Electric Vehicles with a Battery Switching Station: Adoption and Environmental Impact
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ELECTRIC VEHICLES WITH A BATTERY SWITCHING STATION: ADOPTION AND ENVIRONMENTAL IMPACT

Abstract. Widespread adoption of Electric Vehicles can limit the environmental impact of transportation and reduce oil dependence. However, limited range and high upfront battery costs have limited consumer adoption. A novel switching-station-based solution is extensively touted as a promising remedy that resolves range anxiety. Vehicles use standardized batteries that when depleted can be switched for fully charged batteries at switching stations. Further, instead of making an upfront battery purchase, motorists pay for miles driven.

We develop a stylized analytical model that captures the key tradeoffs in the adoption of electric vehicles to assess the effectiveness of this remedy. Our model uses a classical repairable item inventory model to capture switching station operation; we combine it with a moral hazard construct from the contracting literature to capture customer adoption and usage. We find that electric vehicles with switching stations can indeed incent adoption and reduce oil dependence but, paradoxically, we also show that this increased adoption may not necessarily benefit the environment. A profit-maximizing operator increases adoption by limiting motorist range anxiety and the effective marginal costs of driving, which leads motorists to increase their driving, and hence increase electricity consumption. Depending on the source of electricity, this can be more harmful to the environment than the non-adoption of electric vehicles. Further, we show that switching-station electric-vehicle adoption and driving are strategic complements; thus, any policy intervention that increases adoption will also increase driving. Using real data, we calibrate the model and show numerically that with the current generation mix in the USA, switching-station electric vehicles would lead to reduced oil dependence and net environmental benefits, but in just 10 years electric vehicles with switching stations would be harmful to the environment. Further, well-intended policy interventions such as battery purchase subsidies, and seemingly helpful battery technology advances can actually be harmful to the environment.

1. Introduction

The transportation sector is a substantial contributor to carbon dioxide emissions (20%-25%), with its emissions growing faster than any other energy-using sector (here and later, see World Energy Council (2007)). Increasing geopolitical uncertainties have also highlighted the vulnerabilities of a transportation infrastructure based on oil. Sustainable (or green) means of transportation have thus attracted increasing attention from environmentalists, governments, industry and academics. Despite many efforts over the past few decades to make transportation systems sustainable, over

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95% of transport energy still comes from oil. Green vehicles with technologies including electrical, bio-fuel, hydrogen and natural gas, all hold the promise of reducing oil dependence and lessening environmental impact, but, with rare exceptions, implementation of these technologies is currently very limited due to technological, political and other issues (see Struben and Sterman (2008) for historical notes).

Electric vehicles historically predate gasoline vehicles, but have only received mainstream interest in the last decade (see Eberle and Helmolt (2010)). The first mass-produced hybrid gasoline-electric vehicle, the Toyota Prius, was introduced in 2003, and the first mass-use battery-powered electric car, the Nissan Leaf, in 2010. Most other major automakers are in the process of launching their own versions of electric vehicles. However, the adoption of electric vehicles has been minimal, mainly due to two widely accepted limiting factors. The first is range anxiety, a term introduced to mean the fear that a vehicle has insufficient range to reach its destination (Eberle and Helmolt (2010)). While this term equally applies to electric and gasoline vehicles, the former usually have range limitations of about 100 miles on a single charge, and unlike its gas-fueled counterpart, an electric vehicle takes hours to recharge. The second factor is the cost of the battery (around $15,000 at the moment), which is typically the most expensive component of the electric vehicle, driving the cost difference between electric and gasoline vehicles. Although the running cost of an electric vehicle is far lower than that of a gasoline vehicle, the higher upfront costs deter many adopters despite governmental subsidies and tax breaks. Over the last 150+ years, numerous technological advances have been targeted towards making cheaper batteries with longer range but, to this point, their success has been insufficient.

A startup company, Better Place Inc. (Girotra and Netessine (2011a); Mak et al. (2012)) is attempting to address these two limiting factors. Its novel mobility system combines (1) a network of battery switching stations and (2) a payment system such that the motorist is charged per mile driven while the company owns the batteries. The switching stations would be widely accessible and would allow a motorist to exchange a depleted battery for a fully charged one in 90 seconds or less. Since the motorist would potentially have different batteries at different points of time, the batteries would be owned by the firm. Thus, rather than paying the large upfront cost of the battery, the motorist would pay per mile driven. This mobility system still includes the traditional charging stations at a number of locations, with all electricity costs paid by Better Place. Components of this system are not entirely new: the switching stations have been long used for forklift trucks (Timmer (2009)), and pay-per-use contracts have long been used by, for instance, mobile phone companies. The mobility system of Better Place combines the two elements, and companies such as Tesla Motors and Mitsubishi are currently working on similar mobility systems.
The advantages of this switching-station mobility system are apparent: it eliminates the two key barriers to adoption of electric vehicles described above. There are, however, some hidden, as yet unstudied, disadvantages. In addition to the extensive charging infrastructure and the need to standardize batteries to make them swappable, the mobility system must hold more batteries than the number of cars deployed. Presumably, the cost of these (very expensive) extra batteries will depend on the demand dynamics and the service level that the company wants to provide at the switching station. Nevertheless, Better Place has attracted a lot of attention, with venture capitalists valuing it at over $2.25B, the first annual Green Car Breakthrough Award given to the company, and several countries (including Israel, Denmark, China and Australia) signing agreements with Better Place to start service in 2011-2012, with dozens of others negotiating terms right now.

While the advantages of this novel mobility system are evident and the enthusiasm around it is huge, there is, to our knowledge, no rigorous analysis and comparison of this mobility system with more traditional battery-powered electric vehicles or with gasoline vehicles in terms of their adoption and environmental impact, which is exactly the goal of this paper. Thus, our first contribution is in proposing a model for the switching-station mobility system. We posit that charging and storing batteries is similar to a classical repairable items inventory system in which running out of charge is equivalent to “failing” and the recharging process is equivalent to “repair”. We combine this classical inventory model with a principal-agent moral hazard model of consumer and provider-firm behavior in which the amount of driving is uncertain and depends on the contractual arrangement (pay-per-use) between the company and the consumer. Consumers obtain utility from the driving itself and possibly from driving a green vehicle, but must pay for the ability to do so, and they might also incur other costs such as the inconvenience of running out of charge.

Our second contribution is in analyzing this model, characterizing the optimal solution, and identifying some surprising structural properties. On the motorist side, we determine the optimal driving decision and the equilibrium adoption of the technology, and for the mobility system operator, we find the optimal price per mile driven and the inventory of spare batteries. Using these solutions, we obtain several insightful structural results. First, we show that, under very general conditions, the optimal motorist adoption of the mobility system and the optimal driving decisions are strategic complements: that is, any policy intervention (e.g., subsidies, technology advances etc.) would have the same directional effect for both the adoption and the driving. Interestingly, this implies that seemingly environmentally friendly policy changes (e.g., electric vehicle subsidies) would lead to higher adoption, but also to more driving of electric vehicles. While the former should reduce oil dependence and pollution by shifting consumers from gasoline to electric vehicles, the latter increases electricity consumption which, in most countries, is still obtained using non-green technologies. Hence, the
end-effect for the environment is unclear. In fact, we show that, quite often, the mobility system may achieve higher adoption than conventional electric vehicles but, at the same time, lead to higher emissions. In contrast to conventional wisdom, the twin objectives of reducing oil dependence and environmental impact may in fact be in conflict with each other.

Our third contribution is in conducting a large-scale numerical comparison between the switching-station system, conventional electric vehicles and gasoline vehicles. We calibrate our study using 20+ parameters obtained from empirical studies and industry reports. We find that the environmental advantages of the switching-station system critically depend on the mix of technologies used to obtain electricity. For countries in which the majority of electricity is obtained from sustainable sources (e.g., France), we find that the Better Place mobility system is environmentally advantageous now or in the near future. At the same time, countries in which electricity is obtained using a mix of technologies (e.g., USA) would benefit from the adoption of the mobility system now but not in the future when the battery cost decreases and, as a result, the driving of electric vehicles increases. Finally, in certain countries in which almost all electricity comes from dirty sources (e.g., China) adoption of the mobility system would be detrimental now and even more so in the future. Our analysis also illustrates that taxing gasoline (rather than, say, providing battery purchase or development subsidies) is by far the most effective approach to increasing the adoption of electric vehicles.

2. Literature Review

Our work contributes to the growing sustainable operations management literature. Sustainability has become a prominent topic in operations management over recent years, especially given the growing interest in the effects of global warming and corporate social responsibility. Kleindorfer et al. (2005) provide a review of papers integrating sustainability into operations management. Adoption of green practices and associated arrangements is a key topic in the sustainable operations literature. Corbett and Muthulingam (2007) study the adoption of green practices using empirical data to identify the limiting factors. Lobel and Perakis (2011) develop a model for the adoption of solar photovoltaic technology by residential consumers. Akin to the battery contract in this paper, Agrawal et al. (2012) study the environmental impact of pay-per-use contracts (leasing) versus outright purchases in the context of durable products. Taking a life-cycle environmental impact perspective, they identify conditions such that leasing is a superior strategy for the provider firm. These papers study the same adoption and environmental impact issues we do, but for products that do not have any of the demand and use dynamics arising from driving, charging, and battery-switching in our model.

A prominent body of work in the sustainable operations literature focuses on remanufacturing and closed-loop supply chains (cf. Debo et al. (2005) and Flapper et al. (2005) for comprehensive overviews
of this literature). Our work is not directly related to closed-loop supply chains, but rather focuses on the integration of the environmental and resource impacts of a “green” product, in our case an electric vehicle. But since we consider demand-side economics (i.e., the customer’s decision to adopt and use the electric vehicle), our model can be thought of as a closed-loop remanufacturing system (cf. Atasu et al. (2008) and Oraiopoulos et al. (2012)). However, to our knowledge, our paper is the first to use a repairable inventory model in this context.

The research on electric vehicles in operations management is extremely limited. Chocteau et al. (2010) use cooperative game theory to investigate the impact of collaboration and intermediation on the adoption of electric vehicles among commercial fleets, and they determine the conditions under which adoption becomes economically feasible. Worley and Klabjan (2011) and Mak et al. (2012), to the best of our knowledge, are the only papers that study a switching station model. Worley and Klabjan (2011) study the timing and scheduling of charging decisions based on dynamic electricity rates, but they sidestep the battery management and contracting issues that we focus on. Mak et al. (2012) develop models that help the planning process for deploying battery switching network infrastructure and the battery management at the switching stations. Our analysis includes the battery management problem in a similar way, but we use our analysis to examine the effectiveness of a switching station model in reducing dependence on imported oil and the environmental impact of driving. Struben and Sterman (2008) is the only study that we could identify that models the dynamics of alternative fuel vehicle adoption taking the consumer awareness perspective. Another related stream of work is sustainable energy systems (Wu (2011)).

The analysis in this paper uses the results developed in two streams of literature: the repairable item inventory planning and the contracting (principal-agent) literatures. Repairable inventory models such as the METRIC model (Sherbrooke (1968)) have been widely applied to the management of critical spare parts for the aerospace and defense industries (cf. Muckstadt (2005) and Sherbrooke (2004) for recent developments). The switching station in our model can be thought of as an application of this literature to a novel context where an inventory of electric vehicle batteries at switching stations must be managed. Following a vast body of literature in operations management, we use the principal-agent framework to represent the contractual relationship between the provider firm (the principal) and the motorists (the agents) (cf. Bolton and Dewatripont (2005) for a comprehensive review of principal-agent models and Cachon (2003) for their use in the operations management literature). In the sustainable operations literature, Savaskan et al. (2004) apply these models to study a reverse-channel structure for the collection of used products. Like our paper, some recent models have combined contracting mechanisms with repairable parts inventory systems (cf. Kim et al.
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(2010)), albeit in a context with none of the vehicle use, adoption, battery charging and swapping dynamics that exist in our context.

Taken together, to the best of our knowledge, our paper is the first to model the effectiveness of different electric vehicle models for decreasing oil dependence and greenhouse gas emissions.


We develop a model of a population of motorists that make utility-maximizing choices between electric and fossil-fuel vehicles, taking into account subsequent utility from the use of the vehicle. We consider the effectiveness of two alternative electric vehicle systems: 1) The Conventional Electric vehicle system, whereby an electric vehicle with limited range is sold to a motorist, and 2) the Switching-Station system, whereby the motorist has access to a network of "switching stations", which allow the switching of depleted batteries for fully charged batteries, thereby enhancing the range of the vehicle. In the former system, a profit-maximizing firm prices and sells a vehicle with a battery to the motorist. In the latter model, a profit-maximizing firm establishes and stocks the switching stations, and prices the service per mile driven. Motorists make utility-maximizing choices about whether to adopt electric vehicles and how much to drive. We next present a model that captures the most salient features of this setup; an extensive discussion of alternative assumptions and model enhancements is provided in the concluding section.

3.1. Motorist Behavior.

3.1.1. Utility from owning a vehicle. We base our model of motorist behavior on existing empirical work on driving habits. Precise estimates of all parameters in the motorist behavior model described below are provided in the subsequent numerical experiments section. We capture a motorist’s utility through four additive components:

(1) Utility from driving: Motorists derive utility from how much they drive each day. This utility, $u(\cdot)$, is increasing in the miles driven, with diminishing marginal returns, $u'(\cdot) > 0$, $u''(\cdot) < 0$.

(2) Range Inconvenience: Motorists incur a disutility, $M$, whenever their daily driving exceeds the range of the vehicle, $R$.\(^1\) This disutility includes the inconvenience from waiting for the electric vehicle battery to be recharged, excessive depreciation of the battery if using a fast-charge option, and even using an alternate means of transport to reach a destination, etc.

\(^1\)Most electric vehicle batteries can be fully recharged in less than 8 hours, typically during the night. Due to the limited availability of a charging-point network, they are rarely charged during the day (cf. Denholm and Short (2006) for charging characteristics). Thus, our model assumes that each day the full range of the battery is available. With simple modifications, we can also consider cases in which less than the range is available due to an inability to charge during the night, or more than the range is available due to charging during the day. Neither case alters our main insights.
(3) **Green Utility:** Motorists derive additional utility from owning electric vehicles: a "green" utility. This green utility varies in our population of motorists. In particular, we assume that our population's green utility, \(\tilde{U}_{gr}\) is uniformly distributed in the interval \([0, d]\).\(^2\)

(4) **Direct Costs:** Motorists incur the costs of owning and operating the vehicle. Depending on the vehicle type and associated operating model, this potentially includes the initial purchase price of the vehicle, the fuel, the electricity, the battery or per-mile charges, taxes and repairs. We normalize the initial purchase price of a fossil-fuel vehicle and that of an electric vehicles without batteries to zero.\(^3\),\(^4\) Further, we assume that the life-cycle of all vehicles is the same.

We assume that the distance driven per day by a motorist has a planned or controlled component, \(e\), and an unplanned or random component, \(\epsilon\). The planned component captures the average daily distance that a motorist anticipates or plans to drive based on the best information available on driving needs and costs at the time of vehicle purchase. The planned driving is a decision made by the motorist at the time of the purchase because she would like to know how much on the average she expects to drive to quantify the average utility from owning the vehicle. The unplanned component captures any additional driving due to subsequent changes in life situation, driving needs, costs of driving, unexpected detours on any day, traffic conditions, etc. In contrast to the planned component, the unplanned driving is realized on a daily basis and unknown at the time of the purchase. Hence, at the time of purchase, we model this unplanned driving as a random variable, \(\epsilon\), that will be drawn each day from a zero-mean, finite-variance distribution with density \(g(\cdot)\).


With the extensive network of gas stations, fossil-fuel vehicles essentially have an unlimited range and consequently motorists who decide to own such a vehicle never incur range inconvenience, nor do they derive any green utility. The direct costs of use of such a vehicle include the purchase price of the vehicle and the fuel costs for each mile. A utility-maximizing owner of such a vehicle can maximize her ownership utility by planning to drive \(m\) miles per day, such that \(\mathbb{E}[u'(m + \epsilon)] = c_{gas}\), where \(c_{gas}\) is the per-mile fuel cost. If this utility is positive, the motorist purchases the vehicle, else she does not own a vehicle. Specifically, owners earn an expected daily utility, \(U_{gas} \equiv \left(\mathbb{E}[u(m + \epsilon)] - c_{gas}m\right)^+\), and drive \(e_{gas}^* \equiv m \cdot I\{\mathbb{E}[u(m + \epsilon)] > c_{gas}m\}\) miles, where \(I\) is the indicator function.

\(^2\)All results in the subsequent sections also hold for a general distribution of \(\tilde{U}_{gr}\).

\(^3\)90\% or more of the incremental cost of electric vehicles arises out of the battery pack that comprises individual battery modules, an enclosure for the modules, management systems, terminals and connectors, and any other pertinent auxiliaries (Simpson (2006); Pistoia (2010)).

\(^4\)We note that this normalization is not essential, i.e., any difference in the initial purchase prices can be captured by the green utility, \(\tilde{U}_{gr}\), which can be interpreted as the additional utility (or disutility) from owning an electric vehicle, including any purchase, tax subsidies, etc.
3.2. Conventional Electric Vehicles.

3.2.1. Preliminaries. Owners of conventional electric vehicles buy a vehicle with a battery installed and are responsible for all subsequent costs. Specifically, the conventional electric vehicle operating model proceeds along the following steps (Figure 3.1). First, the provider firm offers a selling price, $F_{ce}$, the price premium for the electric vehicle over and above fossil-fuel vehicles. Motorists choose between fossil-fuel or electric vehicles based on their expected utility of ownership, and they then decide on the planned driving, $e^*$, based on the relevant marginal costs and benefits. Finally, for each day of ownership, the random unplanned component of driving is realized. If electric vehicle owners end up driving more than the range of the battery, they incur the range inconvenience penalty.

3.2.2. Electric Vehicle Pricing, Adoption and Use. As is typical in sequential games, we solve for the equilibrium choices using backward induction, starting from the planned driving best response, followed by the adoption response and the pricing decisions. Owners of electric vehicles plan their driving to maximize their expected utility from their use of the vehicle. Specifically, motorists solve the following optimization problem to obtain their optimal driving best response:

$$\max_{e} \left[ E[\epsilon] \left[ u(e + \epsilon) \right] - c_e e - M \cdot \bar{G}(R - e) \right],$$

where $c_e$ is the per-mile operating cost, in this case, the cost of charging and maintaining the battery.\(^5\)

The utility from ownership of the vehicle, denoted by $U_{ce}$, is the above maximized use utility plus the green utility, $\tilde{U}_{gr}$, minus the purchase price, $F_{ce}$.\(^6\) The customers for whom this utility exceeds the utility from a fossil-fuel vehicle (which we assume to be the status quo) will migrate to electric vehicles. The provider firm must decide on the purchase price, $F_{ce}$, to charge for the batteries.\(^7\) Increasing the purchase price increases margins, but reduces the ownership utility and consequently the adoption of and demand for electric vehicles. The firm trades off these two concerns to arrive at

\(^5\)To guarantee a unique solution, we subsequently assume $E[\epsilon^2] + M g'(R - e) < 0$.

\(^6\)All utility and cost values are normalized to a daily level with the daily purchase price $F_{ce}$ leading to a total purchase price of $F_{ce}/i$ with an interest rate of $i$.

\(^7\)Note that in our model, the provider firm has pricing power with respect to the selling price of electric vehicles, but the price of a fossil-fuel vehicle is exogenous (and normalized to zero). This assumption is consistent with the highly competitive fossil-fuel vehicle market and the much less competitive electric vehicle market.
the optimal price. Specifically, the firm solves the following maximization problem:

$$\max_{F_{ce}} E \left[ I \{U_{ce} > U_{gas}\} \cdot (F_{ce} - c) \right],$$

where $c$ is the cost of battery normalized to a daily level.

**Lemma 1. Equilibrium Adoption and Driving of Conventional Electric Vehicles**

a) Owners of conventional electric vehicles plan on driving $e_{ce}^*$ miles, where $e_{ce}^*$ is such that

$$E_{e} \left[ u' \left( e_{ce}^* + \epsilon \right) \right] - M \cdot g \left( R - e_{ce}^* \right) = c_e.$$ 

b) The firm prices the battery such that, in equilibrium, $A_{ce}^*$ fraction of motorists adopts the vehicles

$$2dA_{ce}^* = E_{e} \left[ u \left( e_{ce}^* + \epsilon \right) \right] - c_e e_{ce}^* - M \bar{G} \left( R - e_{ce}^* \right) + d - U_{gas} - c.$$ 

**Proof.** Detailed proofs are provided in the accompanying technical appendix. □

The equilibrium driving decision (Eq. 3.1) is determined by: the trade-off between the motorist’s marginal gain from an extra mile of driving, i.e., the increase in driving utility; the change in the risk of incurring the range-inconvenience penalty; and the marginal cost of driving, the per-mile costs of charging and maintaining the battery. The adoption decision (Eq. 3.2) is driven by the pricing of the firm. Our setup is similar to a monopoly pricing situation in which the uniformly distributed green utility leads to a classical linear downward sloping demand curve. As is typical in such a setting, the profit-maximizing price of the vehicle is such that it attracts half the viable market, that is half of the population for whom the utility of owning the vehicle is higher than the cost. In our setup, this is half of the maximum ownership utility, $E_{e} \left[ u \left( e_{ce}^* + \epsilon \right) \right] - c_e e_{ce}^* - M \bar{G} \left( R - e_{ce}^* \right) + d$, minus the gas utility, $U_{gas}$, and the incremental cost of provisioning electric vehicles, the cost of battery, $c$.

As expected, the adoption and driving decrease in the per-mile cost of battery charging and maintenance and a higher average green utility increases the adoption of electric vehicles. Further, Lemma 1 highlights the two key effects of the limited range of conventional electric vehicles. First, motorists who own such vehicles face the risk of exceeding the battery range. While the direct marginal costs of driving would suggest an average driving level such that $E_{e} \left[ u' \left( e_{ce}^* + \epsilon \right) \right] = c_e$, due to the range inconvenience penalty, $M$, average driving is lower and higher values of the penalty or a smaller range lead to less driving as captured by Eq. 3.1. Second, range anxiety also reduces the adoption of electric vehicles, as is evident from Eq. 3.2. Hence, our model captures the two key features that are relevant in this setting. Owners of electric vehicles are anxious about exceeding the vehicle’s range, so they plan to drive it less, which reduces their utility from owning an electric vehicle. This
observation implies that only those motorists who highly value having a green vehicle would buy it, so fewer motorists switch to electric vehicles than would if electric vehicles had an unlimited range.

3.3. Electric Vehicle with a Switching Station.

3.3.1. Preliminaries. The setup of our model is inspired by and directly follows the operating model of the electric vehicle startup, Better Place (Girotra and Netessine (2011a); Mak et al. (2012)). There are two main points of departure from the conventional electric vehicle model above. 1) Instead of incurring the range-inconvenience penalty, motorists whose driving exceeds the vehicle’s range can now utilize a battery switching station. This switching station has a limited stock of fully charged batteries and the motorist can swap her depleted battery for a fully charged battery. The received depleted batteries are plugged in to charging bays and once charged, they are moved to the stock of fully charged batteries. 2) Instead of paying directly for the electricity consumed to charge the electric vehicle or for the battery, the motorist pays the provider firm for miles driven. The provider firm incurs the cost of charging and maintaining the batteries, be it batteries obtained at the switching station or charged at home. The switching station business model proceeds as follows (Figure 3.2). The provider firm proposes a price for the vehicle, $F_{ss}$, sets a per-mile price, $p_{ss}$, and commits to providing a level of availability for charged batteries. Based on these terms and their idiosyncratic preference for a green vehicle, motorists choose between it and a fossil-fuel vehicle. Based on the fraction of the population that adopts electric vehicles, the provider firm procures batteries both for cars and for the switching station. Motorists decide on their planned driving. Finally, for each day of ownership, the random unplanned component of driving is realized. If electric vehicle owners end up driving more than the vehicle’s range, they visit the switching station. If the station has batteries in stock, the motorist drives away with a replenished battery. If she does not find a battery in stock, she incurs the range inconvenience penalty. We formulate this problem as a sequential game in which the firm decides on the stocking level of batteries and the pricing while motorists select their vehicle types and daily driving. Identifying the equilibria in this game requires us to analyze the operational dynamics of the switching-station model embedded within a pricing and consumption game.
3.3.2. Analysis of the Switching Station. At the heart of this novel operating system for electric vehicles lies the switching station. There are two components to analyzing this system. 1) the demand process that arises from motorists driving and exceeding the vehicle range; and 2) the charging facility.

Demand Process: Demand for a battery occurs when any motorist exceeds the range of the vehicle, i.e., with probability $G(R - e_{ss})$, where $e_{ss}$ is the planned driving. Consider a market with a population of $N$ customers, of which a fraction $A_{ss}$ adopts these electric vehicles. Assuming $N$ is large, the probability $G(R - e_{ss})$ is small and the arrivals are stationary, the demand at the switching station is a Poisson process with a mean arrival rate $A_{ss}N \cdot G(R - e_{ss})$ (Karlin and Taylor (1975)).

Charging Facility: We conceptualize the charging facility (as illustrated in Figure 3.3) as a repairable spare parts facility (see Muckstadt (2005) and Sherbrooke (2004)). Depleted batteries correspond to broken parts, the charging process to the repair process, and charged batteries to the stock of spare parts. We adapt and develop the extensive literature on managing spare parts inventory to our setup. The battery charging process takes a random amount of time, with a mean service time of $\tau$ time units. As is typical in these stations, a large number of (cheap) charging bays is available. Once the battery is fully charged, it is placed in the station’s inventory. As suggested by Mak et al. (2012), we assume that the charged batteries are reused in a first-in-first-out (FIFO) order. This charging process can be modeled as an $M/G/\infty$ queuing system, which is typical in the repairables literature.

The provider firm chooses a spare battery inventory level, $Q$. At any given point in time, $O$ of these batteries are in the process of being charged, while $(Q - O)^+$ others are available for arriving motorists. If a motorist arrives and no battery is available, $Q < O$, she incurs the range inconvenience penalty, and we assume that she waits at the station for a new battery; that is, her demand is back-ordered. From Palm’s theorem, we know that, in a steady state, $O$ is Poisson-distributed (Feeney and Sherbrooke (1966)). Following standard practice for large-scale repairable service parts systems (see Kim et al. (2007)), for all further analysis we analyze $O$ as a continuous random variable that is distributed normally with mean and variance $\tau A_{ss}N \cdot G(R - e_{ss})$. Further, we advance the standard setup by taking a slightly more complex and, we believe, more realistic approach by considering the

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8For a typical US motorist, the probability $G(R - e_{ss})$ is only 6% (based on a battery range of 100 miles (Better Place Blog (2011)) and daily driving distributions provided by the US Department of Transportation Federal Highway Administration (Hu and Reuscher (2004))). See the data-based analysis in Section 5 for details.
9The fixed demand rate assumption is in fact an approximation, because the closed-loop cycle with finite population means $\lambda \tau$ is a function of the number of operating cars. For example, in the case that a replacement is not available and a back-order occurs, the motorist waits at the station until the battery is charged. As the car is not operating in this case, the population size decreases. However, the approximation of the fixed-demand rate is reasonable in our problem context because in practice the expected back-orders at any time are fewer than $\lambda \tau$ and $\lambda \tau \ll N$. This ensures that, on average, the number of motorists waiting at the station at any given time is relatively small, and the correction due to state dependency can be safely ignored.
10Mak et al. (2012) directly uses a Poisson process to model the demand arrival at a switching station.
standard deviation to be a function of the motorist’s decision $e_{ss}$, which allows us to consider even the second-order effect of the motorist’s demand choices, which turns out to be important.

### 3.3.3. Electric Vehicle Pricing, Adoption, Switching-Station Management and Driving

As is typical in sequential games, we solve for the equilibrium using backward induction. We start with the last step, the optimal driving best response, $e_{ss}$, followed by the number of batteries stocked at the switching station, $Q$, the pricing for this service, $(F_{ss}, p_{ss})$, and the resulting adoption $A_{ss}$ of electric vehicles. An owner of a vehicle plans her driving level, $e^*_{ss}$, to maximize her utility. Specifically, her best-response driving is

$$e_{ss} = \arg \max_e \left[ E_e [u(e + \epsilon)] - p_{ss} e - M (1 - r) \cdot G (R - e) \right].$$

Note here that the motorist now incurs the range inconvenience penalty only when the charging facility is out of stock, i.e., with probability $1 - r$, where $r$ is the availability promised by the service provider. This is a lower penalty than that of a conventional electric vehicle. The utility from ownership of the vehicle is the above optimal use utility plus the idiosyncratic green utility, $\bar{U}_{gr}$ minus the purchase price, $F_{ss}$. A fraction, $A_{ss}$, of motorists finds this utility to be higher than the utility from a gasoline vehicle, and they adopt the electric vehicle. Anticipating these driving levels and adoption rates, the firm selects the fixed fee and the per-mile fee for the service to maximize its profits while ensuring that it stocks enough batteries to meet the promised service level. Specifically,

$$\max_{F_{ss}, p_{ss}, \bar{U}_{gr}} E \left[ NA_{ss} (F_{ss} + (p_{ss} - c_e) e_{ss} - c) - c Q \right],$$

11Note that in our setup, the probability that a motorist will find a charged battery in stock (the in-stock probability) corresponds to the steady state probability that the station is in stock (the fill rate), due to the Poisson Arrivals See Time Averages (PASTA) property of our setup (Wolff (1982)).

12Better Place subscribers are guaranteed access to an inventory of batteries with a committed service level agreement (Better Place (2011a)).
where \( e_{ss} \) is the best response described before. For each adopting motorist, the profits for the provider firm include the revenues from the sale of the vehicle, \( F_{ss} \), the profits from the miles driven, \( (p_{ss} - c) e_{ss} \), the costs of batteries in the vehicle, \( c \), and the per-motorist costs of batteries at the station, \( c Q/N_{A_{ss}} \).\(^{13,14}\) In addition to the purchase price of the vehicle, the firm now also has the per-mile price to maximize its profits. The solution is as follows:

**Lemma 2. Equilibrium Outcomes for the Switching-Station Model:** The equilibrium driving, \( e_{ss}^* \), adoption, \( A_{ss}^* \), stocking level, \( Q^* \), and per-mile price, \( p_{ss}^* \), are the solutions to the following system of equations. First, there is the driving equation,

\[
(3.3) \quad E_{\epsilon} \left[ u' \left( e_{ss}^* + \epsilon \right) \right] - M (1 - r) \cdot g (R - e_{ss}^*) = p_{ss}^*.
\]

Next is the stocking-level equation,

\[
(3.4) \quad Q^* = \tau A_{ss}^* N \cdot G (R - e_{ss}^*) + z_r \left( \tau A_{ss}^* N \cdot G (R - e_{ss}^*) \right)^{1/2},
\]

the pricing equation,

\[
(3.5) \quad p_{ss}^* = c_e + c_r \cdot g (R - e_{ss}^*) \cdot \Omega (e_{ss}^*, A_{ss}^*),
\]

and finally the adoption/purchase price equation,

\[
(3.6) \quad 2dA_{ss}^* = E_{\epsilon} \left[ u' \left( e_{ss} + \epsilon \right) \right] - c_e e_{ss}^* - \varphi (e_{ss}^*, A_{ss}^*) \cdot G (R - e_{ss}^*) d - U_{gas} - c,
\]

where \( \Omega (e_{ss}^*, A_{ss}^*) \equiv 1 + 1/2 z_r \left( \tau N A_{ss}^* G (R - e_{ss}^*) \right)^{-1/2} \) and \( \varphi (e_{ss}^*, A_{ss}^*) \equiv M (1 - \alpha) + c_r \Omega (e_{ss}^*, A_{ss}^*), \) \( z_r \) is the standard normal \( z \) value.

Equation 3.3 describes the motorist’s decision regarding planned driving. As with conventional electric vehicles, motorists trade off their utility from driving additional miles, the risk of incurring the range-anxiety penalty, and the per-mile costs of driving. However, there are two departures.

\(^{13}\)To ensure a non-trivial \( (p_{ss} \neq 0) \) and unique pricing solution, we assume that the firm’s profit is concave in the driving level, \( e_{ss} \). Technically, this corresponds to a condition on the shape of the distribution of the unplanned driving, the service level and the battery range: \( \chi (e_{ss}, A_{ss}) = \delta + c r z_r \left( 4 G(x) \sqrt{\tau} \right)^{-1} g^2(x) + c r g'(x) (1 + z_r (2 \sqrt{\tau})^{-1} < 0, \) where \( x = R - e_{ss}, \delta = E_{\epsilon} \left[ u' \left( e_{ss} + \epsilon \right) \right] + M (1 - \alpha) \cdot g (x) \) and \( u = \tau A_{ss} N \cdot G (R - e_{ss}) \). This assumption holds for all distributions with a decreasing failure rate; for example, the Gamma distribution with a shape parameter less than 1. Further, it often holds also for distributions with an increasing failure rate with mild restrictions on other parameters; for example, the triangular and normal distributions also work when the battery in-stock service level, \( r \), is not vanishingly close to 1 and the optimal driving \( e_{ss}^* \) is not too close to the range \( R \).

\(^{14}\)We also assume that this concavity is large enough at the optimum such that the Hessian \( H (e_{ss}^*, A_{ss}^*) \) is positive. Formally, we assume that \( H = \gamma / \delta \left( c r z_r \cdot g (x) \cdot e_{ss} \cdot \left( 4 d \sqrt{\tau} \right)^{-1} + A_{ss} \cdot 2 / \delta \right) - d^{-1} (c r z_r)^2 \cdot g^2 (x) (4 d A_{ss} \sqrt{\tau})^{-2} > 0, \) at \( e_{ss} = e_{ss}^* \) and at \( A_{ss} = A_{ss}^* \), where \( \omega = c r z_r \cdot G (x) (4 d A_{ss} \sqrt{\tau})^{-1} \) and \( \gamma = N / d (-2 + \omega) \).
First, the range-inconvenience penalty is now limited only to instances in which the switching station is out of batteries, which happens with probability $1 - r$. Second, the marginal cost of driving an additional mile is not the cost of maintaining and charging the battery, but the price that the motorist must pay to the provider firm. The stocking-level equation (Eq. 3.4) describes the batteries required to meet the service level constraints. As expected, the constraint is binding and the optimal stocking level follows directly from the amount required to fulfill the service level constraints.

The pricing equation (Eq. 3.5) characterizes the per-mile price. A decrease in the price of miles incentivizes motorists to drive more, hence increasing firm sales, but reducing margins. This trade-off can be managed by using the two-part pricing scheme, i.e., using the purchase and the per-mile prices. In particular, from traditional models of two-part pricing with downward sloping demand curves, one would expect the firm to set the per-mile price equal to the marginal cost of servicing the mile and then to use the purchase price to extract the surplus, with the marginally green customer earning zero utility. This is indeed the case in our setup, but the cost of servicing a mile is very different. There are two components to this cost. First, there is the cost of maintaining and charging the battery, $c_e$, which is the same as that for the conventional electric vehicles (see the first part of the RHS in Eq. 3.5). Second, for each additional mile driven, there is the cost of servicing this mile at the switching station. In particular, the firm is now more likely to see demand for a charged battery at the station and it must increase its stock of charged batteries. These are captured by the second part of the RHS of Eq. 3.5. In equilibrium, the firm sets the per-mile price equal to this total cost.

The per-mile price can be interpreted as the costs of charging and maintaining the battery plus an insurance premium— an additional amount paid to limit the range risk. This insurance premium is captured by the second term in Eq. 3.5. It increases in the battery cost, the charging time and the promised service level. The first part of the expression $\Omega$ reflects the increase in stock due to an increase in the mean demand at the station, while the second reflects the increase in the safety stock.

Finally, Eq. 3.6 describes the adoption level, which is driven by the vehicle’s purchase price. As before, decreasing the price increases adoption but reduces revenues. The optimal purchase price is such that the firm captures half the viable market, that is half the customers with green utility between the utility of gasoline vehicles and electric vehicles with the mid-point customer earning zero utility. We next state a fundamental result that will be of utmost importance for further derivations.

**Theorem 1.** The optimal customer adoption and driving in a switching-station model are strategic complements, formally (with a slight abuse of the notation) $\partial A^*_s / \partial e^*_s > 0$. This implies that any policy intervention, essentially a change in any exogenous parameter, $X$, will have the same directional effect.
on adoption and driving. Formally,

\[ \text{sign} \left( \frac{\partial A^*_ss}{\partial X} \right) = \text{sign} \left( \frac{\partial e^*_ss}{\partial X} \right). \]

The above theorem highlights an important property of the equilibrium outcome in the switching-station model: the relationship between equilibrium adoption and driving. Namely, optimal adoption increases in optimal driving and optimal driving increases in optimal adoption. This observation has important implications for policy-makers trying to create a favorable environment for switching-station vehicles. Policy actions can be thought of as changes to the parameters within which the switching-station model must operate. For example, a battery subsidy can be thought of as a policy intervention that reduces battery prices, \( c \). An electric vehicle purchase subsidy can be thought of as a change to the motorist’s green utility, i.e., \( d \). Our theorem suggests that battery subsidies, electric vehicle purchase subsidies, changes in electricity/gas prices, customer inconvenience and other costs, will all have the same directional effect on adoption and driving. That is, if they increase the adoption of switching-station vehicles, they will also increase the driving of these vehicles. This fundamental property of the switching-station vehicle will be at the root of understanding their effectiveness in decreasing oil dependence and greenhouse gas emissions.

4. The Effectiveness of The Switching-Station Model

There are two main arguments for adopting electric vehicles. First is the reduction in the dependence on oil. Second, electric vehicles have lower carbon emissions on a per-mile basis and thus have the potential to reduce the environmental impact of driving. In this section, we compare the effectiveness of conventional and switching-station electric vehicles in achieving these outcomes.


Theorem 2. 1) Profits: The equilibrium profits of a switching-station vehicle provider, \( \Pi^*_ss \), exhibit economies of scale with respect to the population size \( N \). On the other hand, the equilibrium profits of a conventional electric vehicle provider, \( \Pi^*_ce \), exhibit constant returns to scale. Formally,

\[ \frac{\partial (\Pi^*_ss / N)}{\partial N} > \frac{\partial (\Pi^*_ce / N)}{\partial N} = 0. \]

2) Battery Costs: The equilibrium profits of both electric vehicle providers decrease in the cost of battery, \( c \). Furthermore, the marginal effect is higher for the switching-station vehicle provider with a higher number of total batteries in the equilibrium. Formally,

\[ \frac{\partial \Pi^*_ss}{\partial c} < \frac{\partial \Pi^*_ce}{\partial c} \iff NA^*_ss + Q^* > NA^*_ce, \]
where $A_{ss}^*$, $Q^*$ and $A_{ce}^*$ are given by Lemmas 1 and 2.

Theorem 2 highlights two key differences in the structure of the profits of a switching-station and a conventional electric vehicle provider. First, a switching-station vehicle provider’s profits exhibit economies of scale that arise from the key differentiating feature of the model, the switching station. The costs at a switching station arise from the need to maintain a sufficient inventory of batteries at the station. The inventory required at the station does not increase linearly with the number of adopting motorists. As the population and adopters increase, the statistical economies of scale in inventory kick in, which causes costs to increase in a sub-linear fashion, and profits thus exhibit economies of scale. Furthermore, a key cost for both electric vehicle types is the cost of the battery. Both firms need to invest in batteries, and thus they both benefit when battery costs decrease. The effect is more pronounced for the system with more batteries. For the same adoption level, this is the switching-station system. This is because it gains from both cost saving on batteries in the vehicles and those at the station while the conventional electric vehicle provider only gains on batteries in the vehicles. Further, even if adoption is lower for the switching-station vehicles, the switching-station model could be more sensitive to the cost of batteries.

4.2. Adoption of Electric Vehicles.

Theorem 3. Adoption: Switching-station vehicles achieve a higher adoption than conventional vehicles iff the cost of batteries, $c$, is lower than a threshold $\bar{c}$. Formally, $A_{ss}^* > A_{ce}^*$ iff

$$c < \bar{c} \equiv \frac{Mr}{\tau \Omega(e_{ss}^*, A_{ss}^*)}$$

where, as before, $\Omega(e_{ss}^*, A_{ss}^*)$, is the increase in the switching-station cost for an additional unit of station demand. $A_{ss}^*$, $e_{ss}^*$ and $\Omega(e_{ss}^*, A_{ss}^*)$ are given by Lemmas 1 and 2. Furthermore,

a) The threshold battery cost, $\bar{c}$, decreases in the charging time, $\tau$, and in the per-mile cost of electric vehicle operation, $c_e$.

b) The threshold battery cost, $\bar{c}$, increases in the market size, $N$, and in the per-mile cost of fossil-fuel vehicle operation, $c_{gas}$.

The condition in Theorem 3 illustrates the key difference between conventional electric vehicles and switching-station vehicles. With conventional vehicles, motorists bear the risk of incurring the range-inconvenience penalty. In the switching-station model, customers bear the range-inconvenience penalty only with some probability, $1-r$, a reduction of $r$. The numerator of the expression represents this reduction in risk exposure and, from the customers’ perspective, is a key advantage of the
switching station model. However, this model also has a disadvantage. While the customer transfers a large part of the range-inconvenience risk to the firm, the firm charges the customer for this transfer. Recall from Lemma 2, Equation 3.5, that the firm’s per-mile price is at a premium to the cost of charging and maintaining batteries, which is the per-mile price that the customer pays with a conventional electric vehicle. This premium is the denominator of the above inequality. From the customer’s point of view, if the gains from the transfer of risk are higher than the loss due to the premium price, adoption is higher with the switching-station model; otherwise, conventional electric vehicles dominate. It is instructive to further examine which markets, customer segments or battery technologies are more likely to experience an increased adoption of electric vehicles if the switching-station model is employed.

The threshold battery cost below which switching-station vehicles are more effective in achieving the policy goal of adoption, or electrification of the transportation sector, increases with a decrease in the battery charging time. A shorter charging time decreases the costs of operation for the switching-station model, which in turn decreases the range-risk insurance premium charged to customers, and increases their driving, their utility from ownership, and their consequent adoption of switching-station vehicles. A decrease in charging time has no effect on the ownership utility of conventional vehicles. Taken together, a decrease in charging time makes the switching-station model more effective in electric vehicle adoption. The effect of electricity cost is more involved. The direct effect of a decrease in the cost of electricity is identical in both models— it leads to the same decrease in the marginal cost of driving and it reduces the ownership utility and consequently increases adoption for both electric vehicle types. But with switching-station vehicles there is an additional indirect effect: the increased adoption leads to feedback. Due to the economies of scale in this model, highlighted in Theorem 2, an increased adoption also decreases the per-customer switching-station costs and the per-mile range risk insurance premium that must be charged, bringing down the per-mile costs of driving a switching-station vehicle. Thus, decreases in the costs of electricity increase the effectiveness of switching-station models more than conventional electric vehicle models.

The effects of an increase in the per-mile cost of fossil-fuel vehicle operation are along the same lines. An increase in the price of gasoline increases the adoption of both types of electric vehicles, but switching-station vehicles benefit from an additional feedback effect. With their increased adoption, economies of scale kick in and make them even more desirable. Thus, increases in fossil-fuel prices make switching-station vehicles more effective than conventional electric vehicles. Finally, because of the economies of scale in the switching-station model, an increase in market size increases the relative effectiveness of switching-station vehicles.
The above discussion demonstrates the central premise for the development of the switching-station model. The results illustrate that, indeed, in certain markets, the switching-station model can reduce dependence on fossil fuels. The main mechanism for achieving this is the range-risk transfer from the customer to the firm, while customers pay a premium to the firm that pools this risk and can manage it at a lower cost due to statistical economies of scale. This mechanism operates through the price premium and reduced range risks. However, the operation of this seemingly beneficial mechanism has an unintended, harmful, and as yet overlooked side effect that we demonstrate in the next Theorem.

**Theorem 4.** In markets in which switching-station vehicles achieve higher adoption, customers who own switching-station vehicles drive more than they would had they adopted conventional electric vehicles. Formally, if $A_{ss}^* > A_{ce}^*$, so is $e_{ss}^* > e_{ce}^*$.

The intuition behind the above result is related to the strategic complementarity between driving and adoption in switching-station models (Theorem 1). From the motorist’s point of view, the switching-station model is an improvement over conventional electric vehicles due to a reduction of the range-inconvenience penalty in exchange for a premium that is relatively small and decreasing in market size. But this very reduction in the risk of incurring the range-inconvenience penalty also incentivizes customers to drive more because, in the switching-station model, driving more does not increase the chances of being stuck with a depleted battery as much as it does with conventional electric vehicles. Further, note that the per-mile price for switching-station vehicles is at the cost basis because of the two-part tariff. While this total cost is indeed higher, there is no per-mile margin that the firm is charging. The two-part nature of the pricing scheme definitely helps the firm maximize its profits and increase adoption, but not charging the customer an extra margin has an additional effect: it incentivizes the customer to drive more. Taken together, this discussion illustrates that the two key departures of the switching-station model—reduced range-anxiety and a pricing scheme in which the upfront costs are reduced and the provider firm is also paid for the miles driven—both help incentivize the adoption of electric vehicles, but they both do so by increasing average planned driving. In fact, our analysis illustrates that there is a one-to-one correspondence between adoption and driving which, as we will show shortly, has important implications for the environmental impact of switching-station vehicles.

4.3. **Environmental Impact.** In most countries, electricity is produced using a combination of fossil-fuel, nuclear and renewable sources of energy (Ambec and Crampes (2010)). Given the exact composition of electricity in most countries, vehicles that use contemporary electric power-trains have lower per-mile emissions than vehicles that use fossil-fuel-based power-trains (International
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Energy Agency (IEA) (2010)). Thus, electric vehicles supposedly lie at the heart of establishing a sustainable transportation infrastructure. Total emissions in our model can be computed as the sum of the emissions from electric-vehicle users and those from fossil-fuel-based users, specifically emissions, $EM$, in a universe with an electric-vehicle system $ev, ev \in \{ce, ss\}$ are

$$EM_{ev} = A_{ev}^* \alpha_{ev} + (1 - A_{ev}^*) \alpha_{gas} e_{gas},$$

where $\alpha_e$ and $\alpha_{gas}$ are the per-mile emissions from electric and fossil-fuel vehicles, $\alpha_e < \alpha_{gas}$. Essentially, there are two factors that contribute to emissions: adoption of electric vehicles (a higher adoption leads to lower emissions), and average miles driven (higher average driving leads to higher emissions). However, as we illustrated in Theorem 4, adoption and driving always go in the same direction, which leads to competing effects with respect to environmental impact. A critical question in this context is the relative effectiveness of the two electric vehicle models in reducing total emissions. In particular, are electric vehicles with switching stations more effective than conventional electric vehicles in reducing emissions? Consider $\Delta EM = EM_{ss} - EM_{ce}$ so that a positive $\Delta EM$ indicates that switching-station vehicles are worse at reducing emissions. The next theorem examines these conditions.

**Theorem 5.** Even when the switching-station model achieves higher (lower) adoption of electric vehicles than the conventional model, it may still lead to higher (lower) total carbon emissions if the electric vehicles are not emitting sufficiently less than fossil-fuel vehicles. Formally, despite $A_{ss}^* > (\leq) A_{ce}^*$, $\Delta EM > (\leq) 0$ iff $\alpha_e > \alpha_{gas} \cdot \lambda$, where $\lambda$ is a positive-valued function of the model primitives, $\lambda = \frac{e_{gas} (A_{ss}^* - A_{ce}^*)}{A_{ss}^* e_{ss} - A_{ce}^* e_{ce}}$. Furthermore

1. $\lambda < 1$ if $\max (e_{ss}^*, e_{ce}^*) > e_{gas}^*$.
2. If $\Gamma = \frac{A_{ss}^* e_{ss} - A_{ce}^* e_{ce}}{A_{ss}^* - A_{ce}^*}$ is increasing in $e_{ss}^*$, then $\lambda$ increases in $c$ and $\tau$, and decreases in $c_{gas}$.
3. If $A_{ss}^*$ is concave in $e_{ss}^*$, $\Gamma$ increases in $e_{ss}^*$.
4. If the motorist has a quadratic utility of driving (i.e. $u(e) = \theta e - \frac{e^2}{2}$) and $g(\cdot)$ follows a uniform distribution on the interval $[-a, a]$ with $a \geq R$, $\Gamma$ increases in $e_{ss}^*$.

The above theorem illustrates the key conflict between the dual policy objectives of increasing electric vehicle adoption and reducing greenhouse gas emission: namely, these two goals may not be aligned. In particular, with the use of systems like the switching system model, unless the electricity composition is such that electric vehicles are significantly less polluting than gasoline vehicles, these objectives are exactly in conflict with each other—whenever switching-station models are more...
effective in increasing electric vehicle adoption, the adopters drive more, which leads to higher greenhouse gas emissions. This is an as yet unidentified paradox: the switching-station model limits range anxiety, but as a consequence incentivizes customers to drive more. Since total carbon emissions are determined by both adoption and usage of electric cars, any increase in driving can cause total emissions to increase. This is more likely to happen if the per-mile emissions from electric vehicles are high enough. Following the same logic, switching-station vehicle systems can be better for the environment even if they are adopted by fewer people. In our detailed data-driven numerical analysis in Section 5, we highlight vital policy prescriptions that derive from this analysis.

The Theorem further characterizes how $\lambda$ changes with respect to some problem primitives. A decrease in $\lambda$ makes the key paradoxical results in Theorem 5 more likely. Note that function $\Gamma$ is the ratio of $A^*_ss e^*_ss - A^*_ce e^*_ce$, the difference between total expected driving for switching-station vehicles and conventional electric vehicles, to $A^*_ss - A^*_ce$, the difference in adoption. When $e^*_ss$ increases as a result of a change in any exogenously given property in our model, the increase in total expected electric vehicle driving dominates the increase in adoption; hence the paradoxical results in Theorem 5 become more likely. This result holds if the adoption is concave, i.e., it increases in a diminishing way with respect to optimal driving. More specifically, part 4 illustrates a utility function and a distribution under which the above corollary holds. Consider a change in battery technology that leads to an increase in both adoption and driving (such as a reduction in battery cost or charging time), so that $\lambda$ decreases. In such a case, the condition $\alpha_e > \alpha_{gas} \cdot \lambda$ in Theorem 5 becomes more likely to hold, making the electric vehicle with a higher adoption worse for the environment. If it is the switching-station vehicle that leads to higher adoption, improvements in battery technology can actually hurt the environment.

5. Scenario Analysis and Implications

In this section, we examine the relative effectiveness of switching-station and conventional electric vehicles in achieving the twin objectives of reducing oil dependence and emissions. We use the above developed model and analysis, which we calibrate with real parameters based on existing and forecasted technology, as well as known consumer behavior patterns. We conduct a counterfactual analysis to identify the comparative performance of different electric vehicle business models under plausible current and future scenarios. In particular, we consider changes in battery technology and the likely evolution of energy prices. Tables 5.1 to 5.6 show our calibrated demand parameters and the method and sources employed to come up with the estimates.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Estimation Method/Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>$R = 100$ miles</td>
<td>Standard performance of a 24 kWh battery (Better Place (2011b))</td>
</tr>
<tr>
<td>Charging Time</td>
<td>$\tau = \frac{1}{4}$ days</td>
<td>A battery costs $12500 – $15000 (we take the average $13750) and has a lifetime of 8 years (Better Place Blog (2011)). The daily cost is found based on a fixed annuity amount over the lifetime of a battery with an interest rate of 11.3%. As a proxy for the interest rate, we used the weighted average cost of capital (WACC) for Tesla Motors (Paradise et al. (2010)), another electric car company at a similar stage of development as Better Place.</td>
</tr>
<tr>
<td>Battery Cost</td>
<td>$c = $7.15/ day</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1. Battery Technology Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Estimation Method/Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Gasoline</td>
<td>$c_{gas} = $10.9/mile</td>
<td>The retail price of gasoline was $2.82 per gallon in 2010 (U.S. Energy Information Administration (2011b)). An average passenger car gets 25.8 miles to the gallon based on the U.S. Environmental Protection Agency (EPA) (2010).</td>
</tr>
<tr>
<td>Cost of Electricity</td>
<td>$c_e = $2.5/mile</td>
<td>The U.S. average retail price of electricity for the transportation sector was $10.42 per kWh in 2010 (U.S. Energy Information Administration (2011a)).</td>
</tr>
</tbody>
</table>

Table 5.2. Energy Cost Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Estimation Method/Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Vehicle Emissions</td>
<td>$\alpha_{gas} = 341$ gr/mile</td>
<td>Based on 8,788 grams of CO$_2$/gallon (U.S. Environmental Protection Agency (EPA) (2005)) and a fuel economy of 25.8 miles/gallon of an average passenger car.</td>
</tr>
<tr>
<td>EV Emissions (USA)</td>
<td>$\alpha_e = 138$ gr/mile</td>
<td>Based on 600 grams of CO$_2$/kWh of electricity generated based on the US electricity mix in 2009 (U.S. Energy Information Administration (2010b)) and a range of 100 miles of a 24 kWh battery. This value is found by dividing total CO$_2$ emissions by total net electricity generation in 2009, the most recent year reported.</td>
</tr>
</tbody>
</table>

Table 5.3. Carbon Emission Parameters

5.1. Results. First, we investigate the relative effectiveness of switching-station and conventional electric vehicles based on the parameters of existing technology and known consumer behavior patterns in Tables 5.1–5.6. In the absence of any data on the range-inconvenience penalty, $M$, we make no assumptions and present our results for many different values of $M$. Given $c_{gas} = $10.9, a penalty of $M = $1 is equivalent to a gasoline customer’s total gasoline cost for 9.17 miles. Further, we
Parameter | Estimated Value | Estimation Method/Sources
--- | --- | ---
Utility Function | \( u(Y) = \frac{1}{b} \left( \theta Y - \frac{Y^2}{2} \right) \) | A quadratic utility function with satiation utility fits well the consumption of driving miles (see Singh and Vives (1984) and Farahat and Perakis (2010) for a similar use of the quadratic function.)
Satiation Level | \( \theta = 58.54 \text{ miles} \) | A vast literature on the estimation of gasoline demand (Dahl (1979) and Espey (1998)) has focused on estimating the price elasticity \( \epsilon_p \) of gasoline demand by using a log-linear model specification. Brons et al. (2008) estimate the long-run price elasticity of gasoline demand as \( \epsilon_p = -0.84 \) by using meta-analytical techniques that unify all other studies in the literature. To find the intercept value, we use the mean driving level \( e^*_\text{gas} = 37.14 \) miles from the 2001 National Household Travel Survey and a gasoline price of $1.78 per gallon in 2001. \( \theta \) is the average satiation level for motorists and \( b \) is a scaling factor that captures the utility from driving. The implied demand function from our utility model fits the demand function from the literature extremely well.
Scaling Factor | \( b = 277.74 \text{ miles/$} \) |

Table 5.4. Driving Utility Parameters

Parameter | Estimated Value | Estimation Method/Sources
--- | --- | ---
Distribution | \( g() \equiv \text{Shifted Gamma} \) | We use the distribution of US motorists’ daily driving distances (Hu and Reuscher (2004)) to estimate the distribution. We use the Kolmogorov-Smirnov test to check the fit of various distributions, and we find that the gamma distribution fits total driving distance well (p-value of 0.88). The mean and standard deviation of distribution are calculated as 37.14 and 35.61 miles respectively. This is the distribution of total daily driving distance and includes the customer’s decision \( e \) as well as the noise term \( \epsilon \). In order to isolate \( \epsilon \) from \( e \), we shift the distribution to the left by its mean.
Gamma Shape | \( k = 1.0876 \) |
Gamma Scale | \( m = 34.147 \) |
Gamma Location | \( p = 37.14 \) |

Table 5.5. Unpredictable Demand Variability Parameters

perform counterfactual analysis to identify the relative effectiveness of two business models under multiple plausible scenarios including technological evolution and energy cost evolution.

5.1.1. The Impact of Technology Evolution. We first investigate how the two most important improvements in battery technology affect the adoption and emissions of switching-station and conventional electric vehicles: a reduction in battery cost and an extension in battery range. Figure 5.1 compares the status quo with future scenarios. The middle row shows the results for the current state of the world with a “high” cost of battery ($7.15) and a “low” range (100 miles). The top row illustrates a scenarios with a lower battery cost ($2.86 as projected for 2020 by the Boston Consulting Group (2010)). The bottom row illustrates a scenario with a higher range, 200 miles.
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Estimation Method/Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switch Availability</td>
<td>$r = 99%$</td>
<td>Marketing Materials and website of Better Place.</td>
</tr>
<tr>
<td>Green Utility-Maximum Willingness to Pay (WTP)</td>
<td>$d = $5.30/day</td>
<td>Hidrue et al. (2011) estimate the WTP for some EV attributes based on a survey of 3029 respondents. We found how much extra a customer is willing to pay for an EV that more or less mimics a contemporary gasoline vehicle (with a range of 300 miles, a charging time of 10 minutes, etc.), but emits 60% less CO$_2$ than a gasoline car. We came up with an estimate of $10,192 for the maximum WTP over the vehicle’s 8 years of lifetime. Daily WTP is found based on a fixed annuity amount over the lifetime with an interest rate of 11.3%.</td>
</tr>
</tbody>
</table>

Table 5.6. Other Model Parameters

![Graph](image-url)

The black lines represent the conventional (dashed) and switching-station models (solid); the gray lines illustrate the differences in emissions (solid) and adoption (dashed). Emissions are in kg of CO$_2$.

Figure 5.1. The Impact of Technological Evolution on Adoption and Emissions

First, note from Figure 5.1, panels (d) and (e), that the adoption and emissions of conventional electric vehicles are more sensitive to the range-inconvenience penalty $M$ given the current range of 100 miles. This is natural given that the incidence of range inconvenience is much higher with...
conventional electric vehicles. Comparing panels (a) and (d), we see that the effect of a decrease in battery cost on adoption is substantial both for conventional and switching-station vehicles. Adoption can increase to as high as 55% (from panel (a)) for both electric vehicles, which is almost four times today’s maximum adoption value of 14% (from panel (d)). On the other hand, when we look at Figure 5.1 (b) and (e), we see that a decrease in battery cost has a much smaller effect on total emissions— it corresponds to a decrease of about 11% from the current emissions values.

A comparison of panels 5.1 (c) and (f) suggests that switching-station vehicles become superior to conventional vehicles in the future in terms of adoption. That is, a reduction in the cost of batteries favors switching-station vehicles. However, this also leads switching-station vehicles to emit more in the future, which reiterates the central paradox of switching-station vehicles that superiority in terms of adoption does not necessarily mean lower emissions. As batteries are expected to become much cheaper in the future, the price premium paid by customers will be much lower, hence usage of switching-station vehicles will be elevated, leading to higher emissions.

Finally, we see from panel (f) that today the dual policy objectives of increasing electric vehicle adoption and reducing greenhouse gas emission are aligned. Switching-station vehicle models are, in fact, environmentally better when they are adopted by more people. However, based on our model and counter-factual analysis, we expect that these two goals will not be aligned in the future, if, as expected, battery costs go down (panel (c)). When batteries become cheaper, switching-station models will become more effective in increasing electric vehicle adoption, but this also leads to higher emissions. Note that all of the above effects can arise out of a reduction in battery costs due to technology advancements, but the same effects and misaligned policy objectives will exist if battery costs are reduced due to (misguided) policy interventions such as offering a battery subsidy.

Note from Figure 5.1 panels (g) and (h) that an improvement in battery range leads the two electric vehicle models to become similar both in terms of adoption and emissions. Not surprisingly, if battery range is high enough, range anxiety ceases to be an issue and switching-station vehicles become similar to conventional electric vehicles. When we look at Figure 5.1 (d) and (g), we see that the effect of an extension in battery range on adoption is quite limited both for conventional and switching-station vehicles. It increases adoption up to 2–3% maximum, depending on the level of range inconvenience.

As a result of this limited change in adoption, the change in emissions is also quite limited, less than 0.5% from the current emissions values. Note that this and the above analysis imply that battery cost reductions have a much larger impact on adoption and emissions than range extension. Finally, note in panel (i) that, as opposed to the battery cost reduction effect, the dual objectives of increasing electric vehicle adoption and reducing greenhouse gas emission are still aligned when
there is an extension in battery range. In other words, the policy-makers’ paradox does not arise here; however, the switching-station model does not help increase electric vehicle adoption either.

5.1.2. The Impact of Changing Energy Costs. Geopolitical events, economic integration and global growth have made energy prices highly volatile, which is a key consideration in examining the utility of any transportation and environmental policy. In our next set of counter-factual analyses, we investigate how a change in the costs of electricity and gasoline influence the adoption and emissions of switching-station and conventional electric vehicles. In Figure 5.2, the middle row shows the current state of the world (as before). The top row illustrates our analysis under a scenario with the cost of gasoline at 50% more than the current value (i.e., $c_{gas} = 16.35$) and the bottom row with the cost of electricity at 50% more than the current value (i.e., $c_e = 3.75$).

The black lines represent the conventional (dashed) and switching-station models (solid); the gray lines illustrate the differences in emissions (solid) and adoption (dashed). Emissions are in kg of CO$_2$.

**Figure 5.2.** The Impact of Energy Cost Evolution on Adoption and Emissions

First, note from Figure 5.2 panels (a) and (d) that an increase in the cost of gasoline favors both electric vehicles in terms of adoption, but it favors the switching-station vehicles more. Thus, the expected rise in oil prices in the future (U.S. Energy Information Administration (2010a)) will increase
the attractiveness of switching-station vehicles, which is in line with our results in Theorem 3, a result that is driven by the economies of scale in the switching-station model. More importantly, comparing panels (b) and (e) shows that the reduction in emissions is quite substantial due to an increase in the cost of gasoline: the decrease is almost 50%, much larger than all other comparisons examined by our study. Based on our analysis, we believe that a policy of gasoline taxes is the most effective tool in reducing emissions. The effect of an increase in the cost of electricity is similar to the effect of the cost of gasoline, but in the opposite way. It hurts both electric vehicle models, but hurts switching-station vehicles more due to the economies of scale effect. However, the change in emissions is also quite limited, as seen in Figure 5.2 (h), which suggests that making electricity cheaper would not be an effective tool for reducing emissions.

5.1.3. The Impact of the Electricity Mix. As Theorem 5 suggests, the extent of the environmental advantage of electric vehicles over fossil-fuel vehicles, $\alpha_e/\alpha_g$, is a key parameter that influences the effectiveness of switching-station vehicles in reducing emissions. This environmental advantage depends crucially on the mix of sources used to produce electricity. Further, the electricity mix is also a key variable that determines whether the dual policy objectives of increasing electric vehicle adoption and reducing carbon emissions are aligned. In this section, we investigate the relationship between the electricity mix and the policy objective alignment. We compare three different countries with very different electricity mix. First, we consider France, which has a generation mix that leads to low carbon emissions due to the widespread use of nuclear sources (with $\alpha_e/\alpha_g = 0.06$). Next is the United States, which uses a mix of nuclear, wind and fossil-fuel-based production (with $\alpha_e/\alpha_g = 0.4$). Finally, we consider China, in which the generation mix is dominated by coal, and which is associated with high carbon emissions (with $\alpha_e/\alpha_g = 0.5$).

Figure 5.3 shows our results. The top row compares the alignment of dual policy objectives in different countries with the current state of battery technology, and the bottom row shows results in a future scenario in which batteries cost less.

Figure 5.3 panels (a) and (d), show that for an economy like France, there is no misalignment in the policy objectives, with current technology or improved technology increasing adoption of electric vehicles and leading to lower emissions. In the USA, the objectives are aligned today, but misalignment is expected to happen in the future as a result of battery cost improvements. Finally, in China, the electricity generation mix is highly polluting and increased electric vehicle adoption is associated with increased greenhouse gas emissions today, an effect that will be exacerbated in the future (panels (c) and (f)), suggesting that switching-station vehicles would not be a good alternative to gasoline vehicles in countries with a coal-based electricity mix.

$^{15}$We find $\alpha_e^{FR} = 19.1$ gr/mile and $\alpha_e^{CH} = 171$ gr/mile based on 83 and 745 grams of $CO_2$/kWh of electricity generated, with the 2008 French and Chinese electricity mix (International Energy Agency (IEA) (2010)).
The Current State of the World (High Cost of Battery ($7.15))

France

USA

China

2020 Projections (Low Cost of Battery ($2.86))

France

USA

China

The solid and dashed lines represent the emissions (in kg CO₂) and adoption differences respectively.

Figure 5.3. Emissions in France, USA and China

6. Discussion

Our model had the ambitious goal of capturing the salient features of electric vehicle adoption decisions by modeling range anxiety and the impact of different ownership structures (selling miles vs. selling batteries). Although we believe our model captures these two key factors in the electric vehicle adoption decision, naturally, it does so at the expense of other considerations. Clearly, the adoption of electric vehicles is a very complex decision so, in order to focus on key tradeoffs between the two business models we discuss, we had to make a number of simplification and assumptions.

Given that our paper is one of the very few to study this question from a modeling point of view, there is little literature available to guide our efforts, so we had to make some choices. Some of the obvious tradeoffs that we omitted include:

- The adoption process of new technology is clearly dynamic with multiple feedback loops. An analytical model of such a feedback process is analytically intractable (see, e.g., Struben and Sterman (2008) for a systems dynamics approach). Our model considers adoption as a one-time decision, but it permits the use of non-stationary and correlated distributions describing the driving realizations on different days. These non-stationary and serially correlated distributions can also be used to capture any changes in driving due to feedback loops. Further, some of these feedback loops might lead to changes in gas and electricity prices over time but, since we numerically investigate a large range of options, we do not believe that our findings would be greatly affected except in cases of extreme changes, which are hard to forecast at this point.
While we consider customer heterogeneity in the preference for owning a green vehicle, we do not consider heterogeneity with respect to driving distance (e.g., different average driving for commercial and non-commercial vehicles). We were able to find only aggregate data on driving habits rather than detailed data divided by segment. However, we do not expect our findings to change substantially, although the attractiveness of electric vehicles would improve for, say, a high-driving commercial fleet. On the other hand, if there are multiple customer types, the company may want to “screen” them by using a menu of driving contracts with different per-mile charges and subscription fees. Due to efficiency losses inherent in such contracts (similar to efficiency losses arising due to other reasons: e.g., customer risk-aversion etc.), we would expect the switching-station model to perform marginally worse. Exploring a model with adverse selection in this setting would be a fruitful avenue for future research (see Bakshi et al. (2012) for related work), but at present there is no indication that Better Place has such intentions, nor are customer screening practices employed in the sale of any other existing electric vehicles.

We chose to focus on the decision to adopt electric vehicles with a gasoline vehicle as a base case. One could envision more complex models in which other green modes of transportation are considered (e.g., public transportation, bicycling). Likewise, one could attempt to model a tripartite competition among gasoline, conventional electric and switching-station vehicles. Naturally, we would not expect this model to lead to many tractable results. An alternative could be to focus on the high-level adoption decision at the expense of operational details (cf. Chocteau et al. (2010)), which we intend to do in a follow-up paper focused on public policy.

There are other potential benefits of electric vehicles due to, for instance, lower maintenance costs (cf. Chocteau et al. (2010)). Moreover, the switching-station model could have an advantage due to battery recycling or reuse by the company which owns the battery or, for instance, due to better battery maintenance by the company rather than by the individual consumers. As long as these and other costs that we do not account for are fixed, they can be easily incorporated into the model with predictable results.

Our model concerns a single switching station while, in practice, one expects to see multiple stations covering some geographical area. This is a fruitful avenue for future research that could consider such issues as the optimal location and number of switching stations (see, e.g., Cachon (2011) for research on a related issue) as a function of population density and/or traffic patterns. These issues clearly merit a separate study and, if anything, modeling the network of stations would further increase the economies of scale effect and exacerbate the key paradox in our model.

It is likely that the government would intervene into any large-scale electric car project by proposing subsidies and tax breaks, along with standards and legal requirements. Similarly, a mobility project
on a large scale could cause intense competition among electricity providers, car and battery manufacturers and infrastructure builders. Once again, these issues are clearly outside the scope of our paper but, at present, they do not seem to play a major role: e.g., Better Place uses a single type/size of battery, or cars come from a single manufacturer (Renault-Nissan) and it does not have much competition.

There are numerous issues on the electricity supply side which we sidestep in this paper. For instance, batteries at the switching stations can be used to store electricity and give it back to the network at peak demand, or the charging process can be otherwise optimized to take advantage of fluctuating electricity prices. If this is done, we expect to see the switching-station model become even more attractive. At present, however, it is difficult to estimate the potential impact of such optimization since the business model is not yet tested on the electricity supply side.

Overall, we are confident that the key result of our paper— that there is an overlooked inherent tension between the twin goals of decreasing oil dependence and reducing carbon emissions with the use of a switching-station vehicle model— would survive most of the above proposed additional factors. Our paper draws attention to this overlooked conflict and starkly illustrates that efforts in encouraging adoption of switching-station vehicles can be environmentally harmful if the effects of the economic interactions between the providers, and motorists are ignored. Thus, policy-makers should proceed with caution when considering deployment of these novel mobility systems. Further, we highlight how these inherent conflicts may lead to counter-intuitive impacts of policy interventions such as battery subsidies. While we believe that, overall, the switching-station model offers a promising supply-chain solution to the issue of limited electric vehicle adoption (see Girotra and Netessine (2011b)) and can be an effective tool to reduce carbon emissions for some countries, we advocate a cautious examination of the effects of such a system based on a rigorous analysis of its operational dynamics.

**References**


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† Appendix

A.1. Proof of Lemma 1 (Page 9).

Analysis of a Conventional Electric Vehicle Customer’s Problem. Customers set their optimal driving best response \( e_{ce}^* \) such that \( e_{ce}^* = \arg \max_e \left[ E \left[ u(e + \epsilon) \right] - c_e e - M \cdot \mathcal{G}(R - e) \right] \). The first-order condition with respect to \( e \) is given below:

\[
(A.1) \quad E \left[ u'(e_{ce}^* + \epsilon) \right] - c_e - M \cdot g(R - e_{ce}^*) = 0.
\]

For \( e_{ce}^* \) that solves Eq. A.1 to be a unique optimum, we assume that the following is true:

Assumption 1: \( E \left[ u''(e + \epsilon) \right] + M \cdot g'(R - e) < 0 \) for \( \forall e \).

Analysis of the Conventional Electric Vehicle Firm’s Problem. The firm solves the maximization problem \( \max_{F_{ce}, U_{gr}} E \left[ I \{U_{ce} > U_{gas}\} \cdot (F_{ce} - c) \right] \) where \( U_{ce} = E \left[ u(e_{ce}^* + \epsilon) \right] - c_e e_{ce}^* - M \cdot \mathcal{G}(R - e_{ce}^*) - F_{ce} + \tilde{U}_{gr} \). Given \( E \left[ I \{U_{ce} > U_{gas}\} \right] = P \left( \tilde{U}_{gr} > U_{gas} - E \left[ u(e_{ce}^* + \epsilon) \right] + c_e e_{ce}^* + M \cdot \mathcal{G}(R - e_{ce}^*) + F_{ce} \right) \) and \( \tilde{U}_{gr} \sim \text{Uniform} [0, d] \), the first-order condition with respect to \( F_{ce} \) is given below:

\[
E \left[ u \left( e_{ce}^* + \epsilon \right) \right] - c_e e_{ce}^* - M \cdot \mathcal{G}(R - e_{ce}^*) + d - U_{gas} + c - 2F_{ce}^* = 0.
\]

\( F_{ce}^* \) is the unique optimum, as the second-order condition is \(-2 < 0\). Given \( F_{ce}^* \), the fraction of customers who adopt the electric vehicle \( A_{ce}^* \) is given by

\[
(A.2) \quad A_{ce}^* = \frac{E \left[ I \{U_{ce} > U_{gas}\} \right]}{E \left[ I \{U_{ce} > U_{gas}\} \right]} = \frac{E \left[ u \left( e_{ce}^* + \epsilon \right) \right] - c_e e_{ce}^* - M \cdot \mathcal{G}(R - e_{ce}^*) + d - U_{gas} - c}{2d}.
\]

Equations A.1 and A.2 characterize Lemma 1.


Analysis of a Switching-Station Vehicle Customer’s Problem. Customers set their optimal driving best response \( e_{ss} \) such that \( e_{ss} = \arg \max_e \left[ E \left[ u(e + \epsilon) \right] - p_{ss} e - M (1 - r) \cdot \mathcal{G}(R - e) \right] \). The first-order condition with respect to \( e \) is given below:

\[
(A.3) \quad E \left[ u'(e_{ss} + \epsilon) \right] - p_{ss} - M (1 - r) \cdot g(R - e_{ss}) = 0.
\]

For \( e_{ss} \) that solves Eq. A.3 to be a unique optimum, we need \( E \left[ u''(e_{ss} + \epsilon) \right] + M (1 - r) \cdot g'(R - e) < 0 \) for \( \forall e \). Given Assumption 1, this automatically holds.

Analysis of the Switching-Station Vehicle Firm’s Problem. The firm solves the maximization problem

\[
\max_{F_{ss}, p_{ss}} \Pi_{ss} = \mathbb{E} \left[ \Pi_{ss} \left( F_{ss} + (p_{ss} - c_e) e_{ss} - c \right) - cQ \right]
\]

s.t. \( \Pr(Q > O) \geq r \)
where $A_{ss} = P \left( U_{gr} > U_{gas} - E[u(e_{ss} + \epsilon)] + p_{ss} e_{ss} + M (1 - r) \cdot \mathcal{G}(R - e_{ss}) + F_{ss} \right)$. Given that $O$ is distributed normally with mean and variance $\tau A_{ss} N \cdot \mathcal{G}(R - e_{ss})$, at the optimal driving and adoption levels $e^*_{ss}$ and $A^*_{ss}$, $Q^*$ simply solves

$$Q = \tau A^*_{ss} N \cdot \mathcal{G}(R - e^*_{ss}) + z_r \left( \tau A^*_{ss} N \cdot \mathcal{G}(R - e^*_{ss}) \right)^{1/2},$$

where $\Phi$ is the cdf of the standard normal distribution and $z_r$ is the standard normal $z$ value. The first-order conditions with respect to $F_{ss}$ and $p_{ss}$ are given below:

$$\frac{\partial \Pi_{ss}}{\partial F_{ss}} = \frac{N}{d} \left( E[u(e^*_{ss} + \epsilon)] - 2p^*_{ss} e^*_{ss} + c_e e^*_{ss} - M (1 - r) \cdot \mathcal{G}(R - e^*_{ss}) + \frac{c_r}{r} \mathcal{G}(R - e^*_{ss}) \right) + d - U_{gas} + c - 2F^*_{ss} = 0,$$

$$\frac{\partial \Pi_{ss}}{\partial p_{ss}} = e^*_{ss} \frac{\partial \Pi_{ss}}{\partial F_{ss}} + NA^*_{ss} (p^*_{ss} - c_e - c_r \cdot g(R - e^*_{ss}) \cdot \mathcal{G}(e^*_{ss}, A^*_{ss})) \frac{\partial e_{ss}}{\partial p_{ss}} = 0,$$

where $\Omega(e^*_{ss}, A^*_{ss}) = 1 + \frac{1}{2} z_r \left( \tau N A^*_{ss} \mathcal{G}(R - e^*_{ss}) \right)^{1/2}$ and $\frac{\partial e_{ss}}{\partial p_{ss}} = \frac{1}{\left( \left[ E[u''(e^*_{ss} + \epsilon)] + M (1 - r) g(R - e^*_{ss}) \right] + \mathcal{G}(R - e^*_{ss}) \right)} < 0$. Next, we show that there exist unique maximizers $F^*_{ss}$ and $p^*_{ss}$ that solve Equations A.5 and A.6. Because $\frac{\partial \Pi_{ss}}{\partial F_{ss}} = 0$ and $\frac{\partial e_{ss}}{\partial p_{ss}} < 0$ in Eq. A.6 and defining $\beta(p_{ss}) = p_{ss} - c_e - c_r \cdot g(R - e^*_{ss}) \cdot \Omega(e^*_{ss}, A^*_{ss})$, we need $\beta(p_{ss}) = 0$ at the optimality. Given

$$\frac{\partial^2 \beta}{\partial p_{ss}} = NA_{ss} \left( 1 + c_r \mathcal{G}(e_{ss}, A_{ss}) \right) \frac{\partial e_{ss}}{\partial p_{ss}} \frac{\partial e_{ss}}{\partial p_{ss}}$$

with $\xi(e^*_{ss}, A^*_{ss}) = \left( \frac{z_r}{4G(x)} \left( \tau A_{ss} N \cdot \mathcal{G}(x) \right)^{1/2} g^2(x) + g'(x) \left( 1 + \frac{z_r}{\tau A_{ss} N \cdot \mathcal{G}(x)} \right) \right)$ and $x = R - e_{ss}$, the following assumption guarantees that there exists a unique solution $p^*_{ss}$ that solves $\beta(p^*_{ss}) = 0$.

**Assumption 2:** $\chi(e_{ss}, A_{ss}) = E[u''(e_{ss} + \epsilon)] + M (1 - \alpha) \cdot g(R - e_{ss}) + c_r \mathcal{G}(e_{ss}, A_{ss}) < 0$ for $\forall e_{ss}$ and $\forall A_{ss}$.

Under Assumption 2, the following equation characterizes the unique solution $p^*_{ss}$:

$$p^*_{ss} - c_e - c_r \cdot g(R - e^*_{ss}) \cdot \Omega(e^*_{ss}, A^*_{ss}) = 0.$$

The fact that $\frac{\partial^3 \Pi_{ss}}{\partial F_{ss}^2} = \frac{N}{d} \left( - \frac{3c_r z_r}{8(\mathcal{A} N)^2} \mathcal{G}(R - e_{ss}) \right)^{1/2} < 0$ means that the first-order condition $\frac{\partial \Pi_{ss}}{\partial F_{ss}}$ is concave in $F_{ss}$. Therefore there can be at most two solutions that solve $\frac{\partial \Pi_{ss}}{\partial F_{ss}} = 0$. Let these solutions be $F^\text{min}_{ss}$ and $F^\text{max}_{ss}$ with $F^\text{min}_{ss} < F^\text{max}_{ss}$. As $\frac{\partial^2 \Pi_{ss}}{\partial F_{ss}^2} = 0$ is concave in $F_{ss}$, we have $\frac{\partial^2 \Pi_{ss}}{\partial F_{ss}^2} \bigg|_{F^\text{min}_{ss}} > 0$ and $\frac{\partial^2 \Pi_{ss}}{\partial F_{ss}^2} \bigg|_{F^\text{max}_{ss}} < 0$. Therefore only $F^\text{max}_{ss}$ can be a maximizer, whereas $F^\text{min}_{ss}$ is a minimizer. Hence there is a unique maximizer $F^*_{ss} = F^\text{max}_{ss}$ that solves Eq. A.5.

For the optimality of $F^*_{ss}$ and $p^*_{ss}$, we also need $H(F^*_{ss}, p^*_{ss}) = \left[ \frac{\partial^2 \Pi_{ss}}{\partial F_{ss}^2} \frac{\partial^2 \Pi_{ss}}{\partial p_{ss}^2} - \left( \frac{\partial^2 \Pi_{ss}}{\partial F_{ss} \partial p_{ss}} \right)^2 \right] \bigg|_{F^*_{ss}, p^*_{ss}}$ to be positive evaluated at $F^*_{ss}$ and $p^*_{ss}$. The following assumption guarantees this:

**Assumption 3:** $H(F^*_{ss}, p^*_{ss}) = \frac{N}{d} \frac{\partial^2 \Pi_{ss}}{\partial F_{ss}^2} \left( \frac{N}{d} \frac{\partial e_{ss}}{\partial p_{ss}} \frac{c_r z_r \cdot g(R - e^*_{ss})}{4 (\tau A^*_{ss} N \cdot \mathcal{G}(R - e^*_{ss}))^{1/2}} + \frac{\partial^2 \beta}{\partial p_{ss}^2} \right) > 0.
where \( \frac{\partial^2 \Pi_{ss}}{\partial F_{ss}^2} = \frac{N}{d} \left( -2 + \frac{cT \cdot \overline{G}(R - e_{ss}^*)}{4dA_{ss}^* \left( \tau A_{ss} N \cdot \overline{G}(R - e_{ss}^*) \right)^{1/2}} \right) \). Finally, given \( F_{ss}^* \), the fraction of customers who adopt the electric vehicle \( A_{ss}^* \) solves the following equation:

\[
A_{ss}^* = \frac{E \left[ u\left( e_{ss}^* + \epsilon \right) \right] - c_e e_{ss}^* - \left( M (1 - r) + cT \cdot \Omega \left( e_{ss}^*, A_{ss}^* \right) \right) \overline{G}(R - e_{ss}^*) + d - U_{gas} - c}{2d}.
\]

Equations A.3, A.4, A.7 and A.8 characterize Lemma 2.

### A.3. Proof of Theorem 1 (Page 14).

Given \( p_{ss}^* \) defined by A.7, the customer's optimal driving \( e_{ss}^* \) solves

\[
\Theta_1 = E \left[ u' \left( e_{ss}^* + \epsilon \right) \right] - c_c - \left( M (1 - r) + cT \cdot \Omega \left( e_{ss}^*, A_{ss}^* \right) \right) g \left( R - e_{ss}^* \right) = 0.
\]

Given \( \Theta_1 \) and Eq. A.8, the fraction of customers who adopt electric vehicle \( A_{ss}^* \) solves the following equation:

\[
\Theta_2 = A_{ss}^* - \sqrt{2d} \left( E \left[ u \left( e_{ss}^* + \epsilon \right) \right] - c_c e_{ss}^* - U_{gas} + d - c - \left( E \left[ u' \left( e_{ss}^* + \epsilon \right) \right] - c_c \right) \overline{G}(x) \right) = 0
\]

with \( x = R - e_{ss}^* \). Then \( e_{ss}^* \) and \( A_{ss}^* \) would be solutions to the simultaneous equations A.9 and A.10. Then using the implicit function theorem, we have

\[
\frac{\partial \Theta_2}{\partial A_{ss}^*} \cdot \frac{\partial A_{ss}^*}{\partial e_{ss}} + \frac{\partial \Theta_2}{\partial e_{ss}} = 0 \quad \text{and} \quad \frac{\partial \Theta_1}{\partial e_{ss}} = 0.
\]

With \( \frac{\partial \Theta_2}{\partial A_{ss}^*} = -1 < 0 \) and \( \frac{\partial \Theta_2}{\partial e_{ss}} = -\overline{G}(R - e_{ss}^*) \chi(e_{ss}^*, A_{ss}^*) > 0 \), we have \( \frac{\partial A_{ss}^*}{\partial e_{ss}} > 0 \). With \( \frac{\partial \Theta_1}{\partial e_{ss}} = \chi(e_{ss}^*, A_{ss}^*) < 0 \) and

\[
\frac{\partial \Theta_1}{\partial A_{ss}^*} = \frac{cT \cdot \cdot \cdot (R - e_{ss}^*)}{4dA_{ss}^* \left( \tau A_{ss} N \cdot \overline{G}(R - e_{ss}^*) \right)^{1/2}} > 0,
\]

we have \( \frac{\partial e_{ss}^*}{\partial A_{ss}^*} > 0 \). Hence \( e_{ss}^* \) and \( A_{ss}^* \) are strategic complements.

### A.4. Proof of Theorem 2 (Page 15).

1) **Profits:** Note that \( \Pi_{ss}/N = A_{ss} \left( F_{ss} + (p_{ss} - c_e) e_{ss} - c \right) - cQ/N \) and \( A_{ss}/N = A_{ss} \left( F_{ss} - c \right) \). Then

\[
\frac{\partial \left( \Pi_{ss}/N \right)}{\partial N} \bigg|_{A_{ss}^*, e_{ss}^*} = \frac{cz_r \left( \tau A_{ss} N \cdot \overline{G}(R - e_{ss}^*) \right)^{1/2}}{2N^2} > 0 \quad \text{and} \quad \frac{\partial \left( \Pi_{ss}/N \right)}{\partial N} \bigg|_{A_{ss}^*, e_{ss}^*} = \frac{\partial \left( \Pi_{ss}/N \right)}{\partial N} = 0.
\]

2) **Battery Costs:** Given \( \frac{\partial \Pi_{ss}^c}{\partial c} \bigg|_{A_{ss}^*, e_{ss}^*} = \frac{cz_r \left( \tau A_{ss} N \cdot \overline{G}(R - e_{ss}^*) \right)^{1/2}}{2N^2} > 0 \) and \( \frac{\partial \Pi_{ss}^c}{\partial c} \bigg|_{A_{ss}^*, e_{ss}^*} = -NA_{ss}^* e_{ss} - Q^* < 0 \). Hence \( \frac{\partial \Pi_{ss}^c}{\partial c} < \frac{\partial \Pi_{ss}^c}{\partial c} \) if and only if \( NA_{ss}^* + Q^* > NA_{ss}^* \).

### A.5. Proof of Theorem 3 (Page 16).

Given Eqs. A.1 and A.9 by defining \( \omega(c) = Mr - cT \cdot \Omega \left( e_{ss}^*, A_{ss}^* \right) = 0 \), we have \( e_{ss}^* = e_{ss}^* \) iff \( \omega(\tau) = 0 \). The fact that

\[
\frac{\partial \omega(c)}{\partial c} = -cT \cdot \Omega \left( e_{ss}^*, A_{ss}^* \right) + \frac{cT z_r}{4 \left( \tau A_{ss}^* N \cdot \overline{G}(R - e_{ss}^*) \right)^{1/2}} \left( \frac{\partial A_{ss}^*}{\partial c} + \frac{\partial e_{ss}^*}{\partial c} \right) \frac{g \left( R - e_{ss}^* \right)}{\overline{G}(R - e_{ss}^*)} < 0
\]

with \( \frac{\partial A_{ss}^*}{\partial c} = \frac{\Theta_1}{H(\tau, p_{ss}), p_{ss}} < 0 \) and \( \frac{\partial e_{ss}^*}{\partial c} = \frac{\Theta_1}{H(\tau, p_{ss}), p_{ss}} < 0 \) proves that \( \tau \) is unique.

For the rest of the proof, we use the implicit function theorem \( \frac{\partial \omega(c)}{\partial y} \bigg|_{y=c, N} = 0 \) for \( y = \tau, c_e, N \) and \( e_{gas} \). Hence given \( \frac{\partial \omega(c)}{\partial c} < 0, \text{sign} \left( \frac{\partial \omega(c)}{\partial y} \right) = \text{sign} \left( \frac{\partial \omega(c)}{\partial y} \right) \).
Part a:

\[
\frac{\partial \omega(c)}{\partial \tau} = -c \left(1 + \frac{1}{4z_r} \left(\tau N A_{ss}^* \tilde{G}(x)\right)^{-1/2}\right) + \frac{c \tau z_r}{4 \left(\tau A_{ss}^* N \cdot \tilde{G}(x)\right)^{1/2}} \left(\frac{\partial A_{ss}^*}{\partial \tau} \frac{1}{A_{ss}} + \frac{\partial e_{ss}^*}{\partial \tau} \frac{g(x)}{G(x)}\right) < 0
\]

where

\[
\frac{\partial e_{ss}^*}{\partial \tau} = \frac{-c \cdot g(R - e_{ss}^*)}{H(F_{ss}^*, p_{ss}^*)} \left(1 + \frac{1}{4z_r} \left(\tau N A_{ss}^* \tilde{G}(x)\right)^{-1/2}\right) < 0 \quad \text{and} \quad \frac{\partial A_{ss}^*}{\partial \tau} = \frac{\partial e_{ss}^*}{\partial \tau} \frac{\partial \Theta_2}{\partial e_{ss}^*} < 0 \quad \text{Hence} \quad \frac{\partial \tau}{\partial \tau} < 0.
\]

\[
\frac{\partial \omega(c)}{\partial \epsilon_c} = \frac{c \tau z_r}{4 \left(\tau A_{ss}^* N \cdot \tilde{G}(x)\right)^{1/2}} \left(\frac{\partial A_{ss}^*}{\partial \epsilon_c} \frac{1}{A_{ss}} + \frac{\partial e_{ss}^*}{\partial \epsilon_c} \frac{g(x)}{G(x)}\right) < 0
\]

where

\[
\frac{\partial e_{ss}^*}{\partial \epsilon_c} = \frac{1}{2} \left(-e_{ss}^* + \frac{\tilde{c}(\epsilon_c)}{g(\epsilon_c)}\right) \frac{\partial \Theta_1}{\partial A_{ss}^*} - 1 < 0 \quad \text{and} \quad \frac{\partial A_{ss}^*}{\partial \epsilon_c} = \frac{1}{2} \left(e_{ss}^* \frac{\partial \Theta_1}{\partial e_{ss}^*} - \frac{c \tau z_r \cdot g(x)}{4(\tau A_{ss}^* N \cdot \tilde{G}(x))^{1/2}}\right) < 0 \quad \text{Hence we have} \quad \frac{\partial \tau}{\partial \epsilon_c} < 0.
\]

Part b:

\[
\frac{\partial \omega(c)}{\partial N} = \frac{c \tau z_r}{4 \left(\tau A_{ss}^* N \cdot \tilde{G}(x)\right)^{1/2}} \left(\frac{1}{N} + \frac{\partial A_{ss}^*}{\partial N} \frac{1}{A_{ss}} + \frac{\partial e_{ss}^*}{\partial N} \frac{g(x)}{G(x)}\right) > 0
\]

where

\[
\frac{\partial e_{ss}^*}{\partial N} = \frac{\frac{e_{gas}^*}{H(F_{ss}^*, p_{ss}^*)}}{4 \left(\tau A_{ss}^* N \cdot \tilde{G}(x)\right)^{1/2}} \left(\frac{\partial A_{ss}^*}{\partial e_{gas}^*} \frac{1}{A_{ss}} + \frac{\partial e_{gas}^*}{\partial e_{gas}^*} \frac{g(x)}{G(x)}\right) > 0
\]

where

\[
\frac{\partial e_{gas}^*}{\partial e_{gas}^*} = \frac{1}{2} \frac{e_{gas}^*}{H(F_{ss}^*, p_{ss}^*)} \frac{\partial \Theta_1}{\partial A_{ss}^*} > 0 \quad \text{and} \quad \frac{\partial A_{ss}^*}{\partial e_{gas}^*} = \frac{-\frac{e_{gas}^*}{H(F_{ss}^*, p_{ss}^*)}}{\frac{\partial \Theta_1}{\partial e_{gas}^*}} > 0 \quad \text{Hence} \quad \frac{\partial \tau}{\partial e_{gas}^*} > 0.
\]

A.6. Proof of Theorem 4 (Page 18). Let \( e_{ss}^* = e_{cc}^* + t \). Then using Eqs. A.2 and A.10, we can write

\[
A_{ss}^* - A_{cc}^* = \frac{1}{2d} \left( E \left[ u(t) \right] - e_{cc}^* + t \right)
\]

\[
= \left( \frac{\tilde{c}(\epsilon_c^*)}{g(\epsilon_c^*)} \right) \frac{g(R - e_{cc}^*)}{G(R - e_{cc}^*)} \frac{\tilde{c}(R - e_{cc}^*) - t}{g(R - e_{cc}^*) - t}.
\]

Taking the derivative with respect to \( t \), we have

\[
\frac{\partial (A_{ss}^* - A_{cc}^*)}{\partial t} = -\frac{\tilde{c}(R - e_{cc}^*)}{2dg(R - e_{cc}^*)} \chi(e_{ss}^*, A_{ss}^*) > 0 \quad \text{under Assumption 2.}
\]

Given \( A_{ss}^* = A_{cc}^* \) when \( t = 0 \) and \( \frac{\partial (A_{ss}^* - A_{cc}^*)}{\partial t} > 0 \) imply that \( A_{ss}^* > A_{cc}^* \) if \( e_{ss}^* > e_{cc}^* \) and \( A_{ss}^* < A_{cc}^* \) if \( e_{ss}^* < e_{cc}^* \).

A.7. Proof of Theorem 5 (Page 19). Consider \( \Delta EM = EM_{ss} - EM_{cc} = \alpha_a (e_{ss}^* - e_{cc}^*) - \alpha_g e_{gas}^* (A_{ss}^* - A_{cc}^*) \). Then \( \Delta EM < 0 \) if \( \alpha_a < \alpha_g \cdot \lambda \) when \( A_{ss}^* - A_{cc}^* > 0 \) and \( \Delta EM < 0 \) if \( \alpha_a > \alpha_g \cdot \lambda \) when \( A_{ss}^* - A_{cc}^* < 0 \).

1. \( \lambda < 1 \) if \( A_{ss}^* (e_{ss}^* - e_{cc}^*) > A_{cc}^* (e_{cc}^* - e_{gas}^*) \). Given Theorem 4, this holds if \( e_{ss}^* > e_{gas}^* \) when \( A_{ss}^* - A_{cc}^* > 0 \) and if \( e_{cc}^* > e_{gas}^* \) when \( A_{ss}^* - A_{cc}^* < 0 \). Hence \( \lambda < 1 \) if \( \max(e_{ss}^*, e_{cc}^*) > e_{gas}^* \).

2. Note that \( \lambda = \frac{e_{gas}^*}{\Gamma} \) with \( \Gamma > 0 \). Then

\[
\frac{\partial \Gamma}{\partial e_{ss}^*} = \left( \frac{e_{ss}^* - e_{cc}^*}{A_{ss}^* - A_{cc}^*} \frac{\partial A_{ss}^*}{\partial e_{ss}^*} + \frac{e_{ss}^*}{A_{ss}^* - A_{cc}^*} \frac{\partial A_{ss}^*}{\partial e_{ss}^*} \right) / (A_{ss}^* - A_{cc}^*)^2.
\]

For an arbitary parameter \( y \), we have

\[
\frac{\partial \Gamma}{\partial y} = \left( \frac{e_{ss}^* - e_{cc}^*}{A_{ss}^* - A_{cc}^*} \frac{\partial A_{ss}^*}{\partial e_{ss}^*} + \frac{e_{ss}^*}{A_{ss}^* - A_{cc}^*} \frac{\partial A_{ss}^*}{\partial e_{ss}^*} \right) / (A_{ss}^* - A_{cc}^*)^2.
\]
where \( \frac{\partial(A_{ss}^+ - A_{ss}^-)}{\partial y} = \frac{\partial e_{ss}^+}{\partial y} \frac{\partial \Theta_2}{\partial e_{ss}} \) and \( \frac{\partial e_{ss}^-}{\partial y} = 0 \). For \( y = c, \tau \), we have \( \frac{\partial e_{ss}^+}{\partial y} < 0 \) and \( \frac{\partial A_{ss}^+}{\partial y} \leq 0 \); and for \( y = c_{gas} \), we have \( \frac{\partial e_{ss}^+}{\partial y} > 0 \) and \( \frac{\partial A_{ss}^+}{\partial y} \geq 0 \). Also note that \( \frac{\partial A_{ss}^+}{\partial e_{ss}} = \frac{\partial \Theta_2}{\partial e_{ss}} \). Then if \( \frac{\partial \Gamma}{\partial e_{ss}} > 0 \), for \( y = c, \tau \), we have \( \frac{\partial \Gamma}{\partial y} < 0 \) and for \( y = c_{gas} \), we have \( \frac{\partial \Gamma}{\partial y} > 0 \). As \( \frac{\partial \lambda}{\partial y} = \frac{\partial e_{gas}^+}{\partial y} 1 / \Gamma - \frac{\partial \Gamma}{\partial y} T^2 \), we have \( \frac{\partial \lambda}{\partial y} > 0 \) given \( \frac{\partial e_{gas}^+}{\partial y} \leq 0 \) for \( y = c, \tau \) and we have \( \frac{\partial \lambda}{\partial y} < 0 \) given \( \frac{\partial e_{gas}^+}{\partial y} < 0 \) for \( y = c_{gas} \).

3. We can rewrite \( \frac{\partial \Gamma}{\partial e_{ss}} \) as \( \frac{\partial \Gamma}{\partial e_{ss}} = \left( \left( A_{ss}^+ - A_{cc}^+ \right)^2 + A_{cc}^+ \left( A_{ss}^+ - A_{cc}^+ - (e_{ss}^+ - e_{cc}^+) \frac{\partial A_{ss}^+}{\partial e_{ss}} \right) \right) / \left( A_{ss}^+ - A_{cc}^+ \right)^2 \). If \( A_{ss}^+ \) is concave, it satisfies \( A_{ss}^+(e_{ss}^+) - A_{ss}^+(e_{cc}^+) > (e_{ss}^+ - e_{cc}^+) \frac{\partial A_{ss}^+}{\partial e_{ss}} \bigg|_{e_{ss}^+} > 0 \). As \( A_{ss}^+(e_{cc}^+) = A_{cc}^+(e_{cc}^+) \) from Theorem 4, \( \frac{\partial \Gamma}{\partial e_{ss}} > 0 \) if \( A_{ss}^+ \) is concave in \( e_{ss}^+ \).

4. Given \( u(e) = \theta - \frac{e^2}{2} \), \( g(x) = \frac{1}{2a} \) and \( \overline{G}(x) = \frac{a - x}{2a} \), we have \( A_{cc}^+ = \frac{1}{2} \left( \frac{e_{cc}^2}{2} - \frac{M(a - R)}{2a} - T \right) \) and \( A_{ss}^+ = \frac{1}{2} \left( \frac{e_{ss}^2}{2} - \frac{M(1 - r)}{2a} + \theta \cdot \Omega(e_{ss}^+, A_{ss}^+) \right)(a - R) - T \) where \( T = \frac{\text{Var}(e)}{2} + U_{gas} + c - d > 0 \). Hence \( A_{ss}^+ - A_{cc}^+ = \frac{1}{2} \left( e_{ss}^+ - e_{cc}^+ \right)(a - R + e_{ss}^+) \) and \( \frac{\partial A_{ss}^+}{\partial e_{ss}} = \frac{1}{2}(a - R + e_{ss}^+) \). Then

\[
(A_{ss}^+ - A_{cc}^+) \frac{\partial \Gamma}{\partial e_{ss}} = \frac{1}{4} (e_{ss}^+ - e_{cc}^+) \left( a - R \right) \left( a - R + e_{ss}^+ + e_{cc}^+ + \frac{M}{3a} + \frac{e_{ss}^+ (e_{ss}^+ + 2e_{cc}^+)}{4} + \frac{T}{4} \right) > 0.
\]
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