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Optimal Sequential Investments in New Product Development with Emerging Technologies and Learning

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We address the challenge of determining the optimal investments for a firm with limited new product development (NPD) resources, when the product development opportunities come over time from two distinct technologies. Upfront investment in a product platform gives higher returns from opportunities based on the platform technology in the future, due to the associated learning effects. Contingent on the order that technologies arise, we formulate the resource allocation problem and characterize the optimal development investments that determine the firm product development roadmap. We show that the firm should invest more resources in platform development if the returns from subsequent opportunities to leverage the platform are high, and if opportunities arise intermittently. In addition, the investment in platform development decreases in the uncertainty about either the learning effect, or the resources budget. Finally, we analyze the competitive scenario where two firms develop offerings based on the two technologies, and we show that the competing firms benefit from focusing resource investment on the competitively intense product line. Insights are then provided into the properties of the firm NPD roadmaps based on multiple technologies.

Key words: New product development, Platform Learning, Technology Roadmap, Competition, Uncertainty

History:

1. Introduction

Making the right new product development (NPD) investments in product portfolios is crucial to the success of a firm and it has been well recognized in the literature. Across different industrial contexts, choosing the portfolio of technological opportunities to invest in, the timing of the product development projects, and the assets to be shared across products in a platform, are all important parts of the product planning activity (Krishnan and Ulrich 2001, Joglekar and Ford 2005). Product portfolio composition has been studied traditionally under the assumption that all potential technologies are available simultaneously. The resulting product line design problem, then, represents products as subsets of attributes and underlying technologies, and seeks the product investment choices that maximize customer adoption through the ‘right’ levels of the attribute
values included in each product (Belloni et al. 2008, Schon 2010). We extend this stream of literature, to consider the timely dimension of such investment choices. Such a timely consideration of investment choices, also coined as a technology or product roadmap, concerns the sequential development of product designs, and it has received limited attention in the literature (Dickinson et al. 2001, Ramachandran and Krishnan 2008). Thus, we determine the optimal NPD investments to be made by a firm when crafting their product roadmap, based on two different technologies. Within this context we examine the efficacy of a platform-based design strategy, i.e. one of the underlying technologies may result in multiple product designs over time.

An illustration of the problem we address is provided by Ittiam Systems Private Ltd., a Bangalore-based developer of high-technology consumer electronics products. Ittiam Systems is currently designing and developing digital camcorder and portable media player and recorder systems, based on the MP3 (audio) and MPEG4 (video) technology standards. Investments made during these product designs serve as a platform to reduce the development efforts of subsequent products, which will embed advanced versions of these technologies. They also increase the potential returns from future products, due to better functionality and higher quality. At the same time, Ittiam Systems expects, in the near future, to have the opportunity to design WLAN chipsets and WLAN Network Interface Cards based on the 802.11g wireless communication protocol (Wireless LAN technology standard) for the mobile communications market. This is contemplated as a new technology for that market. For the foreseeable, but distant, future, Ittiam Systems expects to have the technological opportunity to develop IP video phone products and IP video surveillance systems based on an advancement of the video compression technology currently used for their digital camcorders (multi-format High Definition Video Decompression based on the High Definition Video standards of MPEG2, H.264 and VC1). As expected, all these development options have to be accommodated within a limited projected budget. Thus, Ittiam Systems considers how to lay out the best possible product roadmap over the coming years, i.e. how to allocate their NPD budget amongst their current product lines, potential products based on the new WLAN wireless technology, and the likely future product extensions of IP video based phone and surveillance systems. The problem is complex, as Ittiam Systems has to lay out a product roadmap that supports those opportunities, when (and if) they become available in the future.

The situation at Ittiam Systems is a common scenario; firms usually consider multiple technologies that are currently available, or will be in the future, for their product roadmaps, even when these base technologies are developed by other parties (Erat and Kavadias 2006, Ramachandran and Krishnan 2008). The managerial tradeoff is straightforward: over- or under- investing in the
current (certain) opportunity may cripple the firm’s ability to exploit other future (but uncertain) opportunities. In addition, as demonstrated by Ittiam’s challenge, several of these opportunities may arise from the same technology as the current opportunity, and therefore, a potential upfront underinvestment may forego the benefit of “learning” (Loch and Kavadias 2002). In other words, the firm can leverage investments made based on the current technology, owing to the invested absorptive capacity, e.g. the implicit knowledge gained by the firm in developing a product based on the current technology (Cohen and Levinthal 1994), and/or better economies of scale for developing product extensions (Ulrich and Ellison 1999, Fisher et al. 1999). In contrast, other opportunities may promise high returns from NPD, without the potential leverage from subsequent advancements, as the WLAN technology example from Ittiam Systems demonstrates.

The problem of determining the optimal investments to be considered for the firm product roadmap over time, is further complicated by a number of other factors. The investments to be made in the design of the product platforms, or stand alone products, depend on the order of arrival for the underlying technological improvements. Firms tend to start with a product plan or roadmap, which outlines the timing of planned development projects, and includes the portfolio of technological opportunities to be pursued (Krishman and Ulrich 2001, Martino 1980, Phaal et al. 2004). Ittiam Systems, for instance, expects the technological opportunities based on the WLAN technology to arise earlier than the video compression based technology. However, the order of technology emergence may be reversed. In addition to the expected order of technological opportunities, firms may also be uncertain about the resource budget available for new product development (Loch and Kavadias 2002), as it depends on the source of funding (either external, or based on corporate level decisions if internal). The degree of learning (leveraging of the investments on product platform design) may also be uncertain; finally, since the base technologies are being developed externally, competitors also have access to these technologies and may also invest in developing related products (Erat and Kavadias 2006).

In this paper, we study the impact of platform-based design on the properties of a firm’s product roadmap, when alternative technologies may emerge, and the firm can benefit from the learning effects associated with the platform technology. We also examine the roadmap impact from the order of arrival of technological opportunities, uncertainty in the NPD budget, the degree of learning, and competition. Our research questions addressed are as follows: (i) What is the impact of future opportunities on the optimal NPD investments to be made on the product roadmap of a firm? How do they affect potential product platform development? (ii) What is the impact of
budget uncertainty and uncertain learning effects on the roadmap investments? (iii) What is the impact of competition on the roadmap investments?

We show that a firm optimally invests more resources in developing a product platform based on the current technology to take advantage of the learning effect in the future (by building a larger amount of absorptive capacity). If the firms invest their NPD resources in a competitive (duopolistic) environment, then their optimal investments depend on the competition intensity (as defined by the ratio of the marginal return from a firm’s own investment to its marginal loss due to the competing firm’s investment). Surprisingly, both firms should optimally invest more resources in developing products based on the technology where the competition is more intense. If the firm is uncertain about the future benefits of the current (platform) technology, it should optimally invest less in developing a product platform. Interestingly, if the budget available is uncertain, the firm should optimally invest less in the development of a product platform. In other words, uncertainty in the timely attributes of the product roadmap (learning, available budget) prompts the firm to back-load their product roadmap, a result that runs counter to previous studies (Loch and Kavadias 2002).

In the event where the returns from NPD investments are increasing in the amount of investments, the firm optimally concentrates their entire NPD budget on one product. Under competition, both firms focus their budget investments on one product, but they may choose to mitigate the competition intensity by investing in different underlying technologies.

The rest of the paper is organized as follows. In the following subsection, we briefly discuss the contribution of the paper to the current literature on resource allocation in new product development. Section 2 is devoted to the conceptualization of the model and Section 3 to the model formulation and analysis, and the analytical results are presented and the contributions and limitations of this paper and possible directions for future research are outlined in Section 4.

1.1. Extant Literature

Our research contributes to the literature on product platform design, the impact of order of arrival of technological opportunities, uncertainty in resource budget and learning, and competitive assessment on product platform design. The literature on each of these issues, and our contributions to the literature are highlighted below.

The literature on product platform design has been studied in the areas of product line design and product selection as well as the literature on component commonality. In the literature on product line design and product selection, the focus is on identifying the optimal product assortment and related prices in the presence of economies of scale from shared components, design costs, and
cannibalization as one of the revenue drivers (Hopp and Xu 2005, Ramdas and Sawhney 2001, Mu et al. 2011). These papers find that if technologies are static, platform-based modular design increases product variety, but the variety cannot be too high owing to market cannibalization (Ramdas and Sawhney 2001, Mu et al. 2011). Other studies have investigated the impact of cost-reducing design effort and the choice of common components (Heese and Swaminathan 2006), the rates of market growth, margin decline and NPD cost (Druehl et al. 2009), and the impact of production schedules (Netessine and Taylor 2007). Relevant to our paper, Loch and Kavadias (2002) study optimal R&D investments in multiple product lines with a fixed budget simultaneously, and examine the impact of “learning” effects in R&D over the horizon.

The component commonality feature is a part of the product line design and selection problem, as sharing components in the same assortment reduces the cost of designing components, and gives the benefit of economies of scale. This stream of literature identifies which components to be made common, based on variability in product volumes and product variety (Fisher et al. 1999), their effect on product quality (Ramdas and Randall 2008), whether customer requirements are holistic and need new specific components (Ulrich and Ellison 1999), and the effect of commonality on inventory costs with lead times (Bernstein et al. 2011, Song and Zhao 2009). Our research contributes to the above stream of literature by considering technological opportunities arriving dynamically in the platform design problem.

In the literature on the order of technology arrival in product family design, Ramachandran and Krishnan (2008) consider a problem that is closely related to this paper, by studying the serial commercialization of improving technologies. They find that platform-based design policies based on modular design and pricing yield superior profits to integral design choices. Similarly, in an environment with improving technologies, Bhattacharya et al (2003) show that the platform approach to product family design is always better than the non-platform approach. Our research contributes to this stream of literature by considering new technologies and technology extensions simultaneously in the dynamic problem. If technologies are static, models of sequential product introduction show that the platform approach is better than the non-platform approach (Krishnan and Gupta 2001), and overlapped design approaches (differentiation along additional vertical quality dimensions) is better than completely subsumed design (low-end products with subtracted value) (Krishnan and Zhu 2006).

In the literature on the impact of uncertainty on product platform design investments, Dogan et al. (2011) study the effort to be invested in platform design in software development with uncertain future demand that is influenced by word-of-mouth effects. They find that the platform-design
investment is lower when the uncertainty of future demand is higher (firms should follow the wait-and-see approach), and the word-of-mouth effect is low. In contrast, if the word-of-mouth effect is high, the firm should invest more in platform-based design. There is also a substantive literature that provides decision support models for platform-based product design in the presence of uncertainty if technologies have interdependence (Cho and McCardle 2009), using real options approaches (Gonzalez-Zugasti et al. 2001), and the evaluation of alternate designs (Li and Azarm 2000). We contribute to this stream of literature by showing the applicability of the wait-and-see or the early commitment approaches in platform-based design in sequential product development.

In the literature on competitive effects of product design, Klastorin and Tsai (2004) show that when two firms enter the market sequentially with competing products, product differentiation always arises at equilibrium due to the joint effects of resource utilization, price competition and the product lifecycle. Recently, studies have also captured the impact of the retail competitive structure on product design (Luo 2011, Shiau and Michalek 2009, Williams et al. 2011). Luo (2011) consider a product line design problem with multiple manufacturers and one retailer, and finds that the manufacturers use the platform approach with shared components in the product design to maximize their profits. Williams et al. (2011) find that the optimal quality levels tend to be higher and prices of the products offered decrease with an increasing amount of retail competition. Our paper contributes to this stream of literature by assessing the impact of competitive response on platform-based design investments in a dynamic technology setting.

2. Model Description, Notation and Assumptions

In this section, we describe the model setting and state our assumptions. The firm has a technology opportunity roadmap which recognizes that certain technological opportunities may arise over time in a given order with some pre-specified probability (Martino 1980, Krishnan and Ulrich 2001, Phaal et al. 2004). The firm needs to decide now on the product roadmap, i.e., the investments on the development of a product platform based on the current technology (referred to as technology $C$), which may present opportunities for advancements in the foreseeable future, and future investments. We consider for illustrative purposes that the firm has two technological opportunities in the foreseeable future: (i) developing a product platform based on a current technology $C$ with a potential extension of the technology in the future, and (ii) developing a product based on a new technology $N$ (the product based on a new technology $N$ (this technology does not have potential opportunities for follow-up products in the foreseeable future). It is important to note that returns on technology $N$ and the advancement of technology $C$ can also incorporate streams
of revenue from multiple opportunities based on either of the underlying technologies. The firm considers either of two technology roadmaps: intermittent roadmap (where the new technology $N$ arrives before the extension of technology $C$), and the rapid roadmap (where the extension of technology $C$ arrives immediately after technology $C$, and before technology $N$). The roadmaps are summarized in Figure 1. The results of the model generalize to the case where technology $N$ arrives before technology $C$ and its extension (proof available with the authors).

**Figure 1** Timeline and sequence of events in the model

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platform</td>
<td>Technology $N$</td>
<td>Extension of Technology $C$</td>
</tr>
<tr>
<td>Prob = 1</td>
<td>Prob = $p_N$</td>
<td>Prob = $p_E$</td>
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</tbody>
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**Figure 1 (a): Intermittent roadmap**

<table>
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</tbody>
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**Figure 1 (b): Rapid roadmap**

The investment to be made in the development of the product platform based on the current technology $C$ is denoted as $x_P$, in the extension of technology $C$ as $x_E$, and in new technology $N$ as $x_N$. We make the following assumptions:

**A1.** The technological opportunity $N$ arises with a probability of $p_N$, and the technological opportunity of the extension of $C$ arises with probability $p_E$, $p_N$ and $p_E$ are known by the firm (Krishnan and Ulrich 2001, Phaal et al. 2004).

**A2.** The firm has a set of resources denoted by $B_1$ for the entire roadmap horizon, $B_1$ is deterministic$^2$. $B_1$ is a surrogate metric for any binding resource constraint the firm may face, like cash, NPD facilities, and specialized experimentation equipment$^3$. Any resources committed to one product represents an irreversible investment, in that the firm cannot use the same resources for the development effort of later products.

**A3.** We assume that the return from the NPD investment is a concave increasing function of the resources invested, where the returns could be either revenue-increasing, cost-decreasing, or both (Gilbert and Cvsa 2003). We model the returns through an increasing logarithmic function of the

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$^1$ In the technical appendix, we also analyze the cases where the technologies arrive in random order.

$^2$ We relax this assumption in the next section, where we consider that the budget $B_1$ is uncertain.

$^3$ We assume that even if the resources available $B_1$, is a financial support constraint, the firm prefers to invest it in R&D on the opportunities available rather than hold it as cash at the end of the horizon.
investment made by the firm\textsuperscript{4}. Hence, \( R(x_I) = \text{Max}\{0, \mu_I \log x_I\}, I = C, N \), where \( \mu_I \) is a scale effect that includes such factors as market size and market valuations. The logarithm function has been used extensively to model the returns from R&D in the literature (Derman et al. 1975, Prastacos 1983), the results of the paper have the same structure with other concave forms of returns as \( R(I) = A\sqrt{I} \) and \( R(I) = AI^\rho \) (the proofs are available on request from the authors).

\textit{A4}. The “learning” effect is modeled as follows: the effective investment on the product extension based on technology \( C \) is \((x_E + lx_P)\), hence, \( R(x_E) = \text{Max}\{0, \mu_E \log(x_E + lx_P)\} \). The learning effect has been modeled as a carry-over effect by Loch and Kavadias (2002), as absorptive capacity (Cohen and Levinthal 1994), and economies of scale (Ulrich and Ellison 1999, Krishnan and Gupta 2001). From a resource productivity perspective, it represents the fact that the firm achieves the same level of returns with fewer resources; from the cost perspective, it also models that common components in the platform reduce the design cost of the product. From a market payoff standpoint, it models a higher return obtained due to the knowledge accumulated during the initial investment, the build-up of brand equity, and the word-of-mouth effect from the initial product. It also models the effect of cannibalization, as the presence of cannibalization will lower the value of \( l \).

\textit{A5}. \( x_P, x_N, x_E \geq 0, x_P + x_N + x_E \leq B_1 \).

The problem formulation is a special version of the dynamic knapsack problem, with the difference that the initial investment in the development of a product based on Technology \( C \) accrues additional benefits in the future.

3. Model Analysis

In the monopoly case, the trade-off faced by the firm is as follows: if it overinvests initially in developing the product platform based on Technology \( C \), it may not have enough resources to pursue the other opportunities considered in the roadmap, in the event they become available. If they underinvest, they risk wasting NPD resources in the event the future opportunities do not materialize. In our analysis we proceed as follows: we first derive the optimal allocations for each technology based on the order of arrival of the opportunities. We analyze the case in the paper where the probabilities of arrival of the opportunities and the order of arrival are discrete as described in Assumption 3 above\textsuperscript{5}.

\textsuperscript{4}The logarithmic return function is negative when the investment is below 1, but by scaling the resource budget appropriately (in man-days as time for people, days if the budget represents time at a constrained resource, or as a dollar value if the budget is a financial support constraint), the returns are ensured to be positive.

\textsuperscript{5}In the Technical Appendix, we show how the consideration of continuous distributions for the probabilities of arrival in the realization of the roadmap scenarios depends on the contingency results in a straightforward manner.
Define the value function of the firm at the beginning of each period as follows: let $B_n$ denote the remaining resources budget available to the firm at the beginning of period $n$, $V_n(B_n)$ denote the value function of the firm at the beginning of period $n$ contingent upon the realization or not of an investment opportunity, and $H_n(B_n)$ be the expected optimal value function from following the optimal resource allocation. The latter includes the situation in which the opportunities do not materialize. The firm solves

$$V_n(B_n) = \max_{I \geq 0} \{ R(I) + H_{n+1}(B_n - I) \}$$

to determine the optimal value of $I = \{ x_P, x_N, x_E \}$. Where $R(I) = \{ \mu_P \log x_P, \mu_N \log x_N, \mu_E \log (x_E + lx_P) \}$ (or $R(I) = 0$ if $I = 0$) is the return from investing an amount of $I$ when the respective opportunity materializes. If no opportunity is available, then $V_n(B_n) = E[H_{n+1}(B_n)]$. The objective for the firm is to maximize the optimal value function in the first period, with the constraints $x_P \geq 0$, $x_N \geq 0$, and $x_E \geq 0$.

As the problem is a dynamic knapsack problem, we solve it backwards. We start off our analysis with the intermittent and rapid roadmaps as described in Figure 1. The expected investments made by the firm by following an optimal strategy are given by the following result.

**Result 1.** (i) If the firm expects the intermittent roadmap, there is a threshold value $l_1$ such that for $l \leq l_1$, $x_N > 0, x_E > 0$, and the optimal platform investment is:

$$x_P^* = \frac{\mu_P}{(1-l)(\mu_P + p_N \mu_N + p_E \mu_E)} B_1$$

When $l > l_1$, $x_E = 0$, and $x_P^*$ is the solution of the following quadratic equation.

$$(1-l)(\mu_P + p_N p_E \mu_E + p_N \mu_N + p_E \mu_E B_1) x_P^2 - B_1 ((2-l)(\mu_P + p_N p_E \mu_E) + p_N \mu_N + (1-l) p_E \mu_E)$$

$$+ (\mu_P + p_N p_E \mu_E) B_1^2 = 0$$

(ii) If the firm expects the rapid roadmap, then if $l \leq l_2$, the optimal platform investment $x_P^*$ satisfies

$$\frac{\mu_E}{x_P} = \frac{p_E (\mu_P + p_N \mu_N + p_E \mu_E)}{B_1 - x_P} + \frac{(1-l) p_N \mu_N}{B_1 - x_P}$$

If $l > l_2$, $x_E = 0$, and $x_P^* = \frac{(\mu_P + p_E \mu_E) B_1}{(\mu_P + p_E \mu_E + p_N \mu_N)}$.

The values of $l_1$ and $l_2$ are in the Appendix. While accounting for the technology opportunity roadmap, the firm trades off the investment to be made in the development of a product platform against the residual budget to be held back for the future opportunities. In a similar vein, if the opportunity to develop a product based on technology $N$ or advancement of technology $C$ materializes, the firm trades off such an investment against the residual budget to be held back again for the future opportunity. The expected optimal investment made by the firm on the product platform is an increasing function of the expected scale of return from the investment ($\mu_P$) and the
learning effect, $l$. Obviously, if the scale of return is high, the firm would make a higher investment on the product platform.

If the learning effect is high, the firm makes a higher investment in the first period to derive an additional benefit in the third period if an opportunity to invest in developing a product based on an advancement of technology $C$ arrives then. Our analysis indicates the firm should make a higher investment in the first period (a “front-loading” effect), in order to build absorptive capacity. This insight is intuitive and in accordance with past results from the analytical literature (Loch and Kavadias 2002) and industrial observations (Thomke and Fujimoto 2000). Our research complements the findings in the literature that if cost complementarities exist owing to common components (Fisher et al. 1999, Ulrich and Ellison 1999, Hopp and Xu 2005), then the investment in platform-based design should be higher. Similarly, if market cannibalization across products is high (resulting in a lower learning factor $l$), then the investment in platform-based design should be lower. If the word-of-mouth effect is high (resulting in a high value of $l$), then the investment in platform-based design should be higher (Dogan et al. 2011). The expected optimal investment made by the firm in the design of the product platform is also decreasing in the expected rewards from the products based on the new technology and the advancement of the current technology respectively ($p_N\mu_N$ and $p_E\mu_E$). This result corroborates the existing literature in the dynamic knapsack problem, that finds that optimal investments in a product are decreasing in the value functions of other products.

It is easy to show that the other product roadmap investments, i.e., the investment in developing products based on the new technology and the advancement of technology $C$ are an increasing function of $p_N\mu_N$ and $p_E\mu_E$, and a decreasing function of $\mu_P$ and the expected returns of the other opportunity, if these technological opportunities arise. These results are based on the dynamic knapsack problem, where the investments made are increasing in the expected values of the rewards from the investment, and decreasing in the expected reward from the other investments.

It is very easy to incorporate discounting in this model, as the discounting factors multiply the scale effects of $\mu_N$ and $\mu_E$ respectively. Including a discounting factor increases the firm’s optimal investment in developing the product platform based on Technology $C$, and reduces the other product roadmap investments. For the rest of the paper, we focus on interior point solutions to obtain managerial insights, hence, we make the assumption that the learning effect, $l$, satisfies $l < \bar{l}_1 = \frac{p_E\mu_E(p_N\mu_N + p_E\mu_E)}{p_P\mu_N + p_P\mu_E(p_N + p_N\mu_N + p_E\mu_E)}$, it is easy to show that $\bar{l}_2 > \bar{l}_1$. 
3.1. Effect of Technology Order on Platform Investment

We now analyze the effect of the order of technology arrival (either the rapid roadmap or the intermittent roadmap) on the product platform investment. In Ittiam Systems’ case, they would use the rapid scenario if they expected the opportunities to develop products based on the IP video decompression technology ahead of the products based on the WLAN technology, and the intermittent scenario in the reverse case. The optimal investments to be made in the development of the product platform are characterized below. We denote the investments in the product platform in the rapid and intermittent cases by $x^R_p$ and $x^I_p$, respectively.

**Proposition 1.** The optimal allocation of resources to the initial development of the product platform in the rapid case is less than that in the intermittent case, $x^R_p < x^I_p$.

We find, surprisingly, that the firm should optimally invest more resources in the product platform in the intermittent case (where the technological advancement of $C$ arrives after technology $N$) than in the rapid case (if $l = 0$, the optimal investment is the same). In the intermittent roadmap, the residual budget after investing in the product platform is used in the end of the horizon to develop the extension of the product based on the advancement of technology $C$ with a probability of $p_E(1 - p_N)$. Even if the opportunity to develop the product based on technology $N$ materializes, the firm holds back some smaller residual budget to be utilized with probability $p_Ep_N$. In the rapid roadmap, the benefit from building absorptive capacity is $p_E\mu_E \log(lx^*_p + x^*_E)$, however, some of the residual resources are kept “free” to be used for developing a product based on technology $N$. Thus, the benefit of the learning effect is higher from the residual budget in the intermittent roadmap. However, in the presence of a discounting factor, if the development times of the products are the same, then the decrease in the optimal platform investment in the rapid roadmap is smaller. It is easy to see that in the rapid roadmap case, the impact of the residual budget for the NPD investment in the end of the horizon has lower weight, as it is multiplied by a discounting factor, hence, the additional residual budget does not play as significant a role.

In the literature on the order of technologies for platform-based product design, when technologies are sequentially improving, Ramachandran and Krishnan (2008) find that platform-based design policies are better than non-platform design choices, which is corroborated by Bhattacharya et al (2003). We contribute to this stream of literature by demonstrating that the order of arrival of technological opportunities affects the optimal investment to be made in product platform design, and show that the platform design investment is higher when the opportunities to leverage the platform into variants arrive later in the horizon, compared to when they arrive earlier.
We now compare the optimal investments numerically and illustrate the lower initial optimal investment in the rapid roadmap compared to the intermittent case. The results presented in Figure 2 are for the values of $\mu_N = \mu_P = \mu_E = 1$, $l = 0.1$, $p_N = p_E = 0.5$, and $l$ varies from 0 to 0.2.

Figure 2  Optimal platform investment as function of learning factor

![Optimal Platform Investment](image)

If the order of arrival of technologies is uncertain, let the firm contemplate the likelihood that the opportunity to develop a product based on Technology $N$ arrives before the opportunity of a product extension based on Technology $C$ to be $p_I$, and let $p_R$ be the estimate of the likelihood that the rapid scenario happens. In that case, the optimal investments in the three periods is a combination of the optimal investments in the above two roadmap scenarios. Obviously $p_F + p_R \leq 1$.

In the technical Appendix, we demonstrate how the results depend on the two contingent roadmaps, and we offer closed form solutions for the situation in which the two likelihoods are realizations of two exponential distributions for the “arrival” of a specific technological opportunity.

We now present the results when the firm is operating in an uncertain environment, where the resource budget $B_1$ is known after the investment in the product platform has been made, or if the learning effect is uncertain, or if the scale effects of the reward functions ($\mu_N$ and $\mu_E$) are uncertain.

3.2. The Effects of Uncertainty

In this section we explore the directional effects of additional sources of NPD uncertainty, beyond the risk of an investment opportunity not materializing, on the optimal resource allocation. Uncertainty is a dominant structural feature of new product development, and it may originate from different sources (Huchzermeier and Loch 2001, Santiago and Vakili 2005). We consider the following three types of uncertainty: first, we consider the uncertainty with respect to the magnitude of the learning effect. In many instances, the future learning benefits of current investments may not
be predicted accurately due to the unknown performance of the underlying technologies or due to the unpredictable behavior of the consumers in the end product market (Dogan et al. 2011). Second, we consider the case where the resource budget is uncertain (Loch and Kavadias 2002). This case also encompasses the case where the efficiency of the resources invested in a particular product cannot be fully predicted. Resource availability uncertainty is encountered in multiple occasions within organizations, and it often results in conflicts among project managers who try to ensure their project viability (Carrillo and Gaimon 2004). At the same time, due to product development uncertainty, the resources put into a specific product development project may deliver less than expected due to failed trials. Third, we extend our analysis to consider the implications of uncertain scale effects on the allocation profile (unknown market size or potential). Such uncertainty is typical in new product markets since forecasting techniques can reduce but not eliminate uncertainty (Dogan et al. 2011).

The analysis proceeds in a step-wise fashion where we consider only one source of uncertainty at a time, and the mean of the uncertain parameter is the value in the case with certainty. The reason for this style of analysis is to retain clarity in the presentation and highlight clearly the effects of each type of uncertainty. We also retain in all three cases the risk of an opportunity not materializing as this is a core phenomenon we want to consider. Finally, we make the simplifying assumption that any existing uncertainty resolves itself at the beginning of the second period. The assumption does not take away from the core phenomenon while it helps us retain the tractability of the derivations. We model these different types of uncertainty as follows: (i) with respect to the uncertainty in the learning factor we assume that $l$ takes the values of either 0 or $2l$ with the probability of 0.5 each, $2l$ is an upper value that guarantees we stay in the solution region within the “interesting” interior solution. (ii) The uncertainty in the resource budget is modeled by the effective resource budget being either $\tilde{B}_1 = B_1 + \epsilon$ or $\tilde{B}_1 = B_1 - \epsilon$ with a probability of 0.5 each, where $\epsilon > 0$. This assumption also models the case when the investments may not have the same impact as expected. (iii) Finally, to model the uncertainty in the scale effects, $\mu_N$ and $\mu_E$, we assume that they are randomly distributed with a known distribution $f(\mu_N)$, and $h(\mu_E)$ and means of $\bar{\mu}_N$ and $\bar{\mu}_E$ respectively.

The following Proposition describes the effects of the different types of uncertainty on the optimal investment in the product platform:

**Proposition 2.** The optimal platform investment $x_P^*$ is

(i) lower in the presence of uncertainty in the learning factor.

(ii) lower in the presence of uncertainty in the resources budget or efficiency.
(iii) independent of the uncertainty of the market potentials.

The insights in Proposition 2 are significant for understanding the impact of uncertainty on the product roadmap investments made by the firm. The concave return structure of a mature market in the future incurs higher losses from a potentially smaller learning effect. Given this loss, the firm should optimally invest less in the product platform and invest more in developing a product based on technology $N$ to offset the impact of this loss. This result is akin to the “wait-and-see” approach described in the literature, where the firm should prefer to retain resources for a later stage when the uncertainty is resolved. As a result, it is more beneficial to invest less currently and save resources for the future in order to account for these potential losses. The effect is similar for the resource efficiency uncertainty or the budget uncertainty. In this context, we can interpret the lower investment as follows: the firm prefers to decrease the investment in “absorptive capacity” and instead, leave more resources for future investments once the uncertainty has been resolved.

We should note here that the result is not intuitive. The uncertainty in the timely attributes of the product roadmap (learning, available budget) prompts the firm to back-load their product roadmap, a result that runs counter to previous studies (Loch and Kavadias 2002). The real options literature has recorded that in the presence of uncertainty, the firm should invest more in the certain outcome and less in the uncertain one. In contrast here we show that due to the fixed total resources and the concave returns structure, the result is reversed. Finally, the uncertainty with respect to the market potential does not change the optimal allocation. However, that does not imply that in the presence of uncertainty the profitability stays unchanged. The profits of the firm decrease in the presence of a higher market potential uncertainty.

In the literature on the impact of uncertainty on product platform design investments, Dogan et al. (2011) find that the platform-design investment should be lower when the uncertainty of future demand is higher (firms should follow the wait-and-see approach), and if the word-of-mouth effect is high, the firm should invest more in platform-based design. Cho and McCardle (2009) also provide regions for following the early commitment versus following the wait-and-see approach in the presence of uncertainty for the adoption of interdependent technologies. Our research contributes to this stream of literature by demonstrating the efficacy of the wait-and-see approach in the context of decreasing returns to scale, and the impact of the learning effect on the product platform investment.

We now analyze the case where two competing firms develop products based on both technologies, and characterize the optimal investments to be made in the development of the product platform.
3.3. The Competitive Case

In this section, we investigate the sequential investments made by two firms under competition, where the technological opportunities to invest in the products for the two markets are available to both firms (Erat and Kavadias 2006). We model the sequential investment game as a two-stage subgame-perfect Nash equilibrium (SPNE), and index the two firms by X and Y. The terminology used in this section is as follows: both firms X and Y have an opportunity to invest in developing products based on technology C and invest an amount of resources of \( x_P \) and \( y_P \) in developing the product platform. They invest in developing products based on technology N, which arrives with a probability of \( p_N \) with amounts of resources of \( x_N \) and \( y_N \) respectively, and in developing product extensions based on an advancement of technology C, which arrives with a probability of \( p_E \) with an amount of resources of \( x_E \) and \( y_E \) respectively. We assume that both firms have the same technology roadmap (intermittent or rapid), and \( p_N \) and \( p_E \) are common knowledge.

To model the returns from the investments made in developing products under competition, we use the well-established standard properties: the firm’s return is increasing in its own investment, and decreasing in the competitor’s NPD investment. Specifically, for products based on technology C, if firm X makes an investment of \( x \) in developing a product and the competing firm Y makes an investment of \( y \) in developing a product, then firm X’s expected return from the investment is given by \( \max[0, \mu \log(\alpha x - \beta y)] \), where \( \alpha \) is a coefficient of market growth due to competition, and \( \beta \) is a coefficient of substitution between the products developed by the two firms\(^6\). As is the standard practice, since the effect of the firm’s own investment should be greater on its profits than the competitor’s investment, we assume \( \alpha > \beta \). Note that \( \alpha \) and \( \beta \) are expected to be the same for the investments based on technology C, owing to the underlying technology being the same (Technology C). The return of firm Y is symmetric, and is given by \( \max[0, \mu \log(\alpha y - \beta x)] \).

We denote the ratio of \( \beta/\alpha \) as \( s \). \( s \) is a metric for the intensity of competition in products based on technology C, if the value of \( s \) is high, then the competition is intense, as a high investment by a competitor in developing a product based on technology C reduces the firm’s return to a higher degree. For products developed by the firm based on technology N, the structure of firm X’s return for an investment of \( x_N \) is given by \( \max[0, \mu_N \log(\delta x_N - \gamma y_N)] \), the return for firm Y similarly is \( \max[0, \mu_N \log(\delta y_N - \gamma x_N)] \). We denote the ratio of \( \gamma/\delta \) as \( t \), \( t \) can be interpreted as the intensity of competition based on Technology N as before. Furthermore, the NPD resource budgets of firms X and Y are denoted by \( B_x \) and \( B_y \) respectively. We index the firm’s value functions by \( x \) and

\(^6\) It is salient to note that the model can accommodate spillover benefits as well, since if spillovers exist, the value of \( \beta \) is lower, and if spillovers do not exist, the value of \( \beta \) is higher.
for firms X and Y respectively. Without loss of generality, we assume that $B_x < B_y$, and that $\frac{B_x}{B_y} > \text{Max}\{\frac{\alpha}{\alpha}, \frac{\gamma}{\gamma}\}$. The last assumption enables us to find interior point solutions, and identify equilibria where the two firms are comparable in terms of resource budget.

Proposition 3 analyzes the impact of the intensity of competition that is measured by the ratio of the coefficients of market growth and substitution of the markets based on the underlying technologies on the expected optimal investments on the product platform by the two firms.

**Proposition 3.** In the presence of competition, the expected optimal investments in the product platform made by the two competing firms in a subgame-perfect Nash equilibrium depend on how the ratio of market growth ($\alpha$ and $\delta$ respectively) and substitution ($\beta$ and $\gamma$ respectively) compare for the two technologies ($s$ and $t$):

Case 1: If $s = t$ ($\frac{\beta}{\alpha} = \frac{\gamma}{\delta}$), then the expected optimal investments in the product platform made by both firms in competition are equal to those made by the monopolist.

Case 2: If $s > t$ ($\frac{\beta}{\alpha} > \frac{\gamma}{\delta}$), then the expected optimal investments in the product platform made by both firms in competition are higher than the monopolist.

Case 3: If $s < t$, then the expected optimal investments made by both firms in the product platform are higher than the monopolist.

**Result 2.** In a symmetric equilibrium ($B_x = B_y = B$), the optimal investments in the product platform by the two firms are given as follows: $x^*_p = y^*_p = \frac{\mu P}{(1-\lambda)\mu P+\mu N(\mu N+\mu E\mu E)} \cdot B$

where $s = \frac{\alpha}{\alpha} = \frac{\beta}{\beta}$, and $\lambda = \frac{s^{-\mu N+\mu E\mu E}+t^{-\mu N+\mu E\mu E}}{1-t^{-\mu N+\mu E\mu E}+s^{-\mu N+\mu E\mu E}+t^{-\mu N+\mu E\mu E}}$

Proposition 3 demonstrates the effect of the degree of competitive intensity (the ratio of market substitution to market growth) of the markets of the underlying technologies on the investments in the product platform by the two firms. In Case 1, because of the symmetric nature of competition between products for the markets based on the two technologies, in the SPNE, both firms invest in the product platform the same proportion of their resource budget as the monopolist (however, both firms earn a lower return in competition, which is consistent with the literature). If competition results in a proportionate increase in the market growth and substitution effects of products in the markets of both technologies ($s = t$), then the firms do not gain any competitive advantage from investing more in the development of products for markets for either technology. Hence, in this case, both competing firms invest their resources in the same fashion as the monopolist.

If the products based on technology C experience a more intensely competitive market ($s > t$ or $\frac{\beta}{\alpha} > \frac{\gamma}{\delta}$), then in the SPNE, the firms prefer to optimally invest more in the development of the product platform compared to the monopolist. This result is counter-intuitive (intuitively, it would seem better to focus NPD resources on products that operate in a less competitive market). In this
case, any investments made in developing products based on technology $C$ and its advancement suffer from a higher degree of substitution compared to market growth, and hence surprisingly in equilibrium, the firms compensate by investing more in the development of these products. When the returns from investment are concave, the potential loss from an additional dollar invested by the competitor in an intensely competitive market is magnified by the presence of higher substitution effects. Thus in equilibrium, both firms invest more in developing products for the market with the higher competitive intensity. Mathematically, this higher loss is shown by the fact that $\log(\alpha x - \beta y) - \log(\alpha x - \beta(y + 1)) > \log(\alpha x - \beta(y - 1)) - \log(\alpha x - \beta y)$, as the logarithm function is concave.

The effect is reversed in Case 3, where the competitive intensity is higher in the market based on technology $N$. In this case, as there is a lower degree of substitution relative to growth in the products based on technology $C$ and its advancement, the firms prefer to invest more in the development of a product platform based on technology $C$ compared to the monopolist.

In a symmetric equilibrium (as shown in Result 2), we see that many of the insights from the monopoly section hold. For instance, the expected optimal investment made by the firm in the product platform is an increasing function of the expected scale of return from the investment $(\mu_P)$ and the learning effect, $l$. Similarly, if the values of $p_N\mu_N$ and $p_E\mu_E$ are high, then both firms make a lower investment in the product platform and keep a higher proportion of the NPD resource budget for developing products based on subsequent technologies if they arrive.

The optimal platform investments are compared numerically for the two firms when their resource budgets are not the same. The results presented in Figure 3 are for the values of $\mu_N = \mu_P = \mu_E = 1$, $l = 0.1$, $p_N = p_E = 0.5$, $s = 0.1$ and $t = 0.2$. Note that since $t > s$ (Case 3 of Proposition 2), the duopolists will invest more in developing products based on technology $N$ and less in developing products based on technology $C$ and its advancement respectively. We use $B_x = 10$ and vary $B_y$ from 10 to 20 to observe the impact on the optimal investments made by both firms on the product platform.

**Figure 3** Competitive platform investments with unequal budgets
We observe that when firm Y’s resource budget is increasing, firm Y increases its investment to a higher degree in developing a product for technology $N$ ($y_N$) than for its product platform based on technology $C$ ($y_P$) (the ratio of $y_N$ to the total budget increases faster than the ratio of $y_P$ to the total budget as $B_y$ increases) as $t = 0.2 > s = 0.1$. Firm X simultaneously responds by increasing the investment for developing a product based on technology $N$ ($x_N$) and decreasing the investment in developing a product platform based on technology $C$ ($x_P$).

The literature on competitive effects of product design shows that product differentiation always arises when two firms enter the market sequentially with competing products (Klastorin and Tsay 2004), and manufacturers increase their quality and lower their prices when faced with an increasing amount of retail competition (Williams et al. 2011), and that the platform-based design approach is better in competition (Luo 2011). We contribute to this stream of literature by showing that if the degree of competitive intensity varies by the underlying technology behind the product, then the investment in the product platform design is affected. The firm should invest more in the development of a product platform for its current technologies if there is a higher intensity of competition for this set of products than for products based on other unrelated technologies. Our results suggest that the impact of competition on the optimal investment in product platforms is nuanced, and is based on the degree of competitive intensity and the resource budget available to the firm.

3.4. Comparative Statics

We performed a series of checks on the model to verify that the insights of the model are robust to the assumptions, and to investigate the effect of alternate assumptions on the results of the model. The results of our analysis are summarized below, and are presented in detail in the Technical Appendix.

**Uncertain Roadmap Contingency:** If the firm is uncertain about the order of arrival of subsequent technological opportunities, we generalize the results of the discrete models presented in the paper, based upon a continuous time model. We assume that technology $C$ is available at time 0, and the time of arrival of the technological opportunity $N$, referred to as $T_N$, is exponentially distributed with parameter $\theta_N$. The time of arrival of the advancement of technology $C$, referred to as $T_C$, is assumed to be exponentially distributed with parameter $\theta_A$. The time of the horizon is given by $T$. We find that the the results of the paper generalize to the case where the order of arrival of opportunities is not known with certainty. Our analysis is presented in Technical Appendix 1.

**Convex returns to scale on investment:** If the expected returns from NPD is convex, then the firm prefers to invest its entire budget in the development of one product. The firm prefers to invest
its entire resource budget in the product platform based on the current technology \( C \) if \((\mu_P)\) is high. If the expected value of returns in investing in future technological opportunities is higher, the firm holds back its entire resource budget for future investment. In a competitive environment, the firms invest their entire resource budget in any one of the products if the returns from NPD in that product from a competitive environment are higher than the return from the other products, else, they invest their entire resource budgets in different products. Hence, in this case, firms may choose to devote their entire set of resources to developing different products and follow a strategy of differentiation. Our analysis is presented in Technical Appendix 2.

4. Model Results, Discussion, Contributions and Future Research

One of the contributions of this paper is to demonstrate the portfolio implications from the intertemporal characteristics of technology and product roadmaps. The accepted wisdom in the NPD literature is to make portfolio investment decisions through generic ranking methods such as the ratio of project NPV to the resources. Our paper complements this literature by accounting for the temporal balance that portfolio investments need to accommodate. At the same time it complements the existing literature on product line design and the associated literature on component commonality by examining the efficacy of a platform-based design approach when technological opportunities arrive sequentially. The main results and contributions of the paper are as follows:

- In a monopolistic environment, the firm benefits from an investment in the product platform based on the current technology when the investment return increases in the learning effect and the scale effect of the current technology, and decreases in the probability of future competing opportunities and their scale effects respectively. This result complements the literature on product line design (Ramdas and Sawhney 2001, Belloni 2008), as well as the literature on building absorptive capacity (Cohen and Levinthal 1994), by providing guidelines for leveraging the product platform investment. Hence, Ittiam Systems should invest a higher amount in developing products based on the MP3 and MPEG4 standards if they expect that these investments will lead to a better development of products based on the IP Communication and Video Technology and the H.264 (video standard), or if they expect the investments in these products to have a higher position on the risk-return matrix. In the future, they should base their residual investments in NPD on the relative expected risk-return positions of new markets like the WLAN technology, against the advancements in the IP Communication and Video technology.

- The knowledge about the technology roadmap has strong implications for the respective product investments and the resulting product roadmap. If new technologies arrive earlier than advancements of the current technology, then counter-intuitively, the firm should invest a higher amount
of resources in developing a product based on the current technology than in the reverse case. This result adds to the existing literature on the impact of order of arrival of technological opportunities (Ramachandran and Krishnan 2008, Bhattacharya et al. 2003) by identifying the roadmap where more resources should be invested in the product platform. Hence, if Ittiam Systems expects the IP Communication and Video technology to arrive in the near future, it should invest less in the development of a product platform based on the video technologies (MP3 and MPEG4), and more if its expects the WLAN technology to arrive sooner.

- In the presence of uncertainty in the resource budget, or in the learning effect, the firm should adopt a “wait-and-see” approach and invest less in the development of a product platform based on the current technology. Surprisingly, uncertainty in the timely attributes of the product roadmap (learning, available budget) prompts the firm to back-load their product roadmap, a result that runs counter to previous studies (Loch and Kavadias 2002). This result contributes to the existing literature on the effect of uncertainty in product design (Dogan et al. 2011, Cho and McCardle 2009) by comparing the efficacies of the “wait-and-see” and the “early commitment” responses to uncertainty. The implication for Ittiam Systems is that if it is unsure about its resources available or the learning effect for future products for the media market, then it should reduce its investment in the development of its current product platform like the digital camcorder and the portable media player and recorder systems. However, if it is uncertain about the market sizes for the product markets in the future, it should ignore that uncertainty and invest its resources in developing products for the current market as in the certainty case.

- In a competitive environment, both firms should invest more resources in the development of the product platform if they expect the competition in the market for the current technology to be more intense, instead of the obvious approach of diversifying their investments. Our result complements prior discussion in the R&D races literature (Cardon and Sasaki 1998). The intensity of competition is measured by the ratio of coefficient of substitution between the two products to the coefficient of market growth under competition. This result augments the literature on the impact of competitive assessment on product design (Klastorin and Tsay 2004, Luo 2011, Williams et al. 2011) by identifying competitive environments in which the effort in designing product platforms should be higher. Therefore, if Ittiam Systems anticipates that the competition in the WLAN format based products will be higher, then it should invest more in its NPD efforts in those products, while if it expects the competition in products based on MP3 and MPEG 4 standards to be higher along with their advancements, it should invest more in the product platform.
The results of the paper are summarized in Figure 4.

In the early phase of this research, we have made a number of assumptions that must be relaxed in future research to develop a more comprehensive understanding of the benefits of sequential investments in technological opportunities that arrive dynamically. In this paper, we assumed that there was no spillover effect in the competition models between the two firms. This can be incorporated easily into the paper by including spillover effects by reducing the coefficient of substitution between the two products from competition.

It is also likely that investing in developing a product for the current technology can enable the firm to estimate better the return from developing a product extension. We conjecture that taking this effect into account will result in a higher investment in the NPD effort in the product for the current technology, and a lower investment in the NPD effort on the product for the new technology. For a risk-averse firm, we conjecture that this effect is more pronounced, as a risk-averse firm will penalize the return from developing a product for the new technology, as the return has a higher degree of uncertainty.

While we made assumptions of functional forms for the concave return structure for the sake of mathematical tractability, we have checked with other functional forms to validate the generalizability of the results. Finally, we have not taken into account the fact that there may be multiple signals available to the firm about the probability of introduction and the returns from future opportunities. Future research should take these factors into account by incorporating information updating in the model.

In summary, this paper makes a contribution to the literature on product development by highlighting the importance of taking future learning into account when investing scarce NPD resources in the development of product platforms based on the current technology, and analyzing the impact of different factors like the probability of arrival of future opportunities and their associated returns,
uncertainty, and the competitive environment. The early accounting of these future opportunities enables the firm to make better decisions about what resources to invest now and what resources to keep in abeyance for future investment.

References


Appendix

Proof of Result 1:

The values of $\Gamma_1$ and $\Gamma_2$ are given by $\Gamma_1 = \frac{\mu E_\log(B_3 + lx_P)}{\mu B_3 + \mu E}$, $\Gamma_2 = \frac{\mu E}{\mu P + \mu P_E}$. We consider the intermittent roadmap first. In the beginning of the third period, the remaining available budget for investing in NPD is $B_3$. If the opportunity arrives to invest in a product extension for technology $C$, and the firm has invested an amount of $x_P$ in the first period on the development of a product platform, then $V_3(B_3) = \mu E_\log(B_3 + lx_P) \text{ trivially, as there are no further opportunities available in the horizon, and the firm earns a return based on the additional investment of } lx_P \text{ due to the learning effect. If no opportunity arrives for investment in the third period, then } V_3(B_3) = 0$.

Hence, $H_3(B_3) = \mu E_\log(B_3 + lx_P)$ \hspace{1cm} ...(R1.1)
In the beginning of the second period, if an opportunity to invest in developing a product for technology $N$ arrives, then the firm solves

$$V_2(B_2) = \max_{x_N \geq 0, x_N \leq B_2} \{\mu_N \log x_N + p_N \mu_E \log(B_2 - x_N + lx_P)\}$$

if the opportunity takes place. The constrained problem can be solved through the Kuhn-Tucker conditions (KKT). The Lagrangean of the equivalent problem is:

$$L(x_N) = \mu_N \log x_N + p_N \mu_E \log(B_2 - x_N + lx_P) + \lambda(B_2 - x_N).$$

The first order derivatives are:

$$\lambda \frac{\partial L}{\partial x_N} = \mu_N x_N - \frac{p_N \mu_E}{B_2 - x_N + lx_P} \leq 0 \quad \text{and} \quad x_N(x_P) = \frac{\mu_N(B_2 + lx_P)}{\mu_N + p_N \mu_E}.$$

However, $x_N(x_P) \leq B_2$ which imposes a natural constraint for the $x_1$ quantity. We get:

$$x_N(x_P) = \begin{cases} \frac{\mu_N}{\mu_N + p_N \mu_E}(B_2 - x_P + lx_P) & \text{if } x_P \leq \overline{x}_{P} \\ \frac{B_2}{B_1 - x_P} & \text{if } x_P > \overline{x}_{P} \end{cases}$$

If an opportunity for investing in the development of a product for technology $N$ does not arrive in the second period, it follows trivially that

$$V_2(B_2) = p_N \mu_E \log(B_2 + lx_P).$$

Hence, in expectation

$$H_2(B_2) = \begin{cases} (p_N \mu_N + p_N \mu_E) \log(B_1 - x_P + lx_P) + p_N \mu_N \log(\frac{\mu_N}{\mu_N + p_N \mu_E}) + p_N \mu_E \log(\frac{p_N \mu_E}{\mu_N + p_N \mu_E}) & \text{if } x_P \leq \overline{x}_{P} \\ p_N \mu_N \log(B_1 - x_P) + p_N p_E \log(lx_P) + (1 - p_N) p_E \log(B_1 - x_P + lx_P) & \text{if } x_P > \overline{x}_{P} \end{cases} \quad (R1.2)$$

In the first period, when the firm has the opportunity to invest in developing a product platform based on technology $C$

$$V_1(B_1) = \max_{x_P \geq 0, x_P \leq B_1} \{\mu_P \log x_P + H_2(B_1 - x_P)\} \quad \text{...(R1.3)}$$

which is once more a constrained optimization problem. The Lagrangean for this problem is:

$$L(x_P) = \mu_P \log x_P + H_2(B_1 - x_P) + \xi(B_1 - x_P).$$

Then,

$$\frac{\partial L}{\partial x_P} = \mu_P x_P - \frac{p_N \mu_N + p_N \mu_E}{B_1 - x_P + lx_P} (1 - l) \frac{p_N \mu_N + p_N \mu_E}{B_1 - x_P + lx_P} \frac{(1 - l)(1 - p_E) p_E \mu_E}{B_1 - x_P + lx_P}.$$

Setting the upper branch equal to zero leads to:

$$x_P^* = \frac{l p_N}{(1 - l)(\mu_P + p_N \mu_E + p_N \mu_N + p_E \mu_E)} B_1,$$

which in order to be a feasible solution it should hold that:

$$x_P^* \leq \overline{x}_{P}.$$ 

The latter is true only if

$$l \leq l = \frac{p_N \mu_E (p_N \mu_N + p_E \mu_E)}{\mu_P \mu_N + p_N \mu_E (\mu_P + p_N \mu_N + p_E \mu_E)} < 1$$

Setting the lower branch equal to zero results in a second degree polynomial

$$(1 - l)(\mu_P + p_N p_E \mu_E + p_N \mu_N + p_E \mu_E) B_1^2 - B_1((2 - l)(\mu_P + p_N p_E \mu_E) + p_N \mu_N + (1 - l) p_E \mu_E) + (\mu_P + p_N p_E \mu_E) B_1^2 = 0$$
Resubstituting the optimal value of \( x_P \) back into the optimal equation for \( x_N(x_P) \), we get the stated expression for the optimal value of \( x_N \) when \( l \leq \tilde{l} \). The optimal value of \( x_E \) is given by \( x^*_E = B_1 - x^*_P + p_N x^*_N \), which can be obtained by substituting \( x^*_P \) and \( x^*_N \). When the learning effect is relatively large we obtain a solution from the quadratic equation and we exhaust the remaining budget upon the arrival of the second market opportunity.

If the firm has the rapid roadmap, the proof of the result follows in an analogous fashion to the previous case.

Hence, \( H_3(B_3) = p_N \mu_N \log B_3 \) ...(P1.1)

In the beginning of the second period, if an opportunity to invest in developing a product extension based on technology \( C \) arrives, then the firm solves

\[
V_2(B_2) = \max_{x_E \in [B_1, x_P]} \mu_E \log (x_E + lx_P) + p_N \mu_N \log (B_2 - x_E)
\]

The constrained optimization problem has a Lagrangean \( L(x_E) = \mu_E \log (x_E + lx_P) + p_N \mu_N \log (B_2 - x_E) + \lambda (B_2 - x_E) \). The first order derivative with respect to the Lagrangean parameter is:

\[
\lambda \frac{\partial L}{\partial \lambda} = 0 \quad \text{that is} \quad \lambda^* = 0 \quad \text{or} \quad x^*_E = B_2. \]

It is straightforward to reject \( x^*_E = B_2 \) as an optimal solution (due to concavity the third period payoff cannot be zero). Thus, \( \lambda^* = 0 \) and \( \frac{\partial L}{\partial \lambda} = \frac{\mu_E - \mu_P}{x_E + lx_P} - \frac{\mu_N \mu_N}{B_2 - x_E} = 0 \) leading to

\[
x_E(x_P) = \frac{\mu_E B_2 - p_N \mu_N x_E}{\mu_E + p_N \mu_N}. \]

Since \( x_E(x_P) > 0 \) we get that \( x_P < \frac{\mu_E B_2}{p_N \mu_N} \), and given that \( B_2 = B_1 - x_P \) we obtain

\[
x_P < \frac{\mu_E}{\mu_E + p_N \mu_N} B_1 (= \bar{P}_T) \]

Eventually

\[
x_E(x_P) = \begin{cases} 
\frac{\mu_E B_2 - p_N \mu_N x_E}{\mu_E + p_N \mu_N} & \text{if } x_P < \bar{P}_T \\
0 & \text{if } x_P > \bar{P}_T
\end{cases} \]

If an opportunity for investing in the development of a product extension for technology \( C \) does not arrive in the second period, it follows trivially that

\[
V_2(B_2) = \max_{x_P \in [0, B_1]} \mu_P \log x_P + H_2(B_1 - x_P) \quad \text{...(P1.3)}
\]

The Lagrangean relaxation of the P1.3 problem results in:

\[
L(x_P) = \mu_P \log x_P + H_2(B_1 - x_P) + \xi (B_1 - x_P). \frac{\partial L}{\partial \xi} > 0 \quad \text{leads to} \quad \xi^* = 0 \quad \text{or} \quad x^*_P = B_1. \]

As before the \( x^*_P = B_1 \) cannot be optimal due to the overall concavity. Thus, \( \xi^* = 0 \) and the first order conditions yield the following quadratic equation for the upper branch:

\[
\frac{\mu_E}{x_P} = \frac{p_E x_P + \mu_E}{B_1 - x_P} + \frac{(1-p_E) p_N \mu_N}{B_1 - x_P} \quad \text{...(P1.4)}
\]

At the same time the FOC for the lower branch of \( H_2(\cdot) \) result in

\[
\frac{\mu_E}{x_P} = \frac{p_N \mu_N}{B_1 - x_P} + \frac{\mu_P}{x_P} = 0. \]

Note that from the latter we need to have \( x_P > \bar{P}_T \) which, given that the optimal solution is \( x^*_P = \frac{\mu_E x_P + \mu_E}{p_E + p_P} B_1 \), result in the following limit for the learning effect: \( \bar{P}_2 = \frac{\mu_E}{p_P + \mu_P} \). When \( l \leq \bar{l}_2 \), we can calculate the optimal initial allocation through P1.4.
Proof of Proposition 1:
The optimal platform investment if \( t < \bar{t} \) is given by Equation P1.4. Let equation (P1.4) be rewritten as:

\[
x_F^2[\mu_F + p_N \mu_N + p_E \mu_E] - x_F[\mu_F(\frac{B_1}{1-t} + B_1) + p_E(p_N \mu_N + \mu_E)B_1 + (1 - p_E)p_N \mu_N \frac{B_1}{1-t}] + \mu_F \frac{B_1^2}{1-t} = 0
\]

This equation is of the form: \( G_R x_F^2 - H_R x_F + J_R = 0 \)
where
\[
G_R = \mu_F + p_N \mu_N + p_E \mu_E
\]
\[
H_R = \mu_F(\frac{B_1}{1-t} + B_1) + p_E(p_N \mu_N + \mu_E)B_1 + (1 - p_E)p_N \mu_N \frac{B_1}{1-t}
\]
\[
J_R = \mu_F \frac{B_1^2}{1-t}
\]

The optimal first-period investment in the intermittent roadmap is given by the solution to the quadratic equation:

\[
G_F x_F^2 - H_F x_F + J_F = 0
\]
where
\[
G_F = G_R = \mu_F + p_N \mu_N + p_E \mu_E
\]
\[
H_F = \mu_F(\frac{B_1}{1-t} + B_1) + p_E(p_N \mu_N + \mu_E)\frac{B_1}{1-t} + (1 - p_E)p_N \mu_N \frac{B_1}{1-t}
\]
\[
J_F = \mu_F \frac{B_1^2}{(1-t)^2}
\]

It is easy to see that \( H_F > H_R \) and \( J_F > J_R \). Since \( x_F = \frac{H - \sqrt{H^2 - 4GJ}}{2G} \), it is increasing in \( H \) and \( J \), hence \( x_F \) in the intermittent roadmap is greater than \( x_F \) in the rapid roadmap. \( \blacksquare \)

Proof of Proposition 2: All the proofs are provided for the intermittent roadmap (technology \( N \) arrives before the extension of technology \( C \)), the proofs for the rapid roadmap are analogous.

(i) The learning factor takes on the value of either 0 or 2 with a probability of 0.5 each, and gets resolved in the second period. We also assume \((2t < \bar{t} = \frac{p_E p_N(\mu_N + \mu_E)}{\mu_F p_N + p_E(\mu_N + \mu_E)} )\), so that the solutions stay in the interesting region, where the firm invests partially in all three opportunities if they are available. If the extension of technology \( C \) arrives, and the firm has invested an amount of \( x_F \) in the development of a product platform, then \( V_4(B_3) = \mu_E \log(B_3 + \tilde{l}x_F) \), where \( B_3 \) is the available budget. Here, \( \tilde{l} \) is the random binary variable for the learning effect. If no opportunity arrives for investment in the third period, then \( V_3(B_3) = 0 \).

Hence, \( H_3(B_3) = p_E \mu_E \log(B_3 + \tilde{l}x_F) \)

In the beginning of the second period, if an opportunity to invest in developing a product based on technology \( N \) arrives, then the firm solves

\[
V_2(B_2) = \max \{ \mu_N \log x_N + p_E \mu_E \log(B_2 - x_N + \tilde{l}x_F) \}
\]
if the opportunity comes up. As shown before, \( x_N(x_F) = \frac{\mu_N(B_2 + \tilde{l}x_F)}{\mu_N + p_E \mu_E} \).

If an opportunity for investing in the development of a product for technology \( N \) does not arrive in the second period, it follows trivially that

\[
V_2(B_2) = p_E \mu_E \log(B_2 + \tilde{l}x_F).
\]

Hence, in expectation

\[
H_2(B_2) = (p_N \mu_N + p_E \mu_E) \log(B_1 - x_F + \tilde{l}x_F) + p_N \mu_N \log(\frac{\mu_N}{\mu_N + p_E \mu_E}) + p_E \mu_E \log(\frac{p_E \mu_E}{\mu_N + p_E \mu_E})
\]

In the first period, when the firm has the opportunity to invest in developing a product platform
\[ V_1(B_1) = \max_{x \geq 0, x \leq B_1} \{ \mu_P x + H_2(B_1 - x) \} \]

where \( H_2(B_1 - x) \) is given by

\[ H_2(B_1 - x) = \frac{1}{2} [(\mu_P x) + p_\mu \mu_N (\log(B_1 - x + 2l x) + \log(B_1 - x))] + p_\mu \mu_N \log(\frac{\mu_N}{\mu_N + p_\mu \mu_E}) + p + E \mu_E \log(\frac{p_\mu \mu_E}{\mu_N + p_\mu \mu_E}) \]

since \( \tilde{I} \) is a binary random variable that takes on values of 0 or 2l with a probability of 0.5 each. The FOC of \( V_1(B_1) \) with respect to \( x \) gives us:

\[ \frac{\partial x}{\partial x} = \frac{1}{2} [(\mu_P x) + p_\mu \mu_N (\log(B_1 - x + 2l x) + \log(B_1 - x))] + p_\mu \mu_N \log(\frac{\mu_N}{\mu_N + p_\mu \mu_E}) + p + E \mu_E \log(\frac{p_\mu \mu_E}{\mu_N + p_\mu \mu_E}) \]

This equation is a quadratic equation of the form \( Gx_P^2 - Hx_P + J = 0 \), where on simplification,

- \( G = (1 - 2l)(\mu_P + p_\mu \mu_N + p_\mu \mu_E) \)
- \( H = (1 - l)(2\mu_P + p_\mu \mu_N + p_\mu \mu_E)B_1 \)
- \( J = \mu_P B_1^2 \)

and \( x_P = \frac{H - \sqrt{H^2 - 4GJ}}{2G} \)

It is easy to see that the FOC for \( x_P \) for the case without uncertainty is of the form \( G_1 x_P^2 - H_1 x_P + J_1 = 0 \), where

\[ G_1 = (1 - l)(\mu_P + p_\mu \mu_N + p_\mu \mu_E) \]
\[ H_1 = (1 - l)(2\mu_P + p_\mu \mu_N + p_\mu \mu_E)B_1 \]
\[ J_1 = \mu_P B_1^2 \]

and \( x_P^* = \frac{H_1 - \sqrt{H_1^2 - 4G_1J_1}}{2G_1} \)

as derived in Result 1. Since \( H = H_1, J = J_1 \) and \( G < G_1 \), it suffices to show that \( x_P^* \) in the uncertain case is smaller than \( x_P^* \) in the certain case if \( \frac{\partial x}{\partial x} > 0 \), since if \( G < G_1 \), \( x_P^* \) in the uncertain case will be smaller than \( x_P^* \) in the certain case.

Since \( x_P^* = \frac{H - \sqrt{H^2 - 4GJ}}{2G} \),

To prove: \( \frac{\partial G}{\partial G} > 0 \)

To prove: \( \frac{\partial G}{\partial G} > 0 \)

To prove: \( \frac{\partial H}{\partial H} > 0 \) or to prove: \( H^2 - 2GJ > H \sqrt{H^2 - 4GJ} \). Squaring both sides,

To prove: \( H^2 - 2GJ^2 > H^2(\sqrt{H^2 - 4GJ}) \) which is true on simplification since \( 4(GJ)^2 > 0 \).

Hence, \( x_P^* \) is increasing in \( G \), and since the \( G \) in the quadratic equation for the FOC for the uncertain case is lower than the \( G \) in the quadratic equation for the certain case, \( x_P^* \) in the case of learning uncertainty is lower than \( x_P^* \) in the case when the learning parameter is known with certainty.

(ii) The resource budget constraint takes on the value of either \( \tilde{B}_1 = \tilde{B}_1 + \epsilon \) or \( \tilde{B}_1 = B_1 - \epsilon \) with a probability of 0.5 each, and gets resolved in the second period. Analogous to the previous case, we have

\[ H_2(B_2) = (p_\mu \mu_N + p_\mu \mu_E) \log(\tilde{B}_1 - x_P + \epsilon x_P) + p_\mu \mu_N \log(\frac{\mu_N}{\mu_N + p_\mu \mu_E}) + p_\mu \mu_E \log(\frac{p_\mu \mu_E}{\mu_N + p_\mu \mu_E}) \]

In the first period, when the firm has the opportunity to invest in developing a product platform based on technology \( C \)

\[ V_1(B_1) = \max_{x \geq 0, x \leq B_1} \{ \mu_P x + H_2(B_1 - x) \} \]

where \( H_2(B_1 - x) \) is given by

\[ H_2(B_1 - x) = \frac{1}{2} [(p_\mu \mu_N + p_\mu \mu_E) \log(B_1 - x - (1 - l)x_P) + \log(B_1 + x - (1 - l)x_P))] + \text{const} \]
since $\tilde{B}_1$ is a binary random variable that takes on values of $\tilde{B}_1 + \epsilon$ or $\tilde{B}_1 - \epsilon$ with a probability of 0.5 each. The FOC of $V_1(B_1)$ with respect to $x_P$ gives us:

$$\frac{\mu_N}{x_P} = \frac{1}{2}(pN\mu_N + pE\mu_E)(1 - \mu_N\frac{1}{\mu_N + \frac{1}{\mu_N + \mu_E}})\right]$$

This equation is a quadratic equation of the form $Gx_P^2 - Hx_P + J = 0$, where on simplification,

$$G = (1 - l)(\mu_P + pN\mu_N + pE\mu_E)$$
$$H = (1 - l)(2\mu_P + pN\mu_N + pE\mu_E)B_1$$
$$J = \mu_P(B_1^2 - \epsilon^2)$$

and $x_P^* = \frac{H - \sqrt{H^2 - 4GJ}}{2G}$.

It is easy to see that the FOC for $x_P$ for the case without uncertainty is of the form $G_1x_P^2 - H_1x_P + J_1 = 0$, where

$$G_1 = (1 - l)(\mu_P + pN\mu_N + pE\mu_E)$$
$$H_1 = (1 - l)(2\mu_P + pN\mu_N + pE\mu_E)B_1$$
$$J_1 = \mu_P(B_1^2 - \epsilon^2)$$

and $x_P^* = \frac{H_1 - \sqrt{H_1^2 - 4G_1J_1}}{2G_1}$, as derived in Result 1. Since $H = H_1, G = G_1$ and $J < J_1$, it suffices to show that $x_P^*$ in the uncertain case is smaller than $x_P^*$ in the certain case if $\frac{H - \sqrt{H^2 - 4GJ}}{2G} < \frac{H_1 - \sqrt{H_1^2 - 4G_1J_1}}{2G_1}$. To prove: $H - \sqrt{H^2 - 4GJ} < H - \sqrt{H^2 - 4GJ}$ which is true trivially since $J < J_1$.

Hence, $x_P^*$ in the case where the resource budget is uncertain is lower than $x_P^*$ in the case where the resource budget is known with certainty.

(iii) In this part of the proposition, $\mu_N$ and $\mu_E$ are assumed to be uncertain where each have the probability density function (pdf) of $f(\mu_N)$ and $h(\mu_E)$ respectively. Let $\mu_{\mu_N}$ denote the mean of the scale factor $\mu_N$, and

$\mu_{\mu_E}$ denote the mean of the scale factor $\mu_E$.

If no opportunity arrives for investment in the third period, then $V_3(B_3) = 0$.

Hence, $H_3(B_3) = pE\mu_E \log(B_3 + lx_P)$

In the beginning of the second period, if an opportunity to invest in developing a product for technology $N$ arrives, then the firm solves

$$V_2(B_2) = \max_{x_N \geq 0} \{\mu_N \log x_N + pE\mu_E \log(B_2 - x_N + lx_P)\}$$

if the opportunity comes up. As shown before,

$$x_N(x_P) = \frac{\mu_N(B_2 + lx_P)}{pN + pE\mu_E}.$$ 

If an opportunity for investing in the development of a product for technology $N$ does not arrive in the second period, it follows trivially that

$$V_2(B_2) = pE\mu_E \log(B_2 + lx_P).$$

Hence, in expectation

$$H_2(B_2) = (pN\mu_N + pE\mu_E)\log(B_1 - x_P + lx_P) + pN\mu_N \log\left(\frac{\mu_N}{\mu_N + pE\mu_E}\right) + pE\mu_E \log\left(\frac{pE\mu_E}{\mu_N + pE\mu_E}\right)$$

In the first period, when the firm has the opportunity to invest in developing a product platform

$$V_1(B_1) = \max_{x_P \geq 0, x_P \leq B_1} \{\mu_P \log x_P + H_2(B_1 - x_P)\}$$

where $H_2(B_1 - x_P)$ is given by
\[ H_2(B_1 - x_P) = \int_{\mu_N} \int_{\mu_E}(p_N \mu_N + p_E \mu_E)\{\log(B_1 - x_P + lx_P)\}h(\mu_E)d\mu_Ef(\mu_N)d\mu_N + p_N \mu_N \log(\frac{p_N}{p_N + p_E \mu_E}) + p_E \mu_E \log(\frac{p_E \mu_E}{p_N + p_E \mu_E}) \]

Since the terms \( \mu_N \) and \( \mu_E \) are in a linear form,
\[ H_2(B_1 - x_P) = (p_N \mu_N + p_E \mu_E)\{\log(B_1 - x_P + lx_P)\} + p_N \mu_N \log(\frac{p_N}{p_N + p_E \mu_E}) + p_E \mu_E \log(\frac{p_E \mu_E}{p_N + p_E \mu_E}) \]

The FOC of \( V_1(B_1) \) with respect to \( x_P \) gives us:
\[ \frac{\partial V_1}{\partial x_P} = (p_N \mu_N + p_E \mu_E)\{\frac{1}{x_P + 1} \} \] which gives \( x_P^* = \frac{\mu_N}{(1-\mu_N)(\mu_E + p_N \mu_N + p_E \mu_E)} B_1 \), which is the same as the case where \( \mu_N \) and \( \mu_E \) are known with certainty, with the means of the random variables replacing the deterministic values. ■

**Proof of Result 2:** To find the optimal investments made by the firms under competition, we start with the intermittent in the last period as usual. If the opportunity arrives to invest in a product extension for technology \( C \), and the firms had invested an amount of \( x_P \) and \( y_P \) respectively in the first period on the development of a product platform, then \( V_{3x}(B_{3x}) = \mu_E \log[\alpha(B_{3x} + lx_P) - \beta(B_{3y} + ly_P)] \) and \( V_{3y}(B_{3y}) = \mu_E \log[\alpha(B_{3y} + ly_P) - \beta(B_{3x} + lx_P)] \). Hence, as there are no further opportunities available in the horizon, and the firms earn a return based on the additional investment of lx_P and ly_P due to the learning effect. If no opportunity arrives for investment in the third period, then \( V_{3x}(B_{3x}) = 0 \) and \( V_{3y}(B_{3y}) = 0 \).

Hence, \( H_{3x}(B_{3x}) = p_E \mu_E \log[\alpha(B_{3x} + lx_P) - \beta(B_{3y} + ly_P)] \) \( \text{(R3.1)} \)
and \( H_{3y}(B_{3y}) = p_E \mu_E \log[\alpha(B_{3y} + ly_P) - \beta(B_{3x} + lx_P)] \) \( \text{(R3.2)} \)
We rewrite equations (R3.1) and (R3.2) as
\[ H_{3x}(B_{3x}) = p_E \mu_E \log \alpha + p_E \mu_E \log[\mu_N (B_{3x} + lx_P) - \frac{\alpha}{s}(B_{3y} + ly_P)] \]
and \( H_{3y}(B_{3y}) = p_E \mu_E \log \alpha + p_E \mu_E \log[\mu_N (B_{3y} + ly_P) - \frac{\alpha}{s}(B_{3x} + lx_P)] \).

Let \( s = \frac{\alpha}{\mu_N} \). Since \( \alpha > \beta \), \( s < 1 \).

In the beginning of the second period, if an opportunity to invest in developing a product for technology \( N \) arises for firms \( X \) and \( Y \), since the term \( p_E \mu_E \log \alpha \) is a constant, the firms solve
\[ V_{2x}(B_{2x}, x_N) = \max_{\pi_N \geq 0} \mu_N \log(\delta (x_N - y_N)) + p_E \mu_E \log [(B_{2x} - x_N + lx_P) - s(B_{2y} - y_N + ly_P)] \]
\[ V_{2y}(B_{2y}, y_N) = \max_{\pi_N \geq 0} \mu_N \log(\delta (y_N - x_N)) + p_E \mu_E \log [(B_{2y} - y_N + ly_P) - s(B_{2x} - x_N + lx_P)] \]

Since the logarithm function is concave in \( x_N \) and \( y_N \), it follows trivially that the optimal value of \( x_N \) and \( y_N \) in the Nash subgame path in period 2 is given by the first order conditions. As \( \frac{1}{s} = t \), solving for the FOC gives us:
\[ x_N = \frac{(\mu_N + p_E \mu_E)}{(1 - \mu_N + p_E \mu_E)} \left[ (1 - s \frac{\mu_N + p_E \mu_E}{\mu_N + p_E \mu_E})(B_{2x} + lx_P) - \frac{\mu_N}{\mu_N + p_E \mu_E} \right] \]
\[ y_N = \frac{(\mu_N + p_E \mu_E)}{(1 - \mu_N + p_E \mu_E)} \left[ (1 - s \frac{\mu_N + p_E \mu_E}{\mu_N + p_E \mu_E})(B_{2y} + ly_P) - \frac{\mu_N}{\mu_N + p_E \mu_E} \right] \]

Substituting this back into the value functions for the 2nd period gives us,
\[ V_{2x}(B_{2x}, x_N) = (\mu_N + p_E \mu_E) \log[D_1(1 + lx_P) - N_1(B_{2y} + ly_P)] + \text{consts} \]
\[ V_{2y}(B_{2y}, y_N) = (\mu_N + p_E \mu_E) \log[D_1(1 + lx_P) - N_1(B_{2y} + ly_P)] + \text{consts} \]
where \( D_1 = \frac{\mu_N}{\mu_N + p_E \mu_E} - \frac{\mu_N + p_E \mu_E}{\mu_N + p_E \mu_E} s \frac{\mu_N}{\mu_N + p_E \mu_E} + t \frac{\mu_N}{\mu_N + p_E \mu_E} \)
and \( N_1 = s \frac{\mu_N}{\mu_N + p_E \mu_E} - \frac{\mu_N + p_E \mu_E}{\mu_N + p_E \mu_E} s \frac{\mu_N}{\mu_N + p_E \mu_E} + t \frac{\mu_N}{\mu_N + p_E \mu_E} - s \frac{\mu_N}{\mu_N + p_E \mu_E} \).
If an opportunity for investing in the development of a product for technology \( N \) does not arrive in the second period, it follows trivially that
\[
V_{2x}(B_{2x}) = p_E\mu_E\log[(B_{2x} + lx_P) - s(B_{2y} + ly_P)] + \text{const} \quad \text{and} \quad V_{2y}(B_{2y}) = p_E\mu_E\log[(B_{2y} + ly_P) - s(B_{2x} + lx_P)] + \text{const}. \quad \text{Hence,}
\]
\[
H_{2x}(B_{2x}) = p_N\left(\left(\mu_N + p_E\mu_E\right)\log[D_1(B_{2x} + lx_P) - N_1(B_{2y} + ly_P)] + (1 - p_N)p_E\mu_E\log[(B_{2x} + lx_P) - \beta(B_{2y} + ly_P)]\right) + \text{const} \quad \text{... (R3.3)}
\]
\[
H_{2y}(B_{2y}) = p_N\left(\left(\mu_N + p_E\mu_E\right)\log[D_1(B_{2y} + ly_P) - N_1(B_{2x} + lx_P)] + (1 - p_N)p_E\mu_E\log[(B_{2y} + ly_P) - \beta(B_{2x} + lx_P)]\right) + \text{const} \quad \text{... (R3.4)}
\]

In the beginning of the first period, when the firms have the opportunity to invest in developing a product platform based on technology \( N \), they solve
\[
V_{1x}(B_x, x_P) = \max_{x_P \geq 0} \left[p_E(x_P - s) + p_N(\mu_N + p_E\mu_E)\log[D_1(B_x - x_P + lx_P) - N_1(B_y - y_P + ly_P)] + (1 - p_N)p_E\mu_E\log[(B_x - x_P + lx_P) - s(B_y - y_P + ly_P)] + \text{const} \right]
\]
\[
V_{1y}(B_y, y_P) = \max_{y_P \geq 0} \left[p_E(y_P - s) + p_N(\mu_N + p_E\mu_E)\log[D_1(B_y - y_P + ly_P) - N_1(B_x - x_P + lx_P)] + (1 - p_N)p_E\mu_E\log[(B_y - y_P + ly_P) - s(B_x - x_P + lx_P)] + \text{const} \right]
\]

Denote \( \frac{N_1}{N_2} \) as \( \lambda_1 \). Since the logarithm function is concave in \( x_P \) and \( y_P \), it follows trivially that the optimal value of \( x_P \) and \( y_P \) in the first stage Nash subgame-perfect equilibrium is given by the first order conditions.

The reaction functions in the SPNE for \( x_P \) and \( y_P \) are given by:
\[
\frac{\mu_E}{x_P - s_P} + \frac{\mu_N(\mu_N + p_E\mu_E)D_1}{(1 - \lambda_1)(\mu_E + p_N\mu_N + p_E\mu_E)D_1} + \frac{(1 - p_E)\mu_E}{(1 - \lambda_1)(\mu_E + p_N\mu_N + p_E\mu_E)} = 0 \quad \text{... (R3.5)}
\]
\[
\frac{\mu_E}{y_P - s_P} + \frac{\mu_N(\mu_N + p_E\mu_E)}{(1 - \lambda_1)(\mu_E + p_N\mu_N + p_E\mu_E)D_1} + \frac{1 - p_E}{(1 - \lambda_1)(\mu_E + p_N\mu_N + p_E\mu_E)} = 0 \quad \text{... (R3.6)}
\]

Setting \( x_P = y_P \) and \( B_x = B_y \) gives us the stated result.

**Proof of Proposition 3:**

**Case 1:** \( s = t \). If \( s = t \), then
\[
\lambda = \frac{s^4 + 2s^3 + s^2 + 2s + 1}{s^4 + 2s^3 + s^2 + s} = \frac{1}{s} - \frac{1}{s^2} = s, D_1 = 1 - s^2, N_1 = s - s^3.
\]

Substituting the above identities into reaction functions (R3.5) and (R3.6) gives us:
\[
x_P = sy_P + \frac{\mu_E(B_x - s_B)}{\mu_E + p_N\mu_N + p_E\mu_E},
\]
\[
y_P = sx_P + \frac{\mu_E(B_y - s_B)}{\mu_E + p_N\mu_N + p_E\mu_E}.
\]

Solving the above two equations simultaneously gives us:
\[
x_P = \frac{(1 - \lambda_1)(\mu_E + p_N\mu_N + p_E\mu_E)}{\mu_E}(B_x) \quad \text{...(P2.1)}
\]
\[
y_P = \frac{(1 - \lambda_1)(\mu_E + p_N\mu_N + p_E\mu_E)}{\mu_E}(B_y) \quad \text{...(P2.2)}
\]

A comparison of Equations (P2.1) and (P2.2) show that when \( s = t \), the optimal investments made by the competing firms are the same as those invested by the monopolist in Result 1, hence, when \( s = t \), both firms invest in the same way as the monopolist.

**Case 2:** \( s > t \). If \( s > t \), then
\[
\lambda = \frac{s^4 + 2s^3 + s^2 + 2s + 1}{s^4 + 2s^3 + s^2 + s} = \frac{1}{s} - \frac{1}{s^2} = s, D_1 = 1 - s^2, N_1 = s - s^3.
\]

We prove that in this case, \( x_P \) and \( y_P \) are greater than the corresponding investments in the platform by the monopolist by contradiction. Denote the monopolist platform investments as \( x_M \) and \( y_M \) respectively (given by Result 1). Let \( x_P = x_M + k \) and \( y_P = y_M - l \), where \( k, l > 0 \). We show that this leads to a contradiction to the condition of \( y_M \) derived above. Rewrite Equation (R3.6) as
\[
\frac{\mu_P}{y_P-x_P} = \frac{\mu_N}{y_M-x_M} - \frac{\mu_E}{y_E-x_E} (1-l)
\]

The LHS of this equation is
\[
\frac{\mu P}{y P-x P} = \frac{\mu P}{y M-x M} > \frac{\mu P}{y M-x M}
\]

The RHS of Equation (R3.6) is
\[
\frac{\mu N}{y N-x N} + \frac{\mu E}{y E-x E} (1-l)
\]
\[
< \frac{\mu N}{y M-x M} + \frac{\mu E}{y E-x E} (1-l)
\]

which is a contradiction of the condition for \(y^*_M\). Hence, \(x^*_P > x_M\) and \(y^*_P < y_M\) cannot be true. The cases of \(x^*_P < x_M\) and \(y^*_P > y_M\), and \(x^*_P < x_M\) and \(y^*_P < y_M\) also result in contradictions. Hence, \(x^*_P > x_M\) and \(y^*_P > y_M\).

**Case 3:** Similar to Case 2.

The proof of the rapid roadmap case is analogous to the intermittent roadmap case. ■