Environmental Policy and Directed Technical Change in a Global Economy: The Dynamic Impact of Unilateral Environmental Policies
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Abstract

This paper builds a two-country (North, South), two-sector (polluting, nonpolluting) trade model with directed technical change, examining whether unilateral environmental policies can ensure sustainable growth. The polluting good is produced with a clean and a dirty input. A temporary Northern policy combining clean research subsidies and a trade tax can ensure sustainable growth but Northern carbon taxes alone cannot. Trade and directed technical change accelerate environmental degradation either under laissez-faire, or if the North implements carbon taxes, yet both help reduce environmental degradation under the appropriate unilateral policy. I characterize the optimal unilateral policy analytically and numerically using calibrated simulations.

Keywords: Climate Change, Environment, Directed Technical Change, Innovation, Trade, Unilateral Policy

JEL Classification: F18, F42, F43, O32, O33, O41, Q54, Q55
1 Introduction

Countries not subject to any binding constraints under the Kyoto protocol account for an increasing fraction of carbon dioxide (CO\textsubscript{2}) emissions: their share in world emissions has risen from a third in 1990 to more than a half in 2008. Meanwhile, climate negotiations have stalled and no global agreement is in sight. In response, several countries either have undertaken unilateral actions or are considering doing so, and these policies increasingly harbor protectionist aspects. For instance, the American Clean Energy and Security Act — which was supposed to set up a cap-and-trade system in the United States — planned to implement trade barriers with countries that did not have a similar system (absent an international agreement) by 2018.\textsuperscript{1} These responses raise two questions. First, can unilateral policies ensure sustainable growth? Second, how necessary is protectionism to achieving this goal?

These questions are fundamentally about the economy’s long-run behavior. Over the time period relevant to climate change, comparative advantages evolve with innovation, which itself responds to environmental policies. Therefore, a dynamic framework is necessary; this paper builds such a framework by integrating directed technical change with a trade model that features a global pollution externality. In doing so, it establishes two main points. First, intervening countries which only use unilateral carbon taxes typically fail to ensure sustainable growth; in fact such taxes are likely to accelerate environmental degradation because of the innovation response of nonintervening countries. Second, intervening countries can achieve sustainable growth without cooperation from the rest of the world by implementing a temporary industrial policy that combines clean research subsidies and a trade tax. Such a policy develops clean technologies in the polluting sector of the intervening countries, which leads to a long-run reduction of emissions not only in the intervening countries but also in nonintervening ones.

More formally, I consider a dynamic Ricardo–Heckscher–Ohlin model with two countries (North and South) and two sectors. The North represents countries willing to implement an environmental policy; the South represents countries that undertake no such policy. One sector never pollutes, whereas the other sector pollutes more or less depending on the country’s balance between dirty and clean technologies. In practice, the polluting sector includes the manufacture of chemicals and chemical products, nonmetallic mineral products and of basic metals. The distinction between clean and dirty technologies might refer to the use of renewable and nuclear instead of fossil fuel energy or to the use of bioplastics instead of traditional petroleum products. Innovation is undertaken in both countries by profit-maximizing firms

\textsuperscript{1}In this case, the trade barrier was an international reserve allowance. The bill passed the House in 2009 but was rejected by the Senate. Trade barriers have also been discussed for the European Union Emissions Trading System (EU ETS) but have yet not been imposed. However, the EU ETS was extended to air transport in January 2012 and affects all airlines, which makes it the first attempt to tax foreign firms for pollution.
that hire scientists, and it can be directed at the polluting or the nonpolluting sector. The allocation of scientists across these two sectors depends on the relative size of both sectors in the country as measured by their revenue share (Acemoglu, 1998). Under laissez-faire, the country exporting the polluting good has a relatively larger market size in the polluting than in the nonpolluting sector, this tends to amplify comparative advantage over time. Within the polluting sector, innovation can be directed at clean or dirty technologies. The allocation of scientists across clean and dirty technologies is tilted toward the most advanced of the two; thus, there is path dependence in innovation. For most of the analysis, innovation is completely local.

So in the laissez-faire regime, if clean technologies are initially less advanced than dirty ones in both countries, then most polluting sector innovations will be directed toward the dirty technologies. Emissions will then continue to increase and the economy eventually faces an “environmental disaster” as the quality of the environment falls below a critical threshold. In other words, economic growth is not sustainable. I assume that the South initially has a comparative advantage in the polluting sector, so that it increasingly specializes in that sector. Now suppose that the North implements a carbon tax. That policy leads to a reallocation of some of polluting good’s production from the North to the South (the “pollution haven effect”), thereby reinforcing the South’s specialization in the polluting sector. Emissions still grow unboundedly in the South, which eventually causes an environmental disaster. Moreover, because reallocating production goes hand in hand with reallocating innovation, a Northern carbon tax actually increases dirty Southern innovations, and thereby may accelerate environmental degradation. The North could instead use a temporary combination of clean research subsidies and a trade tax, thus ensuring that it develops a comparative advantage in the polluting sector while making it cleaner at the same time. Once clean technologies in the North are sufficiently advanced and the initial comparative advantage is reversed, the market forces that previously drove the economy toward a disaster now work to averting it: emissions decrease both in the North (as innovation is directed to clean technologies) and in the South (as it specializes, over time, in the nonpolluting sector). If the initial environmental quality is high enough, then an environmental disaster can be averted. Directed technical change is essential for this result; if technical change were exogenous, unilateral policies in the North would fail to prevent a disaster when the South initially has a sufficiently large comparative advantage in the polluting sector.

I consider two objectives for a social planner. In the first case, the social planner’s objective

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2Such a reversal of comparative advantage cannot always be ensured with clean research subsidies only. Indeed, under free trade, the South may fully specialize in the polluting sector, so that all its innovation is directed toward that sector. This makes it impossible for the North to reverse the pattern of comparative advantage without using a trade tax.
function depends only on world consumption and environmental quality; in the second case, the planner maximizes a weighted sum of the utilities of infinitely lived representative agents in both countries. Therefore, in the second case, but not in the first case, the planner is concerned about the distribution of consumption across the two countries. In both cases, I characterize the first-best policy and the second-best policy under the constraint that no intervention can occur in the South. This second-best policy can be decentralized through a carbon tax and research subsidies in the North along with a trade tax on the polluting good. Absent redistribution concerns, the trade tax typically takes the form of a tariff and then of an export subsidy; its expression reflects two aims of the social planner: reducing emissions in the South and redirecting Southern innovation toward the nonpolluting sector. Yet when the social planner cares about the distribution of income, the optimal trade tax also reflects terms-of-trade considerations.

I carry out a simple calibration exercise to illustrate the paper’s main results. It shows that, for reasonable parameter values, there are high welfare costs to not implementing any policy in the South. In addition, it highlights the double-edged nature of both trade and directed technical change: they accelerate environmental degradation under laissez-faire, but they also reduce environmental degradation when one country intervenes in an appropriate way.

Finally, I relax the assumption that knowledge is purely local by supposing that the more backward country can partially catch up every period. The main results continue to hold: unilateral carbon taxes still fail to prevent an environmental disaster; whereas a combination of clean research subsidies and a carbon tariff can do so for sufficiently high initial environmental quality. In this scenario, however, the diffusion of knowledge can ensure a switch toward clean innovation in the South; hence an environmental disaster can be prevented even though the South still specializes in the polluting good.

This paper can be interpreted as a green version of the “infant industry argument,” which claims that trade can be detrimental to growth if it leads countries to specialize in sectors with poor development prospects (Krugman, 1981; Young, 1991; Matsuyama, 1992; Galor and Mountford, 2008). Here as well, a country risks specializing in the “wrong” sector, not because that sector offers poor growth prospects, but because this country cannot prevent the environmental externality associated with production in that sector. The idea that free trade may amplify comparative advantages and that a temporary trade policy could permanently reverse the trade pattern was previously broached by Krugman (1987), and Grossman and Helpman (1991, ch. 8).
The literature on trade and the environment has long recognized that, in an open world, the effectiveness of unilateral policies for reducing world pollution can be hampered by the pollution haven effect; see, for instance, Pethig (1976). Empirical evidence is reported by Copeland and Taylor (2004) and more recently by Broner, Bustos, and Carvalho (2012). Markusen (1975) and Hoel (1996) show that the optimal instrument for addressing the pollution haven effect is a tariff. In the specific context of global warming, where the pollutant (CO$_2$) enters differently at several stages of the production process, several papers use computable general equilibrium models to track carbon through the global economy; in this way they determine the pattern of trade and compute the carbon leakage rate (the rate at which emissions abroad increase after a domestic reduction). It is generally agreed that the developed countries are net carbon importers. $^5$ Elliott et al. (2010) compute a carbon leakage rate of 20 percent from a reduction in Annex I countries (the countries with binding constraints under the Kyoto protocol) and show that border tax adjustments eliminate half of it. $^6$ The present paper is also related to the literature which addresses trade’s impact on the environment (see Copeland and Taylor, 1995): in the absence of global cooperation, trade is necessary to avert an environmental disaster however, trade needs to be managed in order to deliver the right outcome. $^7$ This literature has focused on static models and has ignored the evolution of comparative advantage over time. $^8$

A growing literature has shown the importance of taking into account directed technical change when designing policies to combat climate change. On the empirical side, Popp (2002) shows that an increase in energy prices leads to more energy-saving innovation; similar results are found by Newell, Jaffe, and Stavins (1999) in the air conditioner industry and by Hassler, Krusell, and Olovsson (2012) using macroeconomic US data. Aghion et al. (2012) focus on the car industry and establish that (a) an increase in fuel prices leads to clean innovation at the expense of dirty innovation and (b) there is path dependence in clean versus dirty innovation — findings in line with the results reported here. Following this literature, several theoretical change; examples include Acemoglu (2003), who studies the impact of trade on the skill bias of technological change, and Gancia and Bonfiglioli (2008), who show that trade amplifies international wage differences.

$^5$ For instance, Atkinson et al. (2011) find that the net US imports of carbon from China in 2004 amounted to 244 million tons of CO$_2$ or 0.9 percent of total world emissions that year.

$^6$ Among others, Babiker and Rutherford (2005), Böhringer, Fischer, and Rosendahl (2010), Böhringer, Carbon, and Rutherford (2011) and Bucher and Schenker (2010) find similar results. Introducing imperfect competition, Babiker (2005) finds a leakage rate greater than 100 percent; however, Gerlagh and Kuik (2007) find a negative rate when introducing the possibility for energy-saving innovation and international knowledge spillovers. There are comparatively few empirical studies. Aichele and Felbermayr (2010) use a gravity model of trade and find that committing to the Kyoto protocol increases the carbon content of imports from not-committed countries by 10 percent. Aichele and Felbermayr (2012) find that countries committing to the Kyoto protocol reduce domestic CO$_2$ emissions by about 7 percent but that their total CO$_2$ consumption does not change.

$^7$ Empirical studies (e.g. Antweiler, Copeland and Taylor, 2001; Frankel and Rose, 2005) suggest there is a positive effect of trade on the environment for local pollutants, but the effect for CO$_2$ emissions is not clear.

$^8$ An exception is Costinot, Donaldson, and Smith (2012), who study how climate change damages will affect the pattern of comparative advantages in agriculture.
papers have integrated directed technical change in the study of climate change policies; here, I build specifically on the model developed by Acemoglu et al. (2012a; henceforth AABH). The final good in AABH and the polluting sector in this paper are both produced with a clean and a dirty input, which are substitutes for each other. Because of knowledge externalities associated with “building on the shoulders of giants,” there is path dependence in the direction of innovation (clean or dirty). The published version of AABH deals with a single country case, but the working paper presents a two-country version of the model in which trade occurs between two substitutable goods, the polluting tradeable good cannot become less pollutive, and the South does not innovate; note that all these assumptions are reversed here. Di Maria and Smulders (2004) and Di Maria and van der Werf (2008) also tackle the issue of modeling the interaction between directed technical change and international trade. These authors study the allocation of innovation between an energy-intensive sector and a non-energy-intensive sector, but they ignore that innovations within the energy-intensive sector could either reduce or increase pollution.

The rest of this paper is structured as follows. Section 2 presents the model, and Section 3 studies the laissez-faire equilibrium and identifies which policies are able to ensure sustainable growth. Section 4 solves for the first- and second-best policies when the South is constrained to be in laissez-faire. Section 5 presents a stylized calibration, and Section 6 discusses how the main results generalize when knowledge flows across countries. Appendix A presents some extensions of the model, Appendix B contains the main proofs, Appendix C gives details on the calibration, and Appendix D contains additional proofs (the latter two appendices are available online).

2 Model

I consider an infinite-horizon version of a two-country (North, N, and South, S), two-sector (E and F), three-factor (capital, labor and scientists) Heckscher–Ohlin–Ricardo model in which sector E is similar to the economy of AABH. Time is discrete. Each country is endowed with a fixed amount of labor and capital, \( L_N, K_N \) and \( L_S, K_S \), and there is a fixed unitary mass of scientists in both countries.

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10In Di Maria and Smulders (2004), the North develops technologies that are imitated by the South and so opening up to trade leads to a reallocation of innovation toward the sector that the North exports. Carbon leakage is reduced when the goods are substitutes and, amplified otherwise. In Di Maria and van der Werf (2008), both countries innovate and carbon leakage is always reduced by the innovation response to a cut in emissions in a single country. Golombek and Hoel (2004) use a static model to study the interaction between environmental policy and innovation in an open world.
2.1 Welfare

I consider two distinct problems. In the first problem, the economy admits, for each period $t$, a representative agent in the North who lives for one period and a like representative agent in the South. The utility of time-$t$ agent in country $X \in \{N, S\}$ is given by $\nu(S_t) C_{ Xt}$, where $S_t$ is the quality of the environment (identical in North and South) and $C_{ Xt}$ is the final good consumption in country $X$. These preferences are aggregated under the following social welfare function:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{(\nu(S_t) (C_{Nt} + C_{St}))^{1-\eta}}{1-\eta};$$

where $\rho > 0$ is the discount rate and $\eta \geq 0$ is the inverse elasticity of intertemporal substitution ($\eta = 1$ corresponds to a logarithmic utility). Here the social planner cares only about the time profile of world consumption and environmental quality.

In the second problem, the economy admits infinitely lived representative agents in each country, whose utilities are given by $\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{(\nu(S_t) C_{ Xt})^{1-\eta}}{1-\eta}$. The social planner maximizes a weighted sum of these utilities:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{\nu(S_t)^{1-\eta}}{1-\eta} \left( \Psi C_{Nt}^{1-\eta} + (1 - \Psi) C_{St}^{1-\eta} \right),$$

where $\Psi \in [0, 1]$ is the weight on North’s representative agent. In this case, the social planner also cares about the distribution of consumption across the two countries.

Consumption, $C_{ Xt}$, and environmental quality, $S_t$, are weakly positive and $\nu$ is increasing in $S_t$. There is an upper-bound on $S_t$, denoted $\bar{S}$, that corresponds to a pristine environment. I define an environmental disaster as an instance of environmental quality reaching zero in finite time. I assume that $\nu(0) = 0$ and $\nu'(\bar{S}) = 0$; hence a disaster is as detrimental to welfare as zero consumption and the marginal damage of the first unit of pollution is zero.

2.2 Production

Final consumption is a CES (constant elasticity of substitution) aggregate of the consumption of two goods, $E$ and $F$:

$$C_{ Xt} = \left( \nu C_{Et}^{\sigma_{-1}} + (1-\nu) C_{Ft}^{\sigma_{-1}} \right)^{\frac{\sigma}{\sigma_{-1}}};$$

where $C_{XYt}$ represents the quantity of good $Y \in \{E, F\}$ consumed in country $X \in \{N, S\}$, $\sigma$ denotes the elasticity of substitution between goods $E$ and $F$. I restrict attention to the

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11 As specified in what follows, only the social planner makes an intertemporal decision.

12 A disaster puts the economy on an unsustainable path because the utility flow cannot be bounded away from the utility flow given by zero consumption.
cases where the two goods are gross complements \((\sigma < 1)\) or where final consumption is Cobb-Douglas \((\sigma = 1)\), so that both goods are essential. Goods \(E\) and \(F\) are the only goods that are traded internationally. Good \(E\) represents the traded goods responsible for most of the emissions of greenhouse gases (in particular, energy-intensive goods), while good \(F\) represents traded goods that do not generate a lot of emissions. When the model is calibrated, good \(E\) is identified with the manufacture of chemicals and chemical products (ISIC code 24), other nonmetallic mineral products (26), and basic metals (27), good \(F\) is identified with the rest of manufacturing.

Good \(F\) in country \(X\) is produced competitively according to

\[
Y_{XFt} = \left( \int_0^1 A_{XF(it)} x_{XF(it)}^{\gamma} dt \right) \left( K_{Xht}^{1-\beta} L_{Xht}^{1-\beta} \right)^{1-\gamma}. \tag{4}
\]

Here \(K_{Xht}\) and \(L_{Xht}\) are the capital and labor employed in the assembly of good \(F\) in country \(X\); \(x_{XF(it)}\) is the quantity of intermediates \(i\) employed in sector \(F\); and \(A_{XF(it)}\) is the productivity of intermediate \(i\), which specific to the country and sector. The parameter \(\gamma\) is the factor share of intermediates. Intermediates are produced monopolistically according to

\[
x_{XF(it)} = \psi K_{XF(it)}^{\beta} L_{XF(it)}^{1-\beta}, \tag{5}
\]

where \(K_{XF(it)}\) and \(L_{XF(it)}\) are the capital and labor employed in the production of intermediate \(i\) for good \(F\) in country \(X\). Intermediates cannot be traded internationally. Since the same factor share is used in the production of intermediates and in the final assembly of the good, it follows that \(\beta \in (0, 1)\) is the overall factor share of capital in sector \(F\).\(^{13}\) I use \(K_{XFt}\) to denote total employment of capital in sector \(F\) in country \(X\):

\[
K_{XFt} \equiv K_{Xht} + \int_0^1 K_{XF(it)} dt; \tag{6}
\]

similarly, \(L_{XFt}\) is total employment of labor in sector \(F\) in country \(X\).

Good \(E\) is produced competitively with a clean input \(Y_{Xct}\) and a dirty input \(Y_{Xdt}\), which is the sole source of pollution, according to

\[
Y_{XEt} = \left( Y_{Xct}^{\varepsilon-1} + Y_{Xdt}^{\varepsilon-1} \right)^{\frac{1}{\varepsilon}}, \tag{7}
\]

where \(\varepsilon > 1\) is the elasticity of substitution between the clean and the dirty input (I will also mention the perfect substitute case, which corresponds to \(\varepsilon = \infty\)). Both inputs are produced competitively in a similar fashion as good \(F\):

\[
Y_{Xzt} = \left( \int_0^1 A_{Xz(it)} x_{Xz(it)}^{\gamma} dt \right) \left( K_{Xzt}^{\alpha} L_{Xzt}^{1-\alpha} \right)^{1-\gamma} \text{ for } z \in \{c, d\}, \tag{8}
\]

\(^{13}\)The Cobb-Douglas structure of production for intermediates is important because it ensures that monopolists get a constant share of the sector’s revenues, which matters for the incentives to innovate. That being said, the analysis can be extended straightforwardly to production functions for which aggregation between capital and labor is not Cobb-Douglas.
where $K_{Xzt}$ and $L_{Xzt}$ are the capital and labor employed in the assembly of input $z$ in country $X$; $x_{Xzt}$ is the quantity of intermediates $i$ employed in sector $z$; and $A_{Xzi}$ is the productivity of intermediate $i$. Both clean and dirty intermediates are produced by monopolists according to

$$x_{Xzt} = \psi K_{Xzt}^\alpha L_{Xzt}^{1-\alpha},$$

so that $\alpha \in (0, 1)$ is the total factor share of capital in sector $E$. The share of intermediates $\gamma$ is the same for both the clean and dirty input and for sector $F$ (so that the monopoly distortion will have only a scale effect and thus not affect the pattern of comparative advantage). I assume throughout that $\alpha > \beta$, which is true empirically: sectors that pollute the most tend to be the most capital intensive. This assumption is without loss of generality, since all results hold when $\alpha < \beta$ and the analysis can also be extended to a pure Ricardian model with $\alpha = \beta$.\(^\text{14}\) I define $K_{XE}^*$ as the total employment of capital in sector $E$,

$$K_{XE}^* \equiv K_{XE} + K_{XF} + \int_0^1 K_{Xc}di + \int_0^1 K_{Xd}di,$$

and similarly define $L_{XE}^*$ as the total employment of labor in sector $E$ in country $X$. In practice, the clean input models nonpolluting inputs that could substitute for polluting inputs—for instance, renewable energies to replace fossil fuel energy or bioplastics to replace traditional petroleum products (the functional form is discussed at greater length in AABH).\(^\text{15}\)

Market clearing for each factor in each country requires that

$$K_{XE}^* + K_{XF} \leq K_X \quad \text{and} \quad L_{XE}^* + L_{XF} \leq L_X,$$

and market clearing for each good requires that

$$C_{NE}^* + C_{SE} \leq Y_{NE} + Y_{SE} \quad \text{and} \quad C_{NF}^* + C_{SF} \leq Y_{NF} + Y_{SF}.$$  

### 2.3 Innovation

At the beginning of every period, one-period monopoly rights are allocated to entrepreneurs (such that each entrepreneur holds monopoly rights on only a finite number of intermediates). Entrepreneurs can hire scientists to increase the productivity of their variety. By hiring $s_{Xzt}$ scientists, the entrepreneur holding the monopoly right on variety $i$ in sector $z = F$ or subsectors $z \in \{c, d\}$ can increase the initial productivity $A_{Xzi(t-1)}$ of her intermediate to

$$A_{Xzi} = \left(1 + \kappa s_{Xzi} \left( \frac{A_{Xzi(t-1)}}{A_{Xzi(t-1)}} \right)^\frac{1}{1-\gamma} \right)^{1-\gamma} A_{Xzi(t-1)} \quad \text{for} \quad z \in \{c, d, F\},$$

\(^\text{14}\)The only issue in this case is that, when the initial difference in comparative advantages is too small, there are multiple equilibria with different patterns of comparative advantages over time.

\(^\text{15}\)I could alternatively assume that good $G$ is produced with capital, labor, and a CES aggregate of clean and dirty intermediates, without affecting the results qualitatively, if the elasticity of substitution $\varepsilon$ were high enough.
where $0 < \iota < 1$. The innovation function $\kappa s^\iota$ is increasing, is concave, and satisfies the Inada conditions. The analysis could be generalized to innovation functions of the form $\kappa ((s_{Xzit} + Y)^\iota - Y^\iota) (A_{Xzi(t-1)}/A_{Xzi(t-1)})^{1/(1-\gamma)}$, where $Y > 0$, that do not satisfy Inada condition; such a generalization will prove useful in Section 6. The concavity of the innovation function reflects the decreasing returns to scale in innovation during a single period (the more scientists innovate on a particular technology during one period, the more they may reproduce the same innovation). The average productivity of (sub)sector $z \in \{c, d, F\}$ at time $t$ is defined as

$$A_{Xzt} \equiv \left(\int_0^1 A_{Xzi(t-1)}^{1/(1-\gamma)} \, dt\right)^{1-\gamma} \text{ for } z \in \{c, d, F\}.$$  

(14)

The factor $A_{Xzi(t-1)}^{1/(1-\gamma)}$ captures decreasing returns to scale in innovation (the more advanced is a technology, the more difficult it is to innovate further), and $A_{Xzi(t-1)}$ denotes knowledge spillovers from all the other intermediates in the same sector in the same country. This formulation ensures that the innovation decision remains symmetric across varieties and that aggregate productivity grows exponentially for a given mass of scientists working in the (sub)sector. Since the mass of scientists is equal to 1 in both countries, the market clearing equation is given by

$$\int_0^1 (s_{XFit} + s_{Xcit} + s_{Xdit}) \, dt \leq 1.$$  

(15)

Because an entrepreneur has monopoly rights for one period only, she will hire scientists so as to maximize current profits instead of the entire flow of profits generated by the innovations of her scientists. The allocation of scientists across (sub)sectors is therefore myopic. One-period monopoly rights are the only inefficiency in innovation and they allow one to model as simply as possible the “building on the shoulder of giants” externality, whose existence has long been recognized by the endogenous growth literature. In the specific context of climate change, this externality plays a crucial role in explaining why clean technologies have so far failed to really take off so far, and why direct research incentives in addition to carbon taxes are welfare improving, a point made by AABH. In the case of permanent monopoly rights, infinitely lived agents, and no environmental externality, the efficient innovation allocation would be an equilibrium although not usually a unique one.

There are no technological spillovers between countries and technologies are country specific. Section 6 analyzes alternative scenarios. A fixed mass of scientists in both countries has the advantages of enabling a focus on only the direction of technical change and ensuring that one country does not become arbitrarily large relative to the other (this assumption is relaxed in Appendix A).

Finally, it is crucial that innovation may occur in all three (sub)sectors. If innovation were limited to clean and dirty technologies within the polluting sector, then the North could not
build a comparative advantage in a specific sector. With clean innovation in the polluting sector only (as in Di Maria and Smulders, 2004; and Di Maria and van der Werf, 2008), the model would ignore all innovations that aim to increase productivity without decreasing emissions; these include not only innovations that make it cheaper to use fossil fuel energy but also innovations in components that are complements to fossil fuel energy and thus increase its demand. In fact, these dirty innovations still make up the bulk of innovation (with respect to the automotive industry, see Aghion et al., 2012). If, on the contrary, only dirty innovations could be made in the polluting sector, then no innovations could replace existing polluting technologies (because the polluting sector and the nonpolluting sector are complements, \( \sigma \leq 1 \)).

### 2.4 Environment

Within the two bounds 0 and \( \bar{S} \), environmental quality evolves according to

\[
S_t = (1 + \Delta) S_{t-1} - \xi (Y_{dNT} + Y_{dSL}).
\]

This law of motion is the same as in AABH, where it is discussed in greater detail. The parameter \( \xi > 0 \) measures the rate of environmental degradation from the production of dirty input, and \( \Delta > 0 \) is the regeneration rate of the environment. Without loss of generality, I assume that \( S_0 = \bar{S} \). Note that the environment’s regeneration capacity decreases with greater environmental degradation — the type of negative feedback that climatologists worry about, e.g. the change in Earth’s albedo and the release of captured greenhouse gases which may occur as the polar ice cap melts. This law of motion matters for the calibration but is adopted in the analytical part only for the sake of simplicity. The analytical results can easily be generalized to different laws of motion; the only assumptions used are that the regenerative capacity of the environment is limited and, for one result on the optimal policy only, that the disaster level is an absorbing state.\(^1\)

The dirty input is directly responsible for environmental degradation. This specification is equivalent to one where a (cheap) fossil fuel resource can be combined with the dirty input in a Leontieff way. Given that most fossil fuel energy in manufacturing comes from coal and natural gas (which are in large supply relative to the time scale of critical environmental degradation), this is a plausible assumption; however, it is not a good approximation for oil, see Hassler and Krusell (2012).

### 2.5 Policy tools

Section 4 will solve the social planner’s problem of maximizing (1) or (2), but Section 3 studies only whether or not an environmental disaster can be prevented with some specific policy.

\(^1\)For instance, all analytical results carry through under the following law of motion: \( S_t = \Delta \bar{S} + (1 - \Delta) S_{t-1} - \xi (Y_{dNT} + Y_{dSL}) \) for \( S_{t-1} > 0 \) and \( S_t = 0 \) for \( S_{t-1} = 0 \).
instruments (the ones that will eventually be used to decentralize the optimal policy). More specifically, I introduce ad valorem taxes on the dirty input \( \tau_{Xt} \), which are the equivalents of a carbon tax, as well as sector-specific ad valorem research subsidies or taxes on scientists’ wages;\(^{17}\) a country may also subsidize the use of all intermediates identically across every subsector in order to correct for the monopoly distortion. In addition, I allow for an ad valorem trade tax on the polluting good \( E \) (by Lerner symmetry, doing so is without loss of generality; the trade tax could also be on the other good). Hence prices in the South are always equal to international prices: \( p_{SEt} = p_{Et} \) and \( p_{SFt} = p_{Ft} \). In the North, the price of good \( F \) is also equal to the international price, \( p_{NFt} = p_{Ft} \), but the price of good \( E \) is given by \( p_{NEt} = p_{Et} (1 + b_t) \), where \( b_t \) is the trade tax. A positive trade tax corresponds to a tariff (resp., export subsidy) when the North imports (resp., exports) good \( E \).\(^{18}\) When the North is the only country intervening, I assume that trade balance must be maintained every period (there is no intertemporal trade); that is,\(^{17}\)

\[
p_{Et} (Y_{SEt} - C_{SEt}) + p_{Ft} (Y_{SFt} - C_{SFt}) = 0. \tag{17}
\]

Note that the trade tax is not explicitly related to the carbon content of imports. If the South does not undertake any policy, then relating the tax to the average carbon content of imports from a given country and in a given sector would not alter the results; since each Southern firm is atomistic, its impact on average emission is infinitesimal and so its behavior will not affect the trade tax it pays. Changing the behavior of Southern firms would require either the North to know the exact carbon content of each individual import (which seems implausible) or the South to implement a policy in response to the North’s tariff. I return to these issues in Section 3.4.

In short, a policy is characterized by a sequence of ad valorem taxes on the dirty input \( \tau_{Xt} \) in each country, a sequence of subsidies for scientists in every subsector, and a sequence of trade taxes \( b_t \) on the polluting good. All subsidies and taxes are financed (or rebated) through lump-sum taxation at the country level.

\(^{17}\)In order to ensure uniqueness of the equilibrium allocation of scientists, I assume that it is possible to subsidize only a given mass of scientists; hence the social planner can use the subsidy to determine the exact allocation. Note that if the subsidy is greater than 100 percent then a monopolist may be willing to hire scientists even if she is not producing any good. This assumption clarifies the exposition; I discuss in Appendix A how the results would be affected if this were impossible.

\(^{18}\)Starting from the situation where the North imports the polluting good under free trade, an increasingly higher trade tax corresponds to a tariff up to the point where it reproduces autarky. Beyond that point, the North begins to export the polluting good and the trade tax becomes an export subsidy.
3 Preventing an Environmental Disaster

This section seeks to identify which policies can prevent an environmental disaster. Section 3.1 details the behavior of the economy under laissez-faire; in particular, it explains the pattern of trade and the allocation of innovation across sectors. Section 3.2 shows that the economy reaches a disaster in laissez-faire and how it can be avoided with policies in both countries. Section 3.3 explains why taxing the North’s polluting sector likely fails to prevent a disaster. Section 3.4 describes how a disaster can be avoided using unilateral policies in the North. Section 3.5 discusses some extensions, and Section 3.6 summarizes the results. For a given policy, the equilibrium is defined as follows.

**Definition 1** A feasible allocation is a sequence of demands for capital ($K_{Xht}$, $K_{XFIt}$, $K_{Xcit}$, $K_{Xdit}$), demands for labor ($L_{Xht}$, $L_{XFIt}$, $L_{Xcit}$, $L_{Xdit}$, $L_{Xdit}$), demands for intermediates ($x_{Xzit}$ for $z \in \{c, d\}$, $F$), demands for inputs ($Y_{Xct}$, $Y_{Xdt}$), goods production ($Y_{XEt}$, $Y_{XFt}$), demands for goods ($C_{XEt}$, $C_{XFt}$), research allocations ($s_{Xzit}$ for $z \in \{c, d\}$, $F$), and quality of the environment $S_t$ such that, in each period $t$ and in each country $X \in \{N, S\}$, factor and good markets clear (i.e., (11), (12), and (15) hold).

**Definition 2** For a given policy, an equilibrium is given by a feasible allocation and sequences of wages of workers ($w_{Xt}$), returns to capital ($r_{Xt}$), wages of scientists ($\varphi_{Xzt}$), consumer prices for intermediates ($\varphi_{Xzit}$ for $z \in \{c, d\}$, $F$), producer prices for clean and dirty inputs ($p_{Xct}$, $p_{Xdt}$), and international prices of goods ($p_{Et}$, $p_{Ft}$) for $X \in \{N, S\}$ such that: (i) ($\varphi_{Xzit}$, $x_{Xzit}$, $s_{Xzit}$, $K_{Xzit}$, $L_{Xzit}$) maximizes profits by the producer of intermediate $i$ in sector $z \in \{c, d, F\}$ in country $X$; (ii) $L_{Xzt}$ and $K_{Xzt}$ maximize the profits of the producer of good $z \in \{c, d, F\}$; (iii) $Y_{Xct}$ and $Y_{Xdt}$ maximize the profits of producer of good $E$; (iv) $C_{XEt}$ and $C_{XFt}$ maximize consumers’ utility under the trade balance constraint; and (v) the trade balance equation (17) is satisfied.

3.1 Laissez-Faire

**Trade pattern.** Here I analyze the laissez-faire equilibrium; the results are derived and generalized in Appendix B.1. In each country, aggregate production in each sector can be written as

$$Y_{XEt} = \zeta_A X_{XEt} K_{XEt}^{\alpha} L_{XEt}^{1-\alpha} \quad \text{and} \quad Y_{XFt} = \zeta_A X_{XFt} K_{XFt}^{\beta} L_{XFt}^{1-\beta},$$

where $\zeta = \frac{\gamma (1-\gamma)}{(1-\gamma+\gamma^2)\rho^{\gamma}}$ and $A_{XEt} \equiv (A_{Xct}^{\gamma-1} + A_{Xdt}^{\gamma-1})^{1/(1-\gamma)}$ is the average productivity of sector $E$. This formulation highlights that, in a given period, the model collapses to a Heckscher–Ohlin model with varying productivity across countries. The South has the comparative advantage
in the polluting good $E$, and it exports $E$ if and only if
\[
\left(\frac{A_{SEt}}{A_{Sft}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NEt}}{A_{Nft}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}.
\] (19)

Trade is then a result of Ricardian forces (relative productivity) as well as Heckscher–Ohlin forces (relative factors endowment). Provided the difference in comparative advantage is not too large, both countries will produce both goods. Yet once that difference becomes sufficiently large, one country fully specializes; if the difference in comparative advantage grows even more, then both countries fully specialize. Emissions are given by $E_{Xt} = \xi \left(\frac{A_{Xdt}}{A_{XEt}}\right)^{\xi} Y_{XEi}$. Thus the emission rate in the polluting sector is increasing in the relative productivity of the dirty technology $A_{Xdt}/A_{Xct}$. Over time, innovation will change the comparative advantage and the emission rate.

**Allocation of innovation.** Entrepreneurs face a two-stage problem. In the second stage, they choose prices in order to maximize their profits given their productivity. Post-innovation profits in sector $z \in \{c, d, F\}$ are given by:
\[
\pi_{Xzt} = (1 - \gamma) \gamma \left(\frac{A_{Xzt}}{A_{Xzt}}\right)^{1-\gamma} p_{Xzt} Y_{Xzt}
\] (20)
(see Appendix B.1). These profits are directly proportional to the revenues of the intermediate’s (sub)sector (this follows from the Cobb-Douglas specification) and they are increasing in the productivity of the intermediate, $A_{Xzt}$. In the first stage, entrepreneurs hire scientists to increase the productivity of their intermediate. Thanks to the knowledge spillovers across varieties, all monopolists in a given (sub)sector hire the same number of scientists and so average productivity evolves according to
\[
A_{Xzt} = (1 + \kappa s_{Xzt})^{1-\gamma} A_{Xz(t-1)} \text{ for } z \in \{c, d, F\}.
\]

**Path dependence in clean versus dirty technologies.** Assume that country $X$ produces good $E$ (otherwise, $s_{Xct} = s_{Xdt} = 0$). Combining the first-order conditions with respect to the number of scientists in the clean and dirty subsector yields the following equation for the allocation of scientists within sector $E$:
\[
\frac{s_{Xct}^{1-\gamma} (1 + \kappa s_{Xzt}^{1-\gamma})}{s_{Xdt}^{1-\gamma} (1 + \kappa s_{Xdt}^{1-\gamma})} = \frac{p_{Xct} Y_{Xct}}{p_{Xdt} Y_{Xdt}} = \frac{A_{Xct}^{1-\gamma}}{A_{Xdt}^{1-\gamma}}.
\] (21)
The second equality follows from the demand equation for both inputs in sector $E$ and because the production technologies differ only by their level of productivity. The ratio of revenues in the clean sector to those in the dirty sector increases with the ratio of clean to dirty technologies. This association reflects two counteracting forces: a larger technology ratio leads to a larger
market share ratio but also to a lower price ratio; the former effect dominates when the inputs are substitutes. Thus, for a sufficiently small innovation size $\kappa$, more scientists are allocated to the dirty than to the clean subsector if and only if $A_{Xd(t-1)} > A_{Xc(t-1)}$ (if $\kappa$ is too large then there may be multiple equilibria when $A_{Xd(t-1)}$ and $A_{Xc(t-1)}$ are close to each other; see Appendix D.1). So in the polluting sector under laissez-faire, innovation tends to be allocated to the sector that is already the most advanced: there is path dependence.

**Amplification of comparative advantage.** Assume that production occurs in both sectors (otherwise, innovation occurs only in the active sector). By combining the first-order conditions with respect to the number of scientists in sector $F$ and in subsectors $c$ and $d$, I obtain

\[
\frac{s^{1-\gamma}_{Xd}(1 + \kappa s^{1-\gamma}_{Xc}) + s^{1-\gamma}_{Xd}(1 + \kappa s^{1-\gamma}_{Xd})}{s^{1-\gamma}_{XF}(1 + \kappa s^{1-\gamma}_{XF})} = \frac{p_{XE}Y_{XE}}{p_{XF}Y_{XF}}.
\]

This equality implies that, for a given ratio of initial productivities within sector $E$ (i.e., for $A_{Xd(t-1)}/A_{Xc(t-1)}$ given), the number of scientists allocated to sector $E$ is increasing in the ratio of sector $E$ to sector $F$ revenues.

In autarky, consumer demand implies that

\[
\frac{p_{XF}Y_{XF}}{p_{XE}Y_{XE}} = \frac{1 - \nu}{\nu} \left( \frac{Y_{XE}}{Y_{XF}} \right)^{\frac{1-\sigma}{\sigma}}.
\]

Therefore, if $\sigma = 1$ then the right-hand side is constant and, asymptotically, a positive mass of scientists innovate in each sector; if $\sigma < 1$ then innovation tends to occur in the smallest sector (i.e., the sector with lowest productivity) and thus becomes balanced as regards the two sectors after a few periods during which the laggard sector catches up. Since the two sectors are complements, innovation will not disappear in one sector over time (as it does in the case of clean versus dirty innovation).

Under free trade, prices are equalized in North and South; hence each country tends to innovate relatively more in the sector it exports (and does so at an equal ratio of initial productivities within sector $E$). As more innovation in a sector results in its greater comparative advantage, which, in turn, prompts more innovation in that same sector, multiple equilibria could arise. Even so, it is possible to prove the following statement.

**Lemma 1** If $\kappa$ is small enough and $\iota \geq 1/2$, the equilibrium is unique.

**Proof.** See Appendix D.1 ■

I shall henceforth assume these conditions to be satisfied, so that the equilibrium is unique. A sufficiently small size of innovation $\kappa$ ensures that changes in productivities during one period remain sufficiently small. The technical assumption $\iota \geq 1/2$ is further necessary to ensure that
the equilibrium is unique when one country is close to a corner of specialization.\footnote{By “corner of specialization,” I refer to a case where a country is nearly fully specialized, that is where a producer of the imported good could break even if and only if he produces an infinitesimal amount of the good. Letting \( \epsilon < 1/2 \) would not affect any other result in this section or in Section 6. The results of Section 4 would also hold as provided that an interior equilibrium is consistently chosen whenever it exists.} Now I can derive the following result:

**Lemma 2** Consider a laissez-faire economy and assume that the South initially has a weak comparative advantage in sector \( E \) (i.e., \( \left( \frac{A_{SE0}}{A_{SF0}} \right)^{1/\beta} \frac{K_S}{L_S} > \left( \frac{A_{NE0}}{A_{NF0}} \right)^{1/\beta} \frac{K_N}{L_N} \)) and that clean and dirty technologies are further apart in the South than in the North (i.e., \( \min \left( A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0} \right) < \min \left( A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0} \right) \)). Then, in every period, more scientists are hired in sector \( E \) in the South than in the North: \( s_{SEt} > s_{NEt} \). Thus, in finite time, the South fully specializes in producing good \( E \) and the North fully specializes in producing good \( F \).\footnote{The lemma extends to the case where \( \left( \frac{A_{SG0}}{A_{SH0}} \right)^{1/\beta} \frac{K_S}{L_S} > \left( \frac{A_{NG0}}{A_{NH0}} \right)^{1/\beta} \frac{K_N}{L_N} \) and \( \min \left( A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0} \right) = \min \left( A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0} \right) \) is sufficiently small. If \( \left( \frac{A_{SG0}}{A_{SH0}} \right)^{1/\beta} \frac{K_S}{L_S} = \left( \frac{A_{NG0}}{A_{NH0}} \right)^{1/\beta} \frac{K_N}{L_N} \) and \( \min \left( A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0} \right) = \min \left( A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0} \right) \), then, there is no trade.}

**Proof.** See Appendix B.2. □

Because countries tend to innovate more in the sector they export, the difference in the allocation of research across sectors builds up over time; the relative productivities of both sectors eventually become so different that the two countries fully specialize. The condition on \( \min \left( A_{Xc0}/A_{Xd0}, A_{Xd0}/A_{Xc0} \right) \) is necessary because both the incentive to innovate in sector \( E \) and the growth rate of the average productivity of sector \( E \) for a given mass of scientists in that sector depend on the initial relative productivities of clean and dirty technologies \( A_{Xc}(t-1)/A_{Xd}(t-1) \) (both decrease when \( A_{Xc}(t-1)/A_{Xd}(t-1) \) is close to 1). Given the lemma’s assumptions, the difference in initial productivity ratios also incentivizes the South to innovate more in the polluting sector \( E \). Initial comparative advantages determine the long-run pattern of trade, just as in the papers that make the infant industry argument.

### 3.2 Avoiding a Disaster with Policy in Both Countries

Under laissez-faire, as long as dirty technologies are more advanced than clean ones in both countries, innovation in the polluting sector will remain directed primarily toward dirty technologies. Since innovation in the polluting sector does not asymptotically vanish,\footnote{The exporting country innovates more in the polluting good than it would under autarky. But even under autarky, innovation in the polluting good does not disappear because goods \( G \) and \( H \) are either complements or Cobb–Douglas.} the production of good \( E \) grows unboundedly and so do emissions. At some point, the regenerative capacity of the environment becomes overwhelmed and the economy reaches a disaster.
A global government could use clean research subsidies, taxes on dirty research and/or carbon taxes to redirect innovation from the dirty toward the clean subsector in countries that produce the polluting good. Once clean technologies acquire a sufficient lead over dirty intermediates, market forces will ensure that most research is directed toward the clean subsector (which is now the most advanced). Eventually, the emission rate of the polluting good approaches zero, and a disaster can be avoided if the initial environmental quality is high enough. This analysis is similar to AABH and can be summed up as follows.

**Remark 1** Irrespective of how high $\overline{S}$ is, a disaster occurs in the laissez-faire equilibrium if clean technologies are less developed than dirty ones ($A_{Nc0} \leq A_{Nd0}$ and $A_{Sc0} \leq A_{Sd0}$). For sufficiently high $\overline{S}$, a disaster can be prevented if both countries have access to temporary clean research subsidies, taxes on dirty research, or carbon taxes.

**Proof.** See Appendix D.2.

### 3.3 Taxes on the Polluting Good in the North only

Assume now that only the North is able to implement some policy (this rules out the case where the North pays the South to implement some policy). Is this alone enough to avoid environmental disaster? Observe that, in autarky and without knowledge spillovers, no policy restricted to the North can prevent a disaster because Southern emissions grow unboundedly regardless of what the North does. Absent international cooperation, trade is necessary to avoid environmental disaster. Now one can show the following statement.

**Lemma 3** If clean technologies are less developed than dirty ones in the South ($A_{Sc0}/A_{Sd0} \leq 1$), then a disaster can be averted only if (i) all factors in the South are asymptotically allocated to the nonpolluting sector $F$ and (ii) in the long run, the North exports the polluting good.

**Proof.** See Appendix B.3.

In other words, the key to avoid environmental disaster with Northern policies only is ensuring that the South asymptotically fully specializes in the nonpolluting sector. If the South continued to allocate a positive share of labor and capital to the polluting sector, then the amount of innovation in the polluting sector in the South would be bounded away from zero. Hence Southern production of the polluting good would become unbounded — and so too would emissions absent a local environmental policy.

Of interest here are taxes on the polluting good in the North (a carbon tax or a tax on dirty research), which have no protectionist aspect. Both can reduce emissions in the North and

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$^{22}$The equilibrium is still unique with these instruments if the tax on dirty research can be restricted to apply only to a fraction of scientists.
both prompt clean innovation there, and both could prevent an environmental disaster if the
North were the only country or if the South undertook the same policy. However, such policies
may be incompatible with a South specializing in the nonpolluting sector and therefore may
be unable to prevent an environmental disaster.

Proposition 1 If innovation size $\kappa$ is small enough then, no matter how high $\bar{S}$ is, no com-
bination of a carbon tax and a tax on dirty research can prevent an environmental disaster if: (i) clean technologies are less developed than dirty ones in the North ($A_{Nc0}/A_{Nd0} \leq 1$); (ii) clean technologies are sufficiently less developed than dirty ones in the South ($A_{Sc0}/A_{Sd0}$ is sufficiently small); and (iii) the South has a weak initial comparative advantage in the polluting
sector (i.e., $(A_{SE0}/A_{SF0})^{1-\beta} K_S/L_S \geq (A_{NE0}/A_{NF0})^{1-\beta} K_N/L_N$).

Proof. See Appendix B.4. ■

This is a consequence of Lemmas 2 and 3. Under laissez-faire and with the assumptions of
the proposition, the South would keep its comparative advantage in the polluting sector or (in
the knife-edge case) there would never be trade. The North government cannot reverse this
pattern simply by using a tax on dirty research or a carbon tax. On the contrary, a tax on dirty
innovation drives scientists away from the polluting sector $E$ toward the nonpolluting sector
$F$; moreover, within the polluting sector it allocates innovation toward the initially backward
clean subsector, which further reduces the growth rate of average productivity $A_{NEt}$. A carbon
tax has the same effect on innovation and also directly reduces the productivity of the polluting
sector in the North. Because both instruments increase the costs of producing the polluting
good in the North, they lead to an increase in its world relative price. This induces an increase
in production of the polluting good $E$ in the South and hence more emissions there, which is
the classic pollution haven effect. As the relative revenues of the polluting sector increase in
the South, innovation is further tilted toward the polluting sector, where it is mostly directed
at the dirty technologies. This explains why Northern taxes on the polluting good can only
accelerate the Southern specialization in the polluting sector.

In fact, the economy tends to grow faster when countries are more specialized because
then there is less overlap in the type of innovations being undertaken by both countries. This
dynamic tends to increase emissions. In addition, the gap between clean and dirty technolo-
gies in the South grows faster, which increases the South’s emissions rate. Such policies are
therefore likely to increase environmental degradation. Of particular interest is the knife-
edge case, where the South has no comparative advantage (i.e, $(A_{SE0}/A_{SF0})^{1-\beta} K_S/L_S =
(A_{NE0}/A_{NF0})^{1-\beta} K_N/L_N$ and $A_{Nc0}/A_{Nd0} = A_{Sc0}/A_{Sd0}$). In this case there would be no
trade under laissez-faire, but the policy intervention tips the balance toward a comparative
advantage for the South in sector $E$, which then builds on itself over time. The result is more
economic growth but the environmental disaster arrives sooner.
Furthermore, although carbon taxes and taxes on dirty research can tilt innovation within the polluting sector toward clean technologies, they typically fail to ensure that such technologies actually catch up. As production of the polluting sector moves to the South, the market size for clean technologies in the North becomes too small to attract much innovation. This could explain why the implementation of the ETS system has led to only a limited number of clean innovations (see Calel and Dechezleprêtre, 2012).

In the proposition, the condition that $A_{Sd0}/A_{Sc0}$ be sufficiently small (and not simply less than 1) is necessary for the same reasons as in Lemma 2: with the ratio of clean to dirty revenues farther from unity in the North than in the South, more innovation in the polluting sector might take place in the former even if the latter exports the polluting good. The condition could be dispensed with if the initial comparative advantage were sufficiently large.

3.4 Introducing Clean Research Subsidies and Trade Taxes

The previous policies could not prevent an environmental disaster — when the South had the initial comparative advantage in the polluting sector — because they could not reverse the pattern of trade. I now allow the North to use clean research subsidies and a trade tax.

Note that both policies have some protectionist aspects, in that the clean research subsidy is a conditional subsidy granted to the polluting sector, which is the sector facing Southern imports.

Proposition 2 A combination of a temporary trade tax and a temporary clean research subsidy in the North can prevent an environmental disaster provided that the initial environmental quality $\bar{S}$ is sufficiently high.

The key difference between clean research subsidies and the carbon tax or the tax on dirty research is that the former can reallocate scientists who were working in the nonpolluting sector $F$ (in addition to scientists working in the dirty subsector) toward the clean subsector. The result is much more innovation in clean technologies, including when the North does not have the comparative advantage in the polluting sector. In the meantime, the trade tax can be used

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23 More specifically: the incentive to innovate in sector $G$ is, ceteris paribus, lower when the revenues in the clean and dirty subsectors are close to each other — that is when $A_{Sc0}^{c-1}/X_{c0}(t-1)$ and $(1 + \tau_{Xc})^{-\varepsilon} A_{Sc0}^{c-1}/X_{d0}(t-1)$ are comparable. Given carbon taxes that are high enough or taxes on dirty research that are of sufficient duration, the ratio of clean to dirty revenue may become farther from unity in the North than in the South. In that event, the assumption on $A_{Sd0}/A_{Sc0}$ ensures that, when this occurs, the difference in comparative advantages is large enough to ensure that there is more polluting innovation in the South.

24 In the perfect substitute case this condition would be replaced by $A_{Sd0}/A_{Sc0} < (1 + \kappa)^{-(1-\gamma)}$, which ensures the unicity of the equilibrium. In the pure Ricardian case ($\alpha = \beta$), the proposition holds for sufficiently large initial comparative advantage.

25 With a trade tax, the equilibrium may not be unique when $\sigma < 1/2$; however, this does not affect the following analysis.
to reduce specialization in the South and thereby decrease the amount of innovation in the polluting sector there. For sufficiently high initial environmental quality, a policy combining these two instruments can prevent a disaster. To see this and prove Proposition 2, consider the following two-phase approach. In the first phase, a social planner implements a tariff large enough to shut down trade, so that innovation in the South must be balanced between the polluting and nonpolluting sectors. In the meantime, the social planner can implement large clean research subsidies so that nearly all Northern scientists innovate in the clean subsector. As a consequence, not only do clean technologies become more advanced than dirty ones in the North but also the North builds a comparative advantage in the polluting sector $E$, since it innovates relatively more in that sector than does the South. Once the North has acquired the comparative advantage in the polluting sector and $A_{Nc(t-1)}/A_{Nd(t-1)}$ is sufficiently large, the social planner can discontinue all policies: by Lemma 2, sector-$E$ production ends up moving entirely to the North (where it has become clean), and a disaster can be avoided.\footnote{In addition, this proof shows that for a sufficiently high initial environmental quality the Northern social planner does not need to impose an export subsidy: a tariff is enough (see also footnote 18)}. From this discussion one might suppose that clean research subsidies alone should be enough to prevent an environmental disaster. However, the following remark stipulates that this is not the case.

**Remark 2** Suppose final consumption is Cobb–Douglas in both the polluting and nonpolluting goods ($\sigma = 1$). Then there exist initial factor endowments and technologies, such that no matter how high $\overline{S}$ is, no combination of a carbon tax, a tax on dirty research, and a subsidy for clean research can prevent a disaster.

**Proof.** See Appendix D.3. ■

Clean research subsidies alone cannot prevent a disaster when the South fully specializes in the polluting sector and clean technologies in the South are sufficiently less advanced than dirty ones. In that case, all Southern scientists are allocated to the polluting sector and, asymptotically, to dirty technologies. So even if the North were to allocate all its scientists to clean technologies, $A_{SEt}$ would grow as fast as $A_{NEt}$. That situation is irreversible in the Cobb–Douglas case and an environmental disaster cannot be avoided. Full specialization in the South occurs in the first place when its initial comparative advantage in the polluting sector is sufficiently large or when clean technologies are sufficiently backward in the North (the average productivity of the polluting sector in the North, $A_{NEt}$, grows slowly during the period when clean technologies are catching up with dirty ones). Yet this reasoning does not carry through to the strict complement case ($\sigma < 1$). In that case, the South cannot continue to specialize fully in the polluting sector if both countries innovate only in that sector because the demand for the nonpolluting good becomes too large. Hence the South’s innovation in dirty goods becomes too large.
technologies is bounded away from unity, and the North can reverse the pattern of trade and prevent an environmental disaster for a sufficiently high level of initial environmental quality.\textsuperscript{27} In general, however, the trade tax helps to reverse comparative advantages. The South is less specialized in the polluting sector, so the North can ensure a reversal in comparative advantage with a more balanced innovation ratio; the North is less specialized in the nonpolluting sector, so a given ratio of clean to nonpolluting innovation can be achieved with a lower subsidy.

3.5 Extensions

\textbf{South’s retaliation}. I have assumed so far that there is no government in the South that could respond to the unilateral policies imposed by the North. Although a full analysis of the strategic interactions between two governments is beyond the scope of this paper, it is worth mentioning briefly the scope for South’s intervention. First, note that the South’s consumption is not always negatively affected by the North’s unilateral policies (moreover the South benefits from better environmental quality). For instance, if the North’s temporary policy reverses the pattern of comparative advantages, both countries fully specialize in the long run. In the Cobb–Douglas case ($\sigma = 1$), income shares are linked to the consumption share of the good that the country exports; therefore, if the income share for the polluting good is smaller than for the nonpolluting one ($\nu < 1 - \nu$), then the South’s income share will be larger under the policy.\textsuperscript{28}

Second, one could consider the case of a Nash equilibrium between a Northern social planner maximizing the welfare of an infinitely lived Northern representative agent and a Southern social planner maximizing the welfare of an infinitely lived Southern representative agent with identical preferences. In this case, the equilibrium will not be the first best and environmental degradation is likely to be greater. Even so, an environmental disaster will still always be averted because consumption does not increase welfare if environmental quality reaches zero.

It would be more interesting to could consider the case of a Southern government that maximizes only current consumption. Such a government implements its own trade tax to improve its terms of trade. As long as the South retains an initial comparative advantage in the polluting good, this trade tax moves both countries closer to autarky and so does not prevent the North from reversing the pattern of comparative advantage. Once the North exports the polluting good, the South implements its own tariff. This tariff slows down the South’s specialization in the nonpolluting sector. However, one can show that if the North has acquired

\textsuperscript{27}See Appendix D.3. In that case, averting a disaster with clean research subsidies relies on the assumption that the North can induce innovation in clean technologies even if there is no production. Otherwise, a trade tax may also be necessary.

\textsuperscript{28}Even in the short run, the South might benefit: a tariff implemented by the North hurts the South when the South exports the polluting good, but a trade tax high enough to reverse the pattern of trade immediately (so that it serves as an export subsidy) may benefit the South.
a sufficiently large comparative advantage, then the South will eventually specialize in the nonpolluting sector and a disaster can still be avoided for sufficiently high initial environmental quality.

**Goods-based carbon content tariff.** As discussed in Section 2, without policy in the South there is no way for the North to affect directly Southern firms’ emission rates directly unless it knows the carbon contents of each import at the firm level — which seems impossible. Yet this case makes for an interesting benchmark. In that case, when the South has a comparative advantage in the polluting good then the dirty input will be taxed for the exports market. If clean technologies are not too backward in the South and if the export market is relatively large (which is possible if the South is small and has a large comparative advantage), then a switch to clean technologies in the South could occur; thus a disaster can be avoided without reversing the pattern of comparative advantage.\(^{29}\) Appendix A.1 describes the role of other instruments (clean input production subsidies and carbon barrier tax adjustments) as well as the case where the mass of scientists is different in the North and in the South.

### 3.6 Taking Stock and Comparison with Undirected Technical Change case

Several lessons can be derived from the preceding analysis. First, the pollution haven effect becomes worse in a dynamic setting. Taxes on the polluting sector in the North risk placing the economy on a path that leads to the South having a comparative advantage in the polluting sector. Because comparative advantage tends to be reinforced over time, the bulk of production of the polluting sector ends up occurring in the South. This dramatically hampers the impact of such an intervention on worldwide emissions. Furthermore, since the market share for the polluting sector becomes small in the North, the incentives to innovate in clean technologies remain limited. To ensure sustainable growth without cooperation from the South, the North must undertake a temporary and somewhat protectionist industrial policy in order to make the polluting sector cleaner and also to secure the comparative advantage in the polluting sector.

Second, trade acts as a double-edged sword. Trade under laissez-faire leads to specialization, which maximizes long-run growth and therefore leads to more rapid environmental degradation. Moreover, the pollution haven effect means that trade make taxes less effective on the polluting sector. Yet trade, if it can be properly managed, is the key to avoiding an environmental disaster in a noncooperative world: once the North has sufficiently developed its clean technologies, trade forces ensure that pollution will also decrease in the South.

Third, directed technical change also acts as a double-edged sword. Relative to a model

\(^{29}\)On the other hand, if the South could undertake some policy then the North could base the tariff on average emissions from a given country in a given sector. The South’s government would then recognize that the tariff depends on its policy and thus be incentivized to undertake some environmental policy on its own. Such an analysis would be interesting but is beyond the scope of this paper.
where technological levels grow exogenously at an equal rate, directed technical change accelerates economic growth and environmental degradation; it also reduces the benefits from taxes on the polluting sector because of the South’s innovation response. That being said, directed technical change allows the pattern of comparative advantage to be reversed which greatly helps in preventing a disaster with unilateral policies. Crucially, for some parameters, avoiding a disaster with unilateral policies is impossible without directed technical change. Indeed, consider the case where technical change is undirected and occurs at the same rate in all sectors. Further, assume that clean technologies are sufficiently less developed than dirty ones in the North and that the South has a large comparative advantage in the polluting sector; in that case — even if the North produces only the polluting good in a clean way and exports all of its production to the South, which can be achieved with high enough trade and carbon taxes — there will not be enough exports to ensure that the South specializes in the nonpolluting sector. Here, the pattern of comparative advantage (relative productivities and factor endowments) does not change over time; hence the situation is permanent and so a disaster cannot be averted. This thought experiment demonstrates that innovation’s ability to affect comparative advantage is essential to deriving the previous results.

4 Optimal policy

In this section I characterize the first-best policy and the second-best policy under the constraint that the social planner cannot intervene in the South.

4.1 First-Best

In this case, the social planner maximizes (1) or (2) subject to the following constraints: the production function equations (3), (4), (5), (7), (8), and (9); the factor market-clearing equations (11) and (15); the goods market-clearing equation (12); the environmental degradation equation (16); and the knowledge accumulation equation (13). I can then prove the following result.

Proposition 3 The first-best policy can be decentralized by combining a carbon tax in both North and South (with the same price for carbon), research subsidies/taxes (in North and South) in both sectors, and a subsidy for the use of all intermediates. When the social planner maximizes (2), international transfers are also required.

Proof. See Appendix B.5. ■

Each instrument allows the social planner to correct for one distortion. First, the environmental externality is corrected by a carbon tax in both countries that equalizes the marginal
cost of the tax (lower current consumption) with the marginal benefit (higher environmental quality in all subsequent periods). Carbon taxes in the North and the South differ in ad valorem values across countries but are identical as a tax per unit of CO$_2$. Second, the social planner corrects for the myopia of monopolists in their innovation decisions by allocating scientists in accordance with the discounted value of the entire stream of additional revenues generated by their innovation. More specifically: \textit{contra} (21) and (22), scientists are now allocated across the dirty, clean, and nonpolluting (sub)sectors according to
\begin{equation}
\frac{s_{XFt}^{t-1}}{1 + \kappa s_{XFt}^t} \sum_{s=t}^{\infty} B_{s,t} \hat{p}_{F} F_{s} = \frac{s_{Xct}^{t-1}}{1 + \kappa s_{Xct}^t} \sum_{s=t}^{\infty} B_{s,t} \hat{p}_{Xcs} Y_{cs} = \frac{s_{Xdt}^{t-1}}{1 + \kappa s_{Xdt}^t} \sum_{s=t}^{\infty} B_{s,t} \hat{p}_{Xds} Y_{ds}, \tag{24}
\end{equation}
where $\hat{p}_{Xcs}$ and $\hat{p}_{Xds}$ denote the shadow price of (respectively) the clean and dirty inputs in country $X$ and $\hat{p}_{F}$ is the shadow price of good $F$; $B_{s,t} = \frac{1}{(1 + \rho)^{s-t} \frac{\partial u}{\partial C}(C_{Wt}, S_t)}$, where $u(C_{Wt}, S_t) = \frac{\nu(S_t) C_{Wt}^{1-\eta}}{1-\eta}$ and $C_{Wt} \equiv C_{Nt} + C_{St}$ is world consumption. Thus $B_{s,t}$ is the effective discount factor between periods $s$ and $t$. Third, the underutilization of intermediates due to monopoly pricing is corrected by a subsidy equal to $1 - \gamma$ to all intermediates. Finally, if the social planner cares about the distribution of consumption, then transfers are used to equalize the marginal social value of consumption in each country (i.e., $\Psi C_{Nt}^{-\eta} = (1 - \Psi) C_{St}^{-\eta}$).

Since utility flow is minimized during a disaster and since the social planner can always reduce world emissions, the optimal policy always avoids disaster. The following remark further characterizes the optimal policy and establishes conditions under which, as in Remark 1, a switch to clean innovation occurs in one country.

\textbf{Remark 3} The social planner always avoids a disaster. If the discount rate $\rho$ is sufficiently small and the inverse elasticity of intertemporal substitution $\eta \leq 1$, then innovation in sector $E$ switches to mostly clean innovation and both countries reach full specialization in finite time.

\textbf{Proof.} See Appendix D.4. \qed

Since consumption is worthless following a disaster, the social planner always avoids that scenario (he can always stop producing dirty inputs to do so).\footnote{This remark still holds if $S = 0$ is not an absorbing state. If there is a lag in the impact of emissions on environmental quality, the conditions under which a disaster is avoided are the same as in Proposition 5 below.} With the inverse elasticity of intertemporal substitution $\eta \leq 1$ and a sufficiently small discount rate, the optimal policy maximizes the long-run growth rate. A switch to clean innovation allows the polluting sector to grow at a positive rate and still avoids a disaster. Moreover, long-run growth is maximized if each country innovates only in its own sector so that there is no overlap in the innovations they undertake. The difference in comparative advantage then becomes so great that both countries end up fully specializing. Since the dirty input becomes a negligible part of the production
process, emissions vanish. With the law of motion (16), the quality of the environment reverts to \( S \) — and the carbon tax reaches zero — in finite time.\(^{31}\)

When the social welfare function is given by (1), so that there are no redistribution concerns, the country exporting the polluting good is not necessarily the one where consumption is most reduced by environmental policy. Indeed, reducing the production of the polluting good creates a terms-of-trade effect which is beneficial to the polluting country. If final consumption is Cobb–Douglas \((\sigma = 1)\) and if the policy intervention does not affect the pattern of specialization, then long-run consumption is reduced proportionally in both countries relative to laissez-faire. When the goods are strict complements \((\sigma < 1)\), the country exporting good \( E \) actually ends up with a larger share of world consumption because the terms-of-trade effect is stronger.

### 4.2 Second-Best

I now turn to the case where the social planner cannot implement any policy in the South (whose the economy is in laissez-faire)\(^{32}\) and cannot transfer income from one country to another (so that trade balance must be maintained at every point in time; there is no intertemporal trade). The second-best policy is defined by the social planner maximizing (1) or (2) subject to the following constraints: (3) for the North and the South; constraints (4), (5), (7), (8), (9), (11), (15) and (13) for the North only; the environmental degradation constraint (16); the goods market-clearing constraints in both countries, which are now written as

\[
C_{NYt} = Y_{NYt} + M_{Yt}, \quad \text{and} \quad C_{SYt} = Y_{SYt} - M_{Yt}, \quad \text{for} \quad Y \in \{E, F\},
\]

where \( M_{Yt} \) denotes net imports of the North of good \( Y \); the trade balance constraint

\[
p_t M_{Et} + M_{Ft} = 0,
\]

where \( p_t \equiv p_{Et}/p_{Ft} \) is the international price ratio; and constraints describing the South’s laissez-faire economy. These latter constraints (detailed in Appendix B.8) are: a consumer demand equation

\[
\frac{\partial C_S}{\partial C_{SE}} = \frac{\nu C_{SFt}^{\frac{1}{2}}}{(1 - \nu) C_{SEt}^{'}} = p_t;
\]

\(^{31}\)More generally: for an alternative law of motion where environmental regeneration decreases as the quality of the environment approaches \( S \) (see, e.g., the law of motion described in footnote 16), then the quality of the environment reaches \( S \) asymptotically. The carbon tax becomes practically irrelevant in this sense: for any \( e > 0 \) there is a \( T \) such that, for \( t > T \), the difference between the discounted sum of utility flows from \( T \) with the optimal policy and the discounted sum of utility flows from \( T \) with the same policy but a carbon tax that expires after \( T \) is smaller than \( e \).

\(^{32}\)To avoid a scale effect when comparing the first-best and second-best policies in the calibration, I assume that the South implements the optimal subsidy for the use of all its intermediates. This assumption has no bearing on any of the theoretical results. Moreover, the South’s equilibrium must be unique given the North’s allocation, so I assume throughout that \( \kappa \) is sufficiently small and \( \iota \geq 1/2 \).
offer equations in the South of the type
\[ Y_{SEt} = ySE(pt, A_{SEt}, A_{SFt}) \quad \text{and} \quad Y_{SFt} = ySF(pt, A_{SEt}, A_{SFt}) ; \] (28)
an emissions equation \( Y_{Sdt} = (A_{Sdt}/A_{SEt})^{\varepsilon} Y_{SEt} \); an equation that specifies the mass of scientists allocated to sector \( E \),
\[ s_{SEt} = sSE(pt, A_{Sdt}, A_{Sc}, A_{SFt}) ; \] (29)
and the resulting law of motion of aggregate productivity in the South:
\[ A_{SFt} = (1 + (1 - s_{SEt})^{\gamma})^{1-\gamma} A_{SF(t-1)} ; \] (30)
\[ A_{Szt} = 1 + s_{Szt} s_{SEt}^{\gamma} A_{Sdt}^{\gamma} \left( \frac{1}{1 + s_{Szt}^{\gamma}} A_{Sct}^{\gamma} \right) \] for \( z \in \{c, d\} \);
where the allocation between clean and dirty innovation is uniquely determined by the total mass \( s_{SEt} \) and the ratio \( A_{Sc(t-1)}/A_{Sdt(t-1)} \). This leads to the following result.\(^{33}\)

**Proposition 4** The second-best policy can be decentralized through a carbon tax in the North, research subsidies/taxes in the North, a subsidy for the use of all intermediates, and a trade tax.

**Proof.** See Appendices B.6 and D.6. \( \blacksquare \)

In this second-best scenario, the social planner uses the same instruments as before to address the inefficiencies of the North’s economy: a subsidy to the use of all intermediates (for the monopoly distortion), a carbon tax, and research subsidies in order to allocate scientists as in (24). The trade tax is the optimal method for affecting Southern prices, which is the only channel through which the social planner can intervene in the South’s economy. In Appendix B.6, I derive an implicit equation for the value of the optimal trade tax (B.58) for the maximization of (1) that takes the following form:
\[ f_t(b_t) = c_{1t}\tau N + c_{2t} \left( s_{SFt+1}^{\mu_{SFt+1}} A_{SFt+1}^{\mu_{SFt+1}}/1 + \kappa s_{SFt}^{\mu_{SFt+1}} a_{SFt+1} - \frac{\partial s_{Sdt}}{\partial s_{SEt}} a_{Sdt+1}/1 + \kappa s_{Sdt}^{\mu_{Sdt+1}} - \frac{\partial s_{Sc}}{\partial s_{SEt}} a_{Sc+1}/1 + \kappa s_{Sc}^{\mu_{Sc+1}} \right) + \text{term}_3. \] (31)
Here \( f_t(b_t) \) is a term with the same sign as \( b_t \), \( c_{1t} \) and \( c_{2t} \) denote coefficients that are positive when the South is not fully specialized, weakly positive when the South is at a corner of full specialization (see footnote 19), and null otherwise; and \( \mu_{Sz(t+1)} \) denotes the social value of a productivity unit in the South in (sub)sector \( z \in \{c, d, F\} \).

\(^{33}\)For \( \sigma < 1/2 \), there may be several equilibria corresponding to a given policy. Hence, the social planner would need the ability to choose directly the amount of imports (via quotas for instance) in order to pick the right equilibrium.
The first term is always positive and represents the environmental motive for the trade tax: pollution in the South escapes direct taxation, and a positive trade tax on the polluting good \( E \) (a tariff or an export subsidy) reduces the relative price of good \( E \) in the South, which reduces its production and hence emissions in the South. The second term is a correction for the myopia of Southern innovators; it represents the difference – for welfare in all subsequent periods – between the social value of an additional scientist in the nonpolluting sector \( F \) and one in the polluting sector \( E \). In principle, this term could be of either sign, but in practice it is positive and pushes toward a positive tariff or export subsidy. Indeed, to avoid disaster, the South must at least asymptotically fully specialize in the nonpolluting sector (see Lemma 3). So provided the optimal policy avoids a disaster, current innovations in the polluting sector are of little future use. A positive trade tax on the polluting good \( E \) tilts innovation in the South away from that sector. The third term \( \text{term}_3 \) has an ambiguous sign but is small if the gap between clean and dirty technologies is large. Overall, the trade tax is generally positive in this case; it takes the form of a tariff when the North imports the polluting good and of an export subsidy when it exports that good.

If instead (2) is being maximized, then there is a fourth term on the right-hand side of (31):
\[
c_t \left( 1 - \frac{\lambda_{Sl}}{\lambda_{Nt}} \right) \left( \frac{C_{SEt}}{C_{NEt}} \frac{Y_{SEt}}{Y_{SFt}} - 1 \right),
\]
where \( c_t \) is positive and \( \lambda_{Nt} \) is the Lagrange multiplier associated with (3) in country \( X \) and represents the marginal social value of a unit of consumption at time \( t \) in that country. This last term represents the terms-of-trade motive, as the trade tax is modified in order to favor the country with the largest social marginal value of consumption. If the social planner cares only about the North \( (\Psi = 1) \), then this motive pushes toward a tariff when the North imports the polluting good and toward an export tax otherwise. If the social planner cares equally about both countries \( (\Psi = 1/2) \) but the South is poorer, then it pushes toward an import or an export subsidy.

To clarify the relation between my analysis and the literature, in Appendix D.5.2, I derive the following condition for the social optimum:
\[
\frac{p_t + M_{Et} \frac{\partial p_t}{\partial M_{EF}}} {1 + M_{Et} \frac{\partial p_t}{\partial M_{EF}}} = \frac{\partial C_{Nt}} {\partial C_{NEt}} + \frac{\partial C_{Nt}} {\partial C_{NFt}} + \frac{\partial C_{St}} {\partial C_{SEt}} + \frac{\partial C_{St}} {\partial C_{SFt}};
\]
where \( \phi_t \) is the social value of moving an infinitesimal mass of Southern scientists to sector \( E \) at time \( t \) and, \( \lambda_{Nt} = \lambda_{St} = \lambda_t \) when (1) is being maximized. The terms-of-trade effect of an additional unit of imports in sector \( E \) and \( F \) is measured by (respectively) \( M_{Et} \frac{\partial p_t}{\partial M_{EF}} \) and \( M_{Et} \frac{\partial p_t}{\partial M_{EF}} \). Equation (32) stipulates that the \( E/F \) ratio for the North’s cost of imports (prices

---

\(^{34}\)This term is fairly technical. It reflects that if more Southern scientists are allocated to sector \( E \) today then, for a given number of scientists allocated to sector \( E \) tomorrow, more will be allocated to dirty than to clean technologies.

\(^{35}\)In that case the sign of the second term could be ambiguous, since the social value of future innovations includes how Southern innovation affects the future terms-of-trade between North and South.
plus terms-of-trade effects) must be equal to that ratio for marginal social benefits. These include more consumption of the imported good in the North and less consumption in the South — as weighted by its relative marginal social value, environmental damage, and the impact on innovation.

The next proposition further characterizes the shape of the optimal policy. It specifies conditions under which the social planner avoids disaster as well as the conditions under which a switch to clean innovation occurs in the North, so that the optimal policy is similar to the policy described in Section 3.4.

Proposition 5 (i) Whenever doing so is feasible, the social planner avoids disaster if the inverse elasticity of intertemporal substitution $\eta \geq 1$; or if $\eta < 1$ and the discount rate $\rho$ is sufficiently low. The South must asymptotically be fully specialized in the nonpolluting sector $F$ if initially clean technologies are less developed than dirty ones there ($A_{S,0} \leq A_{S,d_0}$).

(ii) If $A_{S,c_0} \leq A_{S,d_0}$, if avoiding a disaster is feasible, if $\rho$ is sufficiently small, and if either the inverse elasticity of intertemporal substitution $\eta \leq 1$ or the polluting and nonpolluting goods are strict complements ($\sigma < 1$), then there is a switch toward clean innovation in the North: the mass of scientists allocated to the dirty subsector tends to $0$, and the mass of scientists allocated to clean technologies in the North is positive (asymptotically $1$ for $\eta \leq 1$).

Proof. See Appendix D.7.

The intuition behind these claims is similar to that behind Remark 3, although avoiding a disaster is not always possible if the initial environmental quality is too low. If the inverse elasticity of intertemporal substitution $\eta \geq 1$ then a disaster brings a utility of $-\infty$, so the social planner always tries to prevent it. To avoid such disaster, the South must asymptotically specialize in the nonpolluting sector $F$ if its dirty technologies are more developed initially than its clean ones. Moreover, for a sufficiently low discount rate, the social planner always prefers a path with positive long-run growth to one with bounded consumption. When the two goods are strict complements ($\sigma < 1$), positive long-run growth can be achieved only if there is growth in both sectors; therefore, innovation in the Northern polluting sector must switch to clean technologies. When $\eta \leq 1$ and the discount rate is low enough, the social planner maximizes long-run growth; hence, in this case too, he prevents an environmental disaster whenever possible.\footnote{This is the only proposition affected by the assumption that a disaster is an absorbing state. If not then a temporary disaster could be part of the optimal policy for $\eta < 1$.}

In addition, maximizing long-run growth requires that the North asymptotically innovate only in clean technologies — just as the South asymptotically innovates only in sector $F$. In both cases, the optimal policy resembles the one described in Section 3.4.\footnote{The alternative policy for avoiding disaster is to forgo developing clean technologies in the North but to bound production of the polluting good via a carbon tax.}

28
especially true for maximizing (1). In that case and under the assumptions of Proposition 5 (ii): both countries fully specialize in finite time; innovation in the North is asymptotically all in clean technologies; the trade tax is temporary; and, as the environment fully recovers, the carbon tax reaches zero (as explained in footnote 21, the latter depends on which law of motion is used).\textsuperscript{38}

Alternatively, one could examine a case where income transfers are allowed across countries, so that the constraint (26) is removed from the problem. This case is similar to the one just studied, but the trade tax is not affected by redistributive concerns.

5 Stylized Calibration

In this section I carry out a simple calibration exercise. This exercise should not be viewed as a careful quantitative assessment since the model’s level of aggregation is too high for that purpose but rather as an illustration of the theoretical results described previously. I show in particular how both trade and directed technical change act as a double-edge sword: they each tend to accelerate environmental degradation under laissez-faire or when the North only taxes the polluting good, but they help prevent an environmental disaster when the North undertakes the appropriate policies. Furthermore, this exercise allows me to investigate how the optimal policy depends on planner’s objectives and the model’s parameters.

5.1 Parameter Choices

Here, I provide a brief description of the calibration; further details are given in Appendix C. A period is defined as five years, and initial values are based on the 2003–2007 world economy while assuming laissez-faire in both countries. For simplicity, I assume that the polluting and nonpolluting goods enter final consumption in a Cobb–Douglas way ($\sigma = 1$) and that the elasticity of intertemporal substitution is unity ($\eta = 1$) (cf. Golosov et al., 2011). The annual time discount rate is 0.015, as in Nordhaus (2008). I identify the North with the countries in Annex I of the Kyoto protocol (i.e., those countries that are subject to binding constraints on their emissions) and identify the South with the rest of the world. Data availability restricts the number of countries studied, but the sample includes the most important ones (the North comprises 33 countries and the South 18). To identify the polluting and nonpolluting sectors, I employ International Energy Agency data on sectoral emissions of CO$_2$ from fossil fuel combustion across the world (IEA, 2010a) as well as United Nations Industrial Development Organization data on sectoral value added (UNIDO, 2011). I restrict attention to manufacturing and compute, at the available aggregation level, the world rate

\textsuperscript{38}For (2) to be maximized when the North has all the weight ($\Psi = 1$), the South must be asymptotically just at the corner of full specialization, and the trade tax must tend to $-1$. See Appendix D.7.
of emissions per dollar of value added in each sector. The sectors with the highest rate are identified with sector $E$ and the others with sector $F$ (according to the model, I ignore the emissions from sector $F$). As mentioned previously, the polluting sector corresponds to the manufacture of chemicals and chemical products (ISIC code 24), of other nonmetallic mineral products (26), and of basic metals (27).\footnote{Sector $F$ corresponds to the other sectors in manufacturing except 23, 25, 33, 36, and 37, for which data are not available.} I find that Southern production is tilted toward sector $E$ relative to Northern production ($Y_{NE0}/Y_{SE0} \times Y_{SF0}/Y_{NF0} = 0.77$); this corresponds to the South having a small initial comparative advantage in the polluting sector $E$. I use world production in sectors $E$ and $F$ to compute the consumption share of good $E$ ($\nu = 0.257$) when the economy consists only of sectors $E$ and $F$.

I use the EU KLEMS dataset (Timmer, O’Mahony, and van Ark (2008)) to compute the capital factor share as the ratio of capital compensation to labor compensation in both sectors in the United States; I find a capital share of $\alpha = 0.5$ in the polluting sector and of $\beta = 0.3$ in the nonpolluting sector. A unit of a good is defined as the quantity for which the value added equals 1 billion US (2000) dollars. Factor shares and initial production values are enough to determine the initial productivity-adjusted endowments, which along with the initial ratio $A_{Xc0}/A_{Xd0}$, are all that matter when determining consumption, production, and emissions.\footnote{Nevertheless, I do not assign arbitrary values for endowments; instead I choose $L_X$ as total employment in sectors $E$ and $F$ in country $X$ and choose $K_X$ as total capital formation in both sectors in country $X$ (from the UNIDO database).} I fix $\gamma = 1/3$, which is a common value in endogenous growth models. For the elasticity of substitution between the clean and the dirty input, I choose $\varepsilon = 5$ as the base value and compare it with the case of both higher ($\varepsilon = 10$) and lower ($\varepsilon = 3$) elasticity. The innovation size $\kappa$ is adjusted so that the long-run growth rate of the economy is 2 percent annually. For the concavity of the innovation function I choose $\iota = 0.55$, which exceeds the threshold (of 0.5) that guarantees unicity of the equilibrium for small $\kappa$. The more concave is the innovation function, the more do decreasing returns to scale mitigate the amplification of comparative advantage and the path dependence in clean versus dirty innovation.

The quality of the environment $S_t$ is assumed to be both linearly and negatively related to the atmospheric concentration of CO$_2$ (the previous assumption that $S_0 = \bar{S}$ is relaxed, and the initial environmental quality $S_0$ is set to the current atmospheric concentration of 379 ppm). The parameter $\Delta$ is calibrated such that, at current levels, half of CO$_2$ emissions are absorbed and do not add to atmospheric concentrations. I compute the Northern and Southern emission rates in sector $E$. The South’s rate is nearly 4 times that of the North’s. Such a large difference cannot be accounted for by the model if $A_{Nd0} > A_{Nc0}$ and the emission rate per unit of dirty input ($\xi$) is identical in both countries. Since $A_{Nc0} > A_{Nd0}$ would be extremely unrealistic, I relax the assumption that the emission rates per unit of dirty input ($\xi$) is the
same in both countries. To derive a proxy for $A_{X,0}/A_{X,0}$, I use IEA (2010b) data and identify the ratio $Y_{X,0}/Y_{X,0}$ with the ratio of nonfossil to fossil fuel energy produced for country $X$’s primary energy supply. This ratio is 25 percent larger for the North than for the South. The rest of the difference between the initial Northern and Southern emission rates per unit of good $E$ is accounted for by the difference between $\xi_N$ and $\xi_S$ ($\xi_S > \xi_N$, so dirty inputs are more polluting in the South than in the North).\footnote{Finally, $\xi_N$ and $\xi_S$ are adjusted upward so that the sum of Northern and Southern emissions in sector $E$ correspond to total world emissions.} Changes in atmospheric CO$_2$ concentrations are then mapped against changes in temperature, and $S = 0$ is chosen to correspond to a disaster temperature level of 6°C. The function $\nu(S_t)$ is the same as in AABH and mimics the cost function of Nordhaus (2008) for temperature increases up to 3°C.

### 5.2 Alternative Policies and their Welfare Costs

Figure 1 displays the allocation of innovation, the carbon taxes and trade tax in the first-best and second-best cases for our baseline scenario: the social planner maximizes (1) — and therefore does not care about the allocation of consumption between the North and the South — and $\varepsilon$, the elasticity of substitution between the clean and dirty inputs, is equal to 5. Figure

---

Figure 1: First-best and second-best policies in the baseline case: $\varepsilon = 5$ and the social planner has no distribution concerns. From left to right and top to bottom, figures: 1.A, 1.B, 1.C, and 1.D.
1.A shows the allocation of innovation in the polluting sector $E$ in the first-best case: nearly all sector-$E$ innovation is carried out in the South, and mostly in clean technologies from the first period. In line with Remark 3, specialization in both countries occurs in finite time; here it occurs quite rapidly, which explains why there is hardly any Northern innovation in sector $E$ (and hardly any Southern innovation in sector $F$). Figure 1.B shows the ad valorem carbon taxes in the North and the South (recall that the per-unit of CO$_2$ taxes are identical). The Southern carbon tax decreases as clean technologies catch up with dirty ones. Figure 1.C plots the allocation of innovation in the second-best case. Contrary to the first-best case, the North must now export the polluting good $E$ in the long run. For these parameter values, a large trade tax on good $E$ (see figure 1.D) ensures that, right from the first period, the South specializes in the nonpolluting good $F$. Northern transition from predominantly dirty to clean innovation occurs rapidly, over a period of 65 years, but not immediately. There are two reasons for the delay. First, because the South’s emission rate is higher than the North’s, the temperature increase is initially lower than in the first-best case and so the North can afford to invest in dirty technologies for a while; second, continuing to invest a bit in dirty technologies helps the North build a large comparative advantage in the polluting sector. The amount of clean innovation increases over time and, beyond the time frame of the simulation, eventually reaches one when the North fully specializes in the polluting sector (in line with Proposition 5). The carbon tax in figure 1.D follows a pattern similar to that for the first-best case; however, since clean technologies are developed more slowly, it follows that emissions decrease less rapidly and also that the carbon tax goes to zero more slowly (see figure 2.A for the temperature increase pattern). A relatively large growth rate (2% a year), when combined with a small difference in capital share between the two sectors ($\alpha - \beta = 0.2$) and a small initial comparative advantage, explains why full specialization in one country occurs rapidly. Taking into account the imperfect mobility of factors or the cross-sector and cross-country knowledge spillovers would have the effect of slowing down the specialization process.

Table 1: Disaster and welfare cost in the baseline scenario

<table>
<thead>
<tr>
<th>Policy Type</th>
<th>Years until disaster</th>
<th>Welfare Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laissez-faire</td>
<td>50</td>
<td>100%</td>
</tr>
<tr>
<td>First-best</td>
<td>Never</td>
<td>6.36%</td>
</tr>
<tr>
<td>Second-best</td>
<td>Never</td>
<td>24.64%</td>
</tr>
<tr>
<td>Third-best</td>
<td>Never</td>
<td>24.75%</td>
</tr>
<tr>
<td>North taxes on good $E$ only</td>
<td>50</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1 shows whether a disaster can be avoided under different policies and what the welfare costs of climate change are. The welfare cost is computed as the equivalent percentage loss of world consumption every period relative to the first-best case in a “miracle” scenario.

\footnote{In this case, the initial trade tax is sufficiently large to reverse the pattern of trade (see footnote 18).}
under which the dirty input would cease to pollute (i.e. \( \xi_N = \xi_S = 0 \) from the first period). Under laissez-faire, a disaster occurs after 50 years (the welfare cost is 100% with log utility). There is no disaster in the first-best case (which is always true) and, since initial environmental quality is sufficiently large, there is no disaster either in the second-best case. Yet the inability to intervene in the South sharply increases the welfare costs of climate change policy (they are 4 times as large). The reason is that reversing the pattern of comparative advantages leads to significant static costs in the first periods and to lower productivity levels in subsequent periods. These results confirm an important lesson: it may be possible to avoid disaster with unilateral policies, but doing so has much higher costs than a global intervention.\(^{43}\)

Table 1 also presents the case of a “third” best in which the North can implement a carbon tax and research subsidies/taxes but cannot implement trade, consumption, or production taxes. With the calibrated parameter values it is still possible to avoid disaster under such a policy (which is not generally true) and dispensing with a trade tax does not significantly increase welfare costs provided the other policy instruments are chosen optimally; in particular, the North must allocate more scientists to clean technologies in the first periods (if both the allocation of scientists and the carbon tax were kept at their second-best level then a disaster could not be avoided). Finally, no combination of Northern taxes on the polluting good can prevent an environmental disaster, in fact, with the calibrated parameter values, no such combination can even postpone the disaster (in the last row of table 1, “North taxes on good E only” refers to the combination of Northern carbon tax and tax on dirty research that minimizes \( \text{CO}_2 \) emissions).

5.3 Trade, a Double-Edged Sword

For an open economy, figure 2.A shows increases in temperature: under laissez-faire, with the combination of Northern taxes on the polluting sector that minimizes \( \text{CO}_2 \) emissions, in the second-best case under maximizing (1) and in the first-best case; figure 2.B plots increases in temperature under the same policies in autarky. In line with table 1, in the open economy case, the emission-minimizing combination of Northern taxes in the polluting sector leads to nearly the same increase in temperature as under laissez-faire: the two curves are virtually indistinguishable. In autarky, the disaster occurs later under laissez-faire because economic growth is slower and taxes on the Northern polluting sector can further postpone it for 85 years since there is no pollution haven effect. However, whether or not clean research subsidies are included has no significant effect on the temperature increase. In figure 2.B, the temperature increase in the “second-best” case (which refers here to the North emission-minimizing

\(^{43}\)This increase in cost is almost entirely due to the environmental externality. In the miracle case there would also be some welfare costs from not being able to intervene in the South (since innovation there would not be allocated optimally), but these costs would be very small: 0.03 percent.
combination of carbon taxes, taxes on dirty research and clean research subsidies) cannot be distinguished from the temperature increase when the North only uses taxes on the polluting good. Even in the first-best case temperatures increase more in autarky because the growth rate of clean technologies is lower than in the open economy scenario. Overall, figure 2 illustrates the double-edged nature of trade’s role: without it, unilateral policies cannot prevent a disaster; but opening up to trade accelerates environmental degradation if the North does not undertake the appropriate policy.

5.4 Directed Technical Change, a Double-Edged Sword

Directed technical change (DTC) plays a similar role. To study it, I compare the current scenario with DTC to one in which the allocation of innovation is exogenous and equal in all subsectors (\( s_{Xct} = s_{Xdt} = s_{XFt} = 1/3 \)). With the calibrated values, however, Northern taxes on the polluting good cannot postpone the disaster even in the exogenous growth case. So as to better illustrate the impact of DTC, I perform the same exercise but now assume that \( \alpha = 0.7 \) and \( \beta = 0.1 \) (a larger difference in capital shares limits the pollution haven effect in a static model and therefore better illustrates how it is amplified by the innovation response). Figure 3 shows the increase in temperature for the different policies and under the two scenarios. DTC accelerates the disaster under laissez-faire because it accelerates the economy’s growth rate. With DTC, no combination of Northern taxes on the polluting good can forestall a disaster (in fact, here the emission-minimizing combination of Northern taxes is no taxes at all). Without DTC, such taxes are still unable to prevent a disaster, but they can delay it for 30 years. With a permanent and large trade tax, unilateral policies can still avert an environmental disaster.
Figure 3: Temperature increase with and without directed technical change ($\varepsilon = 5$ and no redistribution concerns for the social planner, but different capital shares than in the baseline scenario: $\alpha = 0.7$, $\beta = 0.1$). From left to right: figures 3.A and 3.B.

with these parameters (although this need not always hold) but the increase in temperature is much larger — despite a much lower growth rate — and temperature increases for a longer time even in the first-best case.

### 5.5 Different Objectives for the Social Planner

Figure 4 shows the optimal second-best policy when the social planner maximizes (2): first (figures 4.A and 4.B) when the social planner cares only about the North ($\Psi = 1$) and second (figures 4.C and 4.D) when he is equally concerned about the welfare of each country ($\Psi = 1/2$). Figures 4.E and 4.F show the consequences of these policies on Northern and Southern consumption relative to the consumption pattern under laissez-faire and in the second-best case with no redistributional motives (which corresponds to the policy of figures 1.C and 1.D). A social planner who cares only about the North’s welfare undertakes less innovation in the polluting sector than a planner who does not care about income distribution because he thereby improves the North’s terms-of-trade. As a result, the switch from predominantly dirty to clean research occurs much later (after 215 years), and the North must implement a large carbon tax to reduce emissions. The trade tax, which is initially positive, becomes negative once the North specializes in the polluting sector (it is now an export tax) because that, too, increases its terms-of-trade. In contrast, if the social planner cares equally about the welfare of both countries then he initially improves the South’s terms-of-trade (since the South is poorer). In that case, the North invests massively in clean technologies from the first period and there is a large positive trade tax as the North immediately starts exporting the polluting
Figure 4: Second-best policy and resulting consumption pattern when the social planner cares about income distribution. From left to right and top to bottom, figures: 4.A, 4.B, 4.C, 4.D, 4.E, and 4.F.
good (this trade tax is thus an export subsidy). Consumption in the South eventually exceeds consumption in the North, so the trade tax turns negative (it becomes an export tax) in order to favor the North’s terms-of-trade. Figures 4.E and 4.F show that the North bears most of the cost of the intervention because the reversal of comparative advantage forces it to specialize more and more in the sector with the lower consumption share. In all cases, there is a fast reversal of comparative advantages prompted by a positive trade tax on the polluting good and more Northern innovation in the polluting sector (especially in the clean subsector) than in laissez-faire, autarky, or first-best case. However, since the reversal of comparative advantage is especially costly for the North, less polluting good is produced and the North specializes less fast when the social planner cares more about Northern than Southern consumption. This dynamic allows for a more gradual environmental policy in the North that relies less on clean technologies in the first periods.

5.6 Impact of the Elasticity of Substitution

Figure 5 investigates how the elasticity of substitution between clean and dirty inputs affects the second-best policy (for the maximization of (1); i.e. when there is no redistribution concern). As in AABH, a greater elasticity of substitution increases the costs of delaying a switch from dirty to clean innovation because the accumulated knowledge in pre-switch dirty technologies is less useful. Yet here an earlier switch also allows, in turn, for more innovation in the polluting sector; hence the North specializes faster, which leads to even more clean innovation.
In addition, an earlier switch implies a much smaller carbon tax. Even though the shape of the second-best policy is affected by the elasticity of substitution, the comparisons emphasized previously are not. Table 2 shows the welfare cost under the first-best, second-best and third-best policies with these alternative values; the results are similar to those reported in table 1.

Table 2: Welfare cost for different elasticities of substitution

<table>
<thead>
<tr>
<th></th>
<th>ε = 3</th>
<th>ε = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-best</td>
<td>8.72%</td>
<td>9.11%</td>
</tr>
<tr>
<td>Second-best</td>
<td>23.71%</td>
<td>25.21%</td>
</tr>
<tr>
<td>Third-best</td>
<td>23.85%</td>
<td>25.32%</td>
</tr>
</tbody>
</table>

A lower discount rate would likewise push toward an earlier switch to clean innovation in the North under the second-best policy (and thereby toward faster specialization); moreover, because economic growth is larger when countries are more specialized, it would also push directly toward faster specialization.\(^{44}\)

6  Knowledge Diffusion

I now relax the assumption that productivity improvements are entirely country specific. In reality, it is probable that some productivity improvements will cross borders, mitigating the amplification of comparative advantage effect, which partly drove the previous results.\(^{45}\) This brings into question the robustness of the previous analysis. Here I consider an extension of the original model whereby the lagging country can benefit from the diffusion of innovations produced in the leading country. Another extension of the model is presented in Appendix A.2, where innovation itself is international and a product of global firms. In both cases the main lessons from Section 3 still hold, even though the underlying intuitions differ.

To model knowledge diffusion in a simple way, I assume that — at the beginning of every period — the country with the less advanced average productivity in a given sector can exogenously catch up (partially) with the other one. That is, before any innovation occurs, the producer of intermediate \(i\) in sector \(z\) \(\in\{c,d,F\}\) obtains access to the technology:

\[
A_{Xzi(t)} = \max \left( \left( \frac{A_{(X)zi(t-1)}}{A_{Xzi(t-1)}} \right)^{\delta}, 1 \right) A_{Xzi(t-1)},
\]

\(^{44}\)A less concave innovation function (a higher \(\iota\)) would make it more difficult for clean technologies to catch up when the mass of scientists working in the polluting sector is small, which would delay the switch to clean innovation and the North’s full specialization.

\(^{45}\)One should not expect all productivity improvement to cross borders easily, because some may be embedded in capital or infrastructure or may depend on local know-how. Dechezleprêtre et al. (2011) suggest that clean technology transfers between developing and developed countries exist but are quite limited: for the period 2000–2005, only 15 percent of the clean innovations were patented in more than one country; this is slightly less than the share (17 percent) of all innovations patented in more than one country.
where $\delta \in [0, 1]$ measures the strength of the technological diffusion. This equality then delivers the following law of motion for aggregate productivity:

$$A_{Xzt} = (1 + \kappa s_{Xzt})^{1-\gamma} \max \left( \left( \frac{A(-X)z(t-1)}{A_{Xzt}(t-1)} \right)^\delta, 1 \right) A_{Xz(t-1)}$$

for $z \in \{c, d, F\}$. Under this formulation, the ratio of the technological levels across countries cannot diverge: as soon as one country achieves a strong advantage over the other, the catching-up process ensures that, regardless of the pattern of innovation, this difference is reduced in the next period.

In particular, Northern policies that foster clean innovation there now also increase the productivity of clean Southern technologies. In fact, they may even put the South on a clean innovation track: if, in some period, pre-innovation clean Southern technologies become more advanced than dirty ones (i.e., for some $t$, $\overline{A}_{Scd} > \overline{A}_{Sdt}$), market forces will induce more clean than dirty innovations in the South from that period onwards. Preventing a disaster does not necessarily involve pushing the South toward specializing in the nonpolluting sector, it can also be achieved by ensuring a switch to clean innovation there. That transition will occur as soon as more scientists are allocated to clean technologies in the North than to dirty technologies in the South for a sufficient amount of time (given that in the long run, Southern clean productivity $A_{Scd}$ grows like Northern clean productivity $A_{Ncd}$). In other words, there is a horse race — between clean innovation in the North and dirty innovation in the South — to determine whether or not in the long run, the polluting sector will be produced in a clean way. Who wins this horse race depends on the policies that the North allows for and on the pattern of comparative advantage, much as in Section 3. Hence the intuitions developed there still apply, and the broad results are less different than one might expect. In particular, I can show the following.

**Proposition 6** Assume that, initially: (i) technologies are sufficiently close to each other across countries, that $\kappa$ is sufficiently small, and that the spillovers $\delta$ are sufficiently strong; (ii) the South is relatively well endowed in capital, $\frac{K_S}{L_S} > \frac{K_N}{L_N}$; and (iii) clean technologies are sufficiently less advanced than dirty ones ($A_{Sc0}/A_{Sd0}$ sufficiently small). Then no combination of a carbon tax and a tax on dirty research in the North can prevent a disaster irrespective of how high $S$ is.

**Proof.** See Appendix D.9.  

This proposition mirrors Proposition 1. The assumptions (i) ensure that technological levels remain sufficiently close to each other across countries; this, when combined with assumption (ii), ensures that the South maintains its comparative advantage in the polluting sector. Assumption (iii) plays the same role as in Proposition 1 and ensures that, when the South has
the comparative advantage in the polluting sector, it innovates there more than does the North (this assumption could be discarded in the perfect substitutes case). As a result, the South keeps its comparative advantage in the polluting sector and, since a carbon tax in the North can only reinforce this comparative advantage, there are more Southern scientists innovating in dirty technologies than Northern scientists innovating in clean ones. Hence Southern clean productivity $\bar{A}_{Scd}$ never catches up, so a switch in the South to clean innovation never occurs. In other words, the Northern market for the polluting good is too small to generate enough clean innovations.

As before, a temporary combination of clean research subsidies and a tariff can prevent a disaster for sufficiently large initial environmental quality (i.e., Proposition 2 still holds). Clean research subsidies can reallocate Northern innovation to clean technologies, and a tariff can limit Southern innovation in dirty technologies. Then $\bar{A}_{Scd}$ grows faster than $\bar{A}_{Sdt}$, and a switch to clean innovation eventually occurs in the South.

Remark 2 is not robust. Sufficiently large clean research subsidies in the North are now enough to avoid disaster if the initial environmental quality is sufficiently high — even when final consumption is Cobb–Douglas in the polluting and the nonpolluting goods ($\sigma = 1$). Because clean technologies in the South grow at the same rate as in the North, the Southern ratio of clean to dirty technologies cannot approach zero if the North allocates all its scientists to clean technologies. In that case, the mass of Southern scientists allocated to dirty technologies remains bounded away from one even if the South specializes in the polluting sector. Eventually, the North wins the horse race; then pre-innovation Southern clean productivity becomes larger than dirty productivity, $\bar{A}_{Scd} > \bar{A}_{Sdt}$, and a switch to clean innovation must occur.

This analysis relies on the innovation function $\kappa s^t$ satisfying the Inada condition. If, the innovation function were instead $\kappa ((s + \Upsilon)^t - \Upsilon^t)$ with $\Upsilon > 0$ then, for clean technologies initially sufficiently less advanced than dirty ones (i.e., for $A_{Scd}/A_{Sdt}$ sufficiently small), all Southern innovation would be devoted to dirty technologies when the South is fully specialized in the polluting sector. The same scenario applies to the perfect substitutes alternative ($\varepsilon = \infty$). In this situation, whether or not clean research subsidies alone can prevent a disaster in the Cobb–Douglas case depends on the size of the difference in relative factor endowments. For sufficiently close initial endowments, knowledge spillovers ensure that the South does not specialize in the long run, and this prevents all Southern scientists from being allocated to dirty technologies. Hence clean technologies can catch up in the South and disaster can be avoided. However, if initial endowments are sufficiently far apart, then the South’s full specialization in the nonpolluting sector can continue maintained indefinitely. Then the South innovates only in dirty technologies, and a disaster cannot be avoided without a trade tax. A formal proposition
is given and proved in Appendix D.9.\textsuperscript{46}

One can add a nontradeable good to the economy without changing the results. This is important because a large share of emissions are related to nontradeable activities (heat, transport, etc.). Assume now that final consumption is a CES aggregate of nontradeable goods and goods $E$ and $F$ (with an elasticity of substitution weakly smaller than 1), where nontradeable goods are also produced with the clean and dirty inputs (possibly combined with other inputs). In the no-spillovers case, it is impossible to prevent a disaster because Southern emissions from nontradeables will increase unboundedly regardless of Northern policy. In the spillover case, however, the same results as before still apply: if Northern clean technologies win the horse race over Southern dirty technologies, then nontradeables in the South will also begin using clean inputs more intensively.

The structure of the optimal policy (constrained or not with respect to intervention in the South) is broadly similar, but the trade tax and subsidies for research must take knowledge spillovers into account. Moreover, a second-best policy that prevents a disaster need not necessarily feature a South that exports the nonpolluting good in the long run. In addition, the welfare costs of unilateral intervention are typically much lower than in the absence of knowledge spillovers. In the absence of knowledge spillovers, the source of the large welfare costs was the necessary reversal in comparative advantages. With knowledge spillovers, such reversal may not happen, and, even if it happens, it is much less costly since the South ends up benefiting from the technologies that the North had developed. Accordingly, table 3 shows the welfare costs in the first-best and the second-best cases for the baseline scenario in the presence of knowledge spillovers ($\delta = 0.4$ and $\delta = 0.8$): the welfare costs of the first-best policy are very similar to the no-spillover case in table 1, but the welfare costs of the second-best policy are now much lower.\textsuperscript{47}

![Table 3: Welfare cost in the presence of knowledge spillovers](image)

To some extent, technological diffusion itself is a parameter that can be affected by policy: laxer intellectual property rights, direct financing of projects abroad, or migrations of skilled workers could all contribute to a faster diffusion of technology. Therefore, according to the analysis presented here, the diffusion of clean technologies from North to South renders a tariff less necessary, and significantly reduces the costs of a unilateral policy intervention.

\textsuperscript{46} This statement holds also when $\varepsilon < \infty$ and $Y = 0$ if the North needs to produce clean intermediate to be able to allocate scientists to clean technologies.

\textsuperscript{47} In the case of our calibration, the reversal of comparative advantage still takes place in the presence of knowledge spillovers because the difference in factor endowments is small.
7 Conclusion

This paper develops a dynamic model of trade and the environment which incorporates directed technical change in a two-country world, in order to identify the type of unilateral policies that can achieve sustainable growth. It also characterizes the optimal unilateral policy. When knowledge is local, a combination of temporary clean research subsidies and a carbon tariff can prevent an environmental disaster; however, unilateral taxes on the polluting sector are unlikely to do so — especially when the South initially has the comparative advantage in the polluting sector. This finding speaks in favor of “protectionist” policy means for environmentally motivated ends. The second-best policy can be decentralized through research subsidies, a carbon tax, and a trade tax. When the social planner does not care about the distribution of consumption, the trade tax typically takes the form of a tariff on the polluting good and then of an export subsidy; this reflects the dual objective of reducing emissions in the South and of redirecting innovation there. Under some assumptions on the social planner’s preferences, the second-best policy involves a switch to clean innovation in the North, and the South (at least asymptotically) fully specializing in the nonpolluting sector. In the presence of knowledge spillovers, a switch to Southern clean innovation can be achieved via Northern policies, so that a disaster can be avoided even if the South does not specialize in the nonpolluting sector. Unilateral taxes on the polluting sector may still fail to prevent disaster. Therefore, some form of protectionism in favor of the polluting industries is necessary to ensure sustainable growth.

In developed countries it may not be realistic to expect the full revival of such industries as metallurgy, which is what the basic model urges. More practically, the paper argues for a Northern industrial policy in the North that aims to clean the polluting sectors — but without losing too much competitiveness to the South, so as to slow down the move of polluting industries there. In particular, I demonstrate the shortcomings of a “carbon tax only” policy (or equivalent policies, such as cap-and-trade) in the North in the presence of imperfect knowledge markets and a noncooperative South. However, this industrial policy could be phased out once clean technologies diffuse to the South or a global agreement is reached. This paper also points out that, without direct incentives to clean research, clean technologies in the polluting industries are unlikely to be developed in the North if the production of polluting goods move to the South.

This analysis is very much a first step and could be enriched in several ways. First, I have only considered a global social planner faced with the constraint of no intervention in the South. A better understanding of climate negotiations would require modeling two competing social planners in a Nash equilibrium. As mentioned in the text, the design of the trade tax (whether or not it is directly related to average carbon content in a country) will then affect the behavior of the South’s planner. Second, to analyze more carefully the actual implementation of carbon
tariffs, one should take into account not only World Trade Organization constraints but also the possibility that firms (or countries) may hide the true level of their emissions or manipulate environmental policies to their own advantage. Third, I have not seriously considered policies that directly boost technological diffusion. Such policies (e.g., “clean development mechanism”) are already part of climate negotiations. Studying technological diffusion would, however, require proper modeling of intellectual property rights (IPR), whose impact on emissions is a priori ambiguous. On the one hand, more lax IPR could lead to more rapid diffusion of clean technologies to the South, which would facilitate the switch to a clean path there. On the other hand, more lax IPR might reduce incentives to develop Northern clean technologies in the first place. Finally, the paper’s results suggest that directed technical change renders Southern emissions much more responsive to Northern policies in the long run. This finding calls into question the existent estimates of the carbon leakage rate, which are obtained from static models. Therefore, in order to properly evaluate the impact of local carbon taxes and carbon tariffs, it would be useful to integrate directed technical change into a full numerical model of the world economy.

References


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UNIDO (2011): “Industrial demand-supply balance database,”.


A Appendix A

A.1 Extensions of Section 3.

Other instruments. Section 3 focused on a restricted set of instruments and combination of instruments (the ones which end up being used in the optimal policy). In practice other instruments are also used, and there could be constraints on policy that have been ignored so far (particularly on the type of trade tax which is allowed). Here, I examine some of these alternatives to see whether they can prevent an environmental disaster or not.

*Carbon tax and trade tax.* A first possibility is that it is impossible to distinguish a priori the nature of research; therefore one cannot use research subsidies. In this case, it is still possible to prevent an environmental disaster for sufficiently large initial environmental quality by using a combination of a carbon tax and a trade tax: a very large carbon tax redirects innovation in the polluting sector in the North toward clean technologies, while a sufficiently large trade tax, which is an export subsidy for the polluting good in the North, is equivalent to the North (nearly) specializing in the polluting sector and giving up its production (nearly) for free. Maintaining such a policy for a sufficiently long time allows the North to develop a comparative advantage in the polluting sector, while clean technologies there become more advanced than dirty ones. Once this is done the same logic as in proposition 2 applies, and a disaster can be avoided without the need for any further policy in the North. If it were impossible to implement a trade tax, this could still be achieved using only a temporary clean input production subsidy.

*Carbon tax and tariff.* Being able to implement a trade tax sufficiently large to reverse the pattern of trade is crucial in the argument above (while it is not if one allows for clean research subsidies): if the North is only able to reproduce autarky, then reversing the pattern of comparative advantage is not always possible, as both countries end up innovating at the same rate in both sectors (since clean innovation in the South eventually disappears as the gap between clean and dirty technologies there widens).

*CBTA.* Carbon border tax adjustments (CBTA) are a specific combination of a carbon tax and a tariff, where carbon in imports is taxed at the same rate as domestic carbon - here since the carbon content of individual goods is not observable, the CBTA taxes carbon depending on the emission rate in the country-sector pair.\(^{48}\) The paragraph above therefore shows that CBTA could not prevent an environmental disaster on its own. They could do so, however, in combination with clean research subsidies.\(^{49}\)

\(^{48}\) Here, I ignore the case of full CBTA where imports are taxed at the same rate as domestic carbon but exports are not taxed.

\(^{49}\) Similarly to the case of carbon-content based carbon tariff, they could also avoid a disaster in some cases, if it were possible to make the adjustment on the basis of the carbon content of the good instead of the average emissions in the country / sector.
Consumption tax and research subsidies. Another possibility is that the North could only implement a consumption tax on the polluting good instead of a tariff. Such a tax can prevent the South from specializing in the polluting good (by reducing its relative price), but may induce the North to (temporarily) specialize in the nonpolluting sector, in which case clean intermediates are no longer produced in the North for some periods. Under the assumption that entrepreneurs can still hire scientists even when they do not produce any intermediates, a combination of the consumption tax and clean research subsidies can avoid disaster for sufficiently large initial environmental quality. This would not generalize to the case where production is necessary for innovation.

Different masses of scientists in the North and the South. If the mass of scientists in the North was much smaller than in the South, the North would eventually become a small economy relative to the South, and the South’s economy will behave as if it were in autarky: regardless of the policies undertaken by the North, a disaster would be unavoidable. On the contrary, if the mass of scientists in the North was much larger than in the South, a disaster could be avoided using clean research subsidies without the need for a tariff: even when the South fully specializes in sector $E$, $A_{NEt}$ would grow faster than $A_{SEt}$, so the North could build a comparative advantage in the polluting sector. In fact, depending on parameters, a disaster may also be avoided using taxes on dirty research or a carbon tax under the assumptions of proposition 1.

A.2 Worldwide Entrepreneurs

So far I have assumed that innovation in the North and in the South only respond to local conditions. However, many innovative firms are global and make their innovation decision based on the entire world market. I now examine this case and, for simplicity’s sake, focus on the case where final consumption is Cobb-Douglas ($\sigma = 1$). I show that clean research subsidies alone can now prevent a disaster but that carbon taxes may still fail to do so. Whether carbon taxes fail or not now depends on the relative size of the polluting sector in the South and the North rather than simply the pattern of comparative advantage.

More specifically, I consider that the producer of intermediate $i$ in sector $z \in \{c, d, F\}$ is the same in the North and in the South and his intermediate has the same productivity in both countries; however, intermediates are still not tradeable. By hiring $s_{Nzit}$ scientists in the North and $s_{Szit}$ in the South, the entrepreneur for variety $i$ in sector $z \in \{c, d, F\}$ with

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50 For instance, in the Cobb-Douglas case $\sigma = 1$, if the consumption share of the polluting good ($\nu$) is close to 1, $A_{Nzit}$ can grow faster than $A_{Szit}$, leading to a reversal of comparative advantage, provided that there is a sufficiently large mass of scientists in the North; if $\nu = 1/2$, on the contrary, the previous analysis obtains.

51 In this case trade balance typically does not hold since there will be income transfers across countries. Under free trade this does not affect the pattern of production and emissions.
initial technology \(A_{zi(t-1)}\) gets access to the technology:

\[
A_{zt} = \left(1 + \kappa \left(s_{Nzit} + s_{Szit}\right) \frac{A_{zt(t-1)}^{\frac{1}{1-\gamma}}}{A_{zi(t-1)}^{\frac{1}{1-\gamma}}}\right)^{1-\gamma} A_{zi(t-1)}.\]

The results extend to the case where the innovation function is of the form \(\kappa (s_{Nzit} + s_{Szit})\).

As before, since profits are proportional to \((A_{zt}/A_{zt})^{1-\gamma}\), for \(z \in \{c, d, F\}\), every entrepreneur hires the same number of scientists, and the law of motion of aggregate productivity can be written as:

\[
A_{zt} = \left(1 + \kappa \left(s_{Xzt} + s_{(X)zt}\right)\right)^{1-\gamma} A_{zt(t-1)}.
\]

In this subsection, the key to preventing a disaster will be to ensure that \(A_{ct}\) grows faster than \(A_{dt}\), since this will bring the emission rate in the polluting sector down to zero even in the South. The next proposition shows that it is still the case that a carbon tax can fail at preventing an environmental disaster:

**Proposition 7** Assume that \(\sigma = 1\), that clean technologies are sufficiently less advanced than dirty ones \((A_{c0}/A_{d0} \text{ is sufficiently small})\), and that the South originally has a weakly larger market share than the North in the polluting good \((Y_{SE0} \geq Y_{NE0})\), then no carbon tax in the North can prevent a disaster, no matter how large the initial environmental quality \(\overline{S}\) is.

**Proof. Appendix D.10**

Without direct research incentives, the innovation allocation is identical across countries. Within the polluting sector, the allocation of innovation across the clean and dirty subsectors favors the one with the largest revenues. In the North, a large carbon tax can ensure that the clean subsector captures the entire polluting sector’s market size, while in the South, if clean technologies are sufficiently less advanced \((A_{c0}/A_{d0} \text{ is small})\), the size of the dirty subsector is close to the size of the South polluting sector. Worldwide, the size of the clean subsector is then close to the size of the polluting sector in the North, while the size of the dirty subsector is close to the size of the polluting sector in the South. If the South has a larger market share in the polluting sector, there would be more dirty than clean innovations, and \(A_{ct}\) would never catch up. When final consumption is Cobb-Douglas, the relative size of both countries in the production of the polluting good does not change with technologies, and, since a carbon tax can only increase the relative size of the South, an initially larger market share ensures that the South remains the biggest country in the polluting sector.\(^{52}\)

\(^{52}\)If the North uses a tax on dirty research on top of the carbon tax, the remark stays true but for \(Y_{NG0}/Y_{SG0}\) sufficiently small. In this case, the Northern social planner can ensure that no scientists in the North innovate in dirty technologies. Yet, if \(Y_{NG0}/Y_{SG0} \text{ is small}\), most innovation in the North will occur in the non-polluting sector anyway, so most sector \(G\) innovation will occur in the South and will be determined by the South market, favoring dirty innovation. See Appendix D.10.
If subsidies to clean research in the North are sufficiently strong, however, nearly all Northern scientists will innovate in clean intermediates, thereby also improving the productivity of clean intermediates in the South. In the meantime, innovation in the South will be prevented from moving fully toward sector $E$ and dirty intermediates: even if the South fully specializes in sector $E$, entrepreneurs having monopoly rights in sector $F$ will hire some scientists from the South to improve the productivity of the variety they own in the North. Therefore, a tariff (which is still part of the optimal policy) is not necessary to prevent a disaster, regardless of initial endowments.

**Proposition 8**  
**Clean research subsidies in the North alone can prevent a disaster if the initial environmental quality is sufficiently high.**

As technological diffusion, the internationalization of the R&D process makes it possible for policy in the North to induce a switch to clean innovation in the South, so that the second best policy needs not feature a reversal of the pattern of comparative advantage. However, even with a very internationalized process, the North needs to undertake a very proactive policy toward the development of clean technologies; otherwise, the direction of innovation within sector $E$ will be dictated by the economic conditions of the South instead of the North.

**Appendix B: Main Proofs of the Paper**

**B.1 Characterization of the equilibrium in a given period**

For this subsection I consider the economy at a given time, after innovation has occurred (I drop the subscript $t$ for simplicity’s sake). To avoid future repetition, I do not impose that the economy is in laissez-faire: there is a common subsidy $q$ on all intermediates (I only need to consider the case where the subsidy is identical in both countries in the paper, as it is trivial to extend the analysis to different subsidies across countries), there a carbon tax $\tau_X$ in each country, and in the first and last subsection, free trade is not imposed. First I derive the aggregate production functions in each sector given local good prices. Second I solve for good prices in autarky and free trade, and then I characterize the pattern of specialization in free trade. Finally, I derive the allocation of innovation in function of local good prices.

**B.1.1 Deriving aggregate production function**

If good $E$ is produced, then both subsectors $c$ and $d$ are active. Assume that good $E$ is produced in country $X$, the maximization problem for producers in subsector $z \in \{c, d\}$ leads to the demand function for capital and labor in assembly of good $z$:

$$r_X K_{Xz} = (1 - \gamma) \alpha p_X Y_{Xz} \quad \text{and} \quad w_X L_{Xz} = (1 - \gamma) (1 - \alpha) p_X Y_{Xz} \quad \text{(B.1)}$$
and the demand for intermediates:

\[ \varphi_{Xzi} = \gamma p_{Xzi} A_{Xzi} x_{zi}^{\gamma-1} \left( K_{Xz}^{\alpha} L_{Xz}^{1-\alpha} \right)^{1-\gamma}, \]  

(B.2)

with \( \varphi_{Xzi} \) the consumer price of intermediate \( i \). From (9), the cost of producing one unit of intermediate is given by \( \psi \left( \frac{r_X}{\alpha} \right) \left( \frac{w_X}{1-\alpha} \right)^{1-\alpha} \). Monopolists maximize profits by imposing a mark-up \( 1/\gamma \) on their costs, so that producer prices are given by:

\[ \frac{\varphi_{Xzi}}{1-q} = \frac{\psi}{\gamma} \left( \frac{r_X}{\alpha} \right) \left( \frac{w_X}{1-\alpha} \right)^{1-\alpha}. \]  

(B.3)

The production of intermediates is then given by:

\[ x_{Xzi} = \left( \frac{p_{Xzi} \gamma^2}{1-q} \right) \psi \left( \frac{r_X}{\alpha} \right) \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} \left( \frac{1\alpha}{\gamma^2} \right) A_{Xzi}^{1-\gamma} K_{Xz}^{\alpha} L_{Xz}^{1-\alpha}, \]  

(B.4)

and factor demands in the production of intermediate \( i \) in sector \( z \) follows:

\[ K_{Xzi} = \left( \frac{\alpha}{r_X} \frac{w_X}{1-\alpha} \right)^{1-\alpha} \psi x_{Xzi} \text{ and } L_{Xzi} = \left( \frac{r_X}{\alpha} \frac{1-\alpha}{w_X} \right)^{1-\alpha} \psi x_{Xzi}. \]  

(B.5)

Plugging in (B.1) and (B.4) into (8), I get the price of good \( z \) as:

\[ p_{Xz} = \frac{1}{A_{Xz} (1-\gamma)^{1-\gamma} \gamma^2 \psi} \left( \frac{r_X}{\alpha} \right) \left( \frac{w_X}{1-\alpha} \right)^{1-\alpha}, \]  

(B.6)

Now, profit maximization by producers of good \( E \) leads to the demand function:

\[ \frac{Y_{Xc}}{Y_{Xd}} = \left( \frac{p_{XE}}{(1+\tau_X) p_{Xd}} \right)^{-\varepsilon}, \]  

(B.7)

and the price of good \( E \) is given by \( p_E = \left( p_{XE}^{1-\varepsilon} + (1+\tau_X)^{1-\varepsilon} p_{Xd}^{1-\varepsilon} \right)^{1-\varepsilon} \), which, using (B.6), translates into:

\[ p_{XE} = \frac{(1-\gamma)^{\gamma-1} \gamma^2 (1-q)^\gamma \psi}{\left( A_{Xz}^{\varepsilon-1} + (1+\tau_X)^{-\varepsilon} A_{Xd}^{\varepsilon-1} \right)^{\varepsilon-1}} \left( \frac{r_X}{\alpha} \right) \left( \frac{w_X}{1-\alpha} \right)^{1-\alpha}. \]  

(B.8)

This relationship holds if country \( X \) produces good \( E \), otherwise the equality is replaced by:

\[ p_{XE} \leq \frac{(1-\gamma)^{\gamma-1} \gamma^2 (1-q)^\gamma \psi}{\left( A_{Xz}^{\varepsilon-1} + (1+\tau_X)^{-\varepsilon} A_{Xd}^{\varepsilon-1} \right)^{\varepsilon-1}} \left( \frac{r_X}{\alpha} \right) \left( \frac{w_X}{1-\alpha} \right)^{1-\alpha}. \]

Similarly in sector \( F \),

\[ p_{XF} \leq \frac{(1-\gamma)^{\gamma-1} \gamma^2 (1-q)^\gamma \psi}{A_{XF}} \left( \frac{r_X}{\alpha} \right)^{\beta} \left( \frac{w_X}{1-\alpha} \right)^{1-\beta}, \]
with equality if good $F$ is produced in country $X$.

Note that (B.7) gives:

$$
Y_{Xd} = \left( \frac{(1 + \tau_X)^{-1} A_{Xd}}{A_{Xc}^{-1} + \left( (1 + \tau_X)^{-1} A_{Xd} \right)^{\varepsilon-1}} \right)^{\varepsilon} Y_{XE},
$$

(B.9)

which directly leads to the expression for the emission rate. Combining (B.1), (B.4), (B.5), (B.7), and (B.8), one gets that total factor employment in sector $E$ satisfies:

$$
K_{XE} = \left( \frac{\alpha - w_X}{r_X (1 - \alpha)} \right)^{1-\alpha} \frac{1}{\zeta} \left( \frac{A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-\varepsilon} A_{Xd}^{\varepsilon-1}}{A_{Xc}^{-1} + \left( (1 + \tau_X)^{-1} A_{Xd} \right)^{\varepsilon-1}} \right)^{\varepsilon-1} Y_{XE},
$$

(B.10)

$$
L_{XE} = \left( \frac{r_X (1 - \alpha)}{\alpha - w_X} \right)^{\alpha} \frac{1}{\zeta} \left( \frac{A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-\varepsilon} A_{Xd}^{\varepsilon-1}}{A_{Xc}^{-1} + \left( (1 + \tau_X)^{-1} A_{Xd} \right)^{\varepsilon-1}} \right)^{\varepsilon-1} Y_{XE},
$$

(B.11)

with $\zeta_X \equiv \frac{\gamma^{2\gamma} (1-\gamma)^{1-\gamma} (1-\beta)^{1-\gamma}}{(1-\gamma)(1-\beta)^{1-\gamma}}$. Combining these two expressions and following the same strategy in sector $F$, one gets:

$$
Y_{XEt} = \frac{1}{\zeta} \left( \frac{A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-1} A_{Xd}}{A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-1} A_{Xd}} \right)^{\varepsilon-1} K_{XEt} L_{XEt}^{1-\alpha} \text{ and } Y_{XFt} = \zeta A_{XFt} K_{XFt}^{\beta} L_{XFt}^{1-\beta},
$$

(B.12)

This equation translates into (18) in laissez-faire. When both sectors are active, taking the ratio of (B.8) and the equivalent expression for $p_{XF}$, one can express the capital rent to wage ratio as

$$
\frac{r_X}{w_X} = \left( \frac{\alpha (1 - \alpha)^{1-\alpha}}{\beta (1 - \beta)^{1-\beta}} \right)^{\frac{1}{\alpha-\beta}} \left( \frac{A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-1} A_{Xd}}{A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-1} A_{Xd}} \right)^{\frac{1}{\alpha-\beta}} \left( \frac{p_{XE}}{p_{XF}} \right)^{\frac{1}{\alpha-\beta}}.
$$

Plugging this expression into (B.10) and (B.11) and the equivalent equations in sector $F$, and using factor market clearing (11), one gets a system of two equations with two unknowns $(Y_{XE}, Y_{XF})$ that can be solved as:

$$
Y_{XE} = \frac{\zeta}{(\alpha - \beta)} \left( \frac{\beta^\alpha (1 - \alpha)(1-\beta)\alpha}{\alpha^\beta (1 - \alpha)(1-\alpha)\beta} \right)^{\frac{1}{\alpha-\beta}} \frac{A_{XE} (\tau_X)}{1 - \delta_X (\tau_X)} \times \left( \frac{\alpha (1 - \alpha)^{1-\alpha}}{\beta (1 - \beta)^{1-\beta}} \right)^{\frac{1}{\alpha-\beta}} \left( \frac{\beta^\alpha (1 - \alpha)(1-\beta)\alpha}{\alpha^\beta (1 - \alpha)(1-\alpha)\beta} \right)^{\frac{1}{\alpha-\beta}} \frac{K_X - \beta L_X \left( \frac{p_{XE} A_{XE} (\tau_X)}{p_{XF} A_{XF}} \right)^{\frac{1}{\alpha-\beta}}}{\left( \frac{p_{XE}}{p_{XF}} \right)^{\frac{1}{\alpha-\beta}} A_{Xc}^{\varepsilon-1} + (1 + \tau_X)^{-1} A_{Xd}^{\varepsilon-1}} Y_{XE}.
$$

(B.13)
\begin{align*}
Y_{XF} &= \frac{\zeta}{(\alpha - \beta)} \left( \frac{\beta^\alpha (1 - \beta)(1-\beta)\alpha}{\alpha^\beta (1 - \alpha)(1-\alpha)\beta} \right)^{\frac{1}{\alpha-\beta}} A_{XF} \\
&\times \left( \alpha \left( \frac{p_{XE}}{p_{XF}} \frac{A_{XE}(\tau_X)}{A_{XF}} \right)^{\frac{\beta}{\alpha-\beta}} L_X - \left( \frac{\alpha^\alpha (1 - \alpha)(1-\alpha)^{\frac{1}{\alpha-\beta}} (1 - \alpha) \frac{p_{XE}}{p_{XF}} A_{XE}(\tau_X)}{\beta^\beta (1 - \beta)(1-\beta) \frac{1}{\alpha-\beta}} \right) K_X \right) .
\end{align*}

(A.14)

where \(A_{XE}(\tau_X) \equiv \left( A_{Xc}^\epsilon + \left( (1 + \tau_X)^{-1} A_{Xd}^\epsilon \right)^{\frac{1}{\epsilon}} \right)\) is a measure of average productivity of sector \(E\) in country \(X\), and \(\delta_X(\tau_X) \equiv \frac{\tau_X A_{Xd}^{\epsilon (1+\tau_X)^{-\epsilon}}}{A_{Xc}^{\epsilon (1+\tau_X)^{1-\epsilon}} A_{Xd}^{1-\epsilon}} \in [0,1]\) is a correction term (measuring the difference between imposing a tax and a decrease in productivity of the dirty input).

### B.1.2 Equilibrium price

Consumer maximization leads to \(\frac{p_{XE}}{p_{XF}} = \frac{\nu}{1-\nu} \left( \frac{C_{XE}}{C_{XE}} \right)^{\frac{1}{2}}\). In autarky this translates into:

\[
\frac{Y_{XE}}{Y_{XF}} = \left( \frac{\nu}{1-\nu} \right)^\sigma \left( \frac{p_{XE}}{p_{XF}} \right)^{-\sigma} ,
\]

which combined with (B.13) and (B.14), defines the equilibrium autarky price uniquely (given technologies) since \(\frac{Y_{XE}}{Y_{XF}}\) is increasing in \(\frac{p_{XE}}{p_{XF}}\), and the right-hand side decreases. More specifically, one gets that the autarky price must satisfy:

\[
\left( \frac{p_{XE}}{p_{XF}} \right)^\sigma A_{XE}(\tau_X) \left( \frac{\alpha^\alpha (1-\alpha)(1-\alpha)}{\beta^\beta (1-\beta)(1-\beta)} \right)^{\frac{1}{\alpha-\beta}} \left( 1 - \beta \right) \left( \frac{p_{XE}}{p_{XF}} A_{XE}(\tau_X) \right)^{\frac{1}{\alpha-\beta}} K_X \right)
\]

\[
-\beta L_X \left( \frac{p_{XE}}{p_{XF}} A_{XE}(\tau_X) \right)^{\frac{1}{\alpha-\beta}} K_X
\]

\[
\left( \frac{\nu}{1-\nu} \right)^\sigma A_{XF} \left( \frac{\alpha^\alpha (1-\alpha)(1-\alpha)}{\beta^\beta (1-\beta)(1-\beta)} \right)^{\frac{1}{\alpha-\beta}} L_X \left( 1 - \alpha \right) \left( \frac{p_{XE}}{p_{XF}} A_{XE}(\tau_X) \right)^{\frac{1}{\alpha-\beta}} K_X
\]

(B.15)

If \(\varepsilon A_{Xc}^{\epsilon - 1} + (1 - (\varepsilon - 1) \tau_X) (1 + \tau_X)^{-\varepsilon} A_{Xd}^{\epsilon - 1} > 0\), \(\frac{A_{XE}(\tau_X)}{A_{Xc}(\tau_X)}\) increases in \(A_{Xc}\) and always increases in \(A_{Xd}\) and decreases in \(\tau_X\). Therefore \(\frac{p_{XE}}{p_{XF}}\) decreases in \(A_{Xc}\) (when \(\varepsilon A_{Xc}^{\epsilon - 1} + (1 - (\varepsilon - 1) \tau_X) (1 + \tau_X)^{-\varepsilon} A_{Xd}^{\epsilon - 1} > 0\), decreases in \(A_{Xd}\) and \(K_X\) and increases in \(A_{XF}\) and \(L_X\). It is direct to check that in the absence of any tax, the relative autarky price of good \(E\) over good \(F\) is higher in the North than in the South if and only if \(\frac{A_{SE}}{A_{SF}} \frac{1}{\alpha-\beta} K_S/L_S > (A_{NE}/A_{NF}) \frac{1}{\alpha-\beta} K_N/L_N\) (so that under free-trade the North imports good \(E\) in this case).

Under free-trade, the equilibrium price ratio is the same in both countries and satisfies

\[
\frac{p_E}{p_F} = \frac{\nu}{1-\nu} \left( \frac{C_{XF}}{C_{XE}} \right)^{\frac{1}{2}} = \frac{\nu}{1-\nu} \left( \frac{Y_{NF} + Y_{SF}}{Y_{NE} + Y_{SE}} \right)^{\frac{1}{2}},
\]

(B.16)

which similarly defines uniquely the price ratio given technologies (as \(Y_{XF}\) is decreasing in the price ratio and \(Y_{XE}\) increasing). When both countries produce both goods, one can use (B.13)
and (B.14) to get:

$$
\left( \frac{p_E}{p_F} \right)^{\sigma} \left( \frac{A_{NE}(\tau_N)}{1-\delta_N(\tau_N)} \right)^{\frac{1-\alpha}{\alpha-\beta}} (1-\beta) \left( \frac{p_E A_{NE}(\tau_N)}{p_F A_{NF}} \right) \frac{1-\alpha}{\alpha-\beta} K_N - \beta \left( \frac{p_E A_{NF}(\tau_N)}{p_F A_{SF}} \right) \frac{1-\alpha}{\alpha-\beta} L_N \right) \\
+ \left( \frac{p_E A_{SE}(\tau_S)}{p_F A_{SF}} \right)^{\frac{1-\alpha}{\alpha-\beta}} (1-\beta) \left( \frac{p_E A_{SF}(\tau_S)}{p_F A_{SF}} \right) \frac{1-\alpha}{\alpha-\beta} K_S - \beta \left( \frac{p_E A_{SF}(\tau_S)}{p_F A_{SF}} \right) \frac{1-\alpha}{\alpha-\beta} L_S 
$$

(17)

$$
= \left( \frac{\nu}{1-\nu} \right)^{\sigma} A_{NF} \left( \frac{p_E A_{NF}(\tau_N)}{p_F A_{NF}} \right) \frac{1-\alpha}{\alpha-\beta} L_N - \left( \frac{p_E A_{NF}(\tau_N)}{p_F A_{NF}} \right) \frac{1-\alpha}{\alpha-\beta} (1-\alpha) \left( \frac{p_E A_{NF}(\tau_N)}{p_F A_{NF}} \right) \frac{1-\alpha}{\alpha-\beta} K_N \\
+ A_{SF} \left( \frac{p_E A_{SF}(\tau_S)}{p_F A_{SF}} \right) \frac{1-\alpha}{\alpha-\beta} L_S - \left( \frac{p_E A_{SF}(\tau_S)}{p_F A_{SF}} \right) \frac{1-\alpha}{\alpha-\beta} (1-\alpha) \left( \frac{p_E A_{SF}(\tau_S)}{p_F A_{SF}} \right) \frac{1-\alpha}{\alpha-\beta} K_S 
$$

(B.17)

### B.1.3 Pattern of specialization in free trade

I now derive the full pattern of specialization in free trade. To simplify expression I introduce the notations $\tilde{K}_X \equiv \left( \frac{A_{XE}(\tau_X)}{A_{XE}(\tau_X)^\sigma} \right)^{\frac{1}{\alpha-\beta}} K_X$ and $\tilde{L}_X \equiv \left( \frac{A_{XF}(\tau_X)}{A_{XE}(\tau_X)^\sigma} \right)^{\frac{1}{\alpha-\beta}} L_X$, which represent “effective endowments”. Using (B.13), (B.14) and (B.17), assuming that both countries produce both goods, the condition $Y_{XE} > 0$ translates into

$$
\frac{\tilde{K}_X}{\tilde{L}_X} > \frac{\beta}{1-\beta} \frac{\left( \frac{p_E A_{NF}(\tau_N)}{p_F A_{NF}} \right)^{\frac{1-\alpha}{\alpha-\beta}} (1-\alpha) \left( \frac{\tilde{K}_N + \tilde{K}_S}{\tilde{L}_N + \tilde{L}_S} \right) \frac{1-\alpha}{\alpha-\beta} \left( \frac{\nu}{1-\nu} \right)^{\sigma} \left( \frac{\tilde{K}_N + \tilde{K}_S}{\tilde{L}_N + \tilde{L}_S} \right) + \beta \left( \frac{\tilde{K}_N}{\tilde{L}_N + \tilde{L}_S} \right)}{\alpha \left( \frac{\nu}{1-\nu} \right)^{\sigma} \left( \frac{\tilde{K}_N}{\tilde{L}_N + \tilde{L}_S} \right) + \beta \left( \frac{\tilde{K}_N}{\tilde{L}_N + \tilde{L}_S} \right)},
$$

(B.18)

and $Y_{XF} > 0$ into:

$$
\frac{\tilde{K}_X}{\tilde{L}_X} < \frac{\alpha}{1-\alpha} \frac{\left( \frac{p_E A_{NF}(\tau_N)}{p_F A_{NF}} \right)^{\frac{1-\alpha}{\alpha-\beta}} (1-\alpha) \left( \frac{\tilde{K}_N + \tilde{K}_S}{\tilde{L}_N + \tilde{L}_S} \right) \frac{1-\alpha}{\alpha-\beta} \left( \frac{\nu}{1-\nu} \right)^{\sigma} \left( \frac{\tilde{K}_N + \tilde{K}_S}{\tilde{L}_N + \tilde{L}_S} \right) + \beta \left( \frac{\tilde{K}_N}{\tilde{L}_N + \tilde{L}_S} \right)}{\alpha \left( \frac{\nu}{1-\nu} \right)^{\sigma} \left( \frac{\tilde{K}_N}{\tilde{L}_N + \tilde{L}_S} \right) + \beta \left( \frac{\tilde{K}_N}{\tilde{L}_N + \tilde{L}_S} \right)},
$$

(B.19)

Therefore, conditions (B.18) and (B.19) define the set of endowments, productivity and taxes for which there is incomplete specialization in both countries.

Assume now that country $X$ fully specializes in sector $E$, but country $-X$, does not fully specializes. Using (B.12), production of good $E$ in $X$ is given by: $Y_{XE} = \frac{\nu}{1-\nu} \tilde{K}_X \tilde{L}_X^{1-\alpha}$. Combining this expression with (B.16) and (B.13) and (B.14) for country $-X$, delivers an implicit equation for the price ratio, which can be used to show that the condition $Y_{(-X)E} > 0$ is equivalent to:

$$
\left( \frac{\tilde{K}_-X}{\tilde{L}_-X} \right)^{\frac{1-\alpha}{\alpha-\beta}} \tilde{K}_-X \tilde{L}_-X > \left( \frac{\beta^\alpha (1-\beta)(1-\alpha)}{\alpha^\alpha (1-\alpha)(1-\alpha)} \right)^{\sigma} \left( \frac{1-\nu}{\nu} \right)^{\sigma} \frac{\tilde{K}_X \tilde{L}_X^{1-\alpha}}{1-\delta_X}.
$$

(B.20)

Therefore country $X$ fully specializes in sector $E$ and country $-X$ produces both goods when the opposite of (B.18) and (B.20) hold.
Similarly if country \( X \) specializes in sector \( F \), one gets \( Y_{XF} = \zeta K_X \beta L_X^{1-\beta} \) and the condition \( Y_{F(-X)} > 0 \) writes as:

\[
\left( \frac{\alpha^\beta (1 - \alpha)^{1-\beta}}{\beta^\beta (1 - \beta)^{(1-\beta)}} \right)^\sigma \frac{\zeta K_X \beta L_X^{1-\beta}}{1 - \delta_X} < \left( \frac{1}{\nu} \right)^\sigma \frac{\left( \frac{K_X}{L_X} \right)^{(\alpha - \beta)(1-\sigma)} K_X L_X^{1-\beta}}{1 - \delta_X}.
\]

(B.21)

Country \( X \) fully specializes in sector \( F \) and country \(-X\) produces both goods when the opposite of (B.19) and (B.21) hold.

Finally the case where country \( X \) fully specializes in \( E \) while country \(-X\) fully specializes in \( F \) corresponds to the opposite of (B.21).and the opposite of (B.20), for future use, it is convenient to express these two conditions with the actual endowments and productivities as:

\[
A_{(-X)F} K_{-X} L_{-X}^{1-\beta} \geq \left( \frac{1}{\nu} \right)^\sigma \frac{K_X}{L_X} \left( \frac{K_X}{L_X} \right)^{(\alpha - \beta)(1-\sigma)} \frac{K_X L_X^{1-\beta}}{1 - \delta_X} \frac{A_{EF} (A_{XE} (\tau_X))^{1-\sigma}}{1 - \delta_X}.
\]

(B.22)

\[
A^{1-\sigma}_{(-X)F} (A_{(-X)E} (\tau_{(-X)}))^{\sigma} \frac{L_X}{K_X} \left( \frac{L_X}{K_X} \right)^{(\alpha - \beta)(1-\sigma)} K_{-X} L_{-X}^{1-\alpha} \leq \left( \frac{\beta^\alpha (1 - \beta)^{(1-\alpha)} 1 - \nu}{\alpha^\alpha (1 - \alpha)^{(1-\alpha)} \nu} \right)^\sigma \frac{A_{XE} (\tau_X) K_X^{1-\alpha} L_X^{1-\alpha}}{1 - \delta_X}.
\]

(B.23)

These endowment sets have no overlap. Moreover, in each case scenario the relative price of good \( E \) over good \( F \) is uniquely defined, therefore in free trade and for given technologies, the equilibrium is unique.

### B.1.4 Equilibrium profits and innovation decision

Using (8), (B.4), I can express intermediates production in sector \( z \in \{c,d\} \) as: \( x_{Xzt} = \frac{p_{X,dz}}{1-q} \left( \frac{\alpha}{w_X} \right)^\alpha \left( \frac{A_{Xzi}}{A_{Xz}} \right)^{1-\alpha} Y_{Xzt} \), combining this with (B.6) and (B.3) gives

\[
\pi_{Xzit} = \frac{(1 - \gamma)}{(1 - q)} \left( \frac{A_{Xzit}}{A_{Xzt}} \right)^{1-\gamma} p_{Xzt} Y_{Xzt},
\]

(B.24)

or (20) when \( q = 0 \). Using (B.7), this translates into:

\[
\pi_{Xcit} = \gamma \frac{(1 - \gamma)}{(1 - q)} \left( \frac{A_{Xcit}}{A_{Xct}} \right)^{1-\gamma} \frac{A_{Xct}^{\varepsilon-1}}{A_{\varepsilon Xct}^{\varepsilon-1}} + \frac{(1 + \tau_{X})^{-\varepsilon} A_{dXt}^{\varepsilon-1}}{(1 + \tau_{X})^{-\varepsilon} A_{\varepsilon Xct}^{\varepsilon-1}} \pi_{EFt} Y_{EFt}.
\]

(B.25)

\[
\pi_{Xdit} = \gamma \frac{(1 - \gamma)}{(1 - q)} \left( \frac{A_{Xdit}}{A_{Xdt}} \right)^{1-\gamma} \frac{(1 + \tau_{X})^{-\varepsilon} A_{Xdit}^{\varepsilon-1}}{(1 + \tau_{X})^{-\varepsilon} A_{\varepsilon Xdt}^{\varepsilon-1}} + \frac{(1 + \tau_{X})^{-\varepsilon} A_{dXt}^{\varepsilon-1}}{(1 + \tau_{X})^{-\varepsilon} A_{\varepsilon Xdt}^{\varepsilon-1}} \pi_{EFt} Y_{EFt}.
\]

(B.26)

The same reasoning in sector \( F \) gives:

\[
\pi_{XFit} = \gamma \frac{(1 - \gamma)}{(1 - q)} \left( \frac{A_{XFit}}{A_{XFt}} \right)^{1-\gamma} p_{XFit} Y_{XFit}.
\]

(B.27)
To avoid repetition, I let both countries implement a tax \(q_{Xt}\) on the wages of scientists in the dirty subsector. Combining the first order conditions with respect to the number of scientists in the clean and dirty subsector (and assuming that some production takes place in sector \(E\) in country \(X\)) delivers the allocation of scientists within sector \(E\) as:

\[
\frac{s_{Xct}^{1-t} (1 + \kappa s_{Xct}^t)}{s_{Xdt}^{1-t} (1 + \kappa s_{Xdt}^t)} = \frac{p_{Xct} Y_{Xct}}{p_{Xdt} Y_{Xdt}} = \frac{(1 - q_{Xt}) (1 + \tau_{Xt})^\varepsilon A_{Xct}^{\varepsilon-1}}{A_{Xdt}^{\varepsilon-1}} A_{Xct}^{\varepsilon-1} A_{Xdt},
\]

where the second equality arises from (B.7) and (B.9). Similarly, combining the first order condition with respect to the number of scientists in sector \(F\) and subsector \(d\), I get:

\[
\frac{s_{Xdt}^{1-t} (1 + \kappa s_{Xdt}^t)}{s_{XFt}^{1-t} (1 + \kappa s_{XFt}^t)} = \frac{(1 - q_{Xt}) (1 + \tau_{Xt})^{-\varepsilon} A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1} + (1 + \tau_{Xt}) A_{Xdt}^{\varepsilon-1}} \frac{p_{XEt} Y_{XEt}}{p_{XFt} Y_{XFt}}.
\]

With \(\tau_{Xt} = 0\), these two last equations write as (21) and (22).

### B.2 Proofs of path dependence and of lemma 2

In this subsection, I first derive the effect of the relative productivities of clean and dirty technologies on the incentive to innovate in sector \(E\) and on sector \(E\) effective productivity’s growth rate, second I establish the conditions under which there is path dependence and third I prove lemma 2.

**B.2.1 Effect of relative productivity of clean and dirty technologies on the incentive to innovate in sector \(E\) and on sector \(E\) productivity’s growth rate**

To simplify notation, I introduce the following notations: \(a_{Xt} \equiv \min \left( \left( \frac{A_{Xct} (t-1)}{A_{Xct} (t-1)} \right)^{\varepsilon-1} \right) \leq 1 \) measures the gap between the least advanced and the most advanced technology in sector \(E\) prior to innovation, \(s_A (a_{Xt}, s_{XEt})\) denotes the amount of innovation at time \(t\) in the subsector which was the most advanced at time \(t-1\) as a function of the gap and of the total amount of scientists working in sector \(E\), similarly \(s_a\) denotes the amount of innovation in the least advanced subsector within sector \(E\). As in Appendix D.1, I will also use the notation \(\bar{\kappa} (s) = \kappa s\). The functions \(s_A\) and \(s_a\) are defined by \(s_A (a, s_E) + s_a (a, s_E) = s_E\) and by (21) which can be rewritten as:

\[
\bar{\kappa}^t (s_A (a_{Xt}, s_{XEt})) (1 + \bar{\kappa} (s_A (a_{Xt}, s_{XEt})))^{(\varepsilon-1)(1-\gamma)-1} =
\bar{\kappa}^t (s_a (a_{Xt}, s_{XEt})) (1 + \bar{\kappa} (s_a (a_{Xt}, s_{XEt})))^{(\varepsilon-1)(1-\gamma)-1} a_{Xt}
\]

where \(s_A (a, s_E)\) is the allocation of scientists to the subsector amongst clean and dirty with the highest productivity level at \(t-1\) and \(s_a (a, s_{XEt})\).
Further I define the function $f$:

$$f(a, s_E) = \frac{1 + \tilde{\kappa} (1 - s_E)}{\tilde{\kappa}' (1 - s_E)} \frac{1}{2} \left( \frac{a}{(1 + \kappa (s_a))^{(\varepsilon - 1)(1 - \gamma) - 1}} + \frac{\tilde{\kappa}' (s_A) (1 + \kappa (s_A))^{(\varepsilon - 1)(1 - \gamma) - 1}}{(1 + \kappa (s_a))^{(\varepsilon - 1)(1 - \gamma) - 1}} \right),$$

which represents the ratio between the marginal benefit of an additional scientist in sector $F$ divided by sector $F$ revenues over the marginal benefit of an additional scientist in sector $E$ divided by sector $E$’s revenues. For $\kappa$ small enough, $s_a < s_A$ (see Appendix D.1). Taking the derivative with respect to $a$, one gets:

$$\frac{\partial f}{\partial a} = \frac{(1 - \ell) (1 + \tilde{\kappa} (1 - s_E))}{\tilde{\kappa}' (1 - s_E) \ell} \left( \frac{1 + \tilde{\kappa} (s_a)}{\tilde{\kappa} (s_a)} + \frac{(1 + \tilde{\kappa} (s_A))^{(\varepsilon - 1)(1 - \gamma) - 1}}{(1 + \tilde{\kappa} (s_a))^{(\varepsilon - 1)(1 - \gamma)}} \right)^2 \left( \frac{1}{s_A} - \frac{1}{s_a} \right) \frac{\partial s_A}{\partial a} < 0,$$

therefore $f$ is decreasing in $a$, since $s_A > s_a$.

Similarly the growth rate of effective productivity in sector $E$ for a given number of scientists as: $g(a, s_E) = \frac{1}{a + 1} (1 + \tilde{\kappa}(s_a))^{(\varepsilon - 1)(1 - \gamma) - 1} (1 + \tilde{\kappa}(s_A))^{(\varepsilon - 1)(1 - \gamma)}$. $g$ is increasing in $s_E$, and $
\frac{\partial g}{\partial a} = \frac{(1 + \tilde{\kappa}(s_a))^{(\varepsilon - 1)(1 - \gamma)} (1 + \tilde{\kappa}(s_A))^{(\varepsilon - 1)(1 - \gamma)}}{(a + 1)^2} < 0$, since $s_A > s_a$. Therefore for a given amount of scientists in sector $E$, average productivity grows faster when the gap between the two subsectors is large.

**B.2.2 Path dependence**

Here I prove the following lemma:

**Lemma 4** Consider a laissez-faire economy and assume, that country $X$ initially has a (weak) comparative advantage in sector $E$ $(\frac{A_{XE0}}{A_{X0}})^{\alpha} = \frac{K_X}{L_X} \geq (\frac{A_{(-X)E0}}{A_{(-X)0}})^{\alpha} = \frac{K_{(-X)}}{L_{(-X)}}$, and that (i) $a_{X0}$ and $a_{(-X)0}$ are sufficiently small and the previous inequality is strict or (ii) $a_{X0} \leq a_{(-X)0}$; then at all points in time: $s_{XEt} \geq s_{(-X)Et}$ (with a strict inequality if one of the previous equalities is strict). Furthermore when one of these two inequalities is strict, $A_{SEt}/A_{NEt}$ and $A_{NFt}/A_{SFt}$ tend toward infinity.

**Proof.** Without loss of generality, I assume that country $X$ is the South, and that in both countries $A_{X0} \geq A_{X0}$. Assume that at time $t \geq 1$, I have $(\frac{A_{SE(t-1)}}{A_{SF(t-1)}})^{\alpha} = \frac{1}{L_{SF(t-1)}} \frac{K_S}{L_S} \geq (\frac{A_{NE(t-1)}}{A_{NF(t-1)}})^{\alpha} = \frac{1}{L_{NF(t-1)}} \frac{K_N}{L_N}$, and either $a_{N(t-1)}, a_{S(t-1)}$ are both negligible (with a strict inequality) or $a_{S(t-1)} \leq a_{N(t-1)}$. Then in both countries (B.30) holds with $s_{Xet} = s_a$ and $s_{Xdt} = s_A$. Moreover, (B.28) and (B.29) imply:

$$f(a(t-1), s_{XEt}) = \frac{p_{E} Y_{XEt}}{p_{Et} Y_{XEt}}.$$
Using the expressions for (B.13) and (B.14), I get that, when neither country is fully specialized, the equilibrium can be summarized by three equations:

\[
f(a_{X(t-1)}, s_{XE(t)}) = (1 - \alpha) \left( \frac{a^\alpha(1-\alpha)^{1-a} \rho \frac{p_{Et}}{p_{Ft}} ((1+\bar{\kappa}(s_{Z(t)})^{(\varepsilon-1)(1-\gamma)} A_{X(t-1)}^{\gamma-1} + (1+\bar{\kappa}(s_{Z(t)})^{(\varepsilon-1)(1-\gamma)} A_{F(t-1)}^{\gamma-1})^{\frac{1}{1-\gamma}})}{(1+\bar{\kappa}(s_{Z(t)})^{\gamma-1} A_{X(t-1)}^{\gamma-1} + (1+\bar{\kappa}(s_{Z(t)})^{\gamma-1} A_{F(t-1)}^{\gamma-1})^{\frac{1}{1-\gamma}})} \right) \frac{1}{a^\beta} \frac{K_X}{L_X},
\]

\[
(1 - \beta) \left( \frac{a^\alpha(1-\alpha)^{1-a} \rho \frac{p_{Et}}{p_{Ft}} ((1+\bar{\kappa}(s_{Z(t)})^{(\varepsilon-1)(1-\gamma)} A_{X(t-1)}^{\gamma-1} + (1+\bar{\kappa}(s_{Z(t)})^{(\varepsilon-1)(1-\gamma)} A_{F(t-1)}^{\gamma-1})^{\frac{1}{1-\gamma}})}{(1+\bar{\kappa}(s_{Z(t)})^{\gamma-1} A_{X(t-1)}^{\gamma-1} + (1+\bar{\kappa}(s_{Z(t)})^{\gamma-1} A_{F(t-1)}^{\gamma-1})^{\frac{1}{1-\gamma}})} \right) \frac{1}{a^\beta} \frac{K_X}{L_X} - \beta
\]

for \( X \in \{N, S\} \) and the equation determining the price ratio \( \frac{p_{Et}}{p_{Ft}} \).

If \( a_{N(t-1)} \geq a_{S(t-1)} \), the left-hand side (LHS) of (B.32) is lower for the North than for the South at equal \( s_{XE(t)} \). Similarly if both \( a_{N(t-1)} \) and \( a_{S(t-1)} \) are negligible (relative to the difference in comparative advantage), I get that \( s_{Z(t)} \), \( f(a_{t-1}, s_{ZE(t)}) \approx \frac{1}{a^\beta} \frac{K_N}{L_N} \) and \( g \) is decreasing in \( a \). For given prices, both the LHS and the right-hand side (RHS) are decreasing in \( s_{XE(t)} \), but for sufficiently small \( \kappa \), the LHS decreases faster, therefore \( s_{XE(t)} \geq s_{ZE(t)} \), with a strict inequality if either \( a_{N(t-1)} > a_{S(t-1)} \) or if \( \frac{A_{SE(t-1)}}{A_{SF(t-1)}} \frac{1}{a^\beta} \frac{K_S}{L_S} > \frac{A_{NE(t-1)}}{A_{NF(t-1)}} \frac{1}{a^\beta} \frac{K_N}{L_N} \).

Similarly if both \( a_{N(t-1)} \) and \( a_{S(t-1)} \) are negligible (relative to the difference in comparative advantage), I get that \( s_{Z(t)} \), \( f(a_{t-1}, s_{XE(t)}) \approx \frac{1}{a^\beta} \frac{K_E}{L_E} \) and

\[
\frac{(1 + \bar{\kappa}(s_{XE(t)})^{(\varepsilon-1)(1-\gamma)} A_{X(t-1)}^{\gamma-1} + (1 + \bar{\kappa}(s_{XE(t)})^{(\varepsilon-1)(1-\gamma)} A_{F(t-1)}^{\gamma-1})^{\frac{1}{1-\gamma}}}{(1 + \bar{\kappa}(s_{XE(t)})^{\gamma-1} A_{X(t-1)}^{\gamma-1} + (1 + \bar{\kappa}(s_{XE(t)})^{\gamma-1} A_{F(t-1)}^{\gamma-1})^{\frac{1}{1-\gamma}}},
\]

so that, following a similar reasoning, \( \frac{A_{SE(t-1)}}{A_{SF(t-1)}} \frac{1}{a^\beta} \frac{K_S}{L_S} > \frac{A_{NE(t-1)}}{A_{NF(t-1)}} \frac{1}{a^\beta} \frac{K_N}{L_N} \) leads to \( s_{XE(t)} > s_{ZE(t)} \).

Therefore, in both cases \( A_{SE(t-1)} > A_{NE(t-1)} \) and \( A_{NF(t-1)} > A_{SF(t-1)} \) (expect when \( \frac{A_{SE(t-1)}}{A_{SF(t-1)}} \frac{1}{a^\beta} \frac{K_S}{L_S} = \frac{A_{NE(t-1)}}{A_{NF(t-1)}} \frac{1}{a^\beta} \frac{K_N}{L_N} \) and \( a_{N(t-1)} = a_{S(t-1)} \), in which case the strict inequalities are replaced by equalities). Note that \( a_{Et} < a_{X(t-1)} \), so if both \( a_{N(t-1)} \) an
are negligible, \(a_{Nt}\) and \(a_{St}\) will be negligible too. Moreover, \(\frac{1 + \xi(s_E(a \times s_E))}{1 + \kappa(s_E(a \times s_E))}\) is increasing in \(s_E\) and decreasing in \(a\), so if \(a_{N(t-1)} \geq a_{S(t-1)}\) and \(s_{SEt} \geq s_{NEt}\) then \(a_{Nt} \geq a_{St}\).

The analysis extends directly to the case where one country specializes. By induction, this is enough to show that \(s_{SEt} \geq s_{NEt}\) and, that \(A_{SEt}/A_{NEt}\) and \(A_{NFt}/A_{SFt}\) are increasing (with a strict inequality and strictly increasing if either \(p_{Et} > s\) in \(Cobb-Douglas\) case \(p_{Et} = A_{SEt}/A_{NFt}\) or \(a_{N0} > a_{S0}\)).

Therefore in the case of strict inequality for at least one of the initial conditions, \(s_{NEt} < s_{SEt}\) every period. This is not enough to conclude that \(A_{SEt}/A_{NEt}\) and \(A_{NFt}/A_{SFt}\) tend to infinity as \(s_{NEt}\) and \(s_{SEt}\) could converge toward each other. However this would require that either \(\left(\frac{A_{SEt}^{(t-1)}}{A_{SFt}^{(t-1)}}\right)^{1-\beta} \frac{K_E}{L_E}\) and \(\left(\frac{A_{NEt}^{(t-1)}}{A_{NFt}^{(t-1)}}\right)^{1-\beta} \frac{K_N}{L_N}\) also converge toward each other (which is impossible as the ratio of this term is initially weakly greater than 1 and strictly increasing), or that both \(s_{NEt}\) and \(s_{SEt}\) tend toward the same corner solution. This requires that in both countries \(\frac{p_{Et} Y_{NEt}}{p_{Et} Y_{SEt}}\) either tends toward 0 or toward infinity, which is impossible as well: in the Cobb-Douglas case \(\frac{p_{Et} Y_{NEt}}{p_{Et} Y_{SEt}} = \frac{1-\nu}{\nu}\), and when \(\sigma < 1\) case, innovation favors the most backward sector preventing all scientists from innovating in the same sector in both countries asymptotically.

### B.2.3 Reaching full specialization in finite time

Now, under the assumptions of lemma 2 and using lemma 4, \(s_{SEt} > s_{NEt}\) and \(A_{SEt}/A_{NEt}\) and \(A_{NFt}/A_{SFt}\) grow unboundedly. Using the expressions (B.13) and (B.14), avoiding full specialization asymptotically requires that \(\frac{p_{Et} A_{SEt}}{p_{Et} A_{NEt}}\) remains bounded (from \(Y_{NEt} \geq 0\)) and similarly \(\frac{p_{Et} A_{SEt}}{p_{Et} A_{SFt}}\) remain bounded. Taking the product of the two, this leads toward \(\frac{A_{NEt} A_{SEt}}{A_{NEt} A_{SFt}}\) bounded which is a contradiction. Therefore at least one country fully specializes. For the sake of the argument assumes that the South fully specializes in sector \(E\). If this is the case, note that asymptotically \(A_{SEt}\) must grow at the rate \((1 + \kappa)^{1-\gamma} - 1\) (since eventually all scientists are in the dirty sector there). Then to avoid full specialization in the North in finite time, one must keep (from (B.20)):

\[
\left(\frac{A_{NFt}}{A_{SEt}}\right)^{1-\sigma} \left(\frac{K_N}{L_N}\right)^{(\alpha-\beta)\sigma} K_N^{\beta} L_N^{1-\beta} > \left(\frac{1}{\alpha^\sigma} \frac{1-\beta}{(1-\alpha)^{\sigma}} \frac{1-\nu}{\nu} \left(\frac{A_{SEt}}{A_{NEt}}\right)^{\sigma} \frac{K_S}{K_N} L_S^{1-\alpha}ight),
\]

where \(0 < \sigma \leq 1\). Now \(A_{SEt}/A_{NEt}\) grows exponentially, while \(A_{NFt}/A_{SFt}\) cannot grow asymptotically since \(A_{SEt}\) asymptotically grows at the fastest rate. Therefore, keeping that inequality is impossible and the North must also fully specialize. A similar reasoning applies to the case where the North specializes first. Therefore I have proved that full specialization
is reached in finite time.53

B.3 Proof of Lemma 3

Since the emission rate per unit of the polluting good cannot decrease in the South when 
\( A_{Sc0} \leq A_{Sd0} \), the South’s production of the polluting good must remain bounded in order to avoid. Therefore a disaster cannot be avoided if the South fully specializes in sector \( E \), it may be avoided (with sufficiently large initial environmental quality) if the South fully specializes in sector \( E \) in finite time (in which case, by definition, all factors in the South are allocated to the polluting sector in finite time and the North exports the polluting good).

Assume now that there is not full specialization in the South, that is there are an infinite number of periods where the South produces both goods (and in the following I restrict attention to those periods). Rewriting (B.13) with \( \delta_{St} = 0 \) and \( p_t = p_{SEt}/p_{Ft} \), production in sector \( E \) is given by

\[
Y_{SEt} = \frac{\zeta A_{SEt}}{(\alpha - \beta)} \left( \frac{\beta^2 \alpha (1 - \beta)(1 - \beta)\alpha}{\alpha^2 \alpha (1 - \alpha)(1 - \alpha)\beta} \right)^{\frac{1}{\alpha - \beta}} \left( p_t \frac{A_{SEt}}{A_{SFt}} \right)^{-\frac{\alpha}{\alpha - \beta}} \left( \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta} K_S - \beta L_S}.
\]

Therefore to keep \( Y_{SEt} \) bounded it must be the case that \( p_t \frac{A_{SEt}}{A_{SFt}} \frac{1}{\alpha - \beta} \) is bounded, with either

\[
\lim_{t \to \infty} p_t \frac{A_{SEt}}{A_{SFt}} \frac{1}{\alpha - \beta} = \left( \frac{\beta^2 \alpha \alpha (1 - \alpha)(1 - \alpha)}{(\alpha^2 \alpha (1 - \alpha)(1 - \alpha)\beta) (1 - \beta) K_S - \beta L_S} \right)
\]

or with \( A_{SEt} \) bounded. Using (B.14), \( Y_{SFt} \) can be rewritten as:

\[
Y_{SFt} = \frac{\zeta A_{SFt}}{(\alpha - \beta)} \left( \frac{\beta^2 \alpha (1 - \beta)(1 - \beta)\alpha}{\alpha^2 \alpha (1 - \alpha)(1 - \alpha)\beta} \right)^{\frac{1}{\alpha - \beta}} \left( p_t \frac{A_{SEt}}{A_{SFt}} \right)^{-\frac{\beta}{\alpha - \beta}} \left( \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta} K_S}.
\]

Combining these two expressions with (B.29) implies that the allocation of innovation in the

53 In principle, it could also have been the case that each country specializes in its sector in turn, but this situation can only happen if during some periods \( \frac{A_{NFt}}{A_{SEt}} \frac{\alpha (1 - \alpha)}{(\alpha^2 \alpha (1 - \alpha)(1 - \alpha)\beta)} K_N L_N = 1 \) and during others \( \frac{A_{SEt}}{A_{SFt}} \frac{\alpha (1 - \alpha)}{(\alpha^2 \alpha (1 - \alpha)(1 - \alpha)\beta)} K_N L_N = 1 \) and during others \( \frac{A_{SFt}}{A_{NFt}} \frac{\alpha (1 - \alpha)}{(\alpha^2 \alpha (1 - \alpha)(1 - \alpha)\beta)} K_N L_N = 1 \). Since these inequalities must be flip for certain periods, it must be the case that all the terms grow at the same rate, therefore \( \left( \frac{A_{SEt}}{A_{SFt}} \right)^{\sigma} \) cannot become permanently bigger than \( \left( \frac{A_{SEt}}{A_{SFt}} \right)^{\sigma} \), which we know happens. Therefore this case is ruled out too.
South must satisfy

\[
\frac{\kappa' (s_{Sdt})}{(1 + \kappa (s_{Sdt}))} = 1 + \kappa (1 - s_{Sdt}) \frac{A_{Sdt} \neq 1}{\kappa' (1 - s_{Sdt})} + A_{Sdt} \neq 1 = \frac{\alpha L_s - \frac{\alpha(1-a)^{1-\alpha}}{\beta(1-\beta)^{1-\beta}} (1 - \alpha) \left(p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} K_s}{\frac{\alpha(1-a)^{1-\alpha}}{\beta(1-\beta)^{1-\beta}} (1 - \beta) \left(p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} K_s - \beta L_s}.
\]

If \( \lim \left( p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} \neq \frac{\beta(1-\beta)^{1-\beta)}{\alpha(1-a)^{1-\alpha}} \left(1 - \beta \right) \left(p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} K_s \), \( s_{Sdt} \) cannot tend toward 0 therefore \( A_{Sdt} \) must become unbounded, which leads to a disaster. Therefore avoiding a disaster in this case requires that \( \lim \left( p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} = \frac{\beta(1-\beta)^{1-\beta)}{\alpha(1-a)^{1-\alpha}} \left(1 - \beta \right) \left(p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} K_s \), so that asymptotically all factors in the South (scientists, capital, labor) must be allocated to sector \( F \).

Moreover, denoting \( M_{Et} \) and \( M_{Ft} \) net imports from the North, (B.16) leads to:

\[
\frac{p_{S} Y_{SEt}}{p_{S} Y_{SFt}} = \frac{\nu}{1 - \nu} \left( \frac{Y_{SEt} - M_{Et}}{Y_{SFt} - M_{Ft}} \right)^{-\frac{1}{\sigma}} Y_{SEt}.
\]

If the North does not export the polluting good \( (M_{Et} \geq 0) \), then the right-hand side (RHS) of the above expression is greater than \( \frac{\nu}{1 - \nu} \left( \frac{Y_{SEt}}{Y_{SFt}} \right)^{1-\frac{1}{\sigma}} \). But avoiding a disaster requires that the LHS tends toward 0, while \( \frac{Y_{SEt}}{Y_{SFt}} \) becomes unbounded: this yields a contradiction, so the North must export the polluting good \( E \).

### B.4 Proof of Proposition 1

The proof is similar to the proof of lemma 2: I show that as soon as a tax on dirty research or a carbon tax is implemented, \( s_{NEt} < s_{SEt} \) and that this eventually leads to full specialization. Without loss of generality, I assume that a combination of a tax on dirty research or a carbon tax from the first period. First I explain how the presence of a tax on dirty research affects the incentive to innovate in sector \( E \), second I show that in the first period \( (t = 1) \), it is necessarily the case that \( s_{NE1} < s_{SE1} \), third I show that in all following periods the same logic applies and finally I show that full specialization is reached.

#### B.4.1 Tax on dirty research and incentive to innovate

First I define the function \( \tilde{f}_N \) which is a generalization for the North of the function \( f \) defined in B.31:

\[
\tilde{f}_N (a_{Nc(t-1)}, s_{NEt}, 1 - q_{Nt})
\]

\[
= \frac{1 + \tilde{\kappa} (s_{NFt})}{\kappa' (s_{NFt})} \frac{1}{2} \left( 1 + \tilde{\kappa} (s_{NFt}) \right) (1 - \gamma) a_{Nc(t-1)} + (1 - q_{Nt}) \tilde{\kappa}' (s_{Nd}) (1 + \tilde{\kappa} (s_{Nd}))^{(1-1)(1-\gamma)-1}
\]

\[
\alpha L_s - \frac{\alpha(1-a)^{1-\alpha}}{\beta(1-\beta)^{1-\beta}} (1 - \alpha) \left(p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} K_s
\]

\[
= \frac{\alpha(1-a)^{1-\alpha}}{\beta(1-\beta)^{1-\beta}} (1 - \beta) \left(p_{t A_{S} F_t} \right)^{\frac{1}{\alpha - \beta}} K_s - \beta L_s
\]
where \( a_{Ne(t-1)} \) is a given number (not necessarily smaller than 1) and \( s_{Nct} \) and \( s_{Nd} \) are defined by

\[
\tilde{\kappa}' (s_{Nct}) (1 + \tilde{\kappa} (s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1} a_{Ne(t-1)} = (1 - q_t) \tilde{\kappa}' (s_{Nd}) (1 + \tilde{\kappa} (s_{Nd}))^{(\varepsilon-1)(1-\gamma)-1}.
\]

When \( \tilde{\kappa}' (s) (1 + \tilde{\kappa} (s))^{(\varepsilon-1)(1-\gamma)-1} \) is decreasing in \( s \), which is the case for \( \kappa \) sufficiently small, the numerator of \( \tilde{f}_N \) is decreasing in \( q_{Nt} \). The denominator is also decreasing in \( q_t \) since for a given mass scientists in sector \( E \), \( q_t = 0 \), maximizes the growth rate of average productivity. However for sufficiently small \( \kappa \), the variations in the denominator are negligible, so that for \( q_t > 0, \tilde{f}_N (a_{Ne(t-1)}, s_{NEt}, 1 - q_t) < f_N (a_{Ne(t-1)}, s_{NEt}, 1) = f \left( \min \left( a_{Ne(t-1)}, a_{Ne(t-1)}^{-1} \right), s_{NEt} \right) \).

**B.4.2 Proof that \( s_{NE1} < s_{SE1} \)**

Equations (B.28) and (B.29) in the North determine the equilibrium allocation of innovation as:

\[
\tilde{f}_N \left( \left( 1 + \tau_{N1} \right) \frac{A_{Nc0}}{A_{Nd0}} \right)^{\varepsilon-1}, s_{NE1}, \frac{1 - q_{1}}{1 + \tau_{N1}} = \frac{p_{F1} \tilde{Y}_{NF1}}{p_{E1} \tilde{Y}_{NE1}}.
\]

Recall that \( \tilde{f}_N \left( \left( 1 + \tau_{N1} \right) \frac{A_{Nc0}}{A_{Nd0}}, s_{NE1}, \frac{1 - q_{1}}{1 + \tau_{N1}} \right) < f (a_{N0}, s_{NE1}) \) with

\[
a_{N0} = \min \left( \left( 1 + \tau_{N1} \right) \frac{A_{Nc0}}{A_{Nd0}}^{\varepsilon-1}, \left( 1 + \tau_{N1} \right)^{-1} \left( 1 - \frac{A_{Nd0}}{A_{Nc0}} \right)^{\varepsilon-1} \right),
\]

moreover, using (B.13) and (B.14), \( \frac{p_{F1} \tilde{Y}_{NF1}}{p_{E1} \tilde{Y}_{NE1}} \) is increasing in \( \tau \) at given price ratio \( p_{F1}/p_{E1} \) and technological levels. As a result, the logic of the proof of lemma 2 fully applies provided that \( a_{N0} \geq a_{S0} \). Since \( \frac{A_{Nc0}}{A_{Nd0}} \geq \frac{A_{Sc0}}{A_{Sd0}} \), this is necessarily satisfies unless \( (1 + \tau_{N1}) \geq \frac{A_{Nc0}}{A_{Nd0}} \frac{A_{Sc0}}{A_{Sd0}} \). However, for \( \frac{A_{Sc0}}{A_{Sd0}} \) sufficiently small, a carbon that satisfies such an inequality must be large, as \( A_{Nc0} \leq A_{Nd0} \), the difference in initial comparative advantage between the North and the South becomes large, therefore, here as well, the logic of lemma 2 applies (now for the case where \( a_{Sc0} \) is negligible relative to the difference in comparative advantage). Therefore, it must be the case that \( s_{NE1} < s_{SE1} \). A tax on dirty research and a carbon tax further distort the allocation of innovation for a given mass of scientists in sector \( E \), therefore

\[
\frac{A_{SE1}}{A_{SF1}} \left( \frac{A_{Nc0}}{A_{Nd0}} \right)^{\varepsilon-1} K_{S}^{1-\gamma} \frac{K_{S}}{L_{S}} > \left( \frac{A_{SE0}}{A_{SF0}} \right)^{\varepsilon-1} K_{N}^{1-\gamma} \frac{K_{N}}{L_{N}} - \text{in fact, one even gets:}
\]

\[
\frac{A_{SE1}}{A_{SF1}} \left( \frac{A_{Nc0}}{A_{Nd0}} \right)^{\varepsilon-1} \frac{1}{A_{NF1}} \frac{1}{A_{NF1}}^{-1} > \frac{A_{SE0}}{A_{SF0}} \left( \frac{A_{SE0}}{A_{NF0}} \right)^{-1}.
\]

**B.4.3 Proof that \( s_{NEt} < s_{SEt} \)**

I have already established that the difference in comparative advantages increases from one period to the other. To establish the proof by induction, I need to show that in all subsequent
periods, it will still be the case that either \( a_{N(t-1)} \geq a_{S(t-1)} \) or \( a_{S(t-1)} \) is negligible relative to the difference in comparative advantages.

First note that as long as \( A_{Nc(t-1)} \geq A_{Nd(t-1)} \), then every period \( \frac{A_{Nc(t-1)}}{A_{Nd(t-1)}} \geq \frac{A_{Sc(t-1)}}{A_{St(t-1)}} \), since less scientists are allocated to sector \( E \) in the North and the allocation is tilted toward the clean subsector and a large tax (necessary to get \( a_{N(t-1)} < a_{S(t-1)} \)) would have a significant impact on the pattern of comparative advantage.

Now assume that \( A_{Nc(t-1)} \geq A_{Nd(t-1)} \), and that \( a_{N(t-1)} < a_{S(t-1)} \). The latter can be achieved if either \( \left( \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} \right)^{\varepsilon-1} \) and \( a_{S(t-1)} \) are close to each other, or, if \( \tau \) is large. Now if \( a_{S0} \) is sufficiently small, and since \( s_{NEt} < s_{SEt} \) for every \( \tau < t \) by induction hypothesis, it will take a large number of periods to \( \left( \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} \right)^{\varepsilon-1} \) and \( a_{S(t-1)} \) close to each other if it is even possible (it might be possible because all sector \( E \)-innovation in the North occurs in clean technologies). Over such a time period, the difference in comparative advantage will have built up and \( a_{S(t-1)} \) will be small relative to it. Moreover, as before, a large \( \tau \) will have a direct impact on the pattern of comparative advantages, unless \( \left( \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} \right)^{\varepsilon-1} \) is small which can only be achieved after a certain number of periods, by which the difference in comparative advantages will be large relative to \( a_{S(t-1)} \). Regardless of the case scenario, \( a_{S(t-1)} \) is small relative to the difference in comparative advantage, therefore, one gets: \( s_{SEt} > s_{NEt} \) every period.

### B.4.4 Reaching full specialization

Here as well, \( \frac{A_{SEt}}{A_{NEt}} \) and \( \frac{A_{NEt}}{A_{St(t-1)}} \) grow unboundedly. From (B.13) and (B.14), this necessarily leads to specialization in at least one country. Assume that there is full specialization in sector \( E \) in the South, so that asymptotically \( A_{SEt} \) must grow at the rate \((1 + \kappa)^{1-\gamma} - 1 \). Then avoiding full specialization in the North in finite time requires to keep (from (B.23)):

\[
A_{NFt}^{1-\sigma} \left( A_{Nct(t-1)}^{\varepsilon-1} \left( 1 - \tau_{Nt} - 1 \right) A_{Nd(t-1)}^{\varepsilon-1} \right)^{\sigma} K_{X}^{1-\alpha} L_{N}^{1-\alpha} \geq \left( \frac{\beta_{X} (1 - \beta)(1-\alpha)}{\alpha^{\alpha} (1 - \alpha)(1-\alpha)} \right)^{\sigma} \left( \frac{1 - \nu}{\nu} \right)^{\sigma} A_{SEt} K_{X}^{\alpha} L_{X}^{1-\alpha},
\]

which is impossible. Similarly if the North fully specializes in sector \( F \), avoiding specialization in the South will also be impossible. Therefore both countries fully specialize, the emissions in the South necessarily grow unbounded and a disaster occurs.

### B.5 Proof of Proposition 3

I first solve for the problem of maximizing (1), I denote the Lagrange parameters (with the corresponding constraints in parentheses): \( \lambda_{Et} \) (3), \( \lambda_{XFt} \) (4), \( \lambda_{XE} \) (7), \( \lambda_{Xt} \) (8), \( \varphi_{Xzt} \) (9), \( \varphi_{XFt} \) (5), \( \eta_{XKt} \) (11) for capital, \( \eta_{XLt} \) (11) for labor, \( \theta_{Et} \) (12) in sector \( E \), \( \theta_{Ft} \) (12) in sector \( F \), \( \omega_{Et} \) (16), \( \mu_{Xzt} \) (13), \( v_{Xt} \) (15), in addition the social planner faces the constraints: \( 0 \leq Y_{XEt} \)}
and $0 \leq Y_{XFt}$, with Lagrange parameters: $\lambda_{XEt}$, $\lambda_{XFt}$. Taking the first order condition with respect to $Y_{XFt}$ and $Y_{XEt}$ gives:

$$\lambda_{XFt} = \theta_{Ft} + \iota_{XFt} \quad \text{and} \quad \lambda_{XEt} = \theta_{Et} + \iota_{XEt}.$$  

Defining $u(C_{Wt}, S_t) \equiv \frac{(\nu(S_t)C_{Wt})^{1-\eta}}{(1-\eta)}$ with $C_{Wt} \equiv C_{Nt} + C_{St}$, first order conditions with respect to $C_{Nt}$ and $C_{St}$ lead to:

$$\frac{1}{(1+\rho)^t} \frac{\partial u}{\partial C}(C_{Wt}, S_t) = \nu (S_t)^{1-\eta} C_{Wt}^{-\eta} = \lambda_{Et} \equiv \lambda_t.$$  

First order conditions with respect to $C_{XEt}$ and $C_{XFt}$ give:

$$\lambda_t \nu C_{XEt}^{-\frac{1}{\sigma}} \left( \nu C_{XFt}^{-\frac{1}{\sigma}} + (1-\nu) C_{XEt}^{-\frac{1}{\sigma}} \right)^{\frac{1}{\sigma-1}} = \theta_{Et}, \quad (B.36)$$

$$\lambda_t (1-\nu) C_{XFt}^{-\frac{1}{\sigma}} \left( \nu C_{XEt}^{-\frac{1}{\sigma}} + (1-\nu) C_{XFt}^{-\frac{1}{\sigma}} \right)^{\frac{1}{\sigma-1}} = \theta_{Ft}. \quad (B.37)$$

$\theta_{Et}/\lambda_t$ and $\theta_{Ft}/\lambda_t$ can be interpreted as consumer prices in terms of units of welfare. To emphasize this interpretation, I denote $\hat{p}_{Et} = \theta_{Et}/\lambda_t$ and $\hat{p}_{Ft} = \theta_{Et}/\lambda_t$. I then get:

$$\frac{\hat{p}_{Et}}{\hat{p}_{Ft}} = \frac{\nu}{1-\nu} \left( \frac{C_{XFt}}{C_{XEt}} \right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu} \left( \frac{C_{Nt} + C_{St}}{C_{Nt} + C_{St}} \right)^{\frac{1}{\sigma}},$$

which is the equivalent to the equilibrium condition (B.16). Taking the first order condition with respect to with respect to $Y_{XFt}$ and $Y_{XEt}$ gives:

$$\lambda_{XFt} = \theta_{Ft} + \iota_{XFt} \quad \text{and} \quad \lambda_{XEt} = \theta_{Et} + \iota_{XEt},$$

so that when production of good $Y \in \{E, F\}$ takes place: $\lambda_{XYt} = \theta_{Yt}$. Defining $\tilde{\varphi}_{Xzt} \equiv \frac{\tilde{\varphi}_{Xzt}}{\lambda_t}$ and $\tilde{p}_{Xzt} \equiv \frac{\lambda_{Xzt}}{\lambda_t}$, which can be interpreted as the price of intermediate $x_{Xzt}$ and of input $Y_{Xz}$, first order condition with respect to $x_{Xzt}$ gives:

$$\tilde{\varphi}_{Xzt} = \gamma \hat{p}_{Xzt} A_{Xzt} x_{Xzt}^{-\gamma} (K_{Xzt}^{\alpha} L_{Xzt}^{1-\alpha})^{1-\gamma},$$

which is the same as (B.2). Combining the first order conditions with respect to $K_{Xzt}$ and $L_{Xzt}$ further gives

$$\tilde{\varphi}_{Xzt} = \frac{\psi \hat{\varphi}_{Xzt} A_{Xzt}^{\frac{1}{\alpha}}}{\alpha^{\gamma} (1-\alpha)^{1-\gamma}} \tilde{w}_{Xzt},$$

where $\tilde{\varphi}_{Xzt} \equiv \frac{\varphi_{Xzt}}{\lambda_t}$ and $\tilde{w}_{Xzt} \equiv \frac{w_{Xzt}}{\lambda_t}$ are the prices of capital and labor in country $X$. This last equation is identical to (B.3) when the optimal subsidy $\hat{q} = 1 - \gamma$ is used. Recovering the equations equivalent to (B.5) is direct. First order conditions with respect to $K_{Xzt}$ and $L_{Xzt}$
allow to recover the equations equivalent to (B.1). Now taking the first order condition with respect to \(Y_X dt\) and \(Y_X ct\), one gets (when \(Y_{XE t} \neq 0\)):

\[
\begin{align*}
\hat{p}_{Et} Y_{X dt}^{\frac{1}{\varepsilon}} & \left( Y_{X dt}^{\frac{1}{\varepsilon - 1}} + Y_{X dt}^{\frac{1}{\varepsilon - 1}} \right)^{\frac{\varepsilon - 1}{\varepsilon - 1}} = \hat{p}_{X dt} + \xi \omega_t \lambda_t, \\
\hat{p}_{Et} Y_{X ct}^{\frac{1}{\varepsilon}} & \left( Y_{X ct}^{\frac{1}{\varepsilon - 1}} + Y_{X ct}^{\frac{1}{\varepsilon - 1}} \right)^{\frac{\varepsilon - 1}{\varepsilon - 1}} = \hat{p}_{X ct}
\end{align*}
\]

this is equivalent to (B.7) with a tax

\[
\tau_t = \xi \frac{\omega_t}{\lambda_X dt} = \xi \frac{(1 + \rho)^t \omega_t}{\hat{p}_{X dt} \frac{\partial u}{\partial C} (C_{Wi}, S_t)}.
\]

Therefore:

\[
\lambda_{XE} = \frac{\psi \eta_{X KL t}^{1 - \alpha}}{(A_{X z}^{\varepsilon - 1} + (1 + \tau_X)^{-1} A_{X d}^{\varepsilon - 1})^{\frac{\varepsilon - 1}{\varepsilon - 1}} (1 - \gamma)^{1 - \gamma} \gamma^\alpha (1 - \alpha)^{1 - \alpha}},
\]

so that, as in equilibrium, country \(X\) specializes in good \(F\) if

\[
\hat{p}_{XE} < \frac{\psi \eta_{X KL t}^{1 - \alpha}}{(A_{X z}^{\varepsilon - 1} + (1 + \tau_X)^{-1} A_{X d}^{\varepsilon - 1})^{\frac{\varepsilon - 1}{\varepsilon - 1}} (1 - \gamma)^{1 - \gamma} \gamma^\alpha (1 - \alpha)^{1 - \alpha}}.
\]

The analysis of sector \(F\) is identical except that there is no tax there.

Because of the environmental dynamic equation for \(S_t\), the quality of the environment will never reach back \(\overline{S}\) in finite time, so it will remain below this bound. Taking the first order condition with respect to \(S_t\) (and taking into account that if \(S_t = 0\), one gets \(S_{t+1} = S_t = 0\)) gives:

\[
\omega_t = \frac{1}{(1 + \rho)^t} \frac{\partial u}{\partial S} (C_{Ns} + C_{Ss}, S_s) + (1 + \Delta) \omega_{t+1},
\]

which achieves to describe the optimal tax.

I now turn to the optimal solution for the innovation part. First as in the equilibrium case, only the average level of technologies (defined in (14)) matter, since the law of motion can be written as

\[
A_{X z it}^{\frac{1}{\varepsilon - 1}} = A_{X z it (t-1)}^{\frac{1}{\varepsilon - 1}} + \kappa(s_{X z it}) A_{X z it (t-1)}^{\frac{1}{\varepsilon - 1}}, \text{ for } z \in \{c, d, F\},
\]

the solution is also symmetric: \(s_{X z it} = s_{X z t}\) for \(z \in \{c, d, F\}\).
Now taking the first order condition with respect to \( A_{Xzt} \), gives:

\[
\begin{align*}
\mu_{Xzt} &= \lambda_{Xft} x_{Xzt}^\gamma \left( K_{Xzt}^{n \beta} f_{-Xzt}^{1 - n \beta} \right)^{1 - \gamma} \\
&+ \mu_{NZ(t+1)} \left( 1 + \bar{\kappa} (s_{Xzt}) \left( \frac{A_{Xzt}}{A_{Xzt}} \right)^{1 - \gamma} \right) - \bar{\kappa} (s_{Xzt}) \left( \frac{A_{Xzt}}{A_{Xzt}} \right)^{1 - \gamma} \left( 1 + \bar{\kappa} (s_{Xzt}) \left( \frac{A_{Xzt}}{A_{Xzt}} \right)^{1 - \gamma} \right)^{-\gamma} \\
&+ \int_0^1 \bar{\kappa} (s_{Xzt}) \frac{A_{Xzt}^{1-\gamma}}{A_{Xzt}^{1-\gamma}} \left( 1 + \bar{\kappa} (s_{Xzt}) \left( \frac{A_{Xzt}}{A_{Xzt}} \right)^{1 - \gamma} \right)^{-\gamma} A_{Xzt} \mu_{Xzt(t+1)} dj,
\end{align*}
\]

(with \( \alpha \beta = \alpha \) if \( z \in \{c, d\} \) and \( \alpha \beta = \beta \) if \( z = F \)), multiplying both sides by \( A_{Xzt}^{\gamma} \), one gets:

\[
\mu_{Xzt} A_{Xzt}^{\gamma} = \lambda_{Xzt} A_{Xzt}^{\gamma} A_{Xzt}^{\gamma} \left( K_{Xzt}^{n \beta} f_{-Xzt}^{1 - n \beta} \right)^{1 - \gamma} + \mu_{NZ(t+1)} A_{Xzt}^{1-\gamma} \bar{\kappa} (s_{Xzt}) \int A_{Xzt}^{1-\gamma} A_{Xzt(t+1)} dj,
\]

since the equivalent of (B.4) also holds for sector \( F \), \( \lambda_{Xzt} A_{Xzt}^{\gamma} A_{Xzt}^{\gamma} \left( K_{Xzt}^{n \beta} f_{-Xzt}^{1 - n \beta} \right)^{1 - \gamma} \) is a constant across varieties \( i \). Therefore, \( \mu_{Xzt} A_{Xzt}^{\gamma} \) is constant across varieties and one can define:

\[
\mu_{Xzt} = \left( \frac{A_{Xzt}}{A_{Xzt}} \right)^{\gamma} \mu_{Xzt(t+1)},
\]

which represents the shadow value of one unit of average productivity in sector \( z \), in country \( X \) at time \( t \). I can then show that \( \mu_{Xzt} \) follows the law of motion:

\[
\mu_{Xzt} A_{Xzt} = \lambda_{Xzt} Y_{Xzt} + \mu_{Xzt(t+1)} A_{Xzt+1}. \tag{B.41}
\]

Now taking the first order condition with respect to \( s_{Xzt} \) one gets:

\[
\nu_{Xt} = \mu_{Xzt} (1 - \gamma) \bar{\kappa} (s_{Xzt}) \left( \frac{A_{Xzt(t-1)}}{A_{Xzt(t-1)}} \right)^{\gamma} \left( 1 + \bar{\kappa} (s_{Xzt}) \left( \frac{A_{Xzt(t-1)}}{A_{Xzt(t-1)}} \right)^{\gamma} \right)^{-\gamma} A_{Xzt(t-1)},
\]

which can then be rewritten as:

\[
\nu_{Xt} = \left( 1 - \gamma \right) \bar{\kappa} (s_{Xzt}) \\
\frac{1}{1 + \bar{\kappa} (s_{Xzt})} \mu_{Xzt} A_{Xzt}.
\]

Or defining \( \tilde{\nu}_{Xt} = \nu_{Xt} / \lambda_{Xt} \), the wage of scientists in terms of utility units, I can rewrite the last equality as

\[
\tilde{\nu}_{Xt} = \left( 1 - \gamma \right) \bar{\kappa} (s_{Xzt}) \\
\frac{1}{1 + \bar{\kappa} (s_{Xzt})} \sum_{s=t}^{\infty} \lambda_s \tilde{p}_{zs} Y_{zs}. \tag{B.42}
\]

Using (B.6), (B.7), (B.8), (B.9) gives:

\[
\frac{p_{Xet} Y_{Xet}}{p_{Xet} Y_{Xet}} = \frac{A_{Xet}^{\varepsilon - 1}}{A_{Xet}^{\varepsilon - 1} + \left( (1 + \tau_{Xt})^{-1} A_{Xdt} \right)^{\varepsilon - 1}}, \quad \frac{p_{Xdt} Y_{Xdt}}{p_{Xet} Y_{Xet}} = \frac{(1 + \tau_{Xt})^{-\varepsilon} A_{Xdt}^{\varepsilon - 1}}{A_{Xet}^{\varepsilon - 1} + \left( (1 + \tau_{Xt})^{-1} A_{Xdt} \right)^{\varepsilon - 1}}.
\]
Combining this last two equations with (B.42) and (B.41), I get (24).

Solving for the maximization of (2) can be done in a very similar way. One gets:

\[
\lambda_t = \frac{\nu (S_t)^{1-\eta}}{(1 + \rho)^t} \Psi C_{Nt}^{-\eta} = \frac{\nu (S_t)^{1-\eta}}{(1 + \rho)^t} (1 - \Psi) C_{St}^{-\eta} \\
= \frac{1}{(1 + \rho)^t} \left( \Psi \frac{1}{2} + (1 - \Psi) \frac{1}{2} \right)^{\eta} \frac{\partial u}{\partial C} (C_{Wt}, S_t),
\]

and all results carry through provided that one replaces \( u \) by \( \left( \Psi \frac{1}{2} + (1 - \Psi) \frac{1}{2} \right)^{\eta} u \) (this does not affect the value of the optimal tax or the optimal allocation of scientists).

**B.6 Proof of Proposition 4**

This proof has two steps, first I specify the equilibrium constraints for the South, second I derive the social optimum for the case of the maximization of (1) - the maximization of (2) is treated in Appendix D.6.

**B.6.1 Step 1: Laissez-faire constraints in the South**

First I recall the explicit equations for the constraints (28) and (29). \( Y_{SEt} \) and \( Y_{SFt} \) are given by (B.33) and (B.34) if

\[
\left( pt \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta}} \leq \left( \frac{\beta(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \right)^{\frac{1}{\alpha - \beta}} \frac{L_S}{K_S}, \quad Y_{SEt} = 0 \quad \text{and} \quad Y_{SFt} = \zeta A_{SFt} K_S^{\alpha} L_S^{1-\beta}
\]

if \( \left( pt \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta}} \geq \left( \frac{\beta(1-\beta)^{(1-\beta)}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \right)^{\frac{1}{\alpha - \beta}} \frac{L_S}{K_S} \). This overall delivers the constraint (28) with the function \( y_{SE} \) increasing in \( pt \) (weakly), and \( A_{SEt} \) and decreasing in \( A_{SFt} \) (weakly), and the function \( y_{SF} \) decreasing in \( pt \) (weakly) and \( A_{SEt} \) (weakly) but increasing in \( A_{SFt} \). \( y_{SE} \) and \( y_{SF} \) are only piecewise smooth (at the corner of full specialization, the functions are not differentiable). Note that since the South economy maximizes GDP:

\[
pt \frac{\partial y_{SE}}{\partial p} + \frac{\partial y_{SF}}{\partial p} = 0. \quad (B.43)
\]

When \( \left( pt \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta}} > \left( \frac{\beta(1-\beta)^{(1-\beta)}}{\alpha^\alpha(1-\alpha)^{1-\beta}} \right)^{\frac{1}{\alpha - \beta}} \frac{L_S}{K_S} \), the allocation of scientists is trivially given by \( s_{dt} = 1 \) and when \( \left( pt \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta}} \leq \left( \frac{\beta(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \right)^{\frac{1}{\alpha - \beta}} \frac{L_S}{K_S} \), by \( s_{dt} = 0 \). When \( \left( pt \frac{A_{SEt}}{A_{SFt}} \right)^{\frac{1}{\alpha - \beta}} \in \)
\[
\left( \frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{\alpha-\beta}} \right) \frac{1}{1-\beta} L_S + \left( \frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\alpha(1-\alpha)^{\alpha-\beta}} \right) \frac{1}{1-\beta} L_S K_S ,
\]
the allocation of scientists is given by (B.35), that is:

\[
\frac{1 + \tilde{\kappa} (1 - s_{SEl})}{\tilde{\kappa}' (1 - s_{SEl})} \frac{1}{1 - \beta} \left( \frac{\tilde{s}_{Sdt}}{s_{SEl}} \left( s_{SEl} \frac{A_{Sdt}}{A_{Scd}} \right) \right) \left( \frac{A_{Sdt}}{A_{SEl}} \right) \frac{1}{1-\beta} L_S K_S - \beta \left( \frac{A_{Sdt}}{A_{SEl}} \right) \frac{1}{1-\beta} L_S \frac{p_t^{1-\beta} K_S}{(1 - \beta) (\alpha^\alpha(1-\alpha)^{(1-\alpha)}) \frac{1}{1-\beta} s_{SEl}^{\frac{1}{1-\beta}}}.
\]

where \( \tilde{s}_{Sdt} \) is itself defined through:

\[
\frac{\tilde{\kappa}' \left( s_{SEl} - \tilde{s}_{Sdt} \left( s_{SEl} \frac{A_{Scd}}{A_{Sdt}} \right) \right)}{1 + \tilde{\kappa} \left( s_{SEl} - \tilde{s}_{Sdt} \left( s_{SEl} \frac{A_{Scd}}{A_{Sdt}} \right) \right)} = \frac{\tilde{\kappa}' \left( \tilde{s}_{Sdt} \left( s_{SEl} \frac{A_{Sdt}}{A_{Scd}} \right) \right)}{1 + \tilde{\kappa} \left( \tilde{s}_{Sdt} \left( s_{SEl} \frac{A_{Sdt}}{A_{Scd}} \right) \right)} \left( \frac{A_{Sdt}}{A_{Scd}} \right) \frac{1}{1-\beta} L_S K_S - \beta \left( \frac{A_{Sdt}}{A_{Scd}} \right) \frac{1}{1-\beta} L_S \frac{p_t^{1-\beta} K_S}{(1 - \beta) (\alpha^\alpha(1-\alpha)^{(1-\alpha)}) \frac{1}{1-\beta} s_{SEl}^{\frac{1}{1-\beta}}}.
\]

This corresponds to the constraint (29). Note that I defined \( \tilde{s}_{Sdt} \) as a function of \( s_{SEl} \) and \( \frac{A_{Sdt}}{A_{Scd}} \) not of \( s_{SEl} \) and \( \frac{A_{Sdt}}{A_{Scd}} \) as I did in Appendices B.2 and D.1 (or in equations (30)), this allows to express \( s_{SEl} \) as a function of the current productivity levels, which simplifies considerably the expression of the optimal tariff. I use the tilde to ensure that the difference between the two functions is explicit. Yet, (B.44) also implicitly define \( s_{SEl} \) as a unique function of \( p_t \) and the previous period technology levels, this function (weakly) increases in \( p_t \), and (weakly) decrease in \( A_{SEl} \). It is possible to show that the function \( s_{SEl} \) is continuously differentiable. Moreover, (B.44) can be rewritten as:

\[
\left( p_t \frac{\partial y_{SE}^l}{\partial A_{SEl}} + \frac{\partial y_{SF}}{\partial A_{SEl}} \right) \frac{\tilde{\kappa}' (s_{SEl})}{(1 + \tilde{\kappa} (s_{SEl}))} A_{SEl} = \frac{\tilde{\kappa}' (\tilde{s}_{Sdt} (s_{SEl} \frac{A_{Sdt}}{A_{Scd}}))}{(1 + \tilde{\kappa} (\tilde{s}_{Sdt} (s_{SEl} \frac{A_{Sdt}}{A_{Scd}})))} \left( \frac{A_{Sdt}}{A_{SEl}} \right) \frac{1}{1-\beta} L_S K_S - \beta \left( \frac{A_{Sdt}}{A_{SEl}} \right) \frac{1}{1-\beta} L_S \frac{p_t^{1-\beta} K_S}{(1 - \beta) (\alpha^\alpha(1-\alpha)^{(1-\alpha)}) \frac{1}{1-\beta} s_{SEl}^{\frac{1}{1-\beta}}},
\]

which stipulates that for given prices, innovation in the South maximizes current GDP \( p_t Y_{SEl} + Y_{SFl} \).

**B.6.2 Step 2: Deriving the social optimum**

To simplify a bit the exposition, I combine (16) and the emission equation for the South \( Y_{Sdt} = (A_{Sdt}/A_{SEl})^\varepsilon Y_{SEl} \) into:

\[
S_t = \max \left( \min (1 + \Delta S_{t-1} - \xi \left( Y_{Nd} + \left( \frac{A_{Sdt}}{A_{SEl}} \right)^\varepsilon Y_{SEl} \right), \bar{S}) , 0 \right),
\]

(B.47)
I then use the following notations for the Lagrange parameters (the corresponding constraints are in parentheses): \( \lambda_{Xt} \) for (3) - both in North and South -; for the North only: \( \lambda_{NFt} \) (4), \( \lambda_{NET} \) (7), \( \lambda_{NZt} \) (8), \( \varphi_{NZt} \) (9), \( \eta_{NXt} \) (11) for capital, \( \eta_{Nlt} \) (11) for labor, \( \mu_{Nzt} \) (13), \( \omega_{t} \) (B.47), \( \theta_{NET}, \theta_{NFT}, \theta_{SET} \) and \( \theta_{SFt} \) -with obvious subscripts- for the equations in (25), \( \chi_{t} \) (26), \( \kappa_{t} \) (27), \( \lambda_{SET} \) and \( \lambda_{SFt} \) (28), \( \phi_{t} \) (29), \( \mu_{SFt}, \mu_{Sdt} \) and \( \mu_{Set} \) (30), in addition, the social planner faces the constraints: \( 0 \leq Y_{NET} \), \( 0 \leq Y_{NFt} \), with Lagrange parameters: \( \iota_{NET}, \iota_{NFT} \).

As specified above, the functions \( y_{SE} \) and \( y_{SF} \) are not everywhere differentiable, in the following I therefore use generalized Karush Kuhn Tucker conditions: at a point of non differentiability the notation \( \frac{\partial y_{SE}}{\partial p_{t}}, \frac{\partial y_{SE}}{\partial A_{SEt}}, \frac{\partial y_{SE}}{\partial A_{SFt}} \) refers to elements of a vector \( \left( \frac{\partial y_{SE}}{\partial p_{t}}, \frac{\partial y_{SE}}{\partial A_{SEt}}, \frac{\partial y_{SE}}{\partial A_{SFt}} \right) \) belonging to the Clarke generalized gradient of \( y_{SE} \). Therefore it is still the case at these points that \( \frac{\partial y_{SE}}{\partial p_{t}} \geq 0, \frac{\partial y_{SE}}{\partial A_{SEt}} > 0, \frac{\partial y_{SE}}{\partial A_{SFt}} \leq 0 \). First order conditions with respect to all the “North” variables, and \( S_{t} \) allow us to recover exactly the same equations as in the first best for the North part of the economy (up to replacing \( \theta_{Et} \) and \( \theta_{Ft} \) by \( \theta_{NET} \) and and \( \theta_{NFT} \)). This shows that the economy in the North is similar to the first best case (with a carbon tax, subsidy to the use of intermediates, and research taxes/subsidies that can be used to decentralize the equilibrium). I am now going to derive that the social planner creates a wedge between relative prices in the North and in the South. To assess the generality of the results, I do not immediately make use of the functional form. Taking first order condition with respect to \( C_{St} \) gives:

\[
\frac{1}{(1 + \rho)^{7}} \frac{\partial u}{\partial C} (C_{Nt} + C_{St}, S_{t}) = \frac{\nu (S_{t})^{1-\eta}}{(1 + \rho)^{\eta}} (C_{Nt} + C_{St})^{-\eta} = \lambda_{St} = \lambda_{Nt} \equiv \lambda_{t}.
\]

Taking the first order condition with respect to \( C_{SFt} \), I get:

\[
\theta_{SFt} + \kappa_{t} \frac{\partial}{\partial C_{SFt}} \frac{\partial C_{S}}{\partial C_{SFt}} = \lambda_{t} \frac{\partial C_{S}}{\partial C_{SFt}} = \lambda_{t} (1 - \nu) C_{SFt}^{-1} \left( \nu C_{SFt}^{\sigma-1} (1 - \nu) C_{SFt}^{\sigma-1} \right)^{\frac{1}{\sigma-1}}, \tag{B.48}
\]

and with respect to \( C_{SET} \):

\[
\theta_{SET} + \kappa_{t} \frac{\partial}{\partial C_{SET}} \frac{\partial C_{S}}{\partial C_{SET}} = \lambda_{t} \frac{\partial C_{S}}{\partial C_{SET}} = \lambda_{t} \nu C_{SET}^{-1} \left( \nu C_{SET}^{\sigma-1} (1 - \nu) C_{SFt}^{\sigma-1} \right)^{\frac{1}{\sigma-1}}. \tag{B.49}
\]

Therefore combining the two:

\[
\frac{\theta_{SFt} + \kappa_{t} \frac{\partial}{\partial C_{SFt}} \frac{\partial C_{S}}{\partial C_{SFt}}}{\theta_{SET} + \kappa_{t} \frac{\partial}{\partial C_{SET}} \frac{\partial C_{S}}{\partial C_{SET}}} = \frac{\frac{\partial C_{S}}{\partial C_{SFt}}}{\frac{\partial C_{S}}{\partial C_{SET}}} = \frac{1}{p_{t}},
\]

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so that:

\[ \kappa_t = \frac{\theta_{SEt} - \theta_{SFt}}{p_t \frac{\partial C_S}{\partial C_{SEt}} - \frac{\partial C_S}{\partial C_{SFt}}} \quad \text{(B.50)} \]

First order conditions with respect to \( Y_{SFt} \) and \( Y_{SEt} \) give:

\[ \lambda_{SEt} = \theta_{SEt} - \omega_t \xi \left( \frac{A_{St}}{A_{SEt}} \right)^e \quad \text{and} \quad \lambda_{SFt} = \theta_{SFt} \quad \text{(B.51)} \]

First order conditions with respect to \( M_{Ft} \) and \( M_{Et} \) give:

\[ p_t \chi_t = \theta_{NEt} - \theta_{SEt} \quad \text{and} \quad \chi_t = \theta_{NFt} - \theta_{SFt} \quad \text{(B.52)} \]

so that

\[ \frac{\theta_{SEt}}{p_t} - \theta_{SFt} = \frac{\theta_{NEt}}{p_t} - \theta_{NFt} \quad \text{(B.53)} \]

Finally the first order condition with respect to \( p_t \) gives:

\[ M_{Et} \chi_t = \lambda_{SEt} \frac{\partial y_{SE}}{\partial p_t} + \lambda_{SFt} \frac{\partial y_{SF}}{\partial p_t} + \kappa_t + \phi_t \frac{\partial s_{SEt}}{\partial p_t} \quad \text{(B.54)} \]

Let us denote by \((1 + b_t)\) an ad valorem tariff (export subsidy) on good \( E \), using (B.36) and (B.37) in the North. One gets

\[ \frac{\partial C_N}{\partial C_{NEt}} = \frac{1}{1 - \nu} \frac{1}{C_{NFt}^{\frac{1}{\nu}}} \quad \text{and} \quad \frac{\hat{p}_{NEt}}{\hat{p}_{Ft}} = p_t (1 + b_t) \quad \text{(B.55)} \]

Now plugging (B.51), (B.50), (B.52) and (B.53) in (B.54), I get:

\[ M_{Et} \frac{(\theta_{NEt} - \theta_{SEt})}{p_t} \quad \text{(B.56)} \]

Further, using (B.43), (B.36) and (B.37) for the North - replacing \( \theta_E \) by \( \theta_{NEt}^{-} \), (B.49), (B.48), (B.50) and (B.55):

\[ M_{Et} \left( \frac{\partial C_N}{\partial C_{NE}} - \frac{\partial C_S}{\partial C_{SEt}} \right) \quad \text{(B.57)} \]

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Let us denote by \((1 + b_t)\) an ad valorem tariff (export subsidy) on good \( E \), using (B.36) and (B.37) in the North. One gets

\[ \frac{\partial C_N}{\partial C_{NEt}} = \frac{1}{1 - \nu} \frac{1}{C_{NFt}^{\frac{1}{\nu}}} \quad \text{and} \quad \frac{\hat{p}_{NEt}}{\hat{p}_{Ft}} = p_t (1 + b_t) \quad \text{(B.55)} \]
This expression shows that the social planner imposes a wedge between relative prices in the North and in the South. With homothetic preferences, this wedge is entirely created by two terms: $-\omega t \xi \left( \frac{A_{\text{SEt}}}{A_{\text{Sdt}}} \right)^{\varepsilon} \frac{\partial y_{\text{SEt}}}{\partial p_t}$ (which is going to be the environmental motive) and $\phi_t \frac{\partial s_{\text{SEt}}}{\partial p_t}$ (which is going to be the innovation motive), indeed if $\omega t = 0$ and $\phi_t = 0$, the solution to that equation would be $b_t = 0$ as long as preferences are homothetic, so that $\frac{\partial C_{\text{N}}}{\partial C_{\text{SEt}}} = \frac{\partial C_{\text{S}}}{\partial C_{\text{SEt}}}$ at equal relative price. There is no terms of trade motives for the tariff since the social planner cares equally about consumption in the North and consumption in the South. From here, I use the specific functional forms to get a more explicit formula for the tariff (see Appendix D.5.1):

\[
\begin{align*}
\left( \frac{\nu^\sigma + (1 - \nu)^\sigma (p_t (1 + b_t))^{\sigma - 1}}{1 + b_t} \right) &= \frac{b_t p_t}{p_t} \frac{\partial y_{\text{SEt}}}{\partial p_t} \\
&= \frac{p_t}{\lambda t} \omega t \xi \left( \frac{A_{\text{Sdt}}}{A_{\text{SEt}}} \right)^{\varepsilon} \left( \frac{\partial y_{\text{SEt}}}{\partial p_t} + \frac{\partial s_{\text{SEt}}}{\partial p_t} \frac{1}{D_t} \left( A_{\text{SEt}} \frac{\partial y_{\text{SEt}}}{\partial A_{\text{SEt}}} A_{\text{Sdt}}^{-\frac{1}{\varepsilon}} \frac{\partial (s_{\text{Sdt}})}{\partial (s_{\text{Sdt}})} \frac{\partial s_{\text{Sdt}}}{\partial s_{\text{Sdt}}} \right) A_{\text{Sdt}} \right)
\end{align*}
\]

\[
D_t \text{ given by}
\]

\[
D_t \equiv (1 - \gamma)^{-1} + \frac{\partial s_{\text{SEt}}}{\partial A_{\text{SEt}}} \frac{\partial A_{\text{Sdt}}}{(1 + \kappa)(s_{\text{Sdt}})} A_{\text{Sdt}} - \frac{\partial s_{\text{SEt}}}{\partial s_{\text{SEt}}} \frac{\partial A_{\text{Sdt}}}{(1 + \kappa) s_{\text{Sdt}}} A_{\text{Sdt}} - \frac{\partial s_{\text{SEt}}}{\partial s_{\text{SEt}}} \frac{\partial A_{\text{Sdt}}}{(1 + \kappa) s_{\text{Sdt}}} A_{\text{Sdt}}
\]

is strictly positive. On the LHS, the first term has the sign of $b_t$ since $p_t \frac{\partial y_{\text{SEt}}}{\partial p_t} \geq 0$, the second term has the sign of $b_t$ except possibly for $b_t$ sufficiently small (close to $-1$) when $\sigma < 1,$
and the third term has the sign of $b_t$ except possibly for $b_t$ sufficiently large when $\sigma < 1$.
The sum of the two terms into brackets, however, always has the sign of $b_t$ and when $b_t$ gets sufficiently small, the South exports good $E$ (so that $\nu^\sigma Y_{SFt} < (1 - \nu)^\sigma p_{SFt} Y_{SET}$) and imports good $E$ when $b_t$ gets sufficiently large. Therefore, for all purposes, the sum of the second and third term will have the sign of $b_t$. The forth term also has the sign of $b_t$. Therefore $b_t$ will have the sign of the RHS. On the RHS, the first term in bracket is weakly positive since
\[
\frac{\partial y_{SE}}{\partial p_t} \geq 0, \quad \frac{\partial s_{SET}}{\partial p_t} \geq 0, \quad \frac{\partial y_{SE}}{\partial A_{SET}} \geq 0, \quad \frac{\partial y_{SE}}{\partial A_{SFt}} \leq 0, \quad \left( \frac{\tilde{r}'(s_{SDt})}{1 + \kappa(s_{SDt})} \right) \frac{\partial s_{SDt}}{\partial s_{SET}} - \left( \frac{\tilde{r}'(s_{SDt})}{1 + \kappa(s_{SDt})} \right) \frac{\partial s_{SDt}}{\partial s_{SET}} \geq 0
\]
and the denominator is positive. This first term therefore pushes toward a positive tariff (this is the environmental term) - with $\omega_t$ and $\tau_t$ related by (B.38), note that the first part of this term represents the effect of the tariff at given technology, while the second term represents the environmental benefits from reducing innovation in sector $E$. The second term has the sign of
\[
(1 + \tilde{r}(s_{SFt+1}))^{1-\gamma} \mu_{SFt+1} \frac{\tilde{r}'(s_{SFt})}{(1 + \tilde{r}(s_{SFt}))} A_{SFt} - \left( (1 + \tilde{r}(s_{SDt+1}))^{1-\gamma} \mu_{SDt+1} \frac{\tilde{r}'(s_{SDt})}{(1 + \tilde{r}(s_{SDt}))} A_{SDt} \right) \frac{\partial s_{SDt}}{\partial s_{SET}} + \left( (1 + \tilde{r}(s_{SEt+1}))^{1-\gamma} \mu_{SEt+1} \frac{\tilde{r}'(s_{SEt})}{(1 + \tilde{r}(s_{SEt}))} A_{SEt} \right) \frac{\partial s_{SEt}}{\partial s_{SET}}
\]
which is the difference between the social value of allocating one scientist in sector $F$ instead of sector $E$, for all the future periods (excluding the current one). The last term, reflects how the allocation between clean and dirty innovation in the future period is affected by the current number of scientists allocated to sector $E$, and has the sign of $\mu_{SE(t+1)} A_{SE(t+1)} \frac{\tilde{r}'(s_{SEt+1})}{1 + \tilde{r}(s_{SEt+1})} - \frac{\mu_{SD(t+1)} A_{SD(t+1)} \tilde{r}'(s_{SDt+1})}{1 + \tilde{r}(s_{SDt+1})}$, this could be positive or negative: on one hand, since the South is not going to switch to clean technologies, developing clean technologies in the South has little value for consumption’s sake but on the other hand, dirty technologies pollute. However this term vanishes as $A_{SE(t-1)} / A_{SE(t-1)}$ goes to 0.

Finally note that when the South is fully specialized- but not at the threshold of specialization $(\frac{p_t A_{SET}}{A_{SET}}) \frac{1}{\alpha - \beta} \not\in \left( \frac{\beta^\alpha (1-\beta)^{(1-\alpha)/\alpha - \beta}}{\alpha (1-\alpha)^{1-\beta}} \frac{1}{\alpha - \beta} L_S K_S, \frac{\beta^\alpha (1-\beta)^{(1-\alpha)/\alpha - \beta}}{\alpha (1-\alpha)^{1-\beta}} \frac{1}{\alpha - \beta} L_S K_S \right)$, $\frac{\partial y_{SE}}{\partial p_t} = 0$ and $\frac{\partial s_{SEt}}{\partial p_t} = 0$, so that the optimal tariff turns out to be $b_t = 0.$