to a seller of low quality to be mimicked. Thus, a seller of high quality makes the menu just unattractive enough to a seller of low quality (by increasing the warranty lengths) so that the seller of low quality is indifferent between mimicking and not mimicking. It should be noted that when the seller of high quality needs to increase the warranty lengths to meet the no-mimic constraint (i.e., the premium exceeds \( z^* \)), the profit of a seller of high quality is reduced because of the existence of sellers who offer low quality (he would be better off if quality were observable).

A second interesting property is that for any value of the premium \( z \), the seller of high quality offers bundles to each type which are more expensive and have longer
warranties.

PROPOSITION 6: A seller of high quality offers a bundle to each type of buyer which is more expensive and longer than that offered by a seller of low quality for all levels of the premium ‘z’.

PROOF: see Appendix F.

The intuition behind this result is that the premium z and cost advantage associated with a seller of high quality have similar effects on observed warranty lengths. Referring back to Figure 1, a premium ‘z’ means that at any warranty length x, the slope of the indifference curve for the high quality product is greater (for both types of buyers). Therefore, the point of tangency between the isoprofit line for a seller of high quality and the indifference curve for the high valuation buyer will occur further to the right (or at a longer warranty length). Similarly, the warranty length in the low valuation buyer’s bundle will also be longer. A lower cost c_q makes the isoprofit lines less steep and this also moves the same point of tangency to the right. Thus, when a seller of high quality is unaffected by the existence of sellers of low quality products, his bundles for each type will be strictly longer. When a seller of high quality is affected by the existence of sellers of low quality (ie. z > z*), the effect is to lengthen all warranties in his menu to make the menu ‘unaffordable’ to a seller of low quality. Consequently, as long as the high quality seller has a cost advantage in terms of repair (ie. c_h < c_l), he will always offer a longer warranty to each type.

In this model, to obtain a separating equilibrium in which high quality sellers offer two distinct bundles to the market, there is an upper limit on the premium z. The upper limit is greater than z* and is caused by the saturation property of buyers’ utility functions.

PROPOSITION 7: When quality is unobservable and z ≥ z_{max}, a seller of high quality offers one bundle to the market which is purchased by both high and low valuation buyers. z_{max} is given by the following equation:

\[ z_{max} = 2(\pi_{ip} + c_l) - \theta_i \]

PROOF: see Appendix G.

The intuition behind this result is that when z > z_{max}, both types of buyers more than willing to pay \( \pi_{ip} + c_l \) for the product with a warranty of 1 (\( \pi_{ip} \) is given by equation 33 in Appendix B). A seller of high quality is prevented from charging more for his menu.
because of the 'no mimic' constraint (a high price would make his menu attractive to a seller of low quality seller). The individual rationality constraints hold for neither buyer type (both the low valuation buyer and the high valuation buyer obtain positive surplus buying a product with a warranty of length 1 and price of $\pi_{\text{inc}}+c_1$). This contrasts with the menu that is chosen when product quality is observable or when product quality is unobservable and $z$ falls in the range $(0, z_{\text{max}})$. In these cases, the high valuation buyer always receives positive surplus and the low valuation buyer's individual rationality constraint is always binding (and hence she receives zero surplus). When $z > z_{\text{max}}$, the adverse selection problem not only distorts the high quality seller's menu, it also leaves the low valuation buyer strictly better off.

Given $\lambda$, $\theta_h$, $\theta_l$, $z$, $c_h$ and $c_l$, it is possible to describe the equilibrium that will be observed in a market subject to double adverse selection using the information provided in propositions 3-7. Given that $c_h$ is low enough such that it is feasible for a seller of high quality products to serve both types of buyers (we have assumed throughout this paper, $c_h < (\theta_l - \theta_h + \lambda(\theta_h + z))/\lambda$), the potential equilibrium can be represented as 9 distinct areas (or regimes) in cost-premium space (see Figure 2).

(Figure 2)

The legend in Figure 2 outlines the mathematical conditions that characterize each of the nine regimes: $z^*$ and $z_{\text{max}}$ are calculated as per the formulae in this paper (which are valid for all combinations of parameter values). Effectively, the observed regime is a function of the levels of $c_1$ and $z$ (relative to $c_h$, $\lambda$, $\theta_l$, and $\theta_h$).

In all market conditions, a seller of high quality must ensure that his menu is unattractive to a seller of low quality (ie. the menu offered by a seller of high quality seller must satisfy a no-mimic constraint). Several of the regimes where the low quality seller offers an incomplete menu are quite interesting. Regimes 2 and 3 represent situations in which a seller of low quality does not participate in the market, but his potential to enter the market disciplines a seller of high quality and forces him to offer menus which are less profitable and signal 'higher quality' to buyers. The mere "threat of entry" affects the behaviour of a seller of high quality.

Figure 2 underlines that one could observe one firm offering only one bundle and the other offering two bundles (regimes 4, 5 and 9), but one cannot infer from the completeness of the menu whether the seller is high or low quality. Only by examining the length of warranties and prices can a buyer infer the quality of the seller.

Propositions 5-7 imply that there are essentially three regions from the perspective of a seller of high quality. In the first region where the premium for high quality ranges from zero to some level ($z^*$), the presence of sellers who offer low quality does not affect
the menu offered by a seller of high quality; in the second region, when the premium for high quality lies between $z^*$ and $z_{\text{max}}$, the high quality seller’s menu is distorted by the existence of the low quality seller (the bundles are more expensive, longer and closer together than the bundles that would be observed under observable conditions); in the third region, where the premium for high quality exceeds $z_{\text{max}}$, the menu chosen by a seller of high quality is distorted and collapsed by the presence of sellers who offer low quality (a seller of high quality offers one bundle to both types of consumers). Thus, when adverse selection is a problem and the premium for quality is high, we should not expect to observe extended warranties. Perhaps that is why certain ‘premium quality’ products (like Swiss Army Knives and Craftsman tools) are sold with lifetime warranties$^{24}$. This finding suggests that adverse selection has a maximum effect when large premiums are associated with high quality.

8.0 Numerical Example

The following numerical example is included to underline the effect that double adverse selection has on the prices, the magnitude of warranty lengths and the length of extended warranties chosen by sellers of high quality$^{25}$. The following parameter values are utilized:

$$\theta_n=15, \theta_f=13.5, c_f=7, c_n=3, \lambda=.19$$

First let us consider how prices charged by sellers of high quality are affected by the ‘non-observability’ of quality. Figures 3 and 4 show the equilibrium prices that are charged by a seller of high quality to both types of consumers when quality is observable and when it is not.

(Figures 3 and 4)

The figures show that below $z^*$, the prices charged by a seller of high quality are unaffected by the existence of sellers of low quality. However, when the premium exceeds $z^*$ but is less than some value greater than $z_{\text{max}}$, distortion due to double adverse selection results in higher prices than when quality is observable. In this region, a seller of high quality charges a higher price to the both buyer types (consistent with the longer

$^{24}$ Lifetime warranties are not infinite because they are limited to the lifetime of the purchaser.

$^{25}$ In equilibrium, the actions of a seller of low quality are unaffected by the premium ‘$z$’ and thus, are the same regardless of whether quality is observable or not.
warranty which both buyers are offered). Once \( z > z_{\text{max}} \), the price charged by a seller of high quality does not depend on \( z \) and both types of buyers pay the same price \((8.73)^{26}\). Beyond \( z_{\text{max}} \), the seller eventually charges prices which are less than the prices observed when quality is observable. In summary, the effect of unobservable quality on the prices in a screening menu for a high quality product can be make to them higher or lower depending on the level of the premium ‘\( z \)’.

Second, let us examine the lengths of warranties offered by sellers of high quality. Figures 5 and 6 show the equilibrium warranty lengths \((x_h, x_l)\) that are chosen by a seller of high quality when quality is observable and when it is not.

(Figures 5 and 6)

For both types of consumers, the effect of unobservable quality on warranty lengths is either zero \((z < z^*)\) or to increase them \((z > z^*)\). A seller of high quality makes his menu unattractive to a seller of low quality (ie. meets the signalling requirement) by increasing \( x_h \) and \( x_l \). In contrast to prices, the effect of unobservable quality on warranty lengths is either zero or to increase them.

Finally, we consider the effect that unobservable quality has on the length of extended warranties offered by sellers of high quality. In Figure 7, the lengths of optimal extended warranties are shown for a seller of high quality under conditions of observable and unobservable quality.

(Figure 7)

When \( z < z^* \), the effect of unobservable quality on extended warranty lengths is zero. However, when \( z > z^* \), the effect of unobservable quality is to make the length of extended warranties shorter. As noted in the previous section, once \( z > z_{\text{max}} \) extended warranties are unavailable and both types of buyers buy the product at the same price with maximum warranty protection. Thus, in markets with problems of double adverse selection, we should expect to observe sellers of high quality offering longer base warranties but shorter extended warranties.

A final issue explored numerically is the relation of \( z^* \) to the ratio of costs of providing warranty protection for low quality versus high quality cars \((c_l/c_h)\). Figure 8

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26 Above \( z_{\text{max}} \), the binding ‘no mimic’ constraint (equation 30) dictates the price that the seller of high quality can charge. The premium \( z \) does not appear in equation 44 and this explains why the price a seller of high quality charges is independent of \( z \) when \( z > z_{\text{max}} \).
shows that $z^*$ is a monotonic increasing function of $c_i/c_h$.

(Figure 8)

The implication of this finding is that the greater the cost advantage for a seller of high quality in providing warranty coverage (versus a seller of low quality), the less likely it is that he will need to distort his menu to distinguish himself from a seller of low quality.

9.0 Conclusion

The objective of this paper has been to provide insight into the diverse warranty policy that is used to support the sales of durable goods in markets characterized by double adverse selection. As noted in the introduction, double adverse selection exists when a product or service is (to some degree) an experience good, sellers have an advantage over buyers in terms of identifying quality and buyers have unobservable preferences for the product.

The key insight of the paper is that warranties can be used to screen and signal simultaneously when sellers have price setting ability. The reason that warranties can play this dual role is that "warranty length" is a positive attribute (of the product) which can be metered and is cheaper for a seller of high quality to provide.

The important findings of the paper are as follows. First, a seller of high quality always needs to account for the existence of low quality sellers in designing his warranty policy. When the premium for higher quality is relatively low (i.e. less than the cutoff point $z^*$), the menu chosen by a seller of high quality is the same regardless of whether quality is observable or not (when $z < z^*$, a seller of low quality has no incentive to mimic the warranty policy of a seller of high quality). In this situation, a high quality seller offers the "high type" buyer an extended warranty which provides an "efficient length" of warranty coverage. The "low type" buyer on the other hand buys the product with no extended coverage (she receives only the standard base warranty) and this provides her with less than efficient warranty coverage.

However, when $z > z^*$, a seller of high quality alters his warranty policy to account for the existence of low quality sellers. The seller of high quality strategically makes his warranty policy "more expensive" for the low quality seller to mimic. The

\[ z^* \text{ is a monotonic increasing function of } c_i/c_h. \]

\[ (\text{Figure 8}) \]

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27 In most used durable goods markets, consumers must make non-negligible investments in time and transportation to find a product with the features they want (colour, age, options). Therefore, once a consumer finds a product she wants, the seller has a degree of price setting ability.

28 The efficient length of warranty coverage for the each type of buyer is the length at which the marginal benefit to the buyer is equal to the marginal cost of providing the coverage.
optimal action for the high quality seller entails both types of buyers purchasing more warranty coverage than they would in the absence of low quality products. This is achieved by offering a longer base warranty and a shorter extended warranty. Although optimal extended warranties are shorter when \( z > z^* \), the net length of warranty coverage purchased (ie. base warranty plus extended warranty) by the "high type" buyer is longer. This "lengthening" of warranty coverage by sellers of high quality is costly but necessary in order for warranty policy to be an effective signal. Not surprisingly, a recent article highlights the efforts of sellers in second hand markets "to rein in fast talkers who promise coddled Cadillacs but deliver clunkers". The high cost of signalling with extended warranties provides an incentive for sellers of high quality to find cheaper signalling alternatives (for example, certification).

A second important finding is that the ability of warranty policy to play a dual role is impaired when the premium for high quality is overly high. Specifically, when the premium for high quality exceeds the cutoff point \( z_{\text{max}} \) \( (z_{\text{max}} > z^* > 0) \), the signalling requirements on the seller are so severe that he cannot screen buyers (ie. he will not offer an extended warranty). In this situation, the optimal action for a seller of high quality is to offer maximum warranty length to both types of buyers (the product is sold with a base warranty which lasts for the expected life of the product).

Finally, there are several equilibrium conditions that can occur in which sellers do not offer a different warranty length for each type of consumer. As noted above, when the premium for quality is high, a seller of high quality may be forced to offer a 'collapsed' menu in which both types of buyers get the same deal. In addition, there are regions where a seller of low quality may choose to serve both types of buyers, only high valuation buyers or nobody. It should be noted that a seller of high quality is constrained by the existence of low quality sellers and not their equilibrium actions (for example, if the premium for quality is high, a seller of low quality could conceivably not serve anybody in equilibrium and nonetheless cause a seller of high quality to offer a collapsed menu).

It is important to mention that the relevance of this paper is broader than simply explaining the warranty policy offered with used durable goods. As indicated earlier, the reason that warranty policy can be used to screen and signal simultaneously is that it is a positive attribute which can be metered. There are many markets where a) the buying situation is characterized by double adverse selection and b) buyers desire an attribute which can be metered. Several notable examples include the service contracts offered by firms on industrial equipment, service contracts offered to home owners on heating or air conditioning systems, different coverage plans offered by health maintenance

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organizations (HMO's) in the United States and different redemption provisions on financial assets of unknown riskiness. In all of these cases, the seller of the product service knows more about the quality of the product (or service) than the buyer. In addition, the seller can offer variable amounts of positive attribute (degree/extent of service contracts, the size of the HMO's network and the number of services provided, or the penalty associated with early redemption of an asset) and these attributes are generally cheaper for a high quality seller to provide. An interesting empirical extension to this paper would be to examine the question of simultaneous signalling and screening in a broad context across several markets with double adverse selection.

A second extension would be to analyze the welfare implications of double adverse selection. At premium levels marginally greater than $z^*$. The high quality seller offers a base warranty which is closer to the efficient length for the "low type" buyer than the base warranty offered in the absence of low quality sellers (this is welfare enhancing). However, the increased warranty length purchased by the "high type" buyer moves that buyer beyond his efficient length (this is welfare decreasing). It is difficult to tell whether the net effect of these two changes is welfare enhancing or welfare decreasing. When the effect of double adverse selection is welfare decreasing, it might be in the interest of society to ban or 'identify' sellers of low quality.

An assumption of the model that warrants discussion is that of setting the marginal (production) costs for both high and low quality products to zero. Clearly, sellers of used durable products do not obtain their inventory at zero cost; however, once a seller has a product to sell, the marginal cost of production (or procurement) does not affect the simultaneous screening and signalling problem examined in this paper (production cost does not affect the first order conditions for warranty length). However, if sellers must make procurement decisions prior to announcing their warranty policy, differences in marginal cost may affect the selection of products purchased by a seller. A two stage game would be needed to analyze this situation. Nonetheless, this paper provides insights into markets in which screening and signalling occur simultaneously. These insights are as relevant to the second stage of a more complex game as they are to the solution of a simpler game (like the one studied here) in which product (dealer) quality is exogenous.

\[\text{In the first stage, sellers would make a decision about which quality to sell. Assuming high and low quality sellers exist in equilibrium, the second stage would be analogous to the situation described in this paper.}\]
Appendix A

Risk Aversion as a Basis for Heterogeneity in θ

Assumptions

1) Failure occurs according to an exponential distribution ie.

\[ f(t) = \lambda e^{-\lambda t} \]  

This results in a constant hazard rate of \( \lambda \).

2) The cost to repair a broken product is \( r \) and this is constant over time.

3) Products are repaired instantaneously. Thus, for the average product, the expected cost of repairs over a time interval \((0,x)\) is a function of the likelihood of one failure plus the likelihood of two failures plus the likelihood of three failures and so on. This can be represented mathematically as follows:

\[ C(x) = r \int_0^x \int_{t_1}^x \int_{t_2}^x \cdots \lambda dt_3 \lambda dt_2 \lambda dt_1 \]  

where \( t_1 \) is the time until the first failure, \( t_2 \) is the time to the second failure, and so on. When \( f(t) \) is exponential \( C(x) \) can be approximated by:

\[ C(x) = r\lambda x \]  

ie. the cost of repairs can be represented as a linear function of time.

4) A risk averse consumer will pay the expected cost of repairs plus a premium for warranty coverage during any interval.

5) The premium will be larger for consumers who are more risk averse.
Appendix A continued

Let $U(x, \sigma, P)$ be the utility a consumer has for a product priced at $P$ with a warranty of length ‘$x$’, where $\sigma$ is a measure of the consumer’s risk aversion. If ‘$a$’ is a constant equal to the consumer’s valuation for the product without warranty protection and ‘$b$’ is a constant equal to $r\lambda$ (as per equation 3) and then assumptions 1-5 imply that:

$$U(x, \sigma, P) = a + bx + Pr(x, \sigma) - P \quad (4)$$

where $Pr(x, \sigma)$ is the premium. Consider a special case of this formulation:

$$Pr(x, \sigma) = \sigma g(x) \quad (5)$$

where $g'(x)>0$, $g''(x)<0$

Then equation 4 can be rewritten as:

$$U(x, \sigma, P) = a \left( \frac{a+bx}{\sigma} + g(x) \right) - P$$

The utility function $U(x, \theta, P)$:

$$U(x, \theta, P) = \theta V(x) - P \quad (7)$$

where $V(0)=0$, $V'(x)>0$, $V''(x)<0$

is a special case of equation 6 where:

$$\frac{a}{\sigma} + g(0) = 0 \quad (8)$$
Appendix B

Solution for Low Quality Seller's Menu under Observability

In this solution, $c$ is understood to be $c_i$, the cost of providing warranty coverage for the low quality seller. The objective function is:

$$\pi_{w} = (1-\lambda)[P_h - cx_h] + \lambda[P_i - cx_i]$$

subject to:

$$\theta_h V(x_h) - P_h \geq \theta_h V(x_i) - P_i$$

$$\theta_i V(x_i) - P_i \geq \theta_i V(x_h) - P_h$$

$$\theta_h V(x_h) - P_h \geq 0$$

$$\theta_i V(x_i) - P_i \geq 0$$

$$0 \leq x_h \leq 1$$

$$0 \leq x_i \leq 1$$

$$P_h, P_i \geq 0$$

To simplify the mathematics, constraints 6, 7 and 8 will be omitted from the derivation (ranges for $\theta_h$ and $\theta_i$ have been specified to ensure that these constraints are satisfied).

This problem can be simplified to the same problem excluding constraint 4. The logic is as follows:

if constraint 5 holds then

$$\theta_h V(x_h) - P_i > 0 \ since \ \theta_h > \theta_i$$
Appendix B continued

Therefore constraint 2 implies that:

\[ \theta_h V(x_h) - P_h > 0 \]  \(\text{so we can drop constraint 3}\)  \(\text{(10)}\)

Rewriting this problem as a Kuhn-Tucker problem I obtain the following objective function:

\[
\mathcal{L} = (1-\lambda)[P_h - cx_h] + \lambda[P_i - cx_i] + \mu_1(\theta_h V(x_h) - P_h - \theta_2 V(x_i) + P_i) + \mu_2(\theta_i V(x_i) - P_i - \theta_1 V(x_h) + P_h)
\]

which generates the following first order conditions for \(x_h, x_i, P_h\) and \(P_i\):

\[ \frac{\delta \mathcal{L}}{\delta x_h} = (1-\lambda)(-\epsilon) + \mu_1 \theta_h V'(x_h) \leq 0 \]

\[ \text{with complementary slackness condition } \frac{\delta \mathcal{L}}{\delta x_h} x_h = 0 \]  \(\text{(12)}\)

\[ \frac{\delta \mathcal{L}}{\delta x_i} = \lambda(-\epsilon) - \mu_1 \theta_i V'(x_i) + \mu_2 \theta_i V'(x_i) + \mu_3 \theta_i V'(x_i) \leq 0 \]

\[ \text{with complementary slackness condition } \frac{\delta \mathcal{L}}{\delta x_i} x_i = 0 \]  \(\text{(13)}\)

\[ \frac{\delta \mathcal{L}}{\delta P_h} = 1 - \lambda - \mu_1 + \mu_2 \leq 0 \]

\[ \text{with complementary slackness condition } \frac{\delta \mathcal{L}}{\delta P_h} P_h = 0 \]  \(\text{(14)}\)

\[ \frac{\delta \mathcal{L}}{\delta P_i} = \lambda + \mu_1 - \mu_2 - \mu_3 \leq 0 \]

\[ \text{with complementary slackness condition } \frac{\delta \mathcal{L}}{\delta P_i} P_i = 0 \]  \(\text{(15)}\)

condition 6 implies \(\mu_1 > 0\) since:

\[ \theta_h V' = \theta_h (1-x) > 0 \text{ for the allowable range of } x_h. \]
Appendix B continued

Assuming non-zero prices, I add conditions 14 and 15 to obtain:

\[ 1 - \lambda - \mu_1 + \mu_2 + \lambda + \mu_1 - \mu_2 - \mu_3 = 0 \]  
\[ \therefore \mu_3 = 1 \]  

The next step in this solution is to prove that equation 3 is not binding (ie. \( \mu_2 = 0 \)).

First, add equations 2 and 3:

\[ \theta_h V(x_h) - \theta_i V(x_i) \geq \theta_i V(x_i) - \theta_i V(x_j) \]  

Evaluating the buyer utility function at \( x \), I know that:

\[ \theta_i V(x) = \theta_i \left[ \frac{1 - (1-x)^2}{2} \right] \]  

Substituting into equation 18, I obtain:

\[ (\theta_h - \theta_i) \left[ \frac{1 - (1-x_h)^2}{2} - \frac{1 - (1-x_i)^2}{2} \right] \geq 0 \]  
\[ (\theta_h - \theta_i) \left[ \frac{(1-x)^2 - (1-x_h)^2}{2} \right] \geq 0 \]  

Because \( \theta_h - \theta_i > 0 \), equation 21 implies that \( x_h \) is greater than or equal to \( x_i \). I now rewrite constraint 1:

\[ P_h - P_i \geq \theta_h V(x_h) - \theta_i V(x_i) \]  
\[ \therefore P_h - P_i \geq \theta_h \int_{x_i}^{x_h} V'(x) \, dx \]  

Since:

\[ V'(x) = 1 - x \]  
\[ \therefore P_h - P_i \geq \theta_h \int_{x_i}^{x_h} 1 - x \, dx \]
Appendix B continued

Clearly:

\[ P_h - P_l > \theta_1 \int_{x_l}^{x_h} V'(x) \, dx \quad \text{(strictly)} \]  \hspace{1cm} (24)

\[ \therefore P_h - P_l > \theta_1 V(x_h) - \theta_1 V(x_l) \]  \hspace{1cm} (25)

Rewriting:

\[ V(x_h) - P_l > V(x_l) - P_h \quad \text{(strictly)} \]  \hspace{1cm} (26)

Thus equation 3 is not binding and \( \mu_2 = 0 \). Equation 12 implies that \( \mu_1 = 1 - \lambda \). Equations 10 and 11 can be rewritten, substituting for the determined values of \( \mu_1, \mu_2, \) and \( \mu_3 \):

\[ 0 = (1-\lambda)(-c) + (1-\frac{\lambda}{\theta_1})V'(x_h) \]  \hspace{1cm} (27)

\[ 0 = \lambda(-c) - (1-\lambda)\theta_x V'(x_h) + \theta_1 V'(x_l) \]  \hspace{1cm} (28)

If I substitute for the marginal utility function \( V'(x) = 1-x \), equation 27 implies the following:

\[ \theta_1 V'(x_h) = c \]  \hspace{1cm} (29)

\[ \therefore x_h = 1 - \frac{c}{\theta_1} \]

Note that the high valuation buyer consumes the socially optimal bundle and \( x_h > 0 \) because \( \theta_1 \geq c \) by assumption. Similar substitutions yield the following expression for \( x_l, x_l \geq 0 \) because \( \theta_l + (\lambda-1)\theta_h > \lambda c > 0 \) by assumption (ie. I assume that sellers of both high and low quality have incentive to serve both types of buyers).

\[ x_l = 1 - \frac{\lambda c}{\theta_l + (\lambda-1)\theta_h} \]  \hspace{1cm} (30)

35
Appendix B continued

To determine $P_h$ and $P_l$, I use equations 2 and 5 which are binding.

\[
P_l = \frac{\theta_l}{2} \left[ 1 - \frac{(\lambda c)^2}{(\theta_l + (\lambda - 1)\theta_h)^2} \right]
\]

(31)

\[
P_h = \frac{\theta_h}{2} \left[ \frac{(\lambda c)^2}{(\theta_l + (\lambda - 1)\theta_h)^2} - \frac{c^2}{\theta_h^2} \right] + \frac{\theta_l}{2} \left[ 1 - \frac{(\lambda c)^2}{\theta_h^2} \right]
\]

(32)

The profit for the low quality seller is obtained by substituting the values for $P_h$, $P_l$, $x_h$, and $x_l$ into the objective function (equation 1):

\[
\pi_{lpr} = (1-\lambda) \frac{\theta_h}{2} \left[ \frac{(\lambda c)^2}{(\theta_l + (\lambda - 1)\theta_h)^2} - \frac{c^2}{\theta_h^2} \right] + \frac{\theta_l}{2} \left[ 1 - \frac{(\lambda c)^2}{(\theta_l + (\lambda - 1)\theta_h)^2} \right]
\]

\[
-\lambda c \left( \frac{1 - \frac{c}{\theta_h}}{\theta_l + (\lambda - 1)\theta_h} \right) - (1-\lambda) (1 - \frac{c}{\theta_h}) - \lambda (1 - \frac{\lambda c}{\theta_l + (\lambda - 1)\theta_h})
\]

(33)
Appendix C

Proof of Proposition 3

When $c_{i} > \theta_{i}$, the marginal value of warranty length (associated with a low quality product) for both types of buyers is less than $c_{i}$ for all feasible $x$. Therefore, when $c_{i} > \theta_{i}$, there are no saleable bundles that would yield positive profit for a seller of low quality.

Proof of Proposition 4

If $\theta_{i} < (1-\lambda)\theta_{h} + \lambda c_{i}$, any solution which meets the incentive compatibility and individual rationality constraints, violates the feasibility constraint (equation 7 in Appendix B). This can be shown by substituting a value lower than $(1-\lambda)\theta_{h} + \lambda c_{i}$ into equation 30 in Appendix B. Therefore, when $\theta_{i} < (1-\lambda)\theta_{h} + \lambda c_{i}$, to serve both types, the best the seller can do is to offer both types of buyers the socially efficient bundle for the low valuation buyer. Therefore, when $\theta_{i} < (1-\lambda)\theta_{h} + \lambda c_{i}$, I must show that a seller of low quality strictly prefers to serve only the high valuation buyer.

Assume $\pi_{\text{both}} > \pi_{\text{high only}}$ then $[P_{i} - c_{i}x_{i}] > (1-\lambda)[P_{h} - c_{h}x_{h}]$

where $(P_{q}x_{q})$ is the socially efficient bundle for type $q$

substituting the appropriate values for $P_{h}$, $P_{i}$, $x_{h}$ and $x_{i}$, I obtain the following:

$$\theta_{i} - \frac{1}{2} \frac{c_{i}^{2}}{\theta_{i}} - c_{i} \left[ 1 - \frac{c_{i}}{\theta_{i}} \right] > (1-\lambda) \theta_{h} - \frac{1}{2} \frac{c_{i}^{2}}{\theta_{h}} - c_{i} (1-\lambda) \left[ 1 - \frac{c_{i}}{\theta_{h}} \right]$$

therefore:

$$\left( 1 - \frac{c_{i}}{\theta_{i}} \right) \frac{\theta_{i}}{2} \left( 1 + \frac{c_{i}}{\theta_{i}} \right) - c_{i} > (1-\lambda) \left( 1 - \frac{c_{i}}{\theta_{h}} \right) \frac{\theta_{h}}{2} \left( 1 + \frac{c_{i}}{\theta_{h}} \right) - c_{i}$$

$$= \left( 1 - \frac{c_{i}}{\theta_{i}} \right) \frac{\theta_{i}}{2} \left( 1 + \frac{c_{i}}{\theta_{i}} \right) - c_{i} > (1-\lambda) \left( 1 - \frac{c_{i}}{\theta_{h}} \right) \frac{\theta_{h}}{2} \left( 1 + \frac{c_{i}}{\theta_{h}} \right) - c_{i} \text{ since } \theta_{h} > \theta_{i}$$
Appendix C continued

which simplifies to:

\[ \frac{\theta_i}{2} - \frac{c_i}{2} > (1-\lambda) \left( \frac{\theta_h}{2} - \frac{c_i}{2} \right) \]

therefore:

\[ \theta_i - c_i > \theta_h - c_i - \lambda \theta_h + \lambda c_i \]

\[ \Rightarrow \theta_i > (1-\lambda) \theta_h + \lambda c_i \]

*but* \( \theta_i < (1-\lambda) \theta_h + \lambda c_i \) *by assumption*

therefore:

\[ \Leftrightarrow Q.E.D. \]
Appendix D

The first order conditions for the objective function (eq. 15) with respect to \( P_h \), \( P_i \), \( x_h \), and \( x_i \) are:

\[
\frac{\delta \mathcal{L}}{\delta x_h} = (1-\lambda)(-c_h) + \mu_1(\theta_h+z)V'(x_h) - \mu_2(\theta_i+z)V'(x_i) + \mu_4(1-\lambda)c_i \leq 0
\]

with complementary slackness condition \( \frac{\delta \mathcal{L}}{\delta x_h} x_h = 0 \) (1)

\[
\frac{\delta \mathcal{L}}{\delta x_i} = \lambda(-c_h) - \mu_1(\theta_h+z)V'(x_h) + \mu_2(\theta_i+z)V'(x_i) + \mu_4(1-\lambda)c_i \leq 0
\]

with complementary slackness condition \( \frac{\delta \mathcal{L}}{\delta x_i} x_i = 0 \) (2)

\[
\frac{\delta \mathcal{L}}{\delta P_h} = 1 - \lambda - \mu_1 + \mu_2 - \mu_4(1-\lambda) \leq 0
\]

with complementary slackness condition \( \frac{\delta \mathcal{L}}{\delta P_h} P_h = 0 \) (3)

\[
\frac{\delta \mathcal{L}}{\delta P_i} = \lambda + \mu_1 - \mu_2 - \mu_3 - \mu_4 \lambda \leq 0
\]

with complementary slackness condition \( \frac{\delta \mathcal{L}}{\delta P_i} P_i = 0 \) (4)

and if \( \mu_4 > 0 \) then:

\[
\pi^\text{max}_{\text{lag}} = (1-\lambda)[P_h-c_p] + \lambda[P_i-c_p]
\]

Assuming non-zero prices, I add conditions 3 and 4 to obtain:

\[
1-\mu_3-\mu_4 = 0 \Rightarrow \mu_4 = 1-\mu_3
\]

as with all Kuhn Tucker problems, the Lagrangean multipliers are restricted to non-negative values. Therefore, equation 6 implies that \( \mu_4 \in [0,1] \). I now rewrite equation 3:

\[
((1-\mu_4)(1-\lambda) + \mu_2) - \mu_4 = 0
\]

The term in braces is clearly non-negative, therefore \( \mu_1 \geq 0 \). Suppose \( \mu_4 = 0 \), then \( \mu_3 = 1 \) and equation 4, implies that \( \mu_1 > 0 \) strictly. Suppose \( \mu_2 = 1 \), then \( \mu_3 = 0 \) and equation 4 implies:
Appendix D continued

\[ -\mu_1 + \mu_2 = 0 \quad (8) \]

therefore, either \( \mu_1 > 0 \) or \( \mu_2 = 0 \). If \( \mu_4 = 1 \) and \( \mu_1 = 0 \), then \( \mu_2 = \mu_3 = 0 \), and I have a degenerate solution where a seller of high quality offers one bundle to everyone and leaves both types of buyers with positive surplus. This may seem implausible but in this model, it actually occurs when the premium ‘z’ is high enough. When satiation is reached for both types of buyers \((x_h = x_i = 1)\), constraint 4 binds and the shadow price is 1. (If the profit restriction on a seller of high quality is relaxed by one dollar, he charges a price one dollar higher to both types of buyers and gains one dollar in profit).

With the exception of this degenerate solution, I can say that \( \mu_1 > 0 \). I now prove that \( \mu_2 = 0 \). I add the incentive compatibility constraints in the optimization problem to obtain:

\[
(\theta_h + z) V(x_h) - P_h + (\theta_i + z) V(x_i) - P_i \geq (\theta_h + z) V(x_h) - P_h + (\theta_i + z) V(x_h) - P_h
\]

\[ \therefore V(x_h)(\theta_h - \theta_i) = V(x_i)(\theta_h - \theta_i) \]

\[ \therefore (V(x_h) - V(x_i))(\theta_h - \theta_i) \geq 0 \]

since \( \theta_h - \theta_i > 0 \) then \( V(x_h) - V(x_i) > 0 \) \( \therefore x_h > x_i \)

since \( \mu_1 > 0 \), the incentive compatibility constraint for the high valuation buyer holds with strict equality:

\[
(\theta_h + z) V(x_h) - P_h = (\theta_h + z) V(x_h) - P_i
\]

\[ \therefore (\theta_h + z)(V(x_h) - V(x_i)) = P_h - P_i \quad (10) \]

clearly \( (\theta_i + z)(V(x_h) - V(x_i)) < P_h - P_i \) strictly

\[ \therefore (\theta_i + z)(V(x_h) - P_i) > (\theta_i + z)(V(x_i) - P_h) \text{ so } \mu_2 = 0 \]

If \( \mu_2 = 0 \), equation 4 implies:

\[ \mu_1 = (1-\lambda)\mu_3 \quad (11) \]
Appendix D continued

Assuming a non-zero $x_h$, I substitute into equation 1 to obtain an explicit solution for $\mu_3$:

$$\mu_3 = \frac{c_h - c_i}{(\theta_h + z)(1-x_h) - c_i} \quad (12)$$

Substitute into equation 11 to obtain an explicit solution for $\mu_4$:

$$\mu_4 = \frac{(1-\lambda)(c_h - c_i)}{(\theta_h + z)(1-x_h) - c_i} \quad (13)$$

and equation 6 to obtain an explicit solution for $\mu_4$:

$$\mu_4 = \frac{(\theta_h + z)(1-x_h) - c_h}{(\theta_h + z)(1-x_h) - c_i} \quad (14)$$

Before proceeding to the explicit solution for prices and lengths, the Lagrangean multipliers provide insight regarding the limits of this solution. Equation 6 implies that for $\mu_4$ to be non-negative, $\mu_4 \geq 0$. When $\mu_3 = 1$, equation 12 implies:

$$c_h = (\theta_h + z)(1-x_h) \quad (15)$$

This equality holds at the socially optimal length of warranty for the high type consumer. When $x_h$ is greater than this socially optimal length, $0 < \mu_3 < 1$ and $\mu_4 > 0$. However, for lengths less than the socially optimal length, $\mu_3 > 1$, and $\mu_4 < 0$. This situation clearly violates the non-negative restriction placed on Lagrangean multipliers.

The explanation for this is that when $x_h$ as derived in this solution is less than the socially optimal length, the no-mimic constraint (equation 14 in the article) is no longer binding. In this situation, the profit a seller of low quality makes by mimicking a seller of high quality is strictly less than the profit associated with his best bundle in the set defined by buyer beliefs ($\phi = 0$).

For any given set of parameter values $\theta_h$, $\theta_i$, $\lambda$, $c_i$, and $c_h$, there is range of $z$ ($0, z^*$) for which equation 20 is not binding. For this range of $z$, the presence of sellers of low quality causes no distortion in the high quality seller's menu.

Assuming a non-zero $x_i$, I substitute the values for $\mu_1$, $\mu_2$, $\mu_3$, and $\mu_4$ into equation 2 to obtain the following identity:

$$x_h = 1 - \frac{(1-x_h)[(\theta_h + z)(\theta_h + z)]}{\lambda(\theta_h + z)} \quad (16)$$
Using the binding constraints (the incentive compatibility constraint for the high valuation buyer and the individual rationality constraint for the low valuation buyer), the following expressions can be derived for $P_h$ and $P_l$:

$$P_l = (\theta_l + z) \frac{1-(1-x_l)^2}{2}$$  \hspace{1cm} (17)$$

$$P_h = (\theta_h + z) \frac{1-(1-x_h)^2}{2} - (\theta_h - \theta_l) \frac{1-(1-x_l)^2}{2}$$  \hspace{1cm} (18)$$

Equations 16, 17, and 18 are substituted into equation 5 to obtain the following quadratic equation in $x_l$:

$$0 = \left[ \frac{1-\lambda}{2} \frac{\tilde{\theta}^2}{\lambda^2(\theta_h + z)} - \frac{\tilde{\theta}}{2} \right] (1-x_l)^2 +$$

$$\left[ \frac{1-\lambda}{\lambda} c_l \frac{\tilde{\theta}}{\theta_h + z} + c_l \lambda \right] (1-x_l) + \left[ \frac{\theta_l + z}{2} - c_l \right] \pi_{\max}$$  \hspace{1cm} (19)$$

where:

$$\tilde{\theta} = \theta_l - \theta_h + \lambda(\theta_h + z)$$  \hspace{1cm} (20)$$

$\pi_{\max}$ is given by equation 30 (Appendix A)

The premium value below which the presence of the low quality has no effect on the menu chosen by a seller of high quality is obtained by finding the value of $z$ for which equation 19 holds when $x_l$ is set to the value that would be chosen (by the high quality seller) when quality is observable.

$$x_l = 1 - \frac{\lambda c_h}{\theta_l - \theta_h + \lambda(\theta_h + z)}$$  \hspace{1cm} (21)$$

$$\therefore 1-x_l = \frac{\lambda c_h}{\theta_l - \theta_h + \lambda(\theta_h + z)}$$
Appendix D continued

substituting into equation 19, the following cubic equation in $z$ is obtained:

\[
0 = \frac{1-\lambda}{2} \frac{c_h^2}{\hat{\theta}(\theta_h+z)} + \frac{\lambda^2 c_h^2}{2\hat{\theta}} \frac{(1-\lambda)c_h}{(\theta_h+z)}
\]

\[-\lambda^2 c_h \frac{c_h}{\hat{\theta}} - c_l + \frac{\theta_h+z}{2} - \eta_{\max}
\]

where:

\[
\hat{\theta} = \theta_l - \theta_h + \lambda(\theta_h+z)
\]

$z^*$ can be obtained by approximation using this equation.

Appendix E

Proof Of Proposition 5

As shown in Appendix D, when $z=z^*$, $x_h$ is the same regardless of whether quality is observable or not. When quality is not observable and equation 14 holds with strict equality, equation 16 in Appendix D implies that $x_h^*$ (the derivative of $x_h$ with respect to $z$) is positive for all feasible $x_h$ (proof available from the author). Therefore, if equation 14 holds when $z < z^*$, $x_h$ is less than the $x_h$ that would be chosen when quality is observable. In this situation, equations 12 and 15 (both in Appendix D) imply that $\mu_3 > 1$ which means that $\mu_3$ is negative (and not binding). If the optimization problem is recast ignoring the 'no mimic' constraint then the solution is the menu of bundles chosen when quality is observable as shown in Table 2.
Appendix F

Proof of Proposition 6

When \( z < z^* \), the menu of bundles chosen by a seller of high quality is the same as when quality is observable. Comparison of the values in Table 2 across seller type confirms the proposition for \( z < z^* \). For \( z > z^* \), because \( x_h' \) (with respect to \( z \)) is greater than zero in the allowable region \( x_h \) is greater than the \( x_h \) that would be observed when quality is observable. The same is implied for \( x_l \) by equation 16 in Appendix D. Equation 17 in Appendix D implies that for the high quality seller, \( P_1 \) is greater than it would be were quality observable (Thus, \( P_1 \) is also greater than the \( P_1 \) that one would observe in the menu chosen by a seller of low quality). Equations 17 and 18 in Appendix D can be used to show the same thing for \( P_h \). Thus, the proposition holds for all values of \( z > 0 \).

Appendix G

Proof of Proposition 7

When \( z = z_{\text{max}} \), equation 19 in Appendix D implies that \( x_l = 1 \). Therefore, equation 16 in Appendix D implies that \( x_h = 1 \). In this situation, substitution of \( z_{\text{max}} \), \( x_h = 1 \) and \( x_l = 1 \) into equations 16 and 19 will demonstrate that all constraints hold with strict equality. For values of \( z > z_{\text{max}} \), \( \mu_1 = \mu_2 = \mu_3 = 0 \) (ie. the individual rationality and incentive compatibility constraints do not hold) and the solution is one in which a seller of high quality offers only one bundle to the market with a warranty length of 1 and a price of:

\[
P = \pi_{\text{opt}} + c_i
\]
Fig. 1

Equilibrium Menus
Quality is Observable and Buyer Type is Unobservable
Figure 2
Potential Market Equilibria

assume that both types of consumers buy ie. \( c_h < \left( \theta_h - \theta_b + \lambda (\theta_h + z) \right) / \lambda \)

Table showing Conditions for Different Regimes that might be Observed

<table>
<thead>
<tr>
<th>Actions of Seller offering a high quality product</th>
<th>Actions of Seller offering a low quality product</th>
</tr>
</thead>
<tbody>
<tr>
<td>offers same bundles as if quality were observable</td>
<td>offers complete menu but bundles are longer to meet signalling requirements</td>
</tr>
<tr>
<td>offers no bundles to buyers</td>
<td>serves high valuation buyers only</td>
</tr>
<tr>
<td>1. ( c_i &gt; \theta_h ) ( z \in (0,z') )</td>
<td>4. ( c_i &lt; \theta_h ) ( c_i &gt; (\theta_r(1-\lambda)\theta_b) / \lambda ) ( z \in (0,z') )</td>
</tr>
<tr>
<td>2. ( c_i &gt; \theta_h ) ( z \in (z',z_{\text{max}}) )</td>
<td>5. ( c_i &lt; \theta_h ) ( \theta_i &gt; (\theta_r(1-\lambda)\theta_b) / \lambda ) ( z \in (z',z_{\text{max}}) )</td>
</tr>
<tr>
<td>3. ( c_i &gt; \theta_b ) ( z &gt; z_{\text{max}} )</td>
<td>6. ( c_i &lt; \theta_h ) ( c_i &gt; (\theta_r(1-\lambda)\theta_b) / \lambda ) ( z &gt; z_{\text{max}} )</td>
</tr>
</tbody>
</table>
Fig. 3
Adverse Selection Analysis: Menu Characteristics
(Prices for High Valuation Buyer at High Quality Seller)

lambda = 19

Fig. 4
Adverse Selection Analysis: Menu Characteristics
(Prices for Low Valuation Buyer at High Quality Seller)

+ P(h) (quality not observable) ——— P(h) (quality is observable)
Fig. 5
Adverse Selection Analysis
Menu Characteristics (Length of Warranty for High Valuation Buyer)

Fig. 6
Adverse Selection Analysis
Menu Characteristics (Length of Warranty for Low Valuation Buyer)
Adverse Selection Analysis:
Length of Extended Warranty for Different Premium Levels

$\lambda = 0.19$

Length of Extended Warranty for Different Premium Levels

$z^*$ as a Function of the Cost Ratio

+ length of EW (quality not observable) --- Length of EW (quality is observable)
References


References continued


