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Online Fresh Grocery Retail: A La Carte or Buffet?

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This paper identifies the best revenue models for firms aspiring to capture the untapped trillion-dollar opportunity in online retail of fresh groceries. We compare the financial and environmental performance of two revenue models: the per-order model, where customers pay for each delivery; and subscription, where customers pay a subscription fee and receive free deliveries. We build a stylized model that incorporates customers with ongoing uncertain grocery needs who choose between shopping offline or online and an online retailer that makes deliveries through a proprietary distribution network. In contrast with practitioners' widely-held views, we find that subscription incentivizes a customer order pattern that reduces total grocery sales on account of lower food waste. Subscription also has higher delivery costs, but these disadvantages are countered by delivery scale economies, lower grocery acquisition costs and potentially higher adoption of the online channel. From an environmental perspective, the per-order model has higher food waste related emissions, while subscription leads to higher travel. Ceteris paribus, the per-order model is both financially and environmentally preferable for retailers with higher margin and higher consumption product assortments, sold in sparsely populated markets spread over large elongated areas with high delivery costs. Based on geographic and demographic data, we find that for typical products subscription yields higher profits in small, dense, circular markets (Paris, Beijing) whereas per-order performs better in elongated or sparse and large markets (Manhattan, Los Angeles). Subscription is always environmentally superior because lower emissions from food waste dominate higher travel-related emissions.

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Abstract. This paper identifies the best revenue models for firms aspiring to capture the untapped trillion-dollar opportunity in online retail of fresh groceries. We compare the financial and environmental performance of two revenue models: the per-order model, where customers pay for each delivery; and subscription, where customers pay a subscription fee and receive free deliveries. We build a stylized model that incorporates customers with ongoing uncertain grocery needs who choose between shopping offline or online and an online retailer that makes deliveries through a proprietary distribution network. In contrast with practitioners' widely-held views, we find that subscription incentivizes a customer order pattern that *reduces* total grocery sales on account of lower food waste. Subscription also has higher delivery costs, but these disadvantages are countered by delivery scale economies, lower grocery acquisition costs and potentially higher adoption of the online channel. From an environmental perspective, the per-order model has higher food waste related emissions, while subscription leads to higher travel. Ceteris paribus, the per-order model is both financially and environmentally preferable for retailers with *higher margin* and *higher consumption* product assortments, sold in *sparsely populated* markets spread over *large elongated areas* with *high delivery costs*. Based on geographic and demographic data, we find that for typical products subscription yields higher profits in small, dense, circular markets (Paris, Beijing) whereas per-order performs better in elongated or sparse and large markets (Manhattan, Los Angeles). Subscription is always environmentally superior because lower emissions from food waste dominate higher travel-related emissions.

1. Introduction

More than a decade has passed since the spectacular failure of Webvan, the heavily funded online grocery retailer. Today, retail-savvy tech companies, ambitious startups, and deep-pocketed investors are again betting on the online grocery opportunity.[1] Just within the last year, Amazon—the world's most successful online retailer—has invested heavily in order to expand its fresh grocery offering Amazon Fresh to San Francisco and Los Angeles,[2] Peapod has invested $65 Million in new

[1]"The Next Big Thing You Missed: Online Grocery Shopping Is Back, and This Time It'll Work", *Wired Magazine*, 4 February 2014, http://bit.ly/wiredOnlineBack
"Next Up For Disruption: The Grocery Business", *Fortune,* 4 April 2014, http://bit.ly/GroceryDisruption
[2]"Amazon Fresh Launches in San Francisco with $299/year 'Prime Fresh' Membership", *Geekwire*, December 12, 2013, http://bit.ly/FreshLaunch

warehouses while expecting to double its current revenues,[3] Fresh Direct has expanded its services, Google is developing its rival Google Shopping Express, and the startup Instacart expanded from one city to 16 cities in just 18 months while raising more than $50 million (US) from investors at valuations rumored to exceed $400 million.[4]

Online grocery retailing has several attractive features: the potential market is estimated to be over $550 billion in the United States alone, groceries are the second biggest consumer expense (after housing), demand is more stable and consumers are more loyal than in most other retail categories, and online shopping has the potential to significantly improve on the offline experience.[5] Further, unlike many other recent innovations from the tech world, the transfer of grocery buying and delivery tasks from consumers to firms can create many new employment opportunities.

How customer's buy groceries is also intimately tied to two major contributors to greenhouse gas emissions and climate change. First, driving to buy groceries is the second biggest reason for use of passenger vehicles (after driving to work); online grocery shopping has the potential to reduce some of this driving by replacing individual trips to grocery stores with a more efficient delivery truck route. Second, the mode of buying fresh groceries also drives food waste by consumers. About 30 to 50% of food production is lost/wasted.[6] In contrast with a popular myth, waste of fresh groceries at the consumer end is more important than those in the supply chain.[7] In the developed world (North America, Oceania, Europe, and industrialized Asia) retail and distribution stage waste accounts for between 7%-11% of the total food loss, while waste at the consumption stage accounts for 46-61% of the total food loss (Lipinski et al., 2013). American families throw out approximately 25 percent of the food and beverages they buy, two-thirds of which is due to food spoilage (Gunders, 2012). The environmental impact of this food waste is very harsh. UK analysts estimate that if food scraps were removed from landfills, the level of greenhouse gas abatement would be equivalent to removing one-fifth of *all* the cars from the roads (Gunders, 2012). By making grocery shopping more convenient,

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[3] "In New Jersey, Launching Pads for Same-Day Shipments" *New York Times,* 5 August 2014, http://bit.ly/PeapodExpands

[4] "On-Demand Grocery Startup Instacart Raises $44 Million from Andreessen Horowitz", *TechCrunch,* 16 June 2014, http://bit.ly/InstaFunds

[5] "Why Groceries Could Be Amazon's Next Big Loyalty Play", *PandoDaily,* 11 April 2014, http://bit.ly/GroceryPando

[6] "Food Wastage Footprint: Impacts on Natural Resources", *Food and Agriculture Organization,* 2013, http://bit.ly/FAO-Waste. In recognition of the crucial role of food waste, the European Parliament pronounced year 2014 as the Year against Food Waste, proposing a 50% prevention target on avoidable food waste by 2025, http://bit.ly/1u9Mvj8

[7] "Budget buster: Food waste a disgrace", 20 July 2014, http://bit.ly/1tmgK2P

online grocery retail has the potential to change customer buying patterns to reduce food waste and its environmental consequences.

Despite the financial and environmental promise of online grocery retailing, most early attempts failed. The failure in 2001 of Webvan/HomeGrocer is the subject of numerous case studies and anecdotal analyses (see, e.g., Himelstein and Khermouch (2001)). The main reasons cited for that failure are customer unfamiliarity with online shopping and payments, attempt to grow too big too fast, and the fact that they did not generate enough revenues to cover the operational costs.[8]

Since the failure of Webvan, customers have become more accustomed to online shopping and contemporary retailers are more cautious in designing their expansion strategies.Yet, almost 15 year later, there is still only a limited understanding of appropriate revenue models their operational consequences. The current generation of online grocery retailers are experimenting with different revenue models. For example, Amazon charges a $7.99 per-order delivery fee in the Seattle area, whereas a subscription model, Amazon Prime Fresh, is offered in the Los Angeles and San Francisco areas. Customers pay $299/year and have access to unlimited free deliveries.[9] It is worth noting that all three cities share similar geography and demographics. Anecdotal accounts of Amazon's strategy suggest that the experiment is motivated by the lack of clarity about the right revenue model.

Online retail of fresh grocery has some special features that make this revenue model choice more involved than the relatively straightforward trade-offs (between sales and margins) that dictate the pricing of generic products. First, an online grocery retailer sells two bundled products, the groceries themselves and the accompanying delivery service. How a firm charges the customer for the delivery affects the sales of groceries. Second, the revenue model regulates how customers batch their ongoing grocery needs into a pattern of online/offline orders. This pattern affects delivery costs and revenues. Finally, on the supply side, offering a fresh grocery delivery service requires the retailer to build its own logistics and delivery network. Hence costs depend on the scale and scope of operations—in particular, the number of deliveries, delivery mode, delivery area geography, and demographics. Therefore, a detailed operational analysis that explicitly considers customer's choice of ordering pattern and the delivery network is necessary in order to identify preferred revenue

[8] "Where Webvan Failed and How Home Delivery 2.0 Could Succeed", *TechCrunch,* 27 September 2013, http://bit.ly/WebVanTechCrunch

[9] "Amazon Fresh: Big Radish or Bad Kiwi?", *AmazonStrategies.com*, 8 May 2014, http://bit.ly/AmazonPricing

models and to avoid the mistakes that doomed the first generation of grocery startups and realize the environmental and financial potential of online grocery retail.

This study presents the first stylized model that compares the two revenue models most commonly used by online fresh grocery retailers: the per-order model, where customers pay for each delivery; and the subscription model, where customers pay once each year and subsequently receive unlimited free deliveries. On the customer side, our analysis includes a stochastic process that models their ongoing grocery needs, the evolution of a customer's perishable grocery inventory, and her choice of offline/online channel, basket size, and order frequency in response to the proposed revenue model. We develop a detailed delivery cost model for the retailer that is based on identifying the cost-minimizing delivery routing that meets all delivery promises and allows us to relate customer order streams to the driving costs, the delivery area geography, and demographics. We use this setup to identify the revenue model that leads to *higher profits* and the one that results in *lower carbon emissions.*

We find that differences in financial and environmental performance are driven by the same three substantive distinctions between the two models. First, even though the subscription model *lowers* the costs of placing an additional order, customers actually order *fewer* fresh groceries with subscriptions than when paying per order. In deciding the amount of groceries to buy in one order (the basket size), customers trade off the risk of having to place additional orders with the risk of buying more groceries than can be consumed within their shelf life. The former risk is associated with higher cost in the per-order model, leading to larger grocery sales. Even more surprising is that this extra grocery volume does not translate into extra revenue for the retailer. Because customers can shop offline, both the subscription and per-order model are limited in terms of the gain they can extract from the customer; and since additional groceries in the per-order model do not deliver customers additional value (on average, these groceries perish before use), the subscription model can make up any revenue loss due to lower grocery volume by increasing the subscription fees. So the per-order model leads to more grocery sales, but the extra grocery volume does not generate additional revenue; instead it leads only to higher grocery acquisition costs for the retailer and more emissions stemming from food waste. Thus, financially and environmentally, the per-order model is at a *food waste disadvantage* to the subscription model.

The second substantive distinction is the more frequent orders in the subscription model. The lower cost of ordering in this model drives customers to order more often, which has the effect of increasing delivery costs. That increase is sub-linear due to volume economies in the delivery process (i.e., the marginal distance traveled to deliver additional orders is decreasing in the number of orders delivered). The delivery area's specific geography and demographics moderate the extent to which these economies play out. Overall, the subscription model leads to more frequent orders and to higher delivery costs and emissions, although the difference (in comparison with the per-order model) is diminishing in the adoption rate and certain delivery area characteristics. Thus, the per-order model has a financial and environmental *order-frequency advantage* over the subscription model.

The final important distinction is the difference in adoption of online grocery retailing with the two models, *an adoption effect.* The previous two effects lead to different cost structures, and the applicable profit-maximizing prices entail different market coverage. In particular, the model with lower costs (as determined by the trade-off between the first two effects) also exhibits a higher adoption. Because delivery involves economies of scale, the respective financial and environmental advantages of the model that achieves higher adoption are in fact further enhanced by the adoption effect. In short, adoption magnifies the first two effects and so increases the preferred model's advantage.

Note that these three effects play out in the same way from both a financial and an environmental standpoint. In other words, greater food waste and higher order frequencies are each bad both financially and environmentally. Furthermore, increased adoption accentuates the financial or environmental advantage of the preferred model.

This analysis allows us to characterize the financially and environmentally preferred revenue model given the retailer's product assortment (the average margin and consumption rate), the delivery area's characteristics (size, shape, and per-mile driving costs), the delivery area's population demographics (population density, distribution of store visit costs, and comfort with ordering online), and the economics of a firm's delivery operations (per-mile driving costs and number of orders a delivery vehicle can carry). We find that, all else being equal, *online retailers should choose the per-order model over the subscription model for selling products with higher margins or higher consumption rates and in markets that are spread over a large area, have a relatively more elongated shape, necessitate higher per-mile delivery costs, are sparsely populated, or are served with faster delivery promises (that require delivery vehicles to carry only a few deliveries at a time).* The same criteria apply from an environmental perspective.

Finally, we calibrate the models with real demographic and geographic data from major cities around the world. We find that for reasonable estimates of input parameters and for a wide range of cities and product categories, the *financial* consequences of the food waste disadvantage and the order frequency advantage are comparable in magnitude and the trade-off at the heart of our analysis has practical relevance. Moreover, which model earns greater revenues depends strongly on product margins and city characteristics. For a city like Los Angeles and a retailer selling fresh product assortments with average gross margins below $\sim$25%, the subscription model is preferred; the per-order model is preferred for higher margins. The critical gross margin is much higher for denser delivery regions (e.g., Manhattan), for more circular cities (e.g., Paris), and for cities with lower per-mile delivery costs (e.g., Beijing).

As regards the environment, results from using our calibrated estimates are even more interesting. The model predicts a trade-off between the two models, yet our estimates suggest that—*for almost all realistic delivery geographies, product assortments, and customer characteristics*—the *subscription model is environmentally preferable even though it entails more driving.* Essentially, environmental costs associated with the food waste of the per-order model are much greater than the environmental costs of extra driving; thus a trade-off that is financially relevant has no practical relevance when the environment is considered.

This paper makes three contributions. First, our analysis provides important insights and prescriptions for the design of viable revenue models for the online retailing of fresh groceries, which is arguably the most lucrative and exciting open opportunity in online retail. Second, our analysis and calibrated numerical study of the grocery value chain's environmental impact show that food waste, a previously overlooked contributor to emissions, is more important than the emissions associated with transportation—the main focus of past work (e.g., Cachon, 2014). This suggests that future work on the environmental impact of grocery supply chains should include a consumer inventory model that accurately accounts for consumer food waste. Third, our detailed operational model of an online grocery retailer value chain leads to predictions that are more precise (and sometimes contradict) those derived from abstract economic models; it also provides a template for the operational analysis of business models.

2. Related Literature

Our paper is related to past work on subscription versus usage-based pricing models, inventory planning for perishables, and delivery networks. We also contribute to the active literature on the environmental impact of operational choices.

Subscription versus Per-Order Revenue Models. These models were first studied as the "theory of clubs". Buchanan (1965) advocates a subscription (membership) model, whereas Berglas's (1976) extension, which incorporates consumer usage choices, argues for the per-order (visit) model. Barro and Romer (1987) build on this approach by including congestion, and they establish that the two models are equivalent. More recent work considers the choice between subscription and per-order models in the context of cellphone access (Danaher, 2002), information goods (Sundararajan, 2004), and Netflix-like DVD rental services (Randhawa and Kumar, 2008; Cachon and Feldman, 2011). Our paper extends this literature by examining the revenue model choice in the rich new context of fresh grocery delivery that involves two bundled products. We compare the models not just from a financial but also an environmental point of view. The subsequent analysis will show that our model exhibits positive consumption externalities (scale economies) as opposed to the congestion effects found in existing work.

Inventory Management of Perishable Products. Consumers in our model make ordering decisions to ensure that they have adequate inventories of groceries. This aspect of consumer decision making builds directly on the vast operations management literature addressing firm-level inventory choice models. In contrast to our setting, most inventory management theory focuses either on single-period decision making or on nonperishable products. The literature analyzing perishable products is less extensive (for an up-to-date review see (Nahmias, 2011)).

The perishable nature of products can be captured with a discrete lifetime (known (Fries, 1975) or random (Kalpakam and Sapna, 1994)), or a continuous exponential decay (Kalpakam and Arivarignan, 1988). While exponential decay is more tractable, it turns out to be a poor approximation (Nahmias, 1975) and very few real systems are captured by this models (Nahmias, 2011). As in traditional inventory management, there can be continuous and periodic review models for perishable inventory. Continuous review models are typically easier to analyze, despite that there are only a few continuous review perishable inventory models with fixed lifetime (Weiss, 1980; Liu and Lian, 1999). For tractability, all fixed-life perishable inventory models assume Poisson demand and negligible lead times. We follow the literature and use the same well-validated assumptions. However, most studies further assume zero ordering costs and that demand can be backlogged. Both these assumptions are unreasonable in our context and we extend the literature on these two minor dimensions. In sum, our perishable inventory consumer model shares the continuous review, fixed lifetime, Poisson demand, and zero ordering time features with existing models, but in addition considers lost sales and strictly positive ordering costs. Note that our model focuses on *consumer* inventories, not on the *firm* inventories that are traditionally studied.

Delivery Networks. Our model includes the firm's optimal choice of delivery routing choice. This involves (i) an optimal division of the delivery area into sectors assigned to different delivery vehicles and (ii) devising the shortest round-trip for each vehicle. The latter task is the well-known traveling salesman problem (TSP), for which an extensive literature provides heuristics and solutions (see, e.g., Lawler et al., 1985; Bramel and Simchi-Levi, 1997). The former task is studied in Daganzo (1984a,b). More recently, (Cho et al., 2014) and (Cachon, 2014) have advanced this literature by using its results to address highly influential operational system design questions such as location of trauma centers (Cho et al., 2014) and in evaluating the environmental impact of retail store location choices (Cachon, 2014). Our work takes inspiration from these recent developments; we draw extensively on the classical results from the delivery networks and vehicle routing literature to address revenue model design.

Environmental Impact of Operational Decisions. A recent high-impact body of literature has studied the environmental impact of operational decisions. Among others, Agrawal et al. (2012) compare the revenue models of leasing and selling, Lim et al. (2014) consider the leasing/selling choice for batteries in electric vehicles, and Daniels and Lobel (2014) study the role of contracts in green energy. Cachon (2014) compares the environmental performance of different supply chains in *offline* retailing. That work is probably the closest to ours, although we consider *online grocery* retail as well as the choice of revenue model and incorporate the role of consumer food waste.

Like this study, recent operations models include increasingly advanced models of customer behavior: consumer inventory buildup (Su, 2010), conspicuous consumption (Tereyağoğlu and Veeraraghavan, 2012), and social comparisons (Roels and Su, 2013).

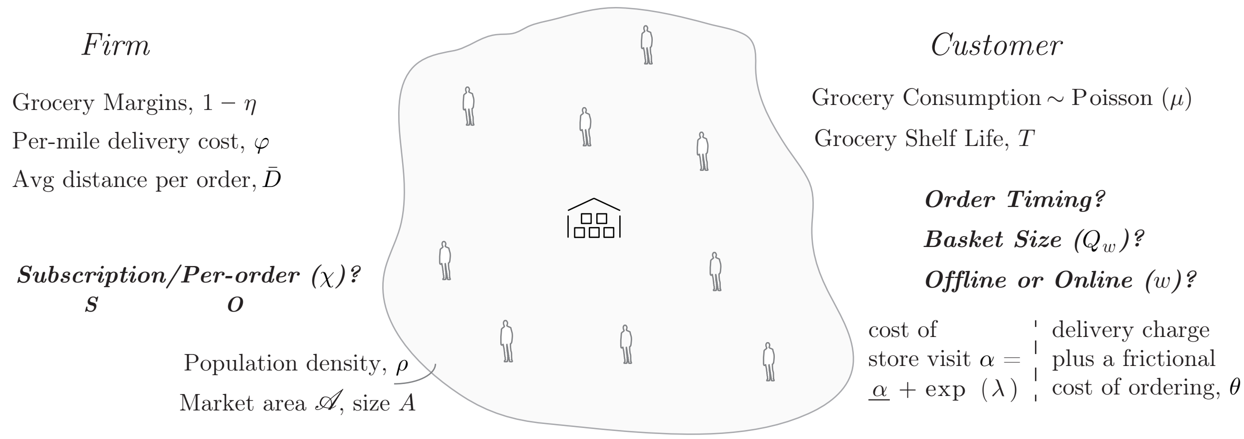


Figure 3.1. The Grocery Delivery Business Model

3. Model Setup

3.1. **The Fresh Grocery Market.** Consider an online fresh grocery retailer that serves a market of area $\mathscr{A}$ with size A (see Figure 3.1). Potential customers are distributed over area $\mathscr{A}$ with uniform density ρ. Fresh groceries purchased by the customer have a limited shelf life; a representative basket expires T days after purchase. The grocery consumption of customers is random owing to unpredictable demand shocks (e.g., unexpected guests, last-minute plans to dine out). Formally, each customer's consumption is generated according to a Poisson process with rate μ monetary units.[10]

Customers choose to buy groceries either offline (by visiting a grocery store) or from an online retailer that offers home delivery. A customer who buys offline incurs a cost α for each visit to the store. Customers are heterogeneous in this cost because of their idiosyncratic marginal costs of time, distances from the grocery store, travel patterns, utility from shopping, and so forth. Accordingly, the per-visit cost α is a random variable observed only by each customer before making her choices. We assume that the store visit cost follows a tractable exponentially distributed form; that is, we assume $\alpha = \underline{\alpha} + x$ with $x \sim \exp(\lambda)$.[11] $G(\alpha)$ denotes its cumulative distribution function, $g(\alpha)$ the density function and $\overline{G}(\alpha) \equiv 1 - G(\alpha)$ is the survival function. Extensive numerical analysis reveals that our qualitative results remain unchanged by using either the uniform or the truncated normal distributions.

[10]For tractability, all fixed-life perishable inventory models assume Poisson demand (Fries, 1975; Nahmias, 1977; Weiss, 1980; Liu and Lian, 1999).

[11]Brown and Borisova (2007) provide empirical support for this assumption, finding that the major component of the store visit cost is time. The value of time can be approximated by income, the distribution of which is known to be roughly exponential (Wikipedia entry on "Household income in the United States", http://bit.ly/HHIncome).

If instead the customer uses the online retailer, she incurs—in addition to the delivery charges—a small frictional ordering cost θ. This term captures the inconvenience of going to the website, selecting groceries, and placing the order. We assume that groceries ordered online are delivered almost instantly (same-day delivery is typically offered by prominent online grocery retailers). Of course, the cost of visiting the store is higher than the frictional cost of ordering online: $\underline{\alpha} > \theta$.

Customers choose not only the timing of orders; each time they order, they also choose whether to buy offline or online as well as the amount of groceries to buy (the basket size).

3.2. **The Online Fresh Grocery Retailer.** The online retailer of fresh groceries builds a Web-based storefront, and a proprietary distribution network to make deliveries.[12] This network consists of a warehouse centrally located in area $\mathscr{A}$ along with a fleet of vehicles, each of which can make as many as K deliveries per run.[13]

Revenues. In addition to generating revenue from the sale of groceries, the firm also charges for delivery. The retailer can choose between two revenue models for its delivery service: the subscription model (S) or the per-order model (O). In the subscription model, customers pay a subscription fee s each year and enjoy free delivery for all orders placed during that year; in the per-order model, the customer pays delivery fee o for each delivery order.

Costs. The firm has two main variable cost heads: the cost of procuring groceries and the cost of delivering them. The firm *procures* groceries at a cost of η times the sale price, where $\eta < 1$ and $1 - \eta$ captures the firm's gross margins. For every order *delivered*, the firm incurs an average direct delivery cost of $\varphi \cdot \bar{D}$. Here $\bar{D}$ is the average distance traveled to deliver an order under an optimal routing scheme and φ is the per-mile delivery cost (which subsumes the costs of fuel, labor, truck purchase, licensing, depreciation, etc.). The average distance traveled ($\bar{D}$) arises from the lowest-cost feasible routing scheme that can fulfill all delivery commitments. Section 5.1 derives the expression for this distance.

In addition to the direct delivery costs, an online firm also incurs other costs associated with delivery; these include picking costs as well as the costs of building a warehouse, buying vehicles,

[12]The use of third-party logistics providers is seldom viable in this context because of the short delivery times, special transit requirements, and perishable nature of the products.

[13]The number of deliveries K is determined by the delivery offer. In particular, K is smaller for faster delivery, for a smaller delivery *time windows*, for a longer time to reach customers, and for smaller delivery vehicles—faster delivery implies that the provider has less time to wait for orders that can be batched; smaller delivery windows require reduced uncertainty which is achieved by carrying fewer orders.

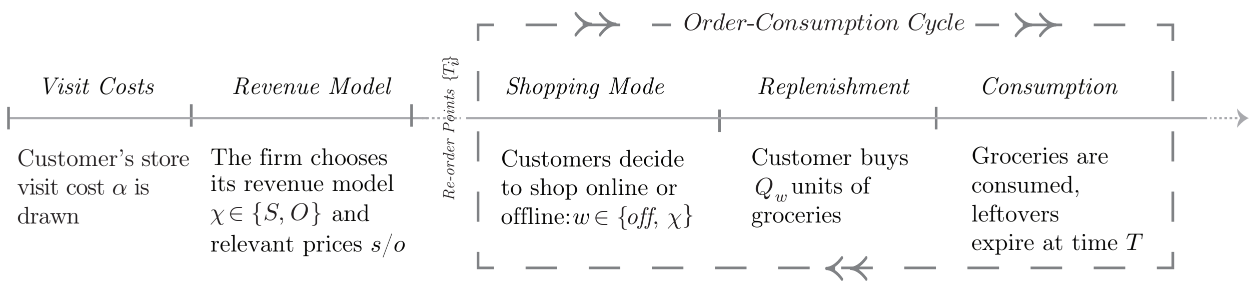


FIGURE 3.2. Sequence of Events

training employees, and so forth. These costs have three potential components: one that depends on the amount of groceries sold, one that depends on the number of orders serviced by the online warehouse, and finally some fixed costs. The first component is subsumed by the cost of groceries (η), and the second is captured by a "picking" cost $c_p > 0$ that the firm incurs for each order. The third component (fixed costs) does not change under the two revenue models in question, so it has been excluded from model comparisons.

3.3. **Sequence of Events.** The sequence of events is illustrated in Figure 3.2. First, nature draws a type for each customer—that is, her cost α of going to the offline store. The draw is known only to the customer, although the distribution is common knowledge. Next, the firm chooses a revenue model χ, $\chi \in \{S, O\}$; recall that S denotes the subscription model and O the per-order model. This firm also chooses the pertinent fees: the yearly subscription fee s or the per-order fee o. Next, at a time of her choosing, the customer decides to order groceries. Ordering groceries involves two further choices. The first is whether to use the online grocery delivery service or to shop offline at the local grocery store. This choice is represented by $w \in \{off, \chi\}$, where *off* signifies buying from an offline store and χ signifies buying online. The second additional choice is that of how many groceries to buy from the chosen store, represented by the basket size Q_w (for notational brevity we express basket size directly in its dollar value). Finally, grocery consumption is realized according to the demand process. Items not consumed by time T from the order time perish and are discarded. The realization of the consumption process—and of the customer's choice of order time, online/offline store, and basket size—continue indefinitely.

We begin our analysis by studying the customer's actions and the firm's distribution network design in (respectively) the two sections that follow. The equilibrium is described in Section 6.

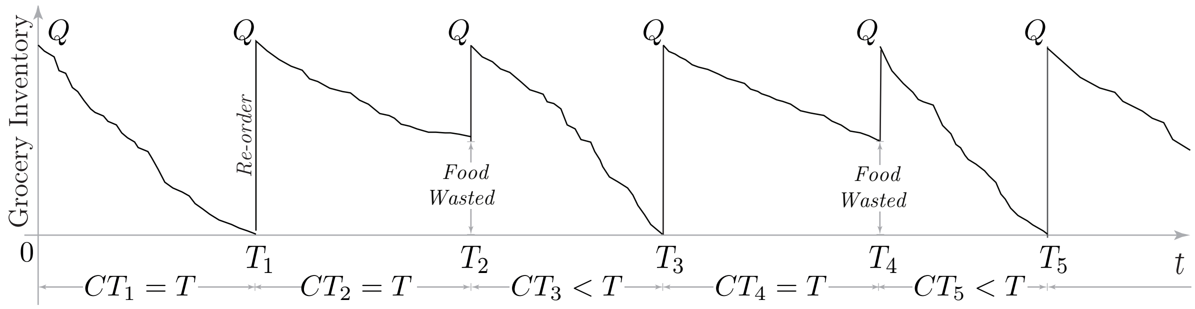


FIGURE 4.1. The Consumer Inventory Process: Possible Realizations of the Consumption Cycle

The choice of revenue model, which is the main focus of our analysis, is discussed in Section 7. Implications for managers and some final remarks are presented in Sections 8 and 9, respectively.

4. CUSTOMER BEHAVIOR

The customer continuously reviews her inventory level and decides whether or not to place a new order—bearing in mind the cost of ordering and the constraint that all grocery demand must be met. Thus the customer decisions are: the timing $\{T_i\}$ of orders and, at each order time, the choice between offline versus online purchase ($w_{T_i} \in \{\mathit{off}, \chi\}$) and the choice of basket size ($Q_{w_{T_i}}$).

The ordering cost a depends on the preceding online/offline choice and the firm's revenue model choice. It is simply the cost of going to the store $a = \alpha$ if the customer chooses an offline store. Suppose the customer decides to shop online. Then, for the per-order model, the ordering cost is $a = \theta + o$, or the sum of the frictional cost of placing the order and the per-order delivery fee charged by the firm. For the subscription model, the ordering cost is just $a = \theta$, the frictional cost of ordering.

Our analysis establishes that the customer's optimal "continuous review" inventory re-order policy is to place an order when she runs out of groceries or when the leftover groceries expire, whichever happens earlier (Lemma 5 in the Appendix). The customer must meet all demand and thus can order no later than the point when she is out of *fresh* groceries. On the other hand, with positive ordering costs, no lead time in delivery, and the perishable nature of groceries, the customer prefers to delay ordering, i.e. order as rarely as possible and no earlier than absolutely necessary. This re-order policy also implies that, at each re-order point, the system returns to the same state. Hence the offline/online choice and the basket size are the same for every order point $w_{T_i} = w$ and $Q_{w_{T_i}} = Q$.

Figure 4.1 illustrates the customer's grocery inventory process (under the optimal policy just described) for a particular sample path of grocery consumption realizations. The inventory level is reduced by grocery consumption and by the discarding of expired items. In ordering cycles 1, 3, and 5, the consumption rate is high enough that all groceries are consumed; in cycles 2 and 4, the consumption rate is low and some groceries expire before they are used (i.e., there is food waste).

The optimal re-order basket size Q must be such that it appropriately trades off ordering costs against food waste. If the customer orders too few groceries, then all items will be consumed before expiration and that would lead to an extra ordering cost. Yet if she orders too many then some of the items will be wasted, leading to unnecessary grocery expenses.

Formally, the customer chooses her basket size to minimize her expected long-term average cost rate of groceries. Since the customer re-orders the same quantity each time, successive cycles are independent and identical. The expected long-term average cost rate is equal to the expected average cost per ordering cycle *divided by* the expected length of one cycle (follows from standard renewal theory arguments; see (Ross, 1970)).

For basket size Q, the expected average cost per ordering cycle is $a+Q$: the sum of the customer's ordering costs a and the direct costs of purchasing groceries Q Length of ith cycle CT_i depends on basket size Q. The expected length of the cycle is (Lemma 6):

$$E[CT_i(Q)] = \frac{1}{\mu}\left(Q - \sum_{j=0}^{Q}(Q-j)\cdot p_j\left(\mu T\right)\right), \qquad p_j\left(\mu T\right) = \frac{e^{-\mu T}(\mu T)^j}{j!}$$

The denominator μ is the average consumption rate, and the "numerator" (in large parentheses) is the expected consumption per cycle. Out of Q units ordered, in expectation, $\sum_{j=0}^{Q}(Q-j)\cdot p_j\left(\mu T\right)$ will be wasted. The probability that consumption during the shelf-life T equals to$j$ is given by p_j.

To accurately incorporate annual subscription costs, we consider all costs on an annual basis. Toward this end, define $n(Q) = \min\{n:\ CT_1(Q) + CT_2(Q) + \cdots + CT_n(Q) \geq 1 \text{ year}\}$. Since n is independent of $CT_{n+1}, CT_{n+2}, \ldots$ for all $n = 1, 2, \ldots$, it follows that $n(Q)$ is a stopping time. We can use Wald's equation to express the expected number of orders in a year induced by ordering Q units at a time: $N(Q) = E[n(Q)] = \frac{1 \text{ year}}{E[CT_i(Q)]}$. Using expressions for the expected length of the cycle, $E[CT_i(Q)]$, we obtain

$$N(Q) = \frac{1}{\frac{1}{\mu}\left(Q - \sum_{j=0}^{Q}\left(Q-j\right)\cdot p_j\left(\mu T\right)\right)}. \tag{4.1}$$

Then the customer's expected long-run average cost rate is $(a+Q)N(Q)$ and the optimal basket size is given by

$$Q^*(a) \equiv \arg\min_{Q} (a+Q)\cdot N(Q)\,. \tag{4.2}$$

Lemma 1. *Customer Basket Size, Order Frequency, and Annual Volume of Groceries Purchased*

i. The customer's optimal basket size $Q^*(a)$ is a solution to

$$Q-\sum_{j=0}^{Q}(Q-j)p_j(\mu T) \approx (a+Q)\left(1-P_j(Q,\mu T)\right), \tag{4.3}$$

$$P_j(Q,\mu T)=\sum_{j=0}^{Q} p_j(\mu T)\,.$$

ii. Higher ordering costs lead to larger basket sizes, fewer annual orders, and a higher annual volume of groceries purchased.

$$\frac{\partial Q^*}{\partial a}>0, \qquad \frac{\partial N(Q^*)}{\partial a}<0, \qquad \frac{\partial\left(N(Q^*)\cdot Q^*\right)}{\partial a}>0.$$

iii. A customer's optimal grocery cost, $(a+Q^*)\cdot N(Q^*)$, is increasing in a.

Proofs for all results are given in the Appendix.

The optimal grocery basket size trades off the risk of ordering too many groceries (which leads to waste) against the risk of ordering too few groceries (which would trigger additional orders and increase ordering costs). A marginal increase in the basket size affects per-cycle costs in two ways. First, the procurement costs are simply higher by one unit: $Q \to Q+1$, which increases the cost rate. Second, the extra unit increases the expected cycle time. If the extra unit was consumed for sure then it would extend the cycle length by $1/\mu$ time units, however the extra unit might end up unconsumed, since there is some likelihood that the last unit would be wasted. In fact the cycle time is actually increased by $(1-P_j(Q,\mu T))\,\mu^{-1}$, where the additional factor captures the increase in waste. This dynamic reduces the cost rate. The optimal quantity Q^* is such that the increase and decrease in cost rate are balanced:

$$\frac{a+Q+1}{\frac{1}{\mu}\left(Q+1-\sum_{j=0}^{Q+1}(Q+1-j)p_j(\mu T)\right)}-\frac{a+Q}{\frac{1}{\mu}\left(Q-\sum_{j=0}^{Q}(Q-j)p_j(\mu T)\right)}\approx 0.$$

Rearranging terms gives us the expression in Equation 4.3.

A higher ordering cost a incentivizes customers to order *larger* basket sizes *less* frequently. Larger basket sizes increase the likelihood that some of the groceries expire before they are consumed, which in turn increases the average waste. Given that annual grocery consumption does not depend on ordering costs, higher waste implies that the customer purchases a higher annual quantity of groceries $N(Q^*) \cdot Q^*$. Finally, the "larger basket size" effect dominates the "less frequent ordering" effect and so the customer's grocery costs are increasing in a.

Given this optimal order quantity choice, the customer's optimal annual costs of buying offline are

$$C_{\mathit{off}} = C_\alpha \equiv (\alpha + Q^*(\alpha)) \cdot N(Q^*(\alpha)) . \tag{4.4}$$

The optimal costs of buying online depend on the online retailer's choice of revenue model χ (which can be subscription S or per-order O) and the corresponding prices, s and o:

$$C_\chi = \begin{cases} (o + \theta + Q^*(o+\theta)) \cdot N(Q^*(o+\theta)) & \text{if } \chi = O, \\ s + (\theta + Q^*(\theta)) \cdot N(Q^*(\theta)) & \text{if } \chi = S. \end{cases} \tag{4.5}$$

Finally, a customer's choice between the offline store (off) and the online store (χ) simply boils down to minimizing the yearly cost: $w^* = \arg\min_{w \in \{\mathit{off}, \chi\}} C_w$.

We simplify notation for the optimal order size by putting $Q_\alpha \equiv Q^*(\alpha)$ as the optimal offline order size, $Q_o \equiv Q^*(o+\theta)$ as the optimal online order size with per-order pricing, and $Q_s \equiv Q^*(\theta)$ as the optimal online order size with subscription pricing. Similarly, for the number of orders per year we set $N_\alpha \equiv N(Q_\alpha)$, $N_o \equiv N(Q_o)$, and $N_s \equiv N(Q_s)$.

5. Firm Decisions

The online grocery firm builds a distribution network to deliver groceries, chooses either the subscription or the per-order revenue model, and determines the relevant price. We start by analyzing the design of the distribution network that meets the delivery requirements at the lowest cost.

5.1. **The Proprietary Distribution Network.**

This section adapts the analysis of Daganzo (1984a,b) and Cachon (2014) to determine the direct delivery costs, which are proportional to the distance traveled when making deliveries.

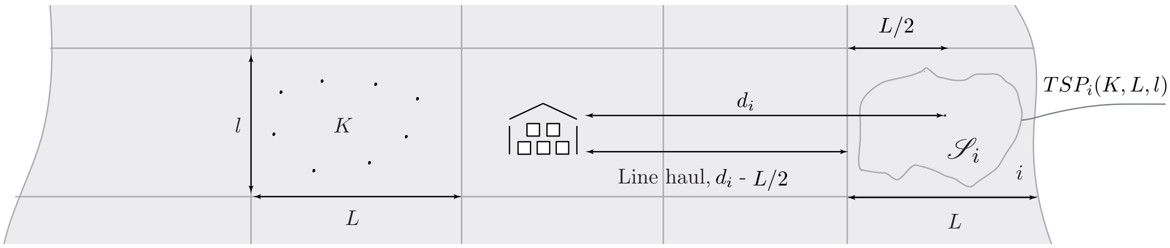


FIGURE 5.1. Distance Traveled to Deliver an Order in Sector i

A distribution network consists of a warehouse centrally located in area $\mathscr{A}$ and a fleet of vehicles, each of which carries K delivery orders. We assume that the number of orders delivered by one vehicle in one delivery period K is significantly smaller than the number to be delivered in the entire market at the same time, $K \ll A \cdot \rho_d$. Here $\rho_d = \bar{\rho}N\delta^{-1}$, $\bar{\rho}$ is the density of the population that adopts the online service (we will define it explicitly as a function of the firm's pricing in Section 5.2), N is the annual number of orders per customer, and δ is a coefficient that converts the annual number of orders into the number of orders per delivery period. For example, if the retailer delivers once per day then δ is 365.

Based on the specific orders to be delivered in a period, the firm devises the following distribution plan. First, it optimally partitions area $\mathscr{A}$ into sectors $\mathscr{S}_i$, $i \in \{1, ..., I\}$, so that each sector has K customers. Each sector is then assigned a vehicle that visits K customers following an optimal route.

The distance traveled to deliver K orders in sector $\mathscr{S}_i$ has two components: the distance between the warehouse and the boundary of the sector, or the "line haul" distance, and the optimal traveling salesman tour within the sector itself (see Figure 5.1). Minor variations in the specific shape of the sectors do not greatly affect either the line-haul distance or the lengths of the traveling salesman tours. We can therefore consider dividing area $\mathscr{A}$ into equal rectangular sectors of length L and height l $(L > l)$ and "slenderness factor" $\beta = \frac{l}{L}$ (Daganzo, 1984a). The distance traveled per order delivered in sector $\mathscr{S}_i$ can then be expressed as

$$D_i \approx \frac{2}{K}\left(d_i - \frac{L}{2}\right) + TSP^*(K, L, l), \tag{5.1}$$

where d_i is the distance traveled in getting to sector $\mathscr{S}_i$'s center of gravity and $L/2$ is the approximate distance from the sector's center of gravity to the edge of the sector where the traveling salesman tour starts. Together these two components constitute the line-haul distance, which must be covered twice (to get to the sector and back) and is distributed over the K deliveries. We use $TSP^*(K, L, l)$ to denote the per-order average length of the optimal traveling salesman tour within the sector. Our quantity of interest, the average distance traveled per order, obtained by averaging over all sectors $\mathscr{S}_i$, $i \in \{1, ..., I\}$, is $\bar{D}(\bar{\rho}, N, \mathscr{A}, K) = \frac{1}{I}\sum_{i=1}^{I} D_i$. Of D_i's three components, L and TSP^* depend on the area's partition into sectors whereas d_i does not. We will first analyze d_i and then the optimal partition of our market area into sectors; this will be followed by deriving the latter two components as well as the optimal total distance.

Average Distance from the Warehouse to the Center of Gravity of Sectors $\bar{d}$. (Daganzo, 1984a) shows that $\bar{d} = \frac{1}{I}\sum d_i$ can also be interpreted as the average distance from the warehouse to a point in market area $\mathscr{A}$. Lemma 7 in the Appendix derives this distance for different shapes of the market area. In general, $\bar{d}$ can be expressed as $\bar{d} = \zeta\sqrt{A}$, where the coefficient ζ is determined by the region's shape. We have $\bar{d}_{\bigcirc} \approx 0.37\sqrt{A}$ for "circular" cities or markets and $\bar{d}_{\square} \approx 0.76\sqrt{A}$ for "square" ones. For a rectangular area with length-to-height ratio $\gamma \geq 1$, we have $\bar{d}_{\sqsubset\!\sqsupset} = \sqrt{A}\cdot\zeta_{\sqsubset\!\sqsupset}$ and $\partial_\gamma \zeta_{\sqsubset\!\sqsupset} > 0$; that is, the higher the ratio γ, the longer the distance traveled. The distance traveled is greater for sector shapes that are more irregular or elongated: $\zeta_{\bigcirc} < \zeta_{\square} \leq \zeta_{\sqsubset\!\sqsupset}$.

Partition of Market into Sectors. The area of individual sectors is predetermined by the per-delivery period order volume ρ_d: $L \cdot l = K\rho_d^{-1}$. Although the area is fixed, we can still choose the shape of its sectors—that is, the slenderness factor β. As we elongate the rectangle toward the warehouse, the distance from the center of the sector to the start of the tour increases (L is increasing); this has the effect of lowering the distance traveled, $d_i - L/2$, yet increasing the length of the traveling salesman tour. The proof of Lemma 2 identifies the optimal sector shape β^*.

Average Distance Traveled to Deliver an Order.

Lemma 2. (i) *When K orders are delivered by one vehicle, the average distance traveled to deliver an order in an area A with uniform customer population density $\bar{\rho}$ and customer yearly order frequency N is*

$$\bar{D}(\bar{\rho}, N, A, K) \cong \frac{2\zeta\sqrt{A}}{K} + \lambda(K)\sqrt{\frac{\delta}{\bar{\rho}N}}, \quad \text{with } \partial_K \lambda(K) < 0. \tag{5.2}$$

(ii) *The per-order average distance is decreasing in the number of orders ($\partial_N \bar{D} < 0$) and in the adopting density ($\partial_{\bar{\rho}} \bar{D} < 0$), but it is increasing in the order costs ($\partial_a \bar{D} > 0$). The total annual* *distance traveled per customer, $N\bar{D}$, is increasing in the number of orders ($\partial_N N\bar{D} > 0$) but it is decreasing in the order costs ($\partial_a N\bar{D} < 0$).*

Part (i) of this lemma is obtained by aggregating Equation 5.1 over different sectors while taking in account the optimal slenderness factor and expression for $\bar{d}$. The proof of Lemma 2 identifies an expression for the coefficient $\lambda(K)$. The economies of scale in the traveling salesman tour are captured by $\lambda(K)$: the per-order average length of the optimal traveling salesman tour decreases as we add more points (orders) to the tour.

Part (ii) of the lemma shows that there are economies of scale in delivery. If there are more customers ($\bar{\rho}\uparrow$) and/or if customers order more often ($N\uparrow$), then the per-order delivery distance decreases. Along the same lines, a higher ordering cost a will lead both to fewer orders and to fewer customers and thus to a higher average distance traveled. Finally, with respect to total annual distance traveled per customer, the order frequency effect dominates the average delivery distance effect; hence a higher frequency leads to more annual travel.

5.2. **Choice of Revenue Model.** Suppose the firm chooses the *per-order* pricing model. Then the customers whose store visit costs are above a certain threshold will choose the online retailer (Lemma 8). In that case, the firm's profits (at the profit-maximizing per-order price) can be written as

$$\pi_o = \max_{o} \left((o + Q_o) N_o - \left\{ \varphi \cdot \bar{D}\left(\rho\bar{G}(\tilde{\alpha}),\, N_o, \mathscr{A}, K\right) + \eta Q_o + c_p \right\} \cdot N_o \right) \cdot A\rho\bar{G}\left(\tilde{\alpha}_o\right), \tag{5.3}$$

where $\tilde{\alpha}_o = \min\{\alpha \in [\underline{\alpha}, \overline{\alpha}$ s.t. $C_{\mathit{off}} \geq C_o\}$. The first term in this profit formulation is the per-customer delivery and grocery revenues. The second term includes the two variable cost components, the per-customer costs of delivery and the per-customer costs of sourcing groceries. Finally, the multiplicative term represents the number of customers buying from the online grocery retailer, i.e. $\bar{G}(\tilde{\alpha})$ captures the fraction of customers that buy online.

If instead the firm chooses the *subscription* model, then again the customer's online/offline choice is a threshold choice in the store visit cost. Analogously, the maximum expected profit under subscription pricing is

$$\pi_s = \max_{s} \left(\{ s + Q_s N_s \} - \left\{ \varphi \cdot \bar{D}\left(\rho\bar{G}(\tilde{\alpha}_s),\, N_s, \mathscr{A}, K\right) + \eta Q_s + c_p \right\} \cdot N_s \right) \cdot A\rho\bar{G}\left(\tilde{\alpha}_s\right), \tag{5.4}$$

where $\tilde{\alpha}_s = \min\{\alpha \in [\underline{\alpha}, \overline{\alpha}$ s.t. $C_{\mathit{off}} \geq C_s\}$. As before, the first terms are the per-customer delivery and grocery revenues, the next terms include the delivery and procurement costs, and the multiplicative term captures the fraction of customers that buy online. Finally, the firm will choose the pricing scheme that maximizes its profit:

$$\chi = \arg \max_{\chi \in \{o,s\}} \pi_\chi.$$

6. Equilibrium Outcomes

We can now combine the analysis of the distribution network from Section 5.1, which gives us the relation between direct delivery costs and consumer order frequency (Equation 5.2), with the firm's best response (Equations 5.3 and 5.4) and the consumer's best response (Equation 4.3) to determine the equilibrium outcomes under each choice of revenue model.

6.1. **Per-Order Revenue Model.**

Lemma 3. *Equilibrium Outcome under the Per-Order Revenue Model*

i. The online retailer charges a per-order delivery fee $o = \alpha_o^* - \theta$, where the optimal market coverage α_o^* is a unique solution to $(N_\alpha - \partial_\alpha h_o(\alpha)) \cdot \bar{G}(\alpha) = (C_\alpha - h_o(\alpha)) \cdot g(\alpha)$; here $h_o(\alpha) = (\theta + \varphi \cdot \bar{D}(\rho \bar{G}(\alpha), N_\alpha, \mathscr{A}, K) + \eta Q_\alpha + c_p) N_\alpha$.

ii. Customers with store visit costs $\alpha > \alpha_o^*$ choose the online firm. These customers all order the same basket size $Q^*(\alpha_o^*)$ from the firm on an ongoing basis, re-ordering every time their inventory runs out or expires. Customers with store visit costs $\alpha < \alpha_o^*$ purchase groceries offline. Each such customer's idiosyncratic basket size is $Q^*(\alpha)$, where α is her individual store visit cost. These customers also re-order when their inventory runs out or expires.

The equilibrium is best understood by examining the effect of a price change on each term in the firm's profits (Equation 5.3) while keeping in mind customer response to this price (i.e., the customer ordering costs; see Lemma 1), delivery costs (Lemma 2) and the adoption.

Recall that an increase in ordering costs increases the customer's basket size, reduces the annual number of orders per customer, and increases the amount of groceries purchased by each customer (since there is more waste); see parts (ii) and (iii) of Lemma 1. Thus an increase in the per-order price increases the direct grocery profits $(1 - \eta)\, Q_o N_o$ but also increases a customer's cost of using the online channel—both directly (owing to higher annual delivery costs oN_o) and indirectly (owing to higher grocery expenses Q_oN_o). As a result, customer adoption $\bar{G}(\tilde{\alpha}_o)$ declines. With regard to delivery, an increase in the per-order price increases *per-order* delivery costs $\varphi\bar{D}$ because of a lower order frequency and lower adoption rate; yet the annual *per-customer* delivery costs $\bar{D}N_o$ actually decrease because the order frequency effect dominates (Lemma 2(ii)). Hence the delivery profits $(o - \varphi\bar{D}) \cdot N_o$ are increasing in o. Altogether, the delivery and grocery profits increase when the firm sets higher per-order prices, though doing so reduces adoption. A per-order delivery price of $o = \alpha_o^* - \theta$ (with α_o^* defined above) optimally trades-off the per-customer profit effect with the adoption or market size effect.

Given an optimal delivery fee, customers with $\alpha \geq \alpha_o^*$ use the online store while all other customers prefer the offline channel. All customers who use the online channel have the same basket size because their ordering costs are now the same; in contrast, customers using the offline channel choose different quantities based on their individual store visit costs (α). Hence the resulting yearly per-customer delivery and grocery purchase cost is captured by $h_o(\alpha_o^*)$.

6.2. **Subscription Pricing.**

Lemma 4. *Equilibrium Outcome under the Subscription Pricing Model*

i. The online retailer charges a yearly subscription fee $s^* = (\alpha_s^* + Q^*_{\alpha_s^*})N^*_{\alpha_s^*} - (\theta + Q_s)N_s$, *where the optimal market coverage* α_s^* *is a unique solution to* $(N_\alpha - \partial_\alpha h_s(\alpha))\bar{G}(\alpha) = g(\alpha)(C_\alpha - h_s(\alpha))$; *here* $h_s(\alpha) = (\theta + \varphi \cdot \bar{D}(\rho\bar{G}(\alpha), N_s) + \eta Q_s + c_p)N_s$.

ii. Customers with store visit costs $\alpha \geq \alpha_s^*$ *choose the online firm. These customers all order the same basket size* Q_s *on an ongoing basis, re-ordering every time their grocery inventory runs out or expires. Customers with store visit costs* $\alpha < \alpha_s^*$ *purchase their groceries offline. Each such customer's idiosyncratic basket size is* $Q^*(\alpha)$, *where* α *is her individual store visit cost. These customers also re-order when their inventory runs out or expires.*

Subscription price affects only the direct delivery revenues and market adoption (which in turn affects delivery costs). Thus the trade-off driving the choice of subscription price is simpler than for the choice of per-order price, since higher subscription prices increase firm revenue but also decrease adoption rates. Lower levels of adoption entail higher delivery costs because in that case there are fewer economies of scale. As before, our marginal customer is the one with store visit cost α_s^*. The resulting yearly per-customer delivery and grocery purchase cost for online customers is captured by $h_s(\alpha_s^*)$.

The equilibrium profits of online retailers depend in a predictable way on the setting. Irrespective of the revenue model employed, online retailing is financially the most rewarding in dense, circular-shaped cities with premium consumers (high consumption rate and high margins), low per-mile delivery costs, and high consumer store visit costs. Each of these factors either decreases costs or increases revenues, lowering the equilibrium price, leading to higher adoption and more economies of scale, which further reinforce the effects. The role played by delivery area is more complex. Larger areas tend to contain more customers but also involve greater travel distances, and generally the former effect dominates.

7. Comparing Revenue Models: Subscription versus Per-Order

7.1. Comparison of Equilibrium Customer Behavior.

Theorem 1. *Customers order larger basket sizes and order less frequently in the per-order model than in the subscription model. Overall, annually more groceries are purchased in the per-order model (larger basket size dominates the lower frequency) and greater delivery distances are traveled in the subscription model.*

Corollary. *The amount of groceries that perish before use (food waste) is higher in the per-order model.*

At equilibrium prices, the ordering costs are higher in the per-order model. It directly follows from Lemma 1 that the per-order model is characterized by larger basket sizes, fewer orders, and more grocery purchased annually. Expected grocery consumption is simply the mean demand rate, which is the same for these two revenue models. Since more groceries are purchased in the per-order model, waste is higher.

Interestingly, in our model the *total* annual grocery purchases are *increasing* in the per-order prices. This finding runs counter to results derived from abstract economic models, which almost always ignore demand uncertainty and consumer inventories while simply assuming that demand declines in response to higher prices. Note also that this result contrasts with the anecdotal wisdom that subscription models lead to higher sales because they remove barriers to purchasing. While, this may be true in settings that involve durable products, this commonly stated observation might not hold in the case of fresh grocery.

A higher annual grocery volume with fewer orders suggests that the per-order firm sells more while spending less money delivering the groceries; hence the per-order model should dominate. But this analysis is incomplete. Besides these effects, there are two others that arise from the customer's adoption decisions. First the two models differ in how much total value they create and how much of this total value is claimed by the firm. In turn, this implies that the two models will have different adoption and consequently different levels of the economies of scale in delivery. Together these effects lead to a drastic departure from the above suggestion.

In terms of environmental impact, the results of Theorem 1 suggest a trade-off between the two models. While the per-order model's higher levels of food waste makes it less eco-friendly, the subscription model's greater driving distances render it less desirable. Yet the subsequent analysis establishes that, in practice, there is no trade-off: one of these two models always dominates.

7.2. **Comparing Equilibrium Outcomes.** The equilibrium profits can be rewritten as maximization problems in which the firm rather than choosing delivery price—which in turn determines market adoption— it directly chooses an optimal adoption level (via the critical store visit costs α^*):

$$\pi_s = \max_{\underline{\alpha} \le \alpha \le \bar{\alpha}} \pi_s(\alpha) \equiv \max_{\underline{\alpha} \le \alpha \le \bar{\alpha}} \left((\alpha + Q_\alpha)N_\alpha - h_s(\alpha)\right) A \cdot \rho\bar{G}(\alpha);$$
$$\pi_o = \max_{\underline{\alpha} \le \alpha \le \bar{\alpha}} \pi_o(\alpha) \equiv \max_{\underline{\alpha} \le \alpha \le \bar{\alpha}} \left((\alpha + Q_\alpha)N_\alpha - h_o(\alpha)\right) A \cdot \rho\bar{G}(\alpha).$$

This formulation subsumes customer behavior and allows for an intuitive decomposition of model differences into (a) per-customer revenues and costs and (b) market adoption.

Both models have the same first term: the online retailer's per-customer delivery and grocery revenue, $\mathscr{R} \equiv (\alpha + Q_\alpha)N_\alpha$. In equilibrium, an online retailer sets its price to make its revenues from each customer equivalent to the offline grocery purchasing costs of the marginal customer, thus leaving the marginal customer with no surplus. If the two models had the same equilibrium adoption (same marginal customer), then their grocery and delivery *revenues* would be the same. Note that the revenues are the *same* even though *more* groceries are sold in the per-order model. The extra groceries are bought only to avoid placing another expensive order; in expectation the customer derives no consumption value from them and so there is no extra value to extract. The

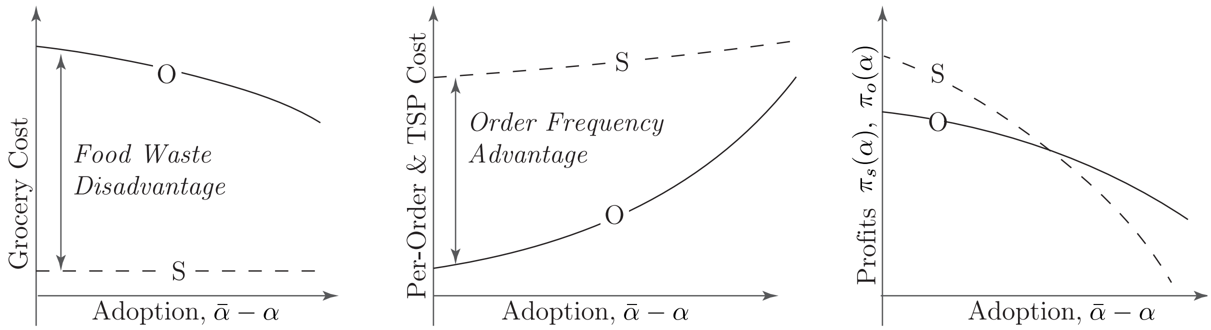


FIGURE 7.1. Per-Order versus Subscription Revenue Model: Comparison of Per-Customer Grocery and Delivery Costs and of Overall Profits

subscription model balances any revenue gains from extra grocery sales simply by adjusting the subscription price.[14]

Now we compare the per-customer costs:

$$h_s(\alpha) = \eta Q_s N_s + (\theta + c_p) N_s + \varphi \cdot \frac{2\zeta\sqrt{A}}{K} N_s + \varphi \cdot \lambda(K) \sqrt{\frac{\delta}{\rho \bar{G}(\alpha)}} \sqrt{N_s};$$

$$h_o(\alpha) = \eta Q_\alpha N_\alpha + (\theta + c_p) N_\alpha + \varphi \cdot \frac{2\zeta\sqrt{A}}{K} N_\alpha + \varphi \cdot \lambda(K) \sqrt{\frac{\delta}{\rho \bar{G}(\alpha)}} \sqrt{N_\alpha}.$$

In both models, these costs have four components. The first is the per-customer grocery procurement cost, $\mathscr{G}_{\cdot} = \eta Q_{\cdot} N_{\cdot}$, which is always higher for the per-order model. As discussed before, both models are capable of generating the same revenues but the per-order model does so by selling more groceries (Theorem 1). So all else equal, the subscription model surprisingly outperforms from the standpoint of grocery cost. This is the first fundamental difference between the two revenue models: the per-order model has a *food waste disadvantage* (see Figure 7.1(a)).

The second component combines the firm's per-order cost $c_p N_{\cdot}$ and the customer's ordering costs $\theta N_{\cdot}$ (in both models, the firm compensates the consumer for her inconvenience by lowering prices). The third components is the per-customer annual line-haul delivery costs $\varphi \cdot \frac{2\zeta\sqrt{A}}{K} N_{\cdot}$. These two cost components are increasing in the order frequency and so are higher under the subscription model: $N_s > N_\alpha$. The fourth and final component is the per-customer annual traveling salesman tour cost: the average per-order traveling salesman tour costs $\varphi \cdot \lambda(K) \sqrt{\frac{\delta}{\rho \bar{G}(\alpha) N_{\cdot}}}$ multiplied by the order frequency $N_{\cdot}$; thus, $\varphi \cdot \lambda(K) \sqrt{\frac{\delta}{\rho \bar{G}(\alpha)}} \sqrt{N_{\cdot}}$. This cost is also increasing in the order frequency,

[14] The two revenue models are equally effective at extracting the maximum possible gains from the marginal customer. So even though the ability to extract gains from customers is a key element of the vast literature on contracting, it plays only a minor role in our model.

but at a slower than linear rate as there are two kinds of scale economies: the order frequency itself ($\sqrt{N.}$) and the adoption level $\bar{G}^{-1/2}(\alpha)$. For the same level of adoption, the traveling salesman tour costs will be higher in the subscription model. Although different equilibrium adoptions might change this, they are usually higher in the subscription model. We refer to the combined effect of these delivery costs (sum of second, third and fourth components) as the per-order model's *order frequency advantage* (Figure 7.1(b)).

In sum, per-order model enjoys the order frequency advantage but suffers from food waste disadvantage. These two effects are on a per-customer basis. Hence, the third and final effect concerns optimal adoption levels. Because adoption scales up the magnitude of both the food waste disadvantage and the order frequency advantage, it determines which of these opposing effects dominates. Both of them are decreasing in the adoption level (see panels (a) and (b) of Figure 7.1). Figure 7.1(c) depicts the firm's profit curves (as a function of adoption level $\bar{\alpha} - \alpha$) under the two revenue models. We can show that if in both models the adoption levels are above (resp., below) intersection point of the two profit curves then the financial impact of the order frequency advantage dominates (resp., is dominated by) the food waste disadvantage. The likelihood of adoption level being above the intersection point is increasing with area size and transportation cost and is decreasing with population density. Thus, the per-order model is preferable for cities that are large, not densely populated, and characterized by expensive transportation. We refer to the role of adoption as the *adoption effect*.

The foregoing discussion illustrates that there are multiple competing effects that interact in a nonlinear fashion and also that it is difficult to obtain intuitive and easy-to-use criteria for determining which model is preferable from a financial and environmental point of view. However, further analysis shows that an overall comparison of the two revenue models can be expressed using a single metric—that allows us to relate the preferred revenue model to the market area's spatial and demographic properties, the product's characteristics, and the economics of delivery.

7.3. **Which Revenue Model Earns Higher Profits?**

Theorem 2. *If the shape parameter of the delivery region is below a threshold level $\zeta < \bar{\zeta}$ then the subscription model is preferred; otherwise, the per-order model is preferred.*

The profit difference between the two revenue models is driven by a nonlinear interaction involving extra grocery costs (the food waste disadvantage), delivery costs (the order frequency advantage),

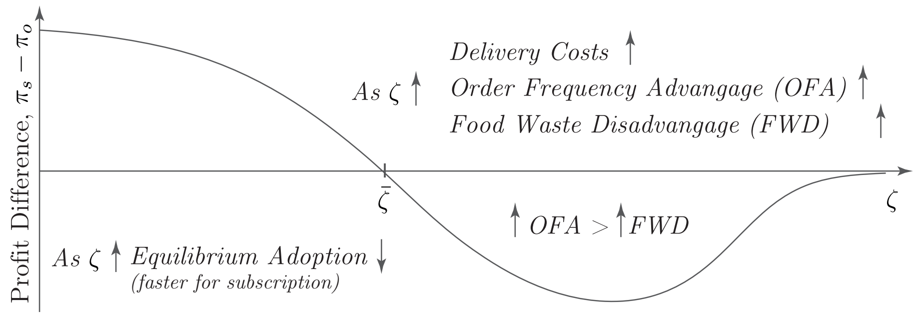


FIGURE 7.2. Retailer Profit: Subscription Model versus Per-Order Model

and the adoption effect. This comparison is involved and hard to characterize, but analysis of the constituent marginals allows us to demonstrate a single-crossing property in the shape parameter.[15] In particular, the difference takes the form shown in Figure 7.2. As the shape parameter ζ increases (delivery region becomes more elongated), two things change: delivery costs increase (because of longer line-haul distances) and adoption declines (in response to higher prices–higher costs make higher prices optimal). The former effect increases the relative value of the order frequency advantage, since each trip becomes more costly; this is the main effect that progressively favors the per-order model. Higher delivery costs mean that the optimal adoption decreases in both models; however, in the subscription model this decline is more rapid because there are more deliveries to make. As a consequence we observe second-order effects: lower adoption rates for the subscription model imply a scaling down of its advantages. This dynamic also favors the per-order model. However, reduced adoption further implies increases in both the order frequency advantage and the food waste disadvantage. This increase in order frequency advantage combined with the direct first-order effect through increase in ζ outpace the increase in the food waste disadvantage (second-order effect) once again favoring the per-order model. Eventually, in both models the adoption level becomes low, reducing not only absolute profits but also the difference *between* the models' respective profits. Hence the profit lines cross only once, which allows us to characterize the preferred revenue model in terms of the threshold shape parameter alone.

One advantage of an intuitive statement of Theorem 2 is that it allows one to characterize the area, product, and operational characteristics that are best suited for each model.

[15]The single-crossing property that drives Theorem 2 also holds for the market area size A, density ρ, per-mile delivery cost φ, and number K of deliveries per truck. The intuition is the same as that for shape parameter (explained next), and both Theorem 2 and the financially preferred model could be equivalently characterized using any of these parameters.

Theorem 3. *Ceteris paribus, the per-order model is preferred over the subscription model if the population is sparse, the delivery area is either large or elongated, and/or retailer delivery commitments require that delivery vehicles carry fewer orders. Formally: the threshold shape parameter $\bar{\zeta}$ is decreasing in the city's area A and in the per-mile delivery costs φ, but it is increasing in the population density ρ and in the number K of deliveries per truck.*

Theorem 3 follows from our previous discussion about the effect of increasing per-mile distances. A sparsely populated area ($\rho\downarrow$), a large delivery area ($A\uparrow$), and high per-mile delivery costs ($\varphi\uparrow$) lead to the same main and second-order effects as an increase in the shape parameter—in particular, an increase in the order frequency advantage, a lower optimal adoption rate, and a consequent increase in both the food waste effect and the order frequency effect. The one difference is that the order frequency–related phenomena in the main and second-order effect appear through the traveling salesman component of the delivery distance for population density (and not through the line-haul component, as in the case of the shape parameter); for the per-mile cost φ, they appear through both the line-haul and traveling salesman components. Finally, less batching of deliveries ($K\downarrow$) induces the same operating phenomena and, like the per-mile delivery cost, operates through both the line-haul and traveling salesman distances. Lower K implies that more trips are needed to the sectors and that the tours within a sector are longer on a per-order basis, which increases the order frequency advantage and lowers the adoption rate.

The preceding comparison of equilibrium profits in the two revenue models does more than tell us which model will help the online retailer earn higher profits; it also provides important guidance on which model gives a retailer the best shot at establishing a financially viable venture. In the interest of brevity, our model does not directly include the fixed costs of setting up the business and the associated costs of capital. Recall, however, that such setup costs are likely to be the same irrespective of the revenue model. Thus, the preferred revenue model (according to the analysis here) is also the one with the best chance for financial viability for a given city, set of product characteristics, and so forth. Another challenge in startup ventures concerns the speed of product diffusion. In other words, even some customers who (as rational agents in equilibrium) *should* adopt the online delivery service nonetheless fail to do so. Often only a small fraction of customers will know about the product and adopt it. The longer the diffusion process, the more "runway" or equity funding a startup venture might need before it can take off and become independently viable. Again, the diffusion process is likely to be independent of the revenue model; hence the above prescriptions will serve also to identify which revenue model is right for a new venture in the fresh grocery space that might be more interested in the length of the gestation period.

7.4. **Environmental Impact of Revenue Models.** In this section we compare the consumer carbon emissions that arise from the online firm's use of subscription versus per-order pricing. We include the emissions that arise from delivery (travel from store/warehouse to home) and those that arise from food waste. To ensure a fair comparison, we include the full population of customers—that is, both online and offline customers. We do not include the differences in travel or food waste in the supply chains from the producer to the store/warehouse, as these can be added with predictable effects.[16]

Before comparing the two online revenue models, it is useful to compare offline and online customers. Emissions differ for the online and offline customers, since in each channel the customers have different basket sizes, order frequencies, and modes of transport. Regardless of the revenue model, customers that adopt the online channel do so in order to reduce their grocery ordering costs; this means that for those customers the online channel is associated with less food waste and its related emissions. Furthermore, orders are now pooled in delivery and so per-order travel is reduced. But online customers will shop more frequently and thus induce more trips (especially in the subscription model), which could cancel out the benefits from their lower food waste and per-order travel. Yet this rarely occurs for reasonable parameter values, and—in accordance with anecdotal beliefs—an online customer typically causes fewer carbon emissions than does an offline customer.

As discussed in Section 7.2, the key differences between the equilibrium outcomes of the two revenue models are in the order frequency, levels of food waste, and rate of adoption. Each of these factors has a direct effect on the carbon emissions associated with each model. The order frequency determines the amount of driving that must be done to make grocery deliveries, and the volume of groceries is directly related to the amount of food waste. Finally, adoption controls the number of customers who actually use the online channel.

[16]Cachon (2014) examines the travel-related component of emissions for offline customers and different supply chains. However, that model does not incorporate online customers, delivery-related travel, or food waste—factors that turn out to be critical in our analysis.

The total emissions for the entire population of the delivery area in the two revenue models are as follows:

$$E_s = \quad A\rho\{\left(e_d \cdot \bar{D}\left(\rho\bar{G}(\alpha_s^*),\, N_s,\, \mathscr{A},\, K\right) N_s \;+\; e_f Q_s N_s\right) \cdot \bar{G}\left(\alpha_s^*\right) \quad + \int_{\underline{\alpha}}^{\alpha_s^*} \left(e_p d_\alpha N_\alpha + e_f N_\alpha Q_\alpha\right) g(\alpha) d\alpha - e_f \mu\};$$

$$E_o = \quad A\rho\{\left(e_d \cdot \bar{D}\left(\rho\bar{G}(\alpha_o^*),\, N_{\alpha_o^*}, \mathscr{A}, K\right) N_{\alpha_o^*} + e_f Q_{\alpha_o^*} N_{\alpha_o^*}\right) \cdot \bar{G}\left(\alpha_o^*\right) \quad + \int_{\underline{\alpha}}^{\alpha_o^*} \left(e_p d_\alpha N_\alpha + e_f N_\alpha Q_\alpha\right) g(\alpha) d\alpha - e_f \mu\}.$$

Here e_d and e_p are the CO_2-equivalent per-mile emissions for (respectively) a delivery truck and a passenger vehicle, e_f signifies the CO_2-equivalent emissions for every dollar's worth of food wasted, and E_s and E_o denote the total emissions in the subscription and per-order models, respectively. The first (resp., second) part of each displayed expression captures the emissions due to online (resp., offline) customers. Each part has two components, emissions from driving and emissions from food waste. The food waste emissions are calculated as the carbon load of all food purchased *minus* the carbon load of the food consumed ($e_f \mu$). Finally, d_α denotes distance to the store for consumers whose cost of going to the store is α.

Comparing the two models in terms of their environmental impact is analogous to comparing them in terms of profits. The per-order model leads to more food waste which has a high environmental cost and therefore per-order model suffers from a food waste disadvantage. On the other hand, it requires less deliveries and hence less driving and less carbon emissions. Because adoption differs across the two models, there is also an adoption effect. Although the preferred model is determined by a nonlinear interaction of these effects, we can more easily characterize the preferred model using a single metric: a threshold level of food waste emissions.

Theorem 4. *The per-order model is more eco-friendly than the subscription model if and only if the emissions e_f for every dollar of food wasted are less than a threshold level $\bar{e}_f$.*

This threshold result is easier to establish than is the threshold result for our profit comparisons. An increase in emissions from a dollar of wasted food increases the relative environmental consequences of the food waste disadvantage while having no influence on the other effects. Increasing unit emissions from food waste render the per-order model progressively less "green", so at some point the subscription model becomes relatively more green despite involving more travel. The result now follows. This analysis suggests that either revenue model could be greener, depending on the parameters. However, in the next section we calibrate our model using realistic parameters and find that subscription revenue model *always* dominates.

8. Managerial Implications: Cities, Product Categories, and Revenue Models

The foregoing analysis can be used by an online grocery provider to choose the revenue model that yields the highest profits and the best chance of financial viability. We now illustrate such use by calibrating our model using indicative values of the input parameters.

8.1. **Financial Considerations.** We proceed by analyzing four different delivery areas: Manhattan, Los Angeles, Paris, and Beijing (old city). Table 1 lists our parameter estimates for these cities, which vary widely in terms of geography. Paris, Beijing, and Manhattan are relatively small in area; Los Angeles is about 10 times their size. Paris and Beijing are roughly circular/oval cities, whereas Los Angeles and Manhattan are rectangular. Note that Manhattan is extremely elongated, so it has a much higher shape parameter. The cities also vary widely with respect to household density, ranging from fewer than 3,000 households per square mile in Los Angeles to about 8 times that density in Manhattan. Finally, the cities have different labor markets. The per-mile delivery costs in China are estimated to be less than a third of those costs in Western cities. In addition

Parameter	***Manhat.***	***L.A.***	***Paris***	***Beijing***	***Sources***
Area, A (sq. mi)	23	500	40	33.63	Wikipedia entries.
Density, ρ (households per sq. mi)	24,137	2,836	18,930	22,168	Population density is divided by an average household size of 2.9 (*Consumer Expenditure Survey* of the Bureau of Labor Statistics, Table 1500: Composition of consumer units, 2012Q3–2013Q2).
Shape parameter, ζ	1.16	0.8	0.4	0.61	Computed based on the following assumptions: Manhattan can be represented by a rectangle whose sides are proportioned as 1:5; Los Angeles, a rectangle of proportion 2:3; Paris, approximately a circle; and Beijing, an oval whose minor and major axes are proportioned as 5:6.
Delivery cost, φ (\$/mi)	1.5	1.5	1.5	0.413	Computed as the cost of operating a truck, including labor and delivery costs. For Western cities, the operating cost is \$1.38/mi ("The Real Cost of Trucking", http://bit.ly/1ojPJaF) plus about \$0.12/mi for other items; for Beijing, costs are estimated to be about 27% of Western costs ("China's E-Commerce Secret Weapon, the Delivery Guy", http://bit.ly/reutersdelguy).

Note: Unless stated otherwise, each city's geographic and demographic data are obtained from its Wikipedia entry.

Table 1. City Characteristics

Parameter	***Value***	***Estimation Method/Source***
Cost of ordering	$\theta = 5\$/\text{order}$	Hann and Terwiesch (2003) estimate mean cost of online ordering to be around \$5.
Cost of store visit	$\alpha = \theta + x$, $x \sim \exp(\lambda)$ $\mathbb{E}[\alpha] = 35\$$ per round-trip	The cost of a store visit is the value of time spent shopping for groceries *plus* other sources of inconvenience (making choices, planning a list, etc.). We assume that these other inconveniences are equivalent to the cost θ of ordering online. Brown and Borisova (2007) report that the average consumer spends about 140 minutes each week shopping for groceries, including travel time. If we value the customer's time at \$13/hr then the mean store visit costs \$35 per round-trip—\$30 of which is the time cost and \$5 of which is the "selection inconvenience".
Product life	$T = 1$ week	Author estimates.
Days in operation	$\delta = 365$	Customers can usually place orders seven days a week (Peapod Corporate Fact-sheet, http://bit.ly/peepod).
Number of deliveries	$K = 15/\text{truck}$	Peapod routing data (Peapod Corporate Fact-sheet, http://bit.ly/peepod).

TABLE 2. Firm and Product Parameters

to city geography and demographics, we estimate other relevant parameters using census data and industry sources (see Table 2).[17]

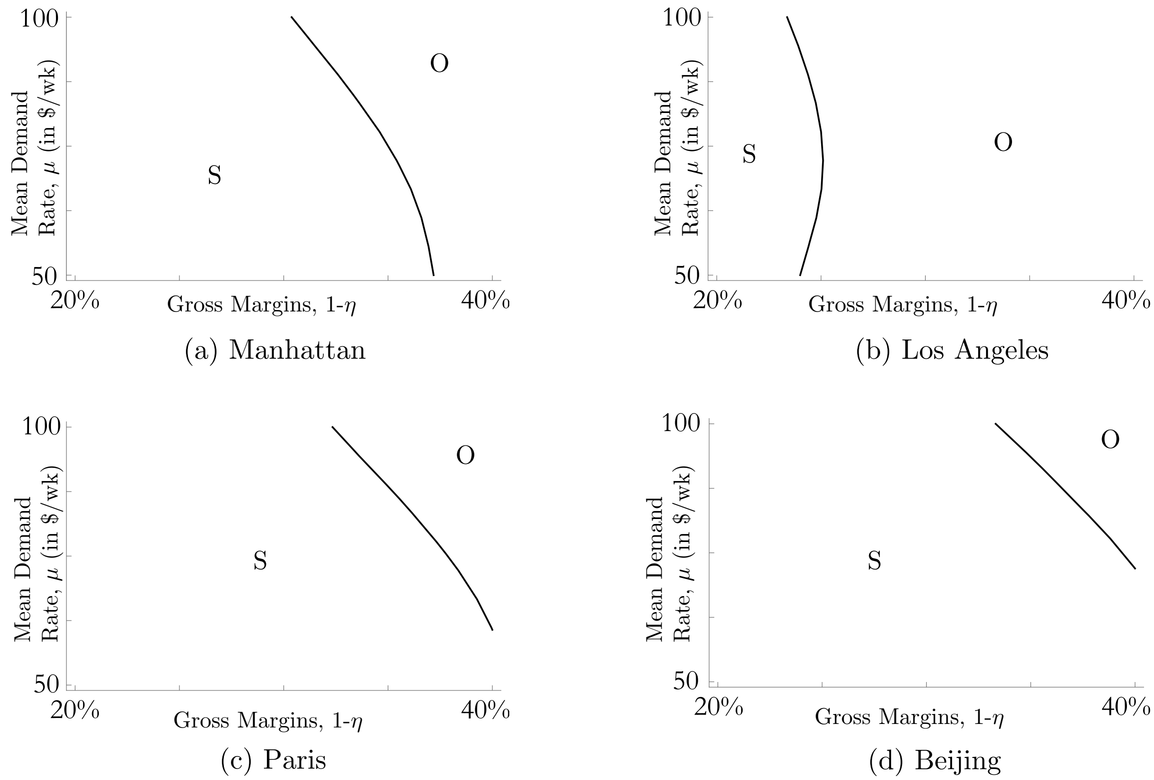
Figure 8.1 shows the prescribed revenue model strategy for the different cities. The horizontal axes of panels (a)–(d) in the figure are the product gross margins. Gross margins $(1-\eta)$ vary considerably for different fresh grocery categories and between premium and basic products. Products in the bulk section generally earn higher margins (around 40%); whereas the margins are only about 20% in the bread aisle.[18] The vertical axes represent the weekly consumption of groceries by consumers. Average US household consumption of groceries is \$86/week,[19] but the exact value varies as a function of the market segment. Large and high–net worth households naturally consume more, as do households that purchase premium, imported, organic, and/or luxury items.

Note first of all that, in each of the cities, firms that sell higher-margin product assortments are better-off taking the per-order approach to pricing. Under all realistic parameter values, firms that target higher–consumption rate households are likewise better-off with the per-order model (although the effect is somewhat weaker in this case).

[17]In addition to the baseline parameter values described in Tables 1 and 2, we compared the two revenue models for many other cities and a wide range of parameter values (for example, see the values used in Figure 8.2). All qualitative results described below continue to hold.

[18]The Reinvestment Fund, "Understanding the Grocery Industry", *Financing Healthy Food Options: Implementation Handbook,* 30 September 2011.

[19]*Consumer Expenditure Survey* of the Bureau of Labor Statistics, Table 1500: Composition of consumer units, 2012Q3–2013Q2.



Note: The per-order revenue model is preferred in the regions marked O, the subscription model in regions marked S.

FIGURE 8.1. Product Characteristics and Revenue Models by City

The differences between cities are more interesting. Manhattan, Paris, and Beijing (panels (a), (c), and (d) in Figure 8.1) are small and densely populated delivery areas. Of these three, Beijing and Manhattan are the most densely populated; however, Beijing has two advantages vis-à-vis Manhattan. First, in contrast to Manhattan's highly skewed rectangle, Beijing has an oval shape; this difference in shape reduces travel distances. Second, because of Beijing's lower labor costs, its per-mile delivery costs are only about 30% of those in Manhattan. Beijing is therefore the city where deliveries are the easiest to make, so the subscription model—with its frequent deliveries and low levels of wasted food—is preferred for most margins and consumption rates.

Comparing Paris and Manhattan, we see that Manhattan's density is higher but that Paris has a more favorable shape. The shape effect is more significant, so for Paris the subscription model is also preferred for the vast majority of items. The choice is a bit more subtle in Manhattan, where there is a significant trade-off between the two models. When choosing a revenue model, firms that operate in Manhattan must carefully examine their product margins as well as the consumption rates of their targeted customers. Finally, Los Angeles (panel (b) in the figure) has a much lower

Parameter	***Values***	***Estimation Method/Source***
Travel emissions (passenger, delivery truck)	$e_p =$ 0.417 kg of CO_2 equivalents per mile $e_d =$ 1.683 kg of CO_2 equivalents per mile	Passenger vehicles travel 21.1 mpg of gasoline, and each gallon of gasoline emits 8.8 kg of CO_2 equivalents (as estimated for 2009 by Cachon, 2014); therefore, $e_p = 8.8/21.1 = 0.417$. The corresponding numbers for delivery vehicles are 10.1 kg of CO_2 per gallon of diesel and diesel mileage of 6 mpg; hence $e_d = 10.1/6 = 1.683$.
Food emissions	$e_f =$ 1.58 kg of CO_2 equivalents per $	An average per person annual food waste accounts for 900 kg of CO_2 equivalents (Food and Agriculture Organization, 2013, http://bit.ly/FAO-Waste). In monetary terms this food waste corresponds to $340-570/year (Gunders, 2012). We use the lower bound estimate $e_f = 900/570 = 1.58$.

TABLE 3. Emission Parameters

density of population and a much larger area. In this city, the subscription model's higher order frequency constitutes more of a disadvantage than does the per-order model's food waste; hence, for most realistic parameter values, the per-order model is preferred there.

These comparisons illustrate that the trade-offs on which we focus are not merely theoretical but in fact are practically relevant and lead to different choices in various realistic contexts. Upcoming online retailers should employ the type of formal analysis described in this paper in order to give themselves the best chance of succeeding in the online grocery market.

8.2. **Environmental Impact.** We also calibrate our results on the environmental impact of different revenue models. This analysis requires three additional parameters: the carbon emissions per mile of travel for passenger and delivery vehicles (e_p and e_d, respectively) and the emissions from each extra dollar of food purchased and not consumed (e_f). Our indicative estimates are presented in Table 3.

The results of our analysis for environmental impact are more stark than those for financial impact (see Figure 8.2). We find that, for all reasonable parameter values, the emissions under the subscription model are *far lower* than those under the per-order model. The carbon impact of extra food waste turns out to be much higher than the impact of extra driving. That is, the negative effects of wasted groceries (in the per-order model) far outweigh the negative effects of extra miles driven when delivering the groceries (in the subscription model). This result holds for all plausible ranges of parameters.[20] Similarly, online grocery consumers are greener than offline consumers for all plausible parameter values—again because food waste effects strongly dominate travel-related

[20]In addition to the baseline parameter values described in Tables 1 and 2, we compared the emissions of the two revenue models for many other cities and a wide range of parameter values (see the values used in Figure 8.2).

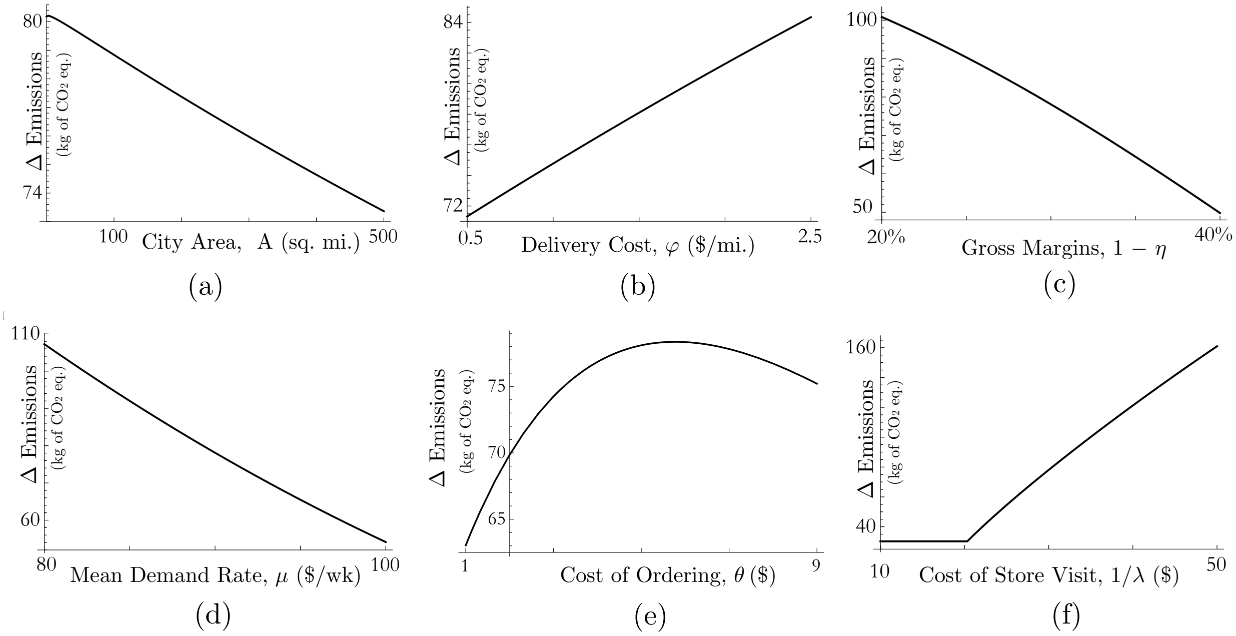


FIGURE 8.2. Annual Per Capita Reduction in Emissions under Subscription Pricing
Note: Unless stated otherwise, $\rho = 12,500$ hh/sq. mi, $A = 150$ sq. mi, $\varphi = 1.5$ \$/mi, $\eta = .75$, $\zeta = 0.76$, $K = 15$, $1/\lambda = 30\$$, $\theta = 5\$$, $T = 1$ wk, $\mu = 90$ \$/wk, and $\delta = 365$.

effects. Importantly, these results stand even if we use very conservative estimates of unit emissions from food waste (that derive from 100% vegetarian diets), values that are as little as one-third of the typical US estimate.

In practical terms, the subscription model's emissions advantage over the per-order model amounts to between 5% and 15% of the food waste emissions created by an average citizen of the Western world. This means that *the combination of subscription pricing and online grocery retailing can significantly reduce emissions from the food supply chain.* Finally, our findings indicate that food waste plays a much more important role in the emissions impact of grocery retail than do the travel-related emissions upon which previous studies focused (Cachon, 2014); hence a reexamination of those results might be warranted.

Further inspection of Figure 8.2 reveals that the emissions advantage of the subscription model is the most pronounced: for small cities, where the driving disadvantage is small; for low margins, high delivery costs, and high store visit costs (all of which lead to waste increases due to lower adoption rates); for high mean demand, which reduces the waste difference; and for moderate costs of ordering (in the subscription model, waste is increasing in θ; eventually, this increase approaches and exceeds the relatively modest effect of θ under the per-order model).

9. Discussion

Much as we do in this paper, the existing literature that compares subscription and per-order pricing considers the two offerings in isolation yet does not consider their combination in a two-part tariff; see, for example, Barro and Romer (1987); Danaher (2002); Randhawa and Kumar (2008); Cachon and Feldman (2011). As this literature highlights, two-part tariffs are rarely used in consumer settings because customers dislike paying twice and a two-part tariff removes the behavioral incentive of "unlimited free usage". These considerations apply also in the case of online grocery delivery. Yet a firm could offer—at least theoretically—a two-part tariff and arguably do no worse than implementing either subscription or per-order pricing in isolation. This statement is almost true in our setup, but with one major caveat. We can show that a two-part tariff capable of outperforming the two revenue models considered in this paper will likely need to charge *negative* subscription fees or per-order prices. It is safe to say that a two-part tariff with negative tariffs is even less practical and would rarely be feasible. Therefore, the firm will resort to choosing either the subscription model or the per-order model, and all of our preceding analysis can be applied directly to help it identify which revenue model is preferable.

Our model considers a fresh-grocery retailer and customers who only order fresh groceries. Customers may prefer to order perishable and durable items in one order and thus even fresh grocery retailers will often stock both. If customers purchase a mix of perishable and durable grocery products from our focal retailer, the relevant customer ordering behavior (frequency and size of orders) is driven only by the consumption dynamics of the perishable products; the durable products are simply batched with the time-dependent perishable orders. Non-perishable products can thus be included in the analysis with minor modifications; the durable items are added to the order streams in both pricing models and the comparison of different revenue models in unaffected.

In our model, the firm has a warehouse whose location is perfectly centered in the delivery area. A centrally located warehouse is ideal, of course, but this is not always possible owing to real estate costs, geographic factors, and so forth. When the warehouse is not centrally located, delivery distances are longer and so the order frequency advantage of the per-order model is enhanced. Our model can also be easily extended to the case where the firm divides the catchment area into multiple zones and builds different warehouses in each of them. Delivery distances are now lower and the order frequency advantage is diminished.

In our model there is no lead time between ordering and receiving the order. Positive lead times will result in more waste, thus increasing the food waste disadvantage of the per-order model. A random shelf life of the grocery basket has a similar effect, it also increases food waste, increasing the food waste disadvantage of the per-order model and, thus, making subscription model more desirable.

We do not explicitly consider issues arising from the retailer's inventory and fleet size management. The pricing model may influence the day-to-day variability in orders to be delivered which affects inventory management related costs. Further, the model that induces lower inter-temporal variability at the warehouse will require less vehicles to provide the same service level for on-time order deliveries. In addition to other factors, the basket size itself might impact the number of orders delivered by one vehicle. The per-order model has larger orders, this might lead to lower number of orders delivered by one delivery vehicle making the order frequency advantage of the per-order model less than estimated by our model.

We have assumed that the fixed costs of building and servicing the per-order and subscription models are the same—that is, costs that depend neither on the volume of groceries sold nor on the number of orders serviced are the same in each model. That being said, incorporating such costs differences would be but a minor extension.

In addition to effects captured in this paper the online grocery retailers might need to consider other advantages of subscription pricing. Cachon and Feldman (2011) and Gilbert et al. (2014) describe a second-order revenue-extracting benefit of subscription pricing that operates in addition to the lower ordering costs already captured in our analysis. Under subscription pricing consumers purchase in advance of the service and, thus, are willing to pay their expected value for the service. In contrast, when consumers spot purchase, i.e., when they know their value for the service, they are naturally willing to pay only their realized value. It might be better to sell in advance to every customer at their expected value than to sell in the spot market to a portion of consumers (i.e., those consumers with a high realized value). In our model this effect would operate when each customer's cost of going to the offline store is randomly drawn every period, in addition to the long-term heterogeneity already captured in our model.

Behavioral phenomena can also alter customer preferences for different pricing schemes. Empirical analysis by Danaher (2002) shows that customers have different sensitivity to subscription and per-order fees. There might be other behavioral differences between the two models–such as the ability to pick groceries, the physical exercise benefits of grocery shopping, etc.; these can be easily incorporated through the adjustment of the store visit costs.

Another simplifying assumption was that the per-unit prices of grocery are the same in both revenue models and also in the offline store. Obviously, that may not be the case. Lower prices can increase adoption, thereby enhancing the relevant model's effect and vice versa. It is also worth mentioning that customers may have different grocery demand rates. Yet because the firm cannot ex ante discriminate among different customers and so must offer different market segments the same pricing model, our analysis provides a close first-order approximation. If there are indeed widely different customer segments, then offering a menu of contracts might help the retailer select which segments to serve and, as needed, help it steer different segments toward different revenue models. Further analysis of this remains an open question and a promising avenue for future work. Further, we considered an online retailer that competes with offline stores. While this is the case in almost all major markets an extended model might consider the competition between different online retailers. Our model captures competitive aspects through the customer's outside option–cost of buying offline. Under online competition this cost could be substituted by the customers outside *online* option. This analysis is another interesting prospect for future work.

An interesting recent development in the online retailing space is the use of logistics structures other than the traditional warehouse–delivery vehicle model considered in this paper. Pioneering firms are now experimenting with crowd-sourced distribution schemes (e.g., Instacart) and the use of autonomous flying vehicles or drones (Amazon). These developments highlight the increasing importance of the grocery retail sector and the need to find viable business models in this large but hitherto untapped category. These new delivery models are certain to have different economic characteristics than the logistics model considered in this paper, and they are intriguing prospects for future study.

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Appendix A. Proofs

All statements are given in their order of appearance in the main text.

Lemma 5. *A (Q, r) policy with $r = 0$ is an optimal continuous review consumer ordering strategy.*

Proof. This follows from Theorem 3 in Weiss (1980) after observing that, in our setting, $h(W) = 0$ and the penalty cost $p = \infty$.

$\square$

Lemma 6. *The expected length of cycles when the customer orders Q units at a time, $E[CT_i(Q)]$, is given by $\frac{1}{\mu}\left(Q - \sum_{j=0}^{Q}(Q-j)p_j\,(\mu T)\right)$.*

Proof. Since $CT_i(Q)$ is a nonnegative random variable, its expectation can be expressed as follows:
$E[CT_i(Q)] = \int_0^{+\infty} \Pr\{CT_i(Q) > t\}dt = \int_0^T \Pr\{CT_i(Q) > t\}dt = \int_0^T \Pr\{\text{Number of events at a time } t < Q\}dt = \int_0^T \sum_0^{Q-1} p_j\,(\mu t)\,dt = \sum_0^Q \int_0^T p_j\,(\mu t)\,dt = \frac{1}{\mu}\left(Q - \sum_{j=0}^{Q}(Q-j)p_j\,(\mu T)\right)$.

$\square$

Proof of Lemma 1 . (i) First, after some algebraic manipulations one can establish that the customer's expected average long-run cost $(a+Q)\left(Q - \sum_{j=0}^{Q}(Q-j)p_j\,(\mu T)\right)^{-1}$ is first decreasing and then increasing in Q. Therefore, the optimal quantity is either the highest Q such that $\frac{a+Q+1}{\frac{1}{\mu}\left(Q+1-\sum_{j=0}^{Q+1}(Q+1-j)p_j(\mu T)\right)} - \frac{a+Q}{\frac{1}{\mu}\left(Q-\sum_{j=0}^{Q}(Q-j)p_j(\mu T)\right)} \leq 0$ or the lowest Q such that $\frac{a+Q+1}{\frac{1}{\mu}\left(Q+1-\sum_{j=0}^{Q+1}(Q+1-j)p_j(\mu T)\right)} - \frac{a+Q}{\frac{1}{\mu}\left(Q-\sum_{j=0}^{Q}(Q-j)p_j(\mu T)\right)} \geq 0$.
For high enough μT, a random variable distributed as Poisson(μT) may (by the central limit theorem) be considered approximately normal. Hence we can rewrite the expected waste during a cycle to obtain

$$\sum_{j=0}^{Q}(Q-j)p_j\,(\mu T) \approx \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Q}(Q-x)e^{-\frac{1}{2}\frac{(x-\mu T)^2}{\sigma^2}}dx \equiv \int_{-\infty}^{Q}(Q-x)f(x)dx$$

with $\sigma = \sqrt{\mu T}$; $\left(1 - \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Q} e^{-\frac{1}{2}\frac{(x-\mu T)^2}{\sigma^2}}dx\right) \equiv \bar{F}(Q)$

(ii) Q^* is given as a solution to $Q - \int_{-\infty}^{Q}(Q-x)f(x)dx = (a+Q)\,\bar{F}(Q)$, which we can simplify as $\int_{-\infty}^{Q} xf(x)dx = a\bar{F}(Q)$. Taking the derivative with respect to a, $Qf(Q)\partial_a Q = \bar{F}(Q) - af(Q)\partial_a Q$, we obtain $\partial_a Q = \frac{\bar{F}(Q)}{f(Q)(Q+a)} > 0$. Similarly, $N(Q) = \mu\left(Q - \sum_{n=0}^{Q}(Q-n)p_n\,(\mu T)\right)^{-1} \approx \mu[Q - \int_{-\infty}^{Q}(Q-x)f(x)dx]^{-1}$. Taking the derivative with respect to Q yields $\partial_Q N(Q) = -\frac{1}{\mu}N(Q)^2\bar{F}(Q) < 0$ and so $\partial_a N(Q) = \partial_Q N(Q)\partial_a Q < 0$. Finally, $\frac{\partial\left(N(Q^*)Q^*\right)}{\partial a} = \partial_a(N(Q^*)Q^*) = \mu\partial_Q[1 - \int_{-\infty}^{Q}(1-\frac{x}{Q})f(x)dx]^{-1}\partial_a Q = \frac{1}{\mu}\cdot[N(Q)]^2 a\bar{F}(Q)\partial_a Q > 0$.

(iii) $\partial_a\,(a+Q^*)\cdot N\,(Q^*) = N(Q^*) + \partial_a Q^* \cdot \partial_Q\,((a+Q)\cdot N\,(Q)) = N(Q^*) > 0$ because, at Q^*, $\partial_Q\,((a+Q)\cdot N\,(Q)) = 0$.

(iv) We further show that $\partial_a(aN_a) > 0$. Note that $\partial_a(aN_a) = N_a + a\frac{\partial N^*}{\partial a} = N_a(1 - \frac{a\bar{F}(Q)}{f(Q)(Q+a)^2})$ and that Q is implicitly defined by $\int_{-\infty}^{Q} xf(x)dx = a\bar{F}(Q)$; therefore, $\frac{a\bar{F}(Q)}{f(Q)(Q+a)^2} \approx \frac{\int_{-\infty}^{Q} x\frac{f(x)}{f(Q)}dx}{(Q+a)^2}$. For μT sufficiently high we have $\int_{-\infty}^{Q} xf(x)dx \approx \int_0^Q xf(x)dx$ (this must be true for the approximation of Poisson to be reasonable). Now, if $Q < \mu T$

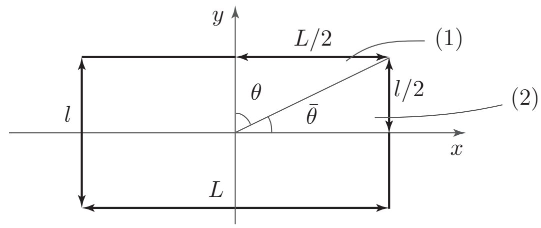

FIGURE A.1. Partitioning of a Rectangle

then $\int_{-\infty}^{Q} x\frac{f(x)}{f(Q)}dx \approx \int_0^Q x\frac{f(x)}{f(Q)}dx \leq \int_0^Q xdx = \frac{1}{2}Q^2$. Thus we obtain $\frac{a\bar{F}(Q)}{f(Q)(Q+a)^2} \leq \frac{\frac{1}{2}Q^2}{(Q+a)^2} < 1$. Furthermore, if $Q > \mu T$ then it follows (from IFR property of normal distribution) that $\frac{f(Q)}{\bar{F}(Q)} > \frac{f(\mu T)}{\bar{F}(\mu T)} = \frac{2}{\sqrt{2\pi\mu T}}$. That is, we need to show $\frac{1}{\sqrt{\frac{\pi}{2}\mu T}} > \frac{a}{(Q+a)}\frac{1}{(Q+a)}$ or $1 > \frac{a}{(Q+a)}\frac{\sqrt{\frac{\pi}{2}\mu T}}{(Q+a)}$. These inequalities hold if $\frac{\pi}{2} < \mu T$, and that expression is true by our assumption $\mu T \gg 0$.

Lemma 7. *The average distance from the warehouse to a point in the market area is given by Equation A.1 for a regular m-gon and by Equation A.2 for a rectangle with length-to-height ratio of $\gamma \geq 1$. Formally,*

$$\bar{d}_{\Diamond} = \sqrt{A}\zeta_{\Diamond};\ \zeta_{\Diamond} = \frac{2}{3}\sqrt{\frac{1}{m\cdot\tan\Phi}}\left(\sqrt{1+\tan^2\Phi} + \ln(\tan\Phi + \sqrt{1+\tan^2\Phi})\cdot\tan^{-1}\Phi\right),\ \Phi = \pi m^{-1}; \tag{A.1}$$

$$\bar{d}_{\square} = \sqrt{A}\zeta_{\square};\ \zeta_{\square} = \frac{1}{3}\sqrt{\frac{1}{\gamma}}\left(\sqrt{\gamma^2+1} + \frac{1}{2}\left(\gamma^2\ln\left(\frac{\sqrt{\gamma^2+1}+1}{\gamma}\right) + \frac{\ln\left(\sqrt{\gamma^2+1}+\gamma\right)}{\gamma}\right)\right). \tag{A.2}$$

Proof. Cachon (2014) gives an expression for a regular m-gon. We can derive an expression for a circle and a square by using Equation A.1 and setting (respectively) $m = \infty$ and $m = 4$.

For a rectangle, the average round-trip distance from the warehouse to a point in market area 1 (see Figure A.1), $\bar{d}_1$, and one in area 2, $\bar{d}_2$, can be calculated as follows:

$$\bar{d}_1 = 2\cdot\frac{\int_0^{\frac{l}{2}}\int_0^{y\tan\theta}\sqrt{x^2+y^2}dxdy}{\frac{1}{2}(\frac{l}{2})^2\tan\theta} \quad \text{and} \quad \bar{d}_2 = 2\cdot\frac{\int_0^{\frac{L}{2}}\int_0^{x\tan\bar{\theta}}\sqrt{x^2+y^2}dydx}{\frac{1}{2}(\frac{L}{2})^2\tan\bar{\theta}}.$$

We have $\tan\theta = \frac{L}{l} = \gamma$ and $\tan\bar{\theta} = \frac{l}{L} = \frac{1}{\gamma}$; moreover, $l = \sqrt{\frac{A}{\gamma}}$ and $L = \sqrt{A\gamma}$. Hence $\bar{d}_1 = \frac{1}{3}\sqrt{\frac{A}{\gamma}}(\sqrt{1+\gamma^2} + \frac{1}{\gamma}\ln[\gamma + \sqrt{1+\gamma^2}])$ and $\bar{d}_2 = \frac{1}{3}\sqrt{\gamma A}(\frac{\sqrt{1+\gamma^2}}{\gamma} + \gamma\ln[\frac{1}{\gamma} + \frac{\sqrt{1+\gamma^2}}{\gamma}])$. Finally, there are an equal number of square miles in area 1 and area 2. This means that $\bar{d} = \frac{1}{2}(\bar{d}_1 + \bar{d}_2)$ and so Equation A.2 follows.

□

Proof of Lemma 2. (i) We are looking to find $\bar{D}(\bar{\rho}, N, \mathscr{A}, K) = \frac{1}{I}\sum_{i=1}^{I} D_i = \frac{2}{K}\frac{1}{I}\sum_{i=1}^{I} d_i - \frac{L}{K} + TSP^*(K, L, l)$ (since all sectors are partitioned in the same fashion and contain K points). Daganzo (1984b) offers the following approximation of the salesman tour:

$$TSP^*(K,L,l) = \frac{\phi(\beta K)}{\sqrt{\rho_d}}, \text{ where } \phi(x) = \begin{cases} 0.9, & x \geq 12; \\ \frac{\sqrt{x}}{6} + \frac{2}{\sqrt{x}}\frac{2}{(x/4)^2}[(1+\frac{x}{4})\log(1+\frac{x}{4}) - \frac{x}{4}], & x < 12. \end{cases} \tag{A.3}$$

Using Lemma 7 and Equation A.3 yields $\bar{D}(\bar{\rho}, N, \mathscr{A}, K) = \frac{2}{K}\bar{d} - \frac{L}{K} + \frac{\phi(\beta K)}{\sqrt{\rho_d}}$; furthermore, $L \cdot l = K\rho_d^{-1}$ and $\beta = \frac{l}{L}$, which means that $L = \sqrt{\frac{K}{\beta\rho_d}}$ and so $\bar{D}(\bar{\rho}, N, \mathscr{A}, K) = \frac{2}{K}\bar{d} - \rho_d^{-\frac{1}{2}}((\beta K)^{-\frac{1}{2}} + \phi(\beta K))$. Optimizing with respect to β, we obtain the *optimal* "slenderness factor":

$$\beta^* = \begin{cases} 1, & K \in [1,7]; \\ \frac{6.7}{K}, & K \geq 7. \end{cases} \tag{A.4}$$

Finally, for $K \leq 4$ the approximation displayed as Equation 5.1 is not very accurate; Daganzo (1984a) provides the more accurate expression $\bar{D}(\bar{\rho}, N, \mathscr{A}, K) = \frac{2}{K}\bar{d} - \rho_d^{-\frac{1}{2}}(\frac{(K-2)^+}{K+1}(\beta K)^{-\frac{1}{2}} + \frac{K-1}{K}\phi(\beta K))$. Combining this equality with the results from Equations A.1, A.2, and A.4 establishes the congruence displayed as Equation 5.2, where the factor $\lambda(K)$ is given as follows:

$$\lambda(K) = \begin{cases} \frac{(K-2)^+}{K+1}\frac{1}{\sqrt{\beta^* K}} + \frac{K-1}{K}\phi(\beta^* K), & K \leq 4; \\ \phi(\beta^* K) - \frac{1}{\sqrt{\beta^* K}}, & K > 4. \end{cases} \tag{A.5}$$

(ii) We have $\partial_N \bar{D} = -\lambda(K)\frac{1}{2N}\sqrt{\frac{\delta}{\bar{\rho}N}} < 0$ and $\partial_{\bar{\rho}} \bar{D} = -\lambda(K)\frac{1}{2\bar{\rho}}\sqrt{\frac{\delta}{\bar{\rho}N}} < 0$. Now $\partial_a \bar{D} = (\partial_N \bar{D})\partial_a N > 0$, since $\partial_a N < 0$ (by Lemma 1); therefore,$\partial_N N\bar{D} = \frac{2\zeta\sqrt{A}}{K} + \lambda(K)\frac{1}{2}\sqrt{\frac{\delta}{\bar{\rho}N}} > 0$ and $\partial_a N\bar{D} = (\partial_N N\bar{D})\partial_a N < 0$.

Lemma 8. *Iff the customer with store visit cost $\tilde{\alpha}$ chooses the online retailer then customers with store visit costs $\alpha \geq \tilde{\alpha}$ will also choose to shop online.*

Proof. Since $C_{off} = C_\alpha$ is an increasing function of α, it follows that all customers for whom $\alpha \geq \tilde{\alpha}$ will purchase online and the rest will purchase offline.

□

Proof of Lemma 3. The firm seeks to maximize its profit:

$$\pi_o \quad = \quad \max_o \left(\left(o + Q_o\right) N_o - h_o(o)\right) \cdot \bar{G}\left(\tilde{\alpha}\right); \ \tilde{\alpha} = \min\left\{\alpha \in [\underline{\alpha}, \bar{\alpha}] \text{ s.t.} \, C_{off} \geq C_o\right\} \tag{A.6}$$

Next we show that the constraint $C_{off} = C_\alpha \geq C_o$ is always binding. When it is binding the firm will charge $o + \theta = \tilde{\alpha}$ and we can reformulate the problem as

$$\max_{\underline{\alpha} \leq \alpha \leq \bar{\alpha}} \left(C_\alpha - h_o(\alpha)\right) \bar{G}(\alpha). \tag{A.7}$$

Otherwise, the firm is solving $\max_{0 \leq o \leq \underline{\alpha} - \theta} \left(\left(o + \theta + Q_o\right) N_o - (y + \eta Q_o + c_p + \theta)N_o - z\sqrt{N_o}\right)$. Recall that $Q_o = Q^*_{o+\theta}$. First observe that $\left(o + \theta + Q_o\right) N_o$ is an increasing concave function because $\partial_o \left(o + \theta + Q_o\right) N_o = N_o > 0$ and

$\partial_o^2 (o+\theta+Q_o) N_o = \partial_o N_o < 0$. Now, for $o \leq \underline{\alpha} - \theta$ both $(y+\eta Q_o + c_p + \theta) N_o$ and $\varphi z \sqrt{N_o}$ are decreasing convex functions (since $\partial_o \sqrt{N_o} = \frac{1}{2} N_o^{-1/2} \partial_o N_o < 0$ and $\partial_o^2 \sqrt{N_o} = -\frac{1}{4} N_o^{-3/2} \partial_o N_o + \frac{1}{2} N_o^{-1/2} \partial_o^2 N_o > 0$. We can therefore conclude that the objective function is an increasing concave function and so its maximum is achieved at $o = \underline{\alpha} - \theta$, which is covered in A.7 formulation.

Since $C_{off} = C_\alpha$ is an increasing function of α, it follows that all customers for whom $\alpha > \alpha_o^*$ will purchase online and the rest will purchase offline. The equation for determining the optimal solution follows from taking the derivative of A.7, and that solution's uniqueness follows from the concavity of A.7. Note that the solution might sometimes be a corner solution—that is,

$$\alpha_o^* = \begin{cases} \bar{\alpha}, & \text{if } (N_\alpha - \partial_\alpha h_o(\alpha)) \cdot \bar{G}(\alpha) - g(\alpha) \cdot (C_\alpha - h_o(\alpha)) > 0 \text{ for all } \alpha; \\ \underline{\alpha}, & \text{if } (N_\alpha - \partial_\alpha h_o(\alpha)) \cdot \bar{G}(\alpha) - g(\alpha) \cdot (C_\alpha - h_o(\alpha)) < 0 \text{ for all } \alpha. \end{cases}$$

Proof of Lemma 4. The proof follows along the lines of that for Lemma 3. As in that case, the solution may turn out to be a corner solution:

$$\alpha_s^* = \begin{cases} \bar{\alpha}, & \text{if } (N_\alpha - \partial_\alpha h_s(\alpha))\bar{G}(\alpha) - g(\alpha)(C_\alpha - h_s(\alpha)) > 0 \text{ for all } \alpha; \\ \underline{\alpha}, & \text{if } (N_\alpha - \partial_\alpha h_s(\alpha))\bar{G}(\alpha) - g(\alpha)(C_\alpha - h_s(\alpha)) < 0 \text{ for all } \alpha. \end{cases}$$

Proof of Theorem 1. The theorem's statements follow from Lemmas 1, 3, and 4 combined with the higher cost of ordering under the per-order model, $\theta < \underline{\alpha} \leq \alpha_o^*$.

Proof of Corollary to Theorem 1. The yearly expected amount of groceries that is spoiled and wasted can be expressed as $(Q_\cdot) \cdot N_\cdot - \mu$. Theorem 1 establishes that $(Q_\cdot) \cdot N_\cdot$ is higher in the per-order model, so the annual amount of wasted groceries is higher in that model.

Lemma 9. *Market coverage under both pricing schemes is decreasing with ζ; that is, α_s^* and α_o^* are increasing in ζ, i.e. $\frac{\partial \alpha_s^*}{\partial \zeta}, \frac{\partial \alpha_o^*}{\partial \zeta} > 0$.*

Proof. We first establish that α_s^* is increasing in ζ, i.e. $\frac{\partial \alpha_s^*}{\partial \zeta} > 0$. To find the optimal α_s^*, the firm solves $\max_{\underline{\alpha} \leq \alpha \leq \bar{\alpha}} (C_\alpha - h_s(\alpha)) \bar{G}(\alpha)$. Since the firm can guarantee a zero profit by choosing $\alpha = \bar{\alpha}$, it follows for any $\alpha_s^* < \bar{\alpha}$ that

$$C_{\alpha_s^*} - h_s(\alpha_s^*) > 0. \tag{A.8}$$

By Lemma 4, α_s^* is implicitly defined by

$$N_{\alpha_s^*} - \partial_{\alpha_s^*} h_s(\alpha_s^*) = \frac{g(\alpha_s^*)}{\bar{G}(\alpha_s^*)} (C_{\alpha_s^*} - h_s(\alpha_s^*)). \tag{A.9}$$

The next is determining the behavior of α_s^* with ζ. We can use the implicit function theorem and our optimality condition to obtain

$$\frac{\partial \alpha_s^*}{\partial \zeta} = \frac{\frac{g(\alpha_s^*)}{\bar{G}(\alpha_s^*)}\frac{1}{2}\varphi \cdot \frac{2\sqrt{A}}{K} N_s}{(\partial_{\alpha_s^*}\frac{g(\alpha_s^*)}{\bar{G}(\alpha_s^*)})(C_{\alpha_s^*} - h_s(\alpha_s^*)) + \frac{g(\alpha_s^*)}{\bar{G}(\alpha_s^*)}\frac{g(\alpha_s^*)}{\bar{G}(\alpha_s^*)}(C_{\alpha_s^*} - h_s(\alpha_s^*)) - \partial_{\alpha_s^*} N_{\alpha_s^*} + \partial^2_{\alpha_s^*} h_s(\alpha_s^*)}.$$

The numerator is positive because $\frac{g(\alpha)}{\bar{G}(\alpha)} = \lambda > 0$, so it remains only to establish that the denominator is positive. In the proof of Lemma 1 we showed that $\frac{\partial N_\alpha}{\partial \alpha} < 0$; note also that the first and second summands are positive because $\frac{g(\alpha)}{\bar{G}(\alpha)} = \lambda > 0$ and $C_{\alpha_s^*} - h_s(\alpha_s^*) > 0$ (see A.8). Finally, the last summand is positive because $\partial_\alpha(\partial_\alpha h_s) \mid_{\alpha_s^*} = \partial_\alpha \left(\frac{1}{2}\frac{g(\alpha)}{\bar{G}(\alpha)}\varphi z\sqrt{\frac{N_s}{\bar{G}(\alpha)}} \mid_{\alpha_s^*}\right) = \frac{1}{4}\frac{g(\alpha)}{\bar{G}(\alpha)}\frac{g(\alpha)}{\bar{G}(\alpha)}\varphi z\sqrt{\frac{N_s}{\bar{G}(\alpha)}} + \frac{1}{2}\frac{\partial \frac{g(\alpha)}{\bar{G}(\alpha)}}{\partial \alpha} \mid_{\alpha_s^*} \varphi z\sqrt{\frac{N_s}{\bar{G}(\alpha)}} > 0$.
Similarly, α_o^* is increasing in ζ, i.e. $\frac{\partial \alpha_o^*}{\partial \zeta} > 0$ and

$$\frac{\partial \alpha_o^*}{\partial \zeta} = \frac{\frac{g(\alpha_o^*)}{\bar{G}(\alpha_o^*)}\frac{1}{2}\varphi \cdot \frac{2\sqrt{A}}{K} N_{\alpha_o^*}}{(\partial_{\alpha_o^*}\frac{g(\alpha_o^*)}{\bar{G}(\alpha_o^*)})(C_{\alpha_o^*} - h_o(\alpha_o^*)) + \frac{g(\alpha_o^*)}{\bar{G}(\alpha_0^*)}\frac{g(\alpha_o^*)}{\bar{G}(\alpha_o^*)}(C_{\alpha_o^*} - h_o(\alpha_o^*)) - \partial_{\alpha_o^*} N_{\alpha_o^*} + \partial^2_{\alpha_o^*} h_o(\alpha_o^*)}.$$

□

Proof of Theorem 2. First, $\frac{\partial}{\partial \zeta}(\pi_o^* - \pi_s^*) = \varphi \cdot \frac{2\sqrt{A}}{K}\{N_s\bar{G}(\alpha_s^*) - N_{\alpha_o^*}\bar{G}(\alpha_o^*)\}$. Next we show that if $N_s\bar{G}(\alpha_s^*) = N_{\alpha_o^*}\bar{G}(\alpha_o^*)$ for some $\hat{\zeta}$ (which entails that $\alpha_s^* > \alpha_o^*$) and if $\pi_o^* > \pi_s^*$, then $N_s\bar{G}(\alpha_s^*) < N_{\alpha_o^*}\bar{G}(\alpha_o^*)$ for all $\zeta > \hat{\zeta}$. To establish these conditions we need only show that $\frac{\partial \alpha_s^*}{\partial \zeta} > \frac{\partial \alpha_o^*}{\partial \zeta}$ for $\zeta > \hat{\zeta}$. Now, for the exponential distribution $\frac{g(\alpha)}{\bar{G}(\alpha)} = \lambda$, thus $\partial_{\alpha_o^*}\frac{g(\alpha_o^*)}{\bar{G}(\alpha_o^*)} = 0$; hence we can rewrite expressions for $\frac{\partial \alpha_s^*}{\partial \zeta}, \frac{\partial \alpha_o^*}{\partial \zeta}$ (derived in Lemma 9) and obtain

$$\frac{\partial \alpha_s^*}{\partial \zeta} = \frac{\lambda\frac{1}{2}\varphi \cdot \frac{2\sqrt{A}}{K}N_s}{\lambda^2(C_{\alpha_s^*} - h_s(\alpha_s^*)) - \partial_{\alpha_s^*}N_{\alpha_s^*} + \partial^2_{\alpha_s^*}h_s(\alpha_s^*)}; \quad \frac{\partial \alpha_o^*}{\partial \zeta} = \frac{1}{\frac{N_s}{N_{\alpha_o^*}}}\frac{\lambda\frac{1}{2}\varphi \cdot \frac{2\sqrt{A}}{K}N_s}{\lambda^2(C_{\alpha_o^*} - h_o(\alpha_o^*)) - \partial_{\alpha_o^*}N_{\alpha_o^*} + \partial^2_{\alpha_o^*}h_o(\alpha_o^*)}.$$

The two numerators are equal and we denote the respective denominators den_s and den_o.

Next we show that $\text{den}_o > \text{den}_s$ for $\zeta > \hat{\zeta}$:

$\text{den}_o - \text{den}_s = \lambda^2\{\frac{N_s}{N_{\alpha_o^*}}C_{\alpha_o^*} - \frac{N_s}{N_{\alpha_o^*}}h_o(\alpha_o^*) - (C_{\alpha_s^*} - h_s(\alpha_s^*))\} + \{\partial_{\alpha_s^*}N_{\alpha_s^*} - \frac{N_s}{N_{\alpha_o^*}}\partial_{\alpha_o^*}N_{\alpha_o^*}\} + \{\frac{N_s}{N_{\alpha_o^*}}\partial^2_{\alpha_o^*}h_o(\alpha_o^*) - \partial^2_{\alpha_s^*}h_s(\alpha_s^*)\}$.
We start by showing that the second summand is positive. We have already established that $\partial_\alpha N_\alpha = -N_\alpha\frac{1}{(\alpha+Q_\alpha)^2}\frac{\bar{F}(Q_\alpha)}{f(Q_\alpha)}$, so $\partial_{\alpha_s^*}N_{\alpha_s^*} - \frac{N_s}{N_{\alpha_o^*}}\partial_{\alpha_o^*}N_{\alpha_o^*} = \frac{N_s}{(\alpha_o^*+Q_{\alpha_o^*})^2}\frac{\bar{F}(Q_{\alpha_o^*})}{f(Q_{\alpha_o^*})} - \frac{N_{\alpha_s^*}}{(\alpha_s^*+Q_{\alpha_s^*})^2}\frac{\bar{F}(Q_{\alpha_s^*})}{f(Q_{\alpha_s^*})} > 0$. The third summand is also positive, $\partial^2_{\alpha_o^*}h_o(\alpha_o^*) - \frac{N_{\alpha_o^*}}{N_s}\partial^2_{\alpha_s^*}h_s(\alpha_s^*) > 0$, as $\frac{N_{\alpha_o^*}}{N_s}\partial^2_{\alpha_s^*}h_s(\alpha_s^*) = \frac{z}{4}\lambda^2N_{\alpha_o^*}\sqrt{\frac{1}{N_s\bar{G}(\alpha_s^*)}}$ and $\partial^2_{\alpha_o^*}h_o(\alpha_o^*) \geq \frac{z}{4}\lambda^2\sqrt{\frac{N_{\alpha_o^*}}{\bar{G}(\alpha_o^*)}}$; therefore, $\partial^2_{\alpha_o^*}h_o(\alpha_o^*) - \frac{N_{\alpha_o^*}}{N_s}\partial^2_{\alpha_s^*}h_s(\alpha_s^*) \geq \varphi\frac{z}{4}\lambda^2N_{\alpha_o^*}(\sqrt{\frac{1}{N_{\alpha_o^*}\bar{G}(\alpha_o^*)}} - \sqrt{\frac{1}{N_s\bar{G}(\alpha_s^*)}}) > 0$. Finally, the first summand is also positive, $\frac{N_s}{N_{\alpha_o^*}}(C_{\alpha_o^*} - h_o(\alpha_o^*)) - (C_{\alpha_s^*} - h_s(\alpha_s^*)) > 0$, and we can rewrite it as follows: $(\alpha_o^* + (1-\eta)Q_{\alpha_o^*})N_s - (\alpha_s^* + Q_{\alpha_s^*})N_{\alpha_s^*} + N_s\eta Q_s + zN_s(\sqrt{\frac{1}{N_s\bar{G}(\alpha_s^*)}} - \sqrt{\frac{1}{N_{\alpha_o^*}\bar{G}(\alpha_o^*)}})$. We can now show that $(\alpha_o^* + (1-\eta)Q_{\alpha_o^*})N_s - (\alpha_s^* + Q_{\alpha_s^*})N_{\alpha_s^*} + N_s\eta Q_s$ is increasing in ζ: $\partial_\zeta(\alpha_o^* + (1-\eta)Q_{\alpha_o^*})N_s - (\alpha_s^* + Q_{\alpha_s^*})N_{\alpha_s^*} = (\partial_\zeta\alpha_o^* + (1-\eta)\partial_\zeta Q_{\alpha_o^*})N_s - \partial_\zeta N_{\alpha_s^*}\partial_\zeta\alpha_s^* > 0$. Furthermore, at $\hat{\zeta}$ $\frac{N_s}{N_{\alpha_o^*}}(C_{\alpha_o^*} - h_o(\alpha_o^*)) - (C_{\alpha_s^*} - h_s(\alpha_s^*)) > 0$ if $\pi_o^* > \pi_s^*$; hence $\frac{N_s}{N_{\alpha_o^*}}(C_{\alpha_o^*} - h_o(\alpha_o^*)) - (C_{\alpha_s^*} - h_s(\alpha_s^*)) > 0$ for all $\zeta > \hat{\zeta}$. This means that, for all $\zeta > \hat{\zeta}$, we have $\partial_\zeta\alpha_s^* > \partial_\zeta\alpha_s^*$ and so $\alpha_s^* > \alpha_o^*$. That is, $\frac{\partial}{\partial \zeta}(\pi_o^* - \pi_s^*) < 0$ for $\zeta > \hat{\zeta}$.

Proof of Theorem 3. By definition, $\bar{\zeta}$ is such that $\pi_s^* = \pi_o^*$; that is,

$$\left(\frac{\mu}{\bar{F}(Q_{\alpha_s^*})} - \tilde{h}_s(\alpha_s^*)\right)\bar{G}(\alpha_s^*) - z\sqrt{N_s}\sqrt{\bar{G}(\alpha_s^*)} = \left((\frac{\mu}{\bar{F}(Q_{\alpha_o^*})} - \tilde{h}_o(\alpha_o^*)\right)\bar{G}(\alpha_o^*) \cdot z\sqrt{N_{\alpha_o^*}}\sqrt{\bar{G}(\alpha_o^*)}$$

where $\tilde{h}_s(\alpha) = (\theta + c_p)N_s + \bar{\zeta}yN_s + \eta Q_s N_s$; $\tilde{h}_o(\alpha) = (\theta + c_p)N_\alpha + \bar{\zeta}yN_\alpha + \eta Q_\alpha N_\alpha$, $z = \varphi\lambda(K)\sqrt{\frac{\delta}{\rho}}$, and $y = \frac{2\varphi\sqrt{A}}{K}$. Using the implicit function theorem yields $\frac{\partial\bar{\zeta}}{\partial A} = -\bar{\zeta}\cdot\frac{\partial y}{\partial A}\frac{\left(\bar{G}(\alpha_o^*)N_\alpha - N_s\bar{G}(\alpha_s^*)\right)}{y(\bar{G}(\alpha_o^*)\cdot N_\alpha - \bar{G}(\alpha_s^*)N_s)} < 0$, since $\frac{\partial y}{\partial A} > 0$; also, $\frac{\partial\bar{\zeta}}{\partial\varphi} = -\bar{\zeta}\cdot\frac{\partial y}{\partial\varphi}\frac{y(\bar{G}(\alpha_o^*)\cdot N_\alpha - \bar{G}(\alpha_s^*)N_s) + z\left(\sqrt{N_{\alpha_o^*}\bar{G}(\alpha_o^*)} - \sqrt{N_s\bar{G}(\alpha_s^*)}\right)}{y(\bar{G}(\alpha_o^*)\cdot N_\alpha - \bar{G}(\alpha_s^*)N_s)} < 0$ because $\frac{\partial y}{\partial\zeta} > 0$ and $\left(\bar{G}(\alpha_o^*)N_\alpha - N_s\bar{G}(\alpha_s^*)\right)$ and $\left(\sqrt{N_{\alpha_o^*}\bar{G}(\alpha_o^*)} - \sqrt{N_s\bar{G}(\alpha_s^*)}\right)$ always have the same sign. Moreover, $\frac{\partial\bar{\zeta}}{\partial\rho} = -\bar{\zeta}\cdot\frac{\partial z}{\partial\rho}\frac{\left(\sqrt{N_{\alpha_o^*}\bar{G}(\alpha_o^*)} - \sqrt{N_s\bar{G}(\alpha_s^*)}\right)}{y(\bar{G}(\alpha_o^*)\cdot N_\alpha - \bar{G}(\alpha_s^*)N_s)} > 0$ because $\frac{\partial z}{\partial\rho} < 0$. Finally, $\frac{\partial\bar{\zeta}}{\partial K} = -\bar{\zeta}\cdot\frac{\frac{\partial z}{\partial K}\left(\sqrt{N_{\alpha_o^*}\bar{G}(\alpha_o^*)} - \sqrt{N_s\bar{G}(\alpha_s^*)}\right) + \frac{\partial y}{\partial K}\left(\bar{G}(\alpha_o^*)N_\alpha - N_s\bar{G}(\alpha_s^*)\right)}{y(\bar{G}(\alpha_o^*)\cdot N_\alpha - \bar{G}(\alpha_s^*)N_s)} > 0$ because $\frac{\partial z}{\partial K}, \frac{\partial y}{\partial K} < 0$.

Proof of Theorem 4. We can express the emission difference as follows: $E_s - E_o = e_d\cdot\left(\bar{G}(\alpha_s^*)yN_s + z\sqrt{\bar{G}(\alpha_s^*)N_s} - \bar{G}(\alpha_o^*)yN_{\alpha_o^*} - z\sqrt{\bar{G}(\alpha_o^*)N_{\alpha_o^*}}\right) + \int_{\alpha_o^*}^{\alpha_s^*}(e_p d_\alpha N_\alpha)\, g(\alpha)d\alpha +$ $+ e_f\{\bar{G}(\alpha_s^*)Q_sN_s - \bar{G}(\alpha_o^*)Q_{\alpha_o^*}N_{\alpha_o^*} + \int_{\alpha_o^*}^{\alpha_s^*}N_\alpha Q_\alpha g(\alpha)d\alpha\}$. If we set $e = e_p/e_d$ then we can write $E_s - E_o = e_d(\dagger) + e_f(*)$ for $(\dagger) \equiv \left(\bar{G}(\alpha_s^*)yN_s + z\sqrt{\bar{G}(\alpha_s^*)N_s} - \bar{G}(\alpha_o^*)yN_{\alpha_o^*} - z\sqrt{\bar{G}(\alpha_o^*)N_{\alpha_o^*}}\right) + \int_{\alpha_o^*}^{\alpha_s^*}(ed_\alpha N_\alpha)\, g(\alpha)d\alpha$; $(*) \equiv \bar{G}(\alpha_s^*)Q_sN_s - \bar{G}(\alpha_o^*)Q_{\alpha_o^*}N_{\alpha_o^*} + \int_{\alpha_o^*}^{\alpha_s^*}N_\alpha Q_\alpha g(\alpha)d\alpha$. We remark that both $(\dagger)$ and $(*)$ are independent of e_f and e_d. There are four possible combinations: (i) $(*), (\dagger) > 0$, in which case $\bar{e}_f = \infty$; (ii) $(*), (\dagger) < 0$, in which case $\bar{e}_f = 0$; (iii) $(*) < 0, (\dagger) > 0$, in which case $\bar{e}_f = -e_d\frac{(\dagger)}{(*)}$; (iv) $(*) > 0, (\dagger) < 0$. Combinations (i)–(iii) are directly covered by the theorem, so we need only rule out combination (iv).

We first consider $\alpha_o^* \geq \alpha_s^*$ which implies $\bar{G}(\alpha_s^*) \geq \bar{G}(\alpha_o^*)$. We will show that $(*) = \bar{G}(\alpha_s^*)Q_sN_s - \bar{G}(\alpha_o^*)Q_{\alpha_o^*}N_{\alpha_o^*} + \int_{\alpha_o^*}^{\alpha_s^*}N_\alpha Q_\alpha g(\alpha)d\alpha < 0$. We have $\bar{G}(\alpha_o^*)(Q_sN_s - Q_{\alpha_o^*}N_{\alpha_o^*}) \leq 0$ since $Q_sN_s < Q_{\alpha_o^*}N_{\alpha_o^*}$ (by Theorem 1). Thus, it only remains to show that $(**) \equiv (\bar{G}(\alpha_s^*) - \bar{G}(\alpha_o^*))Q_sN_s - \int_{\alpha_s^*}^{\alpha_o^*}N_\alpha Q_\alpha g(\alpha)d\alpha < 0$. Since $\int_{\alpha_s^*}^{\alpha_o^*}N_\alpha Q_\alpha g(\alpha)d\alpha > N_{\alpha_s^*}Q_{\alpha_s^*}\int_{\alpha_s^*}^{\alpha_o^*}g(\alpha)d\alpha = N_{\alpha_s^*}Q_{\alpha_s^*}(G(\alpha_o^*) - G(\alpha_s^*))$, it follows that $(**) < (\bar{G}(\alpha_s^*) - \bar{G}(\alpha_o^*))(Q_sN_s - N_{\alpha_s^*}Q_{\alpha_s^*}) < 0$. This inequality allows us to obtain the desired result $(*) < 0$, which rules out the possibility of combination (iv).

Next consider $\alpha_o^* < \alpha_s^*$, from which it follows that $\bar{G}(\alpha_o^*) \geq \bar{G}(\alpha_s^*)$. This means that, under the subscription revenue model, the retailer's coverage will be smaller and so more customers will end up traveling to the (offline) store directly. As self-travel entails higher travel distances $(\dagger) > 0$, which rules out (iv).