Sourcing Innovation: On Feedback in Contests

(forthcoming Management Science)

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It is notoriously difficult to provide outside parties with adequate incentives for innovation. Contests - in which solvers compete for a prize offered by the contest holder - have been shown to be an effective incentive mechanism. Despite considerable interest in this concept, we lack a thorough understanding of important aspects of contests; in particular, feedback from the contest holder to the solvers has received only limited attention. This paper discusses how contest holders can improve contest outcomes by devising an optimal information structure for their feedback policies. We first identify when, and when not, to give feedback as well as which type of feedback to give: public (which all solvers can observe) or private (which only the focal solver can observe). We uncover a nontrivial relationship between contest characteristics and optimal feedback choices. Second, we examine whether the contest holder should mandate interim feedback or instead allow solvers to seek feedback at their own discretion. Third, we discuss how changing the granularity of feedback information affects its value to solvers.

Keywords: Contest; Open Innovation; Innovation Incentives; Feedback and Learning; Research and Development

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1. Introduction

Firms have increasingly found it necessary to source their innovation from beyond their own boundaries (Chesbrough 2003). They often do not have, in house, the expertise needed to solve all the challenges that arise as a result of their ever more complex research and development (R&D) activities. Yet success often eludes innovation initiatives that involve outside parties; much depends on the suitability of the firm’s sourcing mechanism. One mechanism that has garnered widespread interest is the innovation contest. In organizing such a contest, the firm specifies its goals at the outset (and often the metric by which it measures goal achievement) and promises an award to the solver who best fulfills those goals; at the end of the contest, the award is granted to the solver(s) with the best solution(s). The contest mechanism offers two key benefits: (i) it offers considerable flexibility in that the firm can choose a different set of participants for each contest; and (ii) it equips the firm with powerful incentives, since contestants compete fiercely to win the contest holder’s approval and thus the award.
In light of these potential benefits, contests have been widely studied in the context of innovation and also in many other settings (Lazear and Rosen 1981, Moldovanu and Sela 2001, Siegel 2009, Ales et al. 2016). One consequence of this research interest is that a theory of contests has emerged. This theory focuses on how a contest holder can use different aspects of contest design to optimize the intensity of competition among contestants, thereby maximizing the effort exerted by contestants and, by extension, the contest’s effectiveness at providing incentives. The theory of contests has offered solutions for such diverse problems as optimal award structures (Ales et al. 2017), the optimal number of participants (Taylor 1995, Terwiesch and Xu 2008), and the optimal way of nesting contests within contests (Moldovanu and Sela 2006).

However, current theory has some gaps with respect to certain critical aspects. We highlight these gaps by considering Kaggle, an Internet platform that provides firms with the infrastructure for hosting contests on data-driven problems. When setting up such a contest, the firm must define its rules of engagement: the relevant metric (usually, out-of-sample accuracy of the predictions) and the reward(s) offered. After the contest announcement, data scientists compete against each other in developing—at their own expense of time and money—algorithms that perform the required task. The group of scientists that ultimately provides the best-performing algorithm wins the prize. So in those respects, Kaggle’s approach follows the general template of a contest. In one respect, however, it adds a fundamentally new feature: During the competition, data scientists enter preliminary versions of their code and receive feedback on how well it performs (usually in terms of how accurate its predictions are). Furthermore, Kaggle not only provides this performance feedback to the team itself but also maintains a public “leader board” so that each team (or individual participant) can observe its own performance relative to all competing submissions.

Thus Kaggle can be viewed as exemplifying a central question, faced by many contest organizers in practice, that has received but cursory attention in the academic literature: the question of optimal feedback (for notable exceptions, see Aoyagi 2010, Ederer 2010, Marinovic 2015). Performance feedback is a means by which the firm can systematically affect the amount of information held by each contestant—in particular, information about own and rivals’ competitiveness—and thereby influence contestant behavior during the rest of the contest. Put differently, the firm can augment or diminish incentives by redistributing information and in this way can manipulate the contest’s competitiveness. The question that then arises is: How, exactly, should a contest holder influence the information structure during a contest so that contestants are optimally incentivized?

Any comprehensive investigation of this issue must provide answers to the following three questions, which together constitute a feedback policy’s information structure. (i) Which solvers should
receive the feedback information? (ii) Who should decide which solvers receive feedback? (iii) What should be the information content of the performance feedback?

The importance of the first question rests on the fact that a contest holder can freely choose the recipients of feedback. More specifically, the firm may retain all information about the contest’s competitiveness (no feedback), it may inform solvers about their respective individual performance but not about the performance of others (private feedback), or it may provide information about the performance of all contestants (public feedback). These information structures naturally induce different levels of competition and hence provide contestants with different incentives. However, it is not clear which policy is most appropriate for which situation. Real-world contest holders have experimented extensively with different forms. The default mode for Kaggle is to allow all contestants to observe each contestant’s performance feedback. In contrast, the European Union (EU)—which regulates all major infrastructure, architectural design, and civil engineering contests organized within its jurisdiction—introduced in 2004 the “competitive dialogue procedure” (EU 2004) for the specific purpose of establishing a private feedback channel between contest holder and contestants. In 2010, 9% of the EU’s entire public procurement budget was spent via this contest mechanism. The use of private feedback has proven so effective that, in 2016, the World Bank introduced a similar mechanism in its procurement regulations (World Bank 2016).¹

With regard to the second question, it can be either the firm or a contestant who initiates feedback and hence a redistribution of information. In particular, the contest holder might mandate feedback or might simply provide a feedback option. In the latter case, contestants may strategically withhold their performance information to influence the contest’s information structure. Should the contest holder allow for such strategic behavior? Again, companies have devised different approaches. Kaggle, for instance, often (though not always) makes feedback voluntary, whereas performance feedback is mandatory in any contest subject to the EU’s competitive dialogue procedure.

The third question focuses on the accuracy of performance feedback. Clearly, any information about contestants’ relative competitiveness will affect their incentives and thus their behavior. But should the firm divulge all of its available information or only some of it? Kaggle issues exact rankings of contestants’ performance (i.e., their respective prediction accuracy). Yet the annual European Social Innovation Competition, which solicits ideas for building an inclusive economy, tends to provide less fine-grained (and thus merely “indicative”) feedback.

¹ Of course, for public institutions such as the EU or the World Bank, the choice of feedback policy will likely depend also on transparency and compliance rules and thus involve more than pure efficiency considerations.
In practice, the informational impact of different feedback policies is key to designing a successful contest; hence it is imperative for the contest-staging firm to answer each of those three questions. Yet the existing academic literature leaves them largely unanswered by implicitly restricting attention to the role of feedback that is public, mandatory, and fully informative (Aoyagi 2010, Ederer 2010) or at best to a specific form of public, mandatory, and noisy feedback (Marinovic 2015). Thus that literature covers too few of feedback’s dimensions and options within dimensions to have much relevance for most practical settings. Furthermore, it concentrates exclusively on firms interested in promoting the average performance of their solvers. In innovation settings, however, firms are more likely to be interested in the best performance. We contribute to the literature on contests by offering a more complete and practically relevant description of how feedback can be used in contests—whether to improve average performance or to obtain the best performance. In so doing, we consolidate the most relevant feedback policies observed in practice within a broad framework and thereby deepen our theoretical understanding of when and how to use them.

The answers we find to our three guiding questions are as follows. First, and most importantly, contest organizers (and researchers) cannot neglect private feedback. Whereas public feedback always dominates in average-performance settings, a contingency arises for contests that seek to elicit the best performance: private (resp. public) feedback is optimal for contests with high (resp. low) uncertainty. This finding is in stark contrast to the existing literature’s view, based on comparing only the cases of public and no feedback, that the feedback’s role is the same for routine projects as for highly innovative projects. Second, public feedback may be underused when it is voluntary. Contestants always seek performance feedback under a private-feedback policy but never do so under a public-feedback policy, and inducing contestants via monetary incentives to ask for public feedback yields suboptimal results. Third, concerning the effect of information granularity on the value of feedback, we find no evidence that strategically hiding information—either by reducing the information content of feedback (e.g., providing rank-only feedback rather than detailed performance feedback) or by promulgating noise—can be used to improve contest outcomes.

2. Related Literature
The question of how best to motivate innovation and creativity is a central topic of academic inquiry (see, e.g., Erat and Krishnan 2012, Ederer and Manso 2013, Bockstedt et al. 2015, Erat and Gneezy 2016). Contests as a mechanism for eliciting innovation opportunities have become a focal point of attention, figuring prominently in both the economics and the operations management literatures. In the classification of Taylor (1995), this broad literature examines two different types
of contests: (i) innovation races, in which contestants try to achieve a pre-defined and verifiable performance target (see e.g., Bimpikis et al. 2016, Halac et al. 2016); and (ii) innovation contests for solving open-ended problems, in which the firm cannot specify performance targets ex ante and rather tries to induce the best solution.

Our work falls into the second category because the assumption of a pre-defined performance target would be antithetical to our main goal: exploring how feedback can incentivize contestants to achieve optimal output on a given schedule. The literature on contests (in the narrow sense) was initiated by seminal research of Lazear and Rosen (1981), Green and Stokey (1983), and Nalebuff and Stiglitz (1983). Over the last decades, these contests have become an accepted paradigm in the study of settings that include lobbying, litigation, military conflict, sports, education, and of course R&D management (for an overview of applications, see Konrad 2009). The extant literature has addressed many contest design issues. Prominent among these is whether or not the contest should be open for everybody to enter; a larger number of entrants yields a larger number of trials (Terwiesch and Xu 2008), but restricting access increases the effort exerted by individual solvers (Taylor 1995, Fullerton and McAfee 1999, Moldovanu and Sela 2001). Bid caps have been studied as a means of limiting access to a contest (Gavious et al. 2002), and so have more advanced mechanisms such as holding an auction for the right to participate (Fullerton and McAfee 1999). Another prominent issue is the optimal award structure (Che and Gale 2003, Siegel 2009, 2010), which depends on such contingencies as the solvers’ respective cost functions (Moldovanu and Sela 2001), performance uncertainty (Ales et al. 2017), and whether the firm seeks the best solution or only to improve the average solution (Moldovanu and Sela 2006). Another major issue is the contest’s temporal structure. Should the contest designer hold a single, overarching contest or rather a series of smaller, “cascading” contests?—see Moldovanu and Sela (2006) for a discussion. Finally, the literature has also analyzed more dynamic contest formats such as elimination and round-robin contests (Yucesan 2013) as well as repeated contests between the same contestants (Konrad and Kovenock 2009).

All of these models presume that the contest holder is relatively passive during the course of the contest. However, recently, scholarly attention has been shifting toward the actions that a contest holder could take as the contest unfolds (see, e.g., Gürtler et al. 2013). The most prominent of these actions is providing (or not) interim performance feedback.

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2 This generalization is countered by Ales et al. (2016) and Körpeoğlu and Cho (2017), who give examples of contests for which individual solution efforts are increasing in the number of competitors.
The literature on feedback in contests is sparse. Although generally acknowledged to be the first in this area, the paper by Yildirim (2005) does not address feedback per se and focuses instead on information disclosure as a strategic choice made by solvers. Gershkov and Perry (2009) are likewise not primarily concerned with feedback as we understand it here; rather, these authors focus on optimally aggregating scores by combining intermediate and final reviews when the review process itself is noisy. However, there are four papers that do address feedback during contests in a more narrow sense. Goltsman and Mukherjee (2011) explore a setting in which solvers compete for a single prize by fulfilling two tasks at which each solver can either fail or succeed. Closer to our work, Aoyagi (2010), Ederer (2010), and Marinovic (2015) examine settings in which a firm provides feedback to solvers who have to make continuous effort choices.

It is noteworthy that past work on feedback in contests has yielded only preliminary answers to some aspects of the three foundational questions that shape any feedback policy’s information structure. First, all extant research restricts its attention to public feedback and neglects the class of private feedback (which is ubiquitous in practice); hence broader comparisons of different feedback policies have not been made. We solve the challenging case of private feedback and find nontrivial contingencies accounting for when private, public, or no feedback is preferable. Our results confirm the importance of private feedback for highly innovative settings and hence challenge extant research. As an aside, our analysis of private feedback contributes to the mathematical theory of contests by devising—to the best of our knowledge—the first closed-form solution of a stochastic contest with asymmetric private information. Second, previous research has considered only mandatory feedback. In other words, it implicitly assumes that solvers must provide the contest holder with intermediate solutions on which they receive feedback—an assumption often violated in practice. We examine all types of feedback with respect to mandatory versus voluntary feedback and establish the circumstances under which a firm should (or should not) make feedback mandatory. We also investigate the role that intermediate prizes designed to induce voluntary feedback play in this regard. Third, the existing literature simply presumes that the contest holder divulges all available information to contestants; the only exception is Marinovic (2015), who considers a specific form of noisy feedback. Yet feedback may in fact convey less fine-grained information, so we explore the effects of reducing the amount of feedback information conveyed. Finally, the contest literature on feedback has attended solely to the average performance of solvers. We answer each of the three central questions not only for a contest holder aiming to improve average performance but

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3 In the following we concentrate on theoretical work, but it is worth mentioning also the stream of empirical studies (see e.g., Gross 2017, Wooten and Ulrich 2017).
also for one looking for the best possible performance—a goal more typical of innovation settings. We show that the optimality of feedback policies hinges on this distinction.

3. Model Setup

Let us describe in more detail the characteristics of a typical innovation contest in terms of both the firm and the solvers so as to establish our base model (voluntary feedback and reduced information feedback are treated in Sections 6 and 7, respectively). The firm understands its own preference structure well enough that, when presented with a solution, the firm can express how much it values that solution. However, the firm cannot know the effort level expended by a solver in achieving a given performance because the link between performance and effort has a stochastic component. In contrast, each solver knows how much effort he expends and also realizes that expected performance increases with effort. Yet solvers still experience uncertainty about how, exactly, effort links to performance. In addition, solvers are uncertain about the firm’s preference structure and so, even after devising a solution, they cannot truly evaluate their performance. This latter uncertainty reflects that, for any true innovation, the firm cannot fully specify ex ante what criteria it values or how they should be weighted. These modeling requirements are typical for any innovation and R&D setting, and they place the foundation of our model squarely in the contest literature with stochastic effort consequences which postulates a stochastic link between solvers’ actions and contest outcomes (Taylor 1995, Fullerton and McAfee 1999, Ales et al. 2016).

Finally, as a means of dynamically influencing the solvers’ effort provision in the course of a contest, the firm may (partially) resolve the solvers’ uncertainty about their performance by transmitting interim performance feedback. We classify such feedback as public, private, or no feedback. The firm employs whichever feedback policy optimizes the contest’s intended outcome—the highest average performance or best possible performance.

**Formal Description of the Base Model.** In order to create a parsimonious model that nonetheless captures the essence of the scenario just outlined, we consider a firm that hosts a dynamic innovation contest over two rounds, \( t \in \{1, 2\} \), with two risk-neutral solvers, \( i \) and \( j \).\(^4\) The primitives of the contest are common knowledge; its structure is depicted in Figure 1.

The process begins when the firm publicly announces the contest, the fixed award \( A \) for which the two solvers compete, and its feedback policy. In order to concentrate on the role of feedback (and to minimize technical complexity), we treat \( A > 0 \) as a parameter. Our decision variable for the firm at this stage is whether and, if so, how to give feedback. The firm may choose to give

\(^4\)For notational simplicity, we explicitly define only the parameters for solver \( i \); an identical set applies to solver \( j \).
no feedback at all, to offer public feedback (i.e., both solvers receive the same information about their own and their competitor’s performance), or to provide private feedback (i.e., solver \(i\) receives feedback on his own performance but not on the performance of solver \(j\), and vice versa).

Next, solver \(i\) expends effort \(e_{i1} \geq 0\) at private cost \(c e_{i1}^2\), where \(c > 0\). He finds an initial solution of value \(v_{i1} = k c e_{i1} + \zeta_{i1}\); here \(k_c > 0\) is the sensitivity of effort and \(\zeta_{i1}\) is a random shock that follows a uniform distribution, \(\zeta_{i1} \sim \text{Uniform}(-a/2, a/2)\) with \(a > 0\).

After the first round, each solver hands in his solution and the firm perfectly observes \(v_{i1}\). However, solver \(i\)’s effort is unobservable to the firm (and also to solver \(j\)); hence the firm cannot determine whether a high solution value stems from high effort, a large random shock, or both. In contrast, solver \(i\) knows how much effort he has invested; but since he cannot observe the realization of \(\zeta_{i1}\), he is uncertain about the true performance of his solution. To address that uncertainty, the firm provides interim performance feedback in accordance with its own policies. As is customary in the fledgling research field of feedback in contests, we assume that feedback is pre-committed, truthful and accurate (Aoyagi 2010, Ederer 2010)—although the “accurate feedback” assumption is relaxed in Section 7. It is clear that, in the absence of pre-committed truthfulness (i.e., if feedback does not convey a somewhat informative signal in a Bayesian sense), feedback is utterly meaningless. It is easy to prove that the firm would have a strong incentive to provide only feedback that maximizes future efforts irrespective of actual performance; naturally, each solver would anticipate this manipulation and discard the received information as uninformative.

Upon observing the firm’s feedback, solver \(i\) updates his belief about the realization of first-round performances \(v_1 = (v_{i1}, v_{j1})\) in accordance with Bayesian rationality. Then, solver \(i\) expends

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5 An effort of 0 should be interpreted as the normalized minimal effort necessary to participate in the contest.
additional solution effort $e_{i2} \geq 0$ and submits his final solution $v_{i2} = v_{i1} + k_e e_{i2} + \zeta_{i2}$, where $\zeta_{i2}$ is again a random shock that follows the same distributional assumptions as in the first round. Random shocks are independent and identically distributed across solvers and rounds. For notational simplicity we define $\Delta \zeta_t = \zeta_{it} - \zeta_{jt}$ as the difference between the random shocks in round $t$ with associated probability density function $g_{\Delta \zeta_t}$.

Finally, after receiving the final solutions, the firm announces the contest winner by choosing the highest-value solution. Thus solver $i$ wins if $v_{i2} > v_{j2}$ (ties can be broken by invoking any rule).

**Model Implications.** A firm will naturally seek to employ the feedback policy that maximizes its expected profits. The relevant profit function is $\Pi_{\text{best}} = E[\max\{v_{i2}, v_{j2}\}] - A$ if the firm is interested in the performance of the best solution only, or $\Pi_{\text{avg}} = E[v_{i2} + v_{j2}] / 2 - A$ if the firm wishes to maximize the average performance of both solvers.

Whereas the firm—whatever its profit function—is interested in the solvers’ absolute performance, each solver’s sole interest is in winning the contest. The utility that solver $i$ receives from winning is $A - \sum_t c e_t^2 i$; losing the contest yields a utility of $-\sum_t c e_t^2 i$. Hence solver $i$’s expected utility of participating in the contest is $u_i = A \cdot P(v_{i2} > v_{j2}) - \sum_t c e_t^2 i$ (we assume his outside option to be 0), and the effort he invests in the contest is determined by maximizing his expected utility.\(^6\)

We are concerned with Perfect Bayesian Equilibria (PBE) of the contest. To avoid unnecessary technical complications during the analysis, we assume that $\kappa \equiv (a^2 c) / (A k_e^2) > 1$. For technical reasons, similar assumptions on the contest’s inherent performance uncertainty are made in practically the entire literature on contests (see, e.g., Nalebuff and Stiglitz 1983, Aoyagi 2010, Ederer 2010). Clearly, $\kappa$ increases in the variance of the random noise and the costs of effort, and it decreases in the size of the award and the effort sensitivity. Thus, with a higher $\kappa$, improvement effort is more expensive and the solution performance becomes more stochastic.

**4. Solvers’ Solution Efforts**

In this section we analyze, for our base model, the solvers’ solution efforts under each feedback policy. We can do so without specifying the firm’s objectives because—given a particular feedback policy—the solvers’ strategies are independent of whether the firm’s aim is to improve average performance or rather to attain the best performance. Each solver simply tries to win the contest.

We start by re-establishing familiar results in the context of our model, characterizing solvers’ equilibrium efforts in the absence of feedback as a benchmark (Section 4.1); next we describe

\(^6\)Note that an outside option of zero ensures that a solver always participates in the contest because zero effort already guarantees him a nonnegative expected utility and in equilibrium his utility cannot be worse.
how providing public feedback affects the solution efforts of solvers (Section 4.2). In this section’s main contribution, we then determine equilibrium levels of solution effort under a private-feedback policy (Section 4.3). Throughout the text, initial managerial implications are discussed in passing; however, our systematic comparison of feedback policies is deferred until Section 5.

4.1. No Feedback

In the benchmark case of no feedback, the firm does not provide any interim performance information to the solvers. As a result, each solver’s two-stage effort choice problem reduces to a simultaneous, single-stage utility maximization problem.

**Proposition 1 (No-Feedback Policy).** The unique PBE under a no-feedback policy is symmetric, with

\[ e_{no}^1 = e_{no}^2 = \frac{Ak_e}{3ac}. \]

Proposition 1 parallels previous results of Taylor (1995), Fullerton and McAfee (1999), and Ales et al. (2016). Since neither solver receives any interim performance information and since the costs of effort are convex, it follows that solution efforts are identical across rounds. Moreover, because solvers are symmetric at the start of the contest, they always choose the same effort in equilibrium; hence they do not try to leapfrog each other. So under a no-feedback policy, it is the contest’s inherent performance uncertainty that ultimately determines the contest winner.

It is instructive at this juncture to examine how our key contextual parameters affect a solver’s solution efforts. As one would expect, those efforts are increasing in the size of the award \( A \) and in the effort sensitivity \( k_e \) but are decreasing in the costs of effort \( c \) and in the uncertainty involved \( a \). Thus, a solver exerts relatively more effort if effort becomes relatively more rewarding (i.e., if \( A/c \) increases) and/or if effort becomes relatively more important (i.e., if \( k_e/a \) increases).

4.2. Public Feedback

Next we study the implications of public feedback. In this case, after submitting his initial solution, each solver learns his own as well as his competitor’s first-round performance. That is, public feedback perfectly reveals the solvers’ first-round performance difference before the start of the second round, at which point solvers are therefore no longer symmetric.

**Proposition 2 (Public-Feedback Policy).** The unique PBE under a public-feedback policy is symmetric, with

\[ e_{pub}^1 = E_{\Delta \zeta_1} \left[ e_{pub}^2(\Delta \zeta_1) \right] = \frac{Ak_e}{3ac}, \]

\[ e_{pub}^2(\Delta \zeta_1) = \frac{Ak_e}{2a^2c} \left( a - |\Delta \zeta_1| \right). \]
Mirroring Aoyagi (2010) and Ederer (2010), Proposition 2 has two main implications. First, it shows that each solver cares only about his relative performance and completely disregards the absolute performance information embedded in public feedback. Specifically: if the solvers’ first-round performance difference is small (i.e., the contest is competitive), then second-round efforts are substantial and the solvers fight hard to win the contest; but if the first-round performance difference is sizable, then solvers reduce their solution efforts because the contest is de facto decided. Second, despite being asymmetric in the second round, both solvers expend the same amount of effort. In other words, the first-round leader pursues a simple blocking strategy: he tries to keep the follower at a distance but without trying to increase the performance gap. At the same time, the follower tries to not fall farther behind but without attempting to close the gap. The follower just relies on a large positive second-round shock to reverse his fortune.

4.3. Private Feedback

We have just shown that, under a public-feedback policy, solvers set their second-round solution efforts as a function of their relative first-round performance. Yet that solver strategy is not viable under private feedback, since each solver receives information only about his own performance. Thus, only absolute performance information can affect a solver’s solution effort.

The absence of relative performance information fundamentally affects the contest’s information structure. Whereas solvers always possess symmetric and consistent beliefs under no and public feedback, private feedback introduces an asymmetric and inconsistent belief structure which allows for the solvers’ assessments of their chances to win to not be “coherent”. Suppose, for example, that each solver receives the information that he performed extremely well in the first round. Then both solvers believe that their respective chances of winning are much greater than 50%, although in reality those chances are merely 50%. And in contrast with the public-feedback scenario, solvers are never entirely certain whether they are ahead or behind their competitor. It is this asymmetric belief structure that drives asymmetric equilibrium solution efforts.

**Proposition 3 (Private-Feedback Policy).** The unique PBE under a private-feedback policy is symmetric, with

$$ e_{1}^{pri} = E_{z_{i_{1}}} [e_{2}^{pri}(z_{i_{1}})], $$

$$ e_{2}^{pri}(z_{i_{1}}) = \begin{cases} \frac{a + \gamma_{1}}{k_{e}} + \frac{2 a}{k_{e}} \ln \left( \frac{2}{3} \sin \left( \frac{1}{3} \sin^{-1} \left( \frac{3(1+12a) + 6z_{i_{1}}}{47} \right) \right) \right) & \text{if } z_{i_{1}} \in \left[ -\frac{a}{2}, \gamma_{3} x^{2} - a \kappa \right], \\
-\frac{\gamma_{1}}{k_{e}} + \frac{a}{k_{e}} \ln \left( \frac{\gamma_{3} x^{2}}{\gamma_{3}} \right) & \text{if } z_{i_{1}} \in \left[ \gamma_{3} x^{2} - a \kappa, \gamma_{3} y^{2} - a \kappa \right], \\
-\frac{\gamma_{1}}{k_{e}} + \frac{2 a}{k_{e}} \ln \left( \frac{\sqrt{2} (z_{i_{1}})}{6} - 2 \frac{22}{47} \right) & \text{if } z_{i_{1}} \in \left[ \gamma_{3} y^{2} - a \kappa, \frac{a}{2} \right]. \end{cases} $$
Here $z(\zeta_1) = 12[-9(a(1/6 - 2\kappa) - \zeta_1) + (12\gamma_2^2/\gamma_1 + 81(a(1/6 - 2\kappa) - \zeta_1)^2)^{1/2}] / \gamma_1$. The constants are defined as $\gamma_1 = p(ny - x)/(3nx^2o)$, $\gamma_2 = py(n^3x + y)/(nxo)$, and $\gamma_3 = p(n^2x^2 + y^2)/(2x^2o)$, where $m = (1 - 6\kappa)/(1 + 6\kappa)$, $n = e^{1/(2\kappa)}$, $o = 3y^2 - n^2x^2 + 4nxxy$, and $p = a(1 + 6\kappa)$ and where $x \in [e^{-1/(4\kappa)}, e^{-(1-1/\kappa)/(4\kappa)}]$ and $y \in [e^{1/(4\kappa)}, e^{(1+1/\kappa)/(4\kappa)}]$ are the unique solutions to the following system of equations:

\[
\begin{align*}
mn^2x^4 - 4mn^3x^3y - 3(m + n^2)x^2y^2 - 4n^{-1}xy^3 + y^4 &= 0, \quad (6) \\
\frac{1 - 6\kappa^2}{\kappa(1 + 6\kappa)} + m \ln(y) - \ln(x) + \frac{n^2x^4 + 8n^3x^3y + 9(1 + n^2)x^2y^2 + 8n^{-1}xy^3 + y^4}{6x^2(3y^2 - n^2x^2 + 4nxxy)} &= 0. \quad (7)
\end{align*}
\]

This proposition presents—to the best of our knowledge—the first solution of a contest with asymmetric private information but it is rather unwieldy; we offer a more tractable approximation in Corollary 1. Our numerical analyses indicate that the corollary yields an exceptionally good approximation even for low $\kappa$, which makes it a good starting point for reflecting on Proposition 3.

**COROLLARY 1.** Define $\tilde{\gamma}_3 = a(1 + 6\kappa)e^{(\kappa-1)/(2e^2)}/(2(1 + 2e^{1/\kappa}))$, and let

\[
ge_2(\zeta_1) = \frac{-\zeta_1}{k_e} + \frac{ak}{k_e} \ln(\zeta_1 + ak) - \frac{ak}{k_e} \ln(\tilde{\gamma}_3).
\]

Then $\lim_{\kappa \to \infty} e_2^{pri}(\zeta_1) - \tilde{e}_2(\zeta_1) = 0$ for all $\zeta_1$.

Figure 2 plots the equilibrium effort functions $e_2^{pri}$ and $e_2^{pri}(\zeta_1)$ for different first-round shocks. The graph makes salient that Proposition 3 provides striking managerial insights for those staging innovation contests. First, as before, each solver splits his expected solution effort equally between the two rounds. That is: in expectation, the first and second round contribute equally to a solver’s overall performance. Second, a solver’s second-round effort $e_2^{pri}(\zeta_1)$ is not monotonically increasing in $\zeta_1$. In fact, $e_2^{pri}(\zeta_1)$ has an inverted U-shape; it increases with $\zeta_1$ for $\zeta_1 \leq 0$ but decreases with $\zeta_1$ for $\zeta_1 > 0$. Thus solvers with a moderate first-round performance (i.e., $\zeta_1 = 0$) exert substantial efforts in the second round, whereas solvers with a very high or very low first-round performance reduce their second-round efforts. The reason is that a moderately performing solver perceives the contest as being competitive whereas exceptionally good- or ill-performing solvers perceive the contest as more or less decided. Most importantly, however, unlike the public-feedback scenario, under private feedback the bad solvers reduce their efforts to a greater extent than do the good solvers; formally, $e_2^{pri}(-\zeta_1) < e_2^{pri}(\zeta_1)$ for all $\zeta_1 > 0$ (observe the asymmetry in Figure 2). This finding stems from the absence of relative performance information. A solver with a high first-round shock can never be certain that he is ahead, so he invests more effort to maintain his chances of winning in case the competitor is equally strong—even though that is unlikely. Hence, private
feedback induces well-performing solvers to invest relatively more effort; it makes them relatively more risk averse. This asymmetric response to feedback is the central feature that distinguishes private from public feedback.

But does this mean that less fortunate solvers can leapfrog better solvers by increasing their second-round efforts? The answer is No. To see this, note that solver $i$’s final performance $v_{i2}^{pri}$ is increasing in $\zeta_{i1}$. That is: the more fortunate a solver is in the first round (i.e., the higher his shock $\zeta_{i1}$), the better he performs in the contest. More interestingly, this intuitive result also sheds light on the strategic behavior of solvers. In equilibrium, no solver ever allows a less fortunate solver (i.e., one with a lower first-round shock) to overtake him in the second round through effort alone. So once a solver has fallen behind his competitor after the first round, he needs a good random shock in the second round in order to win the contest.

5. The Optimal Feedback Policy

Having characterized the solvers’ equilibrium solution efforts under the different feedback policies, we are now ready to answer our main research question: Which feedback policy is the best for each of the two stipulated objectives? We first discuss the optimal feedback policy for maximizing average performance (Section 5.1); we then shift our focus to maximizing the performance of the best solution (Section 5.2).

5.1. Maximizing Solvers’ Average Performance

Since the firm must set the feedback policy at the outset of the contest and since solvers are ex ante symmetric, it follows that $\Pi_{avg} = E[v_{i2} + v_{j2}]/2 - A = E[\sum_{t} e_{it}] / 2 - A = E[\sum_{t} e_{it}] - A$. That is, maximizing average performance is equivalent to maximizing the sum of a solver’s (ex ante) expected first- and second-round equilibrium efforts. Proposition 4A compares the expected first-
and second-round effort choices of a solver as well as the firm’s expected profits for the cases of no feedback, public feedback, and private feedback.

**Proposition 4A (Optimal Feedback Policy for Average Performance).** The following statements hold:

(i) $e_{1}^{\text{pri}} < e_{1}^{\text{pub}} = e_{1}^{\text{no}}$;
(ii) $E_{\zeta_{1}}[e_{2}^{\text{pri}}(\zeta_{1})] < E_{\Delta \zeta_{1}}[e_{2}^{\text{pub}}(\Delta \zeta_{1})] = e_{2}^{\text{no}}$;
(iii) $\Pi_{\text{avg}}^{\text{pri}} < \Pi_{\text{avg}}^{\text{pub}} = \Pi_{\text{avg}}^{\text{no}}$.

The first noteworthy result of Proposition 4A is that, in each round, the ex ante expected effort of each solver is identical under a no-feedback and a public-feedback policy. This result can be explained by public feedback having two opposed effects on a solver’s second-round effort choice. On the one hand, if the revealed first-round performance difference is low ($|\Delta \zeta_{1}| < a/3$), then each solver understands that the contest is highly competitive and is motivated thereby to expend more effort than under a no-feedback policy. On the other hand, if the performance difference is large ($|\Delta \zeta_{1}| > a/3$), then solvers are discouraged from investing effort because they believe that the contest is practically decided. In equilibrium, these countervailing effects of motivation and de-motivation offset each other; thus, $E_{\Delta \zeta_{1}}[e_{2}^{\text{pub}}(\Delta \zeta_{1})] = e_{2}^{\text{no}}$. Clearly, when deciding on his first-round solution effort, each solver anticipates this balance between motivation and de-motivation effects and therefore chooses to exert the same effort as under a no-feedback policy: $e_{1}^{\text{pub}} = e_{1}^{\text{no}}$.

In contrast, the announcement of private feedback reduces the willingness of solvers to expend solution effort as compared with both the no-feedback and public-feedback policies. Two different effects are responsible for this result. First, much as under a public-feedback policy, private feedback can motivate a solver to expend more effort than in the no-feedback case if his first-round performance was middling. However, this motivation effect is much less pronounced for private than for public feedback. To see why, recall that the motivation effect of public feedback is strongest when the firm communicates a small performance difference. Under private feedback, the firm never releases relative performance information and so each solver can (and will) form only a belief about the performance difference. Yet given the inherent randomness of performance, each solver knows that his competitor is unlikely to have achieved the same performance. For this reason, solvers respond only moderately to the motivation effect of private feedback.

Second, private feedback has a strong de-motivating effect on relatively low-performing solvers. As Figure 2 illustrates, solvers with a bad first-round performance exert less effort in the second

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7 This happens if and only if $-ak(1 + W_{0}(\gamma e^{-1+1/(3\kappa^{2})} / (ak))) < \zeta_{1} < -ak(1 + W_{-1}(\gamma e^{-1+1/(3\kappa^{2})} / (ak)))$, where $W_{0}$ (resp. $W_{-1}$) is the upper (resp. lower) branch of the Lambert $W$ function.
round than do solvers with a good first-round showing. Put differently, the anticipated performance gap between bad and good solvers widens in the second round because of these asymmetric effort choices. As a result, we observe a phenomenon that does not arise under a public-feedback regime—namely, a solver with a relatively bad first-round performance realizes that he may face a competitor that he can never beat. Hence the set of potential competitors against whom the focal solver can win becomes smaller and so he begins to shirk. In short: private feedback reduces the contest’s competitiveness, which in turn leads solvers to reduce their effort.

This phenomenon also has a strong effect on a solver’s effort in the first round. Since effort in the second round is reduced, solvers refrain from wasting effort in the first round; that is why $e_{1\text{pri}}^1 < e_{1\text{pub}}^1$. Thus the mere pre-announcement of private interim performance feedback has a negative effect on the solvers’ expected behavior. This “strategic” effect is not observed in a public-feedback contest.

In sum: since maximizing the solvers’ average performance is equivalent to maximizing the solvers’ average effort provision, it follows that a private-feedback policy always generates the lowest expected profits for the firm. It is therefore optimal for the firm to choose either a no-feedback or a public-feedback policy. And whereas the firm is indifferent between these two policies, solvers strictly prefer a no-feedback policy.

5.2. Finding the Best Solution

In practice, most innovation contests are designed to elicit one exceptional solution that promises significant value upon implementation. In this case, the firm focuses not on maximizing the solvers’ average performance but rather on maximizing the performance of the best solution; that is, the firm maximizes $\Pi_{\text{best}} = E[\max\{v_{i2}, v_{j2}\}] - A$. Proposition 4B establishes that, for certain types of innovation contests, private feedback is the optimal policy.

**Proposition 4B (Optimal Feedback Policy for Best Performance).** (i) $\Pi_{\text{best}}^{\text{pub}} = \Pi_{\text{best}}^{\text{no}}$.

(ii) There exists a $\kappa > 1$ such that $\Pi_{\text{best}}^{\text{pub}} > \Pi_{\text{best}}^{\text{pri}}$ for all $\kappa < \kappa^*$.

(iii) There exists a $\bar{\kappa} < \infty$ such that $\Pi_{\text{best}}^{\text{pri}} > \Pi_{\text{best}}^{\text{pub}}$ for all $\kappa > \bar{\kappa}$.

Irrespective of whether the firm is interested in the solvers’ average or best performance, employing a public-feedback policy generates the same expected profits as does a no-feedback policy. This result reflects the identity of expected efforts under these two feedback policies.

The key result of Proposition 4B is that public (resp., private) feedback is optimal if $\kappa < \kappa^*$ (resp., if $\kappa > \bar{\kappa}$).\(^8\) To better understand this result, recall that $\kappa = (a/k_c)^2/(A/c)$. The numerator is a

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\(^8\)The complexity of the equilibrium emerging under a private-feedback policy makes it difficult to find the dominant strategy for $\kappa \leq \kappa \leq \bar{\kappa}$. However, numerical simulations indicate that $\kappa = \bar{\kappa}$; hence there exists a unique threshold for $\kappa$ above which a private-feedback policy maximizes the firm’s expected profits.
Figure 3  Ex ante Expected Second-Round Efforts under Private and Public Feedback

Note. The graphs compare solver $i$’s (expected) equilibrium second-round effort conditional on $\zeta_i$ under private feedback (solid line; $e^p_{2i}(\zeta_i)$, as stated in Proposition 3) and under public feedback (dashed line; $E_{\zeta_{j1}}[e^p_{2i}(\Delta \zeta_1)]$, with $e^p_{2i}(\Delta \zeta_1)$ as in Proposition 2); we take the expectation over $\zeta_{j1}$ in the public-feedback case in order to make it directly comparable to the private-feedback case. In the left panel, performance uncertainty is low ($\kappa = 1.01$); in the right panel, performance uncertainty is high ($\kappa = 4$). The vertical dotted line marks the unique intersection point of the two curves. Parameters are: $A = 1$, $a = 1$, and $k_e = 1$ (both panels); $c = 1.01$ (left panel); and $c = 4$ (right panel).

Yet how can this result be explained—especially since, according to Proposition 4A(iii), private feedback induces lower average performance than public feedback (or no feedback)? We can answer this question by considering Figure 3, which compares solver $i$’s expected second-round effort conditional on his first-round shock $\zeta_i$ under a private-feedback (solid line) and a public-feedback (dashed line) policy. The figure’s left (resp. right) panel shows the functions for low (resp. high) performance uncertainty $\kappa$. Two key observations can be made here. First, comparing the solid and the dashed lines plotted in each panel reveals that public feedback induces a larger effort than does private feedback for most first-round shocks; this result is consistent with our finding that private feedback induces a lower ex ante expected effort than does public feedback. Comparing the two panels reveals also that, for low performance uncertainty $\kappa$, the reduction in average effort under private feedback is much greater than for high $\kappa$. Second, for top-performing solvers (i.e., solvers...
with a high first-round shock), private feedback increases effort provision: the solid line surpasses
the dashed line for sufficiently high $\zeta_i$. This finding captures the need of top performers to protect
their good position more determinately under private than under public feedback owing to the lack
of relative performance information. Moreover, Figure 3 shows that the fraction of solvers for whom
private feedback increases their effort is small under low performance uncertainty but is large under
high performance uncertainty.

Of course, it is exactly these top performers in whom the firm is interested when maximizing the
performance of the best solution. So when using private feedback, the firm faces a non-trivial trade-
off. On the one hand, private feedback reduces the solvers’ average effort provision; on the other
hand, it encourages greater effort from the best solvers. Thus the optimal feedback policy is the one
that best balances the average effort provision against the likelihood that a top-performing solver
participates in the contest. Consider the left panel of Figure 3. For low $\kappa$, the decline in average
effort under private feedback is relatively pronounced while the likelihood of a top-performing solver
(i.e., a solver with a first-round shock to the right of the dotted vertical line) participating in the
contest is relatively low. As a result, public feedback dominates private feedback. In contrast, the
right panel shows that the decline in average effort is much less pronounced for high $\kappa$. Furthermore,
the chances that a solver exerts more effort under private than public feedback are much greater
(i.e., the solid line crosses over the dashed line much farther to the left). In this case, then, private
feedback is the optimal feedback policy.

This finding—that the optimal feedback policy is tightly linked to the relative importance of
effort and uncertainty—has not been addressed in the extant literature and it has two immediate
managerial implications. First, when setting up an innovation contest, it is crucial that the firm
identifies the extent to which a solver’s performance depends on stochasticity. For instance, there
is seldom much uncertainty associated with contests that seek to foster incremental innovation.
In such contests, the hosting firm should provide public feedback. However, private feedback is
the preferred choice for ideation contests that aim to develop novel concepts, new ideas, or break-
through research (all of which are associated with high levels of uncertainty). Even more, our results
concern communications between solvers: whenever the effort–performance link becomes tenuous,
idea exchange between solvers becomes detrimental to firm goals and so the firm should minimize
any form of communication between competing solvers. Second, if performance uncertainty is sub-
stantial then a firm should not focus on improving the average second-round effort; instead, it
should choose a feedback mechanism that “pampers” potential first-round top performers—even if
in realization such top performers may not be present in the contest.
From the social welfare standpoint, Proposition 4B (in conjunction with Proposition 4A) provides another important result for contests that are inherently uncertain: the private-feedback policy is not only optimal for the firm but can also be socially efficient. More precisely: apart from maximizing the firm’s expected profits, a private-feedback policy also allows solvers to reduce their expected efforts. As a consequence, both the firm and the solvers may well prefer private feedback to either public or no feedback in settings characterized by high performance uncertainty.

6. Voluntary Feedback

So far we have assumed that the firm, by dictating the type of feedback, also obliges solvers to submit intermediate solutions. Those intermediate solutions are the firm’s vehicle for providing solvers with interim performance feedback. In reality, however, instead of enforcing intermediate submissions, a contest holder may also simply offer an option for feedback. In such cases, feedback is voluntary and it is each solver’s deliberate choice whether or not to seek feedback. Clearly, a solver will only do so if he sees a significant benefit in submitting his intermediate solution. We now explore how different potential benefits affect a solver’s decision.

For concreteness, we extend our base model (as depicted in Figure 1) by allowing each solver to decide—after his first-round efforts—whether or not to submit an interim solution. The firm then follows its announced feedback policy and provides the solvers with accurate feedback on any submitted solutions. That is, if feedback is private then solver \( i \) receives feedback on \( v_{i1} \) only if he has submitted his solution beforehand; under a public-feedback policy, solver \( i \) can receive feedback on \( v_{j1} \) also—but only if solver \( j \) has submitted his interim solution. The following proposition characterizes the solvers’ equilibrium behavior.

**Proposition 5 (Voluntary Feedback).** (i) Under a public-feedback policy, no solver voluntarily discloses his first-round solution in equilibrium.

(ii) Under a private-feedback policy, each solver submits his first-round solution in equilibrium.

The key insight here is that a solver’s behavior as regards submitting interim solutions depends critically on the contest holder’s feedback policy. It is intuitive that under a private-feedback policy, each solver always seeks feedback as there are no negative externalities from requesting feedback. Without disclosing any information to his competitors, each solver receives more refined information on his performance, allowing him to optimally adjust his second-round efforts. In contrast, under a public-feedback policy, solvers refrain from submitting their interim solutions because the threat of an escalation of effort provision outweighs any potential benefits. More precisely, public feedback introduces a relative benchmark that induces solvers to invest exceptionally high effort levels if the
contest is competitive. However, such bold effort choices are utterly futile because each solver invests the same amount of effort in equilibrium, and therefore the chances of winning are unaffected. Instead, it is only the costs of effort that skyrocket.

The same logic continues to hold when accounting for another potential benefit of feedback: the reduction of uncertainty in the second round. In fact, as Proposition 2 indicates, the problem of effort escalation becomes even stronger because with a lower performance uncertainty (i.e., a lower $a$) equilibrium efforts are becoming even higher.

We conclude that whereas pure information incentives are strong enough for a solver to reveal his first-round performance under private feedback, they are insufficient under a public-feedback policy. Thus to further strengthen the solvers’ incentives the firm may need to resort to monetary incentives by granting a so-called “milestone” award to the solver with the best first-round solution. It is evident that such financial prospectives will—if large enough—induce each solver to submit his intermediate solution. However, as the next proposition shows, this is never in the firm’s interest.

**Proposition 6 (Milestone Award).** Under a public feedback policy, it is never optimal for the firm to grant a milestone award.

The higher the milestone award the more the firm shifts the solvers’ incentives away from winning the overall contest towards winning the first round. As a result, the introduction of a milestone award drastically dilutes a solver’s second-round incentives, and thus his equilibrium efforts. Per se, however, the firm is not interested in these intermediate submissions, but it only cares about the final solution qualities. Thus, the higher the milestone award the more misaligned are the firm’s and the solvers’ objectives. This is why the firm strictly prefers to not grant any milestone awards.

In conclusion, we have shown that in the absence of monetary rewards, submitting interim solutions and populating a publicly available leader board are not decisions that a rational solver would make. It is thus an important alley for future empirical research to investigate why, in practice, many solvers are nonetheless eager to share their solution quality with competitors.

### 7. Partial Information Feedback

Until now we have assumed that if the firm provides feedback then this feedback is perfect; in other words, the firm always reveals fully accurate information on the solvers’ performance. However, the firm may either prefer or (for practical reasons) be required to provide only partial feedback. Two canonical cases are practically relevant. First, the firm may provide information that is less detailed—for example, by publishing the solvers’ rankings but no specific information about their performance. Second, the firm’s performance feedback may be disturbed by some level of noise.
7.1. Rank-Only Feedback

We first consider the case in which the firm reduces the granularity of feedback by providing only information on the solvers' ranking in the contest. Before delving into the details, it is important to recognize that such rank-only feedback cannot be conceptualized as private information; revealing a rank is inherently public feedback. It is not material for a solver which other solver holds which rank, but rather that one of the other solvers holds each of the ranks above or below him.

As compared with a public-feedback policy, rank-only feedback may change solver behavior in two ways. When first-round performances are extremely close, providing rank-only feedback reduces competition because the “blurriness” of the rank information leaves none of the solvers fully aware of how close the competition actually is. When first-round performances instead vary widely, rank information results in solvers underestimating their relative performance difference and so induces them to exert more second-round effort than actually required by the situation. Our next proposition compares the relative strength of these effects.

Proposition 7 (Rank-Only Feedback). The unique PBE under a rank-only feedback policy is identical to the unique PBE under a no-feedback policy. In particular, \( e_1^r = e_1^{no} \) and \( e_2^r = e_2^{no} \).

Proposition 7 holds a surprise. In comparison with a no-feedback policy, providing rank-only feedback has no effect on the equilibrium behavior of solvers; that is, solvers completely ignore their respective rankings. This outcome is in stark contrast to the fate of accurate performance feedback, which is always used by solvers to adjust their second-round behavior. The implications for practice are striking. If a contest holder wants its feedback to influence the second-round efforts of solvers, then this feedback must include information about each solver’s actual performance; rank-only feedback will not have the desired effect.

7.2. Feedback with Noise

Next we analyze the implications of noisy feedback, which are important for a firm that cannot (or prefers not to) guarantee perfectly accurate feedback. Consider again the case of Kaggle. Some contest holders provide interim performance feedback by using only a sample of the full data set on which to test the solvers’ algorithms; this approach helps prevent overfitting of the data during the contest. However, the final ranking is based not on that sample but rather on the full data set. So from the solvers’ perspective, the interim performance feedback is not entirely accurate: it is disturbed by noise.

Two questions arise. First, does the introduction of noise change the balance between no feedback, public feedback, and private feedback? Second, how does noisy feedback compare to entirely
accurate feedback? Is it possible for the firm to benefit from introducing noise into the feedback mechanism? Here we set out to start answering these questions.

Treating all the possible facets of noise is beyond the scope of this paper. Instead, we investigate a simple example of noisy feedback that nonetheless captures the essence of masking a solver’s true performance. More specifically, we assume that the firm gives perfectly accurate feedback with publicly known probability \( q \), and with probability \( 1 - q \) it transmits absolutely uninformative feedback (i.e., white noise).

**Proposition 8 (Noisy Feedback).** (i) Under a public-feedback policy, the firm’s expected profits \( \Pi_{\text{pub avg}} \) and \( \Pi_{\text{pub best}} \) are invariant with respect to \( q \).

(ii) For any fixed \( q > 0 \), there exists a \( \kappa < \infty \) such that \( \Pi_{\text{pri best}} > \Pi_{\text{pub best}} \) for all \( \kappa > \kappa \).

Part (i) of this proposition establishes that, under a public-feedback policy, noise does not affect the firm’s expected profits in terms of either average performance or best performance. The implication is that, under a public-feedback policy, contest holders cannot use noise strategically to improve contest outcomes. In contrast, noise does affect the firm’s profits under a private-feedback policy; however, exact analytical expressions are difficult to derive. Yet our numerical studies reveal that profits are monotonic in \( q \): for small \( \kappa \), a no-feedback policy (i.e., \( q = 0 \)) maximizes the firm’s profits; for large \( \kappa \), accurate private feedback (i.e., \( q = 1 \)) is optimal. It seems once again that noise cannot be deployed strategically to improve contest outcomes.

Combining Proposition 8(i) for public feedback and part (ii) for private feedback indicates that our results about the preferability of different feedback types are robust to the introduction of noise. That is, noise does not impact the ranking of the different feedback policies and hence the selection of a feedback policy should not be affected by the accuracy of the feedback.

8. **Robustness and Extensions**

We explored the sensitivity of our conceptual results to key modeling choices. Our results are exceptionally robust to changes in those assumptions, as we summarize in this section.\(^9\)

**Solver asymmetry.** Across solvers, the distribution of random shocks may be asymmetric. Two possible sources of such asymmetry are differences in mean and differences in variance; the former (resp., latter) signifies inherent differences in solvers’ capabilities (resp., innovation processes). A closed-form analysis of an extended model establishes that our results are robust to both sources of asymmetry; moreover, it seems that our assumption of solver symmetry is actually conservative with respect to the true value of private feedback.

\(^9\) Full mathematical details are available from the authors upon request.
Alternative feedback policy. Another form of partial information feedback is to inform solvers only of whether (or not) they have surpassed a certain performance threshold. Our formal investigation of such a threshold feedback policy shows that it can never improve on the performance of a fully informative policy—in accordance with our results in Section 7.

Cost of effort. For some contests, solvers may be more concerned with their total expended effort than with their respective individual efforts in each round. In such cases, a more appropriate cost-of-effort function would be \( c(e_{i1} + e_{i2})^2 \). We can demonstrate analytically that allowing for effort interaction effects between the two rounds does not alter our results in any meaningful way.

Performance uncertainty. Depending on the contest’s particular context and the innovation process of solvers, random shocks can follow a multitude of distribution functions. After conducting a large number of numerical experiments with normal and beta distributions, we can report that our results are robust.

9. Conclusions

Contests are a frequently used mechanism for providing incentives when companies and government agencies source innovation. Prize competitions organized via the Internet (as hosted by Kaggle) are contests, and so are sourcing efforts organized via the European Union’s “competitive dialogue procedure” and many of the approaches taken by private companies to source custom-engineered innovations. Feedback has been extensively used in practice to improve both the efficiency and the efficacy of contests. However, a rigorous understanding of when and how to provide which kind of feedback—and of when to refrain from giving feedback—is rather limited. The primary goal of this paper is to begin building a more comprehensive understanding of feedback in contests.

Our main contribution consists of charting a practically relevant landscape—one that determines how feedback can be used in contests—by addressing three questions that together define a feedback policy’s information structure: Who receives the feedback information? Who decides about which contestants receive feedback? What should be the information content of the performance feedback that is given? Answering these questions allows us to analyze many forms of feedback that actually are used in contests and to prescribe beneficial policies for a wealth of settings. In doing so we build new insights and challenge existing ones.

It is remarkable that—almost irrespective of whether feedback is voluntary and of whether feedback includes all or only some of the available information—firms need only focus on two straightforward factors when choosing whom to provide with any feedback: the contest objective (average versus best performance) and the contest’s inherent performance uncertainty. If the firm
is concerned about the solvers’ average performance, then either no feedback or public feedback is preferable to private feedback. The same preference obtains if the firm is interested in the best performance, provided that performance uncertainty is low. However, private feedback is the optimal choice if the firm seeks the best possible performance and performance uncertainty is high. Hence, private feedback is most suitable for innovation settings.

Our findings have immediate managerial implications. Contest holders that aim to raise the overall effort level among all solvers should refrain from giving private feedback; thus, if performance information is released then it should be made public. Incremental innovation contests likewise do not benefit from private feedback; for such contests, the relatively low performance uncertainty makes public feedback the preferred policy. In contrast, contests looking for breakthrough innovation (e.g., completely new algorithmic solutions, novel engineering concepts, any problem that requires the exploration of uncharted territories) should rely solely on private feedback.

As for who should decide on whether feedback is provided, one must bear in mind that voluntary feedback can function only if the solvers have an incentive to seek it. Generally speaking, that incentive may be of two different forms: an informational advantage or a monetary reward. As regards informational advantages, solvers always (resp. never) ask for feedback under a private-feedback (resp. public-feedback) policy. That is, the informational advantage outweighs the informational cost with private but not with public feedback. Monetary rewards intended to induce feedback-seeking behavior do work, but they are never optimal from the firm’s perspective.

Our results on voluntary public feedback have consequences for practice. If the contest holder anticipates benefits from having solvers share information about their performance, then it should find ways to make such feedback mandatory and not leave feedback up to the solver. Under private feedback, however, the opposite is true. Since in this case the solvers will always ask for feedback, contest holders should offer feedback only if they truly want to provide it. Intermediate prizes are never advisable—that is, from an incentive perspective.

Finally, if the feedback recipients have been identified and if the choice of voluntary or mandatory feedback has been made, then there remains the question of how granular the feedback should be. There is no evidence—in the cases examined here—that reducing feedback granularity and/or accuracy benefits the contest holder. Even more interestingly, we show that feedback lacking specific performance-related information (e.g., rank-only feedback) will likely be disregarded by solvers and thus fail to achieve its intended effect.

One aspect that must be considered when interpreting our results is that we did not explicitly incorporate the cost of providing feedback. We made this choice for two reasons. First, in nearly
all practical cases the cost differences among no feedback, public feedback, and private feedback are small compared with the benefits of providing tailored incentives to solvers (in our Kaggle and EU examples, the cost of giving feedback is negligible when compared with the potential benefits of an optimal feedback policy). Second, including such costs would be trivial, both technically and conceptually, because adding a cost term to the firm’s profits has no effect on the equilibrium analysis. Decision makers can simply subtract the cost differences between feedback policies from their respective benefit differences (as derived in this paper) to determine the overall trade-off.

Our model has limitations that should be mentioned. In order to maintain tractability and develop a parsimonious model, we made some assumptions about the purpose of contests; in particular, we focus on the incentive effects of feedback in contests. However, feedback may be used also to guide solvers in terms of where to direct their search efforts. Understanding this directional aspect of contest feedback requires an approach that differs fundamentally from ours and is a promising avenue for future research. Another limitation is our focus on a contest’s rational motivation effects, which are amenable to game-theoretic analysis. Yet real-world contests may also effect their outcomes by way of psychological inducements. Within the game-theoretically rational framework we consider only two competing solvers; this assumption ensures tractability but also introduces symmetries that may be absent in contests with three or more solvers. Hence, there is no guarantee that our results hold for more than two players; thus our results should be interpreted with some caution for large contests. Finally, we assume an additive performance relation between effort and uncertainty. This approach can be viewed as a first-order Taylor approximation of a more general relationship, and adding higher-order terms could capture additional effects—for example, an increase in uncertainty with greater solution efforts.

Previous research on contests has not broadly explored the repercussions of many practically relevant feedback policies. The aim and principal contribution of this paper is to fill several critical gaps in the literature and to build a deeper understanding of feedback in contests. It is only by considering the many variants and aspects of feedback that managers can reasonably hope to make the contest mechanism—a method often relied upon in practice for sourcing innovation—more efficient and effective.

Appendix. Proofs.

**Lemma A1.** There exists a unique pure-strategy second-round equilibrium for any feedback policy.

*Proof of Lemma A1.* Observe that each Nash equilibrium satisfies $e_{i2} \in [0, \sqrt{A/c}]$ because $u_{i2}(0, e_{j2}) \geq 0 \geq u_{i2}(\sqrt{A/c}, e_{j2}) > u_{i2}(e_{i2}, e_{j2})$ for any $e_{j2}$ and all $e_{i2} > \sqrt{A/c}$. Hence to prove existence and uniqueness of the second-round equilibrium, we can replace the original contest by a modified contest where each solver's
effort choice is restricted to \([0, \sqrt{A/c}]\). These two contests have the same Nash equilibria because each Nash equilibrium satisfies \(e_{i2} \in [0, \sqrt{A/c}]\), \(u_{i2}\) is continuous, and each solver’s strategy space is an interval of \(\mathbb{R}\). Also, straightforward differentiation shows that \(u_{i2}\) is strictly concave in \(e_{i2}\) for given \(e_{i2}\). Hence, by Theorem 1.2 in Fudenberg and Tirole (1991), there exists a pure-strategy Nash equilibrium in the original contest.

It remains to show that this equilibrium is also unique. However, once we observe that \(\partial^2 u_{i2}/\partial e_{i2}^2 + 2c = -\partial^2 u_{i2}/\partial e_{i2} \partial e_{i2}\), and \(\partial \mathbb{E}_{\xi_1} [g \Delta_{\xi_2} (v_{i1} + k_i e_{i2} - v_{i2} - k_e e_{j2}) f_i] / \partial e_{i2} \in [-k_e/a, k_e/a]\) for any feedback \(f_i\), this is just a simple application of Theorems 2 and 6 in Rosen (1965).

*Proof of Proposition 1.* Without feedback between round one and two, solver \(i\)’s optimization problem is equivalent to a utility maximization problem with simultaneous decisions on \(e_{i1}\) and \(e_{i2}\). Moreover, since performance is linear in effort, while the costs are strictly convex, equilibrium effort levels must be the same in both rounds. Thus, in equilibrium, \(e_{i1} = e_{i2} = e_i^{no}\), and solver \(i\)’s equilibrium effort has to solve \(e_i^{no} \in \arg \max_{e_i} A \cdot \mathbb{E}_{\xi_1} [G_{\Delta_{\xi_2}} (2k_i e_i + \zeta_{i2} - 2k_e e_j - \zeta_{j2})] - 2ce_i^2\). Since this is a strictly concave maximization problem, \(e_i^{no}\) is given by the following first-order optimality condition:

\[
2Ak_e \cdot \mathbb{E}_{\xi_1} [g_{\Delta_{\xi_2}} (2k_i e_i^{no} + \zeta_{i2} - 2k_e e_j^{no} - \zeta_{j2})] = 4ce_i^{no}.
\]

By the symmetry of \(g_{\Delta_{\xi_2}}\) around zero, it follows readily that the unique solution to the solvers’ optimality conditions is symmetric; that is \(e_i^{no} = e_j^{no}\). Inserting this information in (9) yields \(e_i^{no} = Ak_e/(2c) \cdot \mathbb{E}_{\xi_1} [g_{\Delta_{\xi_2}} (\zeta_{i2} - \zeta_{j2})] = Ak_e/(2c) \cdot \int_{-\infty}^{\infty} g_{\Delta_{\xi_2}} (z) g_{\Delta_{\xi_1}} (z) dz = Ak_e/(2c) \cdot \int_{-\infty}^{\infty} g_{\Delta_{\xi_1}} (z) dz = Ak_e/(3ac)\).

*Proof of Proposition 2.* Given public feedback, solvers perfectly learn \(v_{i1}\) after round one. As such, solver \(i\)’s second-round equilibrium effort solves \(e_{i2}^{pub} \in \arg \max_{e_{i2}} A \cdot \mathbb{E}_{\xi_1} [G_{\Delta_{\xi_2}} (v_{i1} + k_i e_{i2} - v_{j2} - k_e e_{j2}) - ce_{i2}^2\)]\) and the corresponding necessary and sufficient first-order optimality condition is given by

\[
Ak_e \cdot g_{\Delta_{\xi_2}} (v_{i1} + k_i e_{i2}^{pub} - v_{j2} - k_e e_{j2}^{pub}) = 2ce_{i2}^{pub}.
\]

By the symmetry of \(g_{\Delta_{\xi_2}}\) around zero, the unique second-round equilibrium is symmetric: \(e_{i2}^{pub} = e_{j2}^{pub}\).

In the first round, solver \(i\)’s equilibrium effort has to solve \(e_{i1}^{pub} \in \arg \max_{e_{i1}} A \cdot \mathbb{E}_{\xi_1} [G_{\Delta_{\xi_2}} (k_i e_{i1} + \zeta_{i1} - k_e e_{j1} - \zeta_{j1})] - c e_{i1}^2 - \mathbb{E}_{\xi_1} [c(e_{i2}^{pub})^2],\) and the corresponding necessary first-order optimality condition is given by

\[
Ak_e \cdot \mathbb{E}_{\xi_1} [g_{\Delta_{\xi_2}} (k_i e_{i1}^{pub} + \zeta_{i1} - k_e e_{j1}^{pub} - \zeta_{j1})] - 2ce_{i1}^{pub} - \frac{\partial}{\partial e_{i1}} \mathbb{E}_{\xi_1} [c(e_{i2}^{pub})^2] = 0.
\]

Note that the first two terms in (11) capture the direct effect of \(e_{i1}^{pub}\) on solver \(i\)’s expected utility, whereas the third term captures the indirect effect of \(e_{i1}^{pub}\) on \(i\)’s second-round effort \(e_{i2}^{pub}\). In equilibrium, this indirect effect must be zero. To see this, note that (11) reveals that \(e_{i1}^{pub}\) has no strategic effect on \(e_{i2}^{pub}\). By the symmetry of the second-round equilibrium, this implies that, in equilibrium, the strategic effect of \(e_{i1}^{pub}\) on \(e_{i2}^{pub}\) has to be zero as well. Yet, this is true if and only if \(e_{i1}^{pub} = e_{j1}^{pub}\); i.e., first-round equilibrium efforts are symmetric. Inserting this information in (10) and (11) shows that the unique PBE under public feedback is given by \(e_{i2}^{pub} (\Delta_{\xi_1}) = Ak_e/(2c) \cdot g_{\Delta_{\xi_2}} (\Delta_{\xi_1}) = Ak_e/(2a^2c) \cdot (a - |\Delta_{\xi_1}|),\) and \(e_{i1}^{pub} = \mathbb{E}_{\xi_1} \left[ e_{i2}^{pub} (\Delta_{\xi_1}) \right] = Ak_e/(3ac)\).
Proof of Proposition 3. Given private feedback, solver \( i \) perfectly learns \( v_{i1} \) after round one, but receives no additional information on \( v_{j1} \). As such, solver \( i \)'s second-round equilibrium effort solves \( e_{i2}^{\text{pri}} \in \arg \max_{e_{i2}} A \cdot E_{v_{j1}}[G_{\Delta c_2}(v_{i1} + k_v e_{i2} - v_{j1} - k_v e_{j2})|v_{i1}] - c e_{i2}^2 \), and the corresponding necessary and sufficient first-order optimality condition is given by

\[
A_k \cdot E_{v_{i1}}[g_{\Delta c_2}(v_{i1} + k_v e_{i2} - v_{j1} - k_v e_{j2})|v_{i1}] = 2 c e_{i2}^2. \tag{12}
\]

Lemma A1 ensures that the second-round equilibrium defined by (12) is unique. Next, we establish the uniqueness of the first-round equilibrium. In the first round, solver \( i \)'s equilibrium effort has to solve \( e_{i1}^{\text{pri}} \in \arg \max_{e_{i1}} A \cdot E_{e_{j1}}[G_{\Delta c_2}(k_v(e_{i1} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1} + e_{j2}^{\text{pri}}) - \zeta_j)] - c e_{i1}^2 - E_{e_{i1}}[c(e_{i2}^{\text{pri}})^2] \), and the corresponding necessary first-order optimality condition is given by

\[
A_k \cdot E_{e_{i1}}\left[ g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j) \cdot \left( 1 + \frac{\partial e_{i2}^{\text{pri}}}{\partial e_{i1}} - \frac{\partial e_{i2}^{\text{pri}}}{\partial e_{i1}} \right) \right] - 2 e_{i1}^{\text{pri}} - E_{e_{i1}}\left[ 2 c e_{i2}^{\text{pri}} \cdot \frac{\partial e_{i2}^{\text{pri}}}{\partial e_{i1}} \right] = 0. \tag{13}
\]

Clearly, solver \( j \)'s second-round effort cannot be influenced by solver \( i \)'s first-round effort, because solver \( j \) does not receive any information on \( v_{j1} \). Therefore, \( \partial e_{j2}^{\text{pri}}/\partial e_{i1} = 0 \). Rewriting (13) yields \( A_k E_{e_{i1}}[g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j)] - 2 c e_{i1}^2 + E_{e_{i1}}[(A_k g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j) - c e_{i1}^2)] \cdot \partial e_{i2}^{\text{pri}}/\partial e_{i1} = 0 \), where the third term is zero because \( E_{e_{i1}}[(A_k g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j) - c e_{i1}^2] \partial e_{i2}^{\text{pri}}/\partial e_{i1}] = E_{v_{i1}}[E_{e_{j1}}[(A_k g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j) - c e_{i1}^2] \partial e_{i2}^{\text{pri}}/\partial e_{i1}] = E_{v_{i1}}[\partial e_{i2}^{\text{pri}}/\partial e_{i1} \cdot E_{e_{j1}}[A_k g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j) - 2 c e_{i1}^2] \partial e_{i2}^{\text{pri}}/\partial e_{i1}] = E_{v_{i1}}[\partial e_{i2}^{\text{pri}}/\partial e_{i1} \cdot E_{e_{j1}}[A_k g_{\Delta c_2}(v_{i1} + k_v e_{i2} - v_{j1} - k_v e_{j2})|v_{i1}] - 2 c e_{i1}^2] = 0 \). The first equality follows from the law of iterated expectations, the second equality is true because solver \( i \)'s second-round effort choice is independent of \( v_{j1} \), the third equality follows from rearranging terms, and the last equality follows from solver \( i \)'s second-round optimality condition (12). Thus, the first-order optimality condition of solver \( i \) is \( A_k \cdot E_{e_{i1}}[g_{\Delta c_2}(k_v(e_{i1}^{\text{pri}} + e_{i2}^{\text{pri}}) + \zeta_i - k_v(e_{j1}^{\text{pri}} + e_{j2}^{\text{pri}}) - \zeta_j)] - 2 c e_{i1}^2 = 0 \), and by the symmetry of \( g_{\Delta c_2} \) around zero, it follows readily that the unique solution to the solvers' optimality conditions is symmetric; that is \( e_{i1}^{\text{pri}} = e_{j1}^{\text{pri}} \). Moreover, \( e_{i1}^{\text{pri}} = E_{v_{i1}}[e_{i2}^{\text{pri}}(v_{i1})] = E_{e_{i1}}[e_{i2}^{\text{pri}}(\zeta_i)]. \)

We now proceed with deriving the solvers' second-round equilibrium effort. We conjecture that the unique second-round equilibrium is symmetric in the sense that \( e_{i2}^{\text{pri}}(\zeta_i) = e_{j2}^{\text{pri}}(\zeta_i) \) and \( e_{j2}^{\text{pri}}(\zeta_i) = e_{i2}^{\text{pri}}(\zeta_i) \), and that \( v^{\text{pri}}(\zeta_i) = \zeta_i + k_v e_{i2}^{\text{pri}}(\zeta_i) \) increases in \( \zeta_i \). We will demonstrate in retrospective that these claims are true. Together with (12), the above properties imply that the equilibrium effort function \( e_{i2}^{\text{pri}}(\cdot) \) solves the following integral equation: \( A_k \cdot E_{\zeta_i}[g_{\Delta c_2}(v_{i1}^{\text{pri}}(\zeta_i) - \zeta_i - k_v e_{i2}^{\text{pri}}(\zeta_i))|\zeta_i] = 2 c e_{i2}^{\text{pri}}(\zeta_i), \) or equivalently,

\[
A_k^2 \cdot E_{\zeta_i}[g_{\Delta c_2}(v^{\text{pri}}(\zeta_i) - v^{\text{pri}}(\zeta_i))|\zeta_i] = 2 c (v^{\text{pri}}(\zeta_i) - \zeta_i). \tag{14}
\]

Because \( g_{\Delta c_2}(v^{\text{pri}}(\zeta_i) - v^{\text{pri}}(\zeta_i)) \) is positive only if \( v^{\text{pri}}(\zeta_i) - v^{\text{pri}}(\zeta_i) \in [-a, a] \), we distinguish three cases:

(I) If \( v^{\text{pri}}(\zeta_i) - v^{\text{pri}}(\zeta_i) \in [-a, a] \) for all \( \zeta_i \), then \( \zeta_i \in [\zeta_u, \zeta_o] \). In this case, (14) is given by

\[
\int_{\zeta_i}^{\zeta_i + a} (a - v^{\text{pri}}(\zeta_i) + v^{\text{pri}}(\zeta_i)) d\zeta_i + \int_{\zeta_i}^{\zeta_i + a} (a + v^{\text{pri}}(\zeta_i) - v^{\text{pri}}(\zeta_i)) d\zeta_i = 2 a k (v^{\text{pri}}(\zeta_i) - \zeta_i), \tag{15}
\]

where \( \zeta_i \) is the unique root of the integrand in the interval \([\zeta_u, \zeta_o]\).
and differentiating both sides with respect to $\zeta_1$ leads to the first-order ordinary differential equation
\[(v_{\text{pri}}(\zeta_1))' = a\kappa/(\zeta_1 + \alpha),\]
with canonical solution $v_{\text{pri}}(\zeta_1) = a\kappa \ln((a\kappa + \zeta_1)/y)$. It is easy to verify that $v_{\text{pri}}(a/2) - v_{\text{pri}}(-a/2) = a\kappa \ln((2\kappa + 1)/(2\kappa - 1)) > a$, implying that $[\zeta_u, \zeta_o] \subset [-a/2, a/2].$

(II) For $\zeta_1 \in [-a/2, \zeta_u]$, (14) becomes
\[
\int_{-\frac{a}{2}}^{\zeta_1} (a - v_{\text{pri}}(\zeta_1) + v_{\text{pri}}(\zeta_1)) d\zeta_1 + \int_{\zeta_1}^{-a} (a + v_{\text{pri}}(\zeta_1) - v_{\text{pri}}(\zeta_1)) d\zeta_1 = 2a\kappa (v_{\text{pri}}(\zeta_1) - \zeta_1),
\]
and differentiating both sides with respect to $\zeta_1$ leads to the differential equation $(v_{\text{pri}}(\zeta_1))'[2\zeta_1 + 2a\kappa + a/2 - v_{\text{pri}}(\zeta_1) + a] = 2a\kappa$, which is a Bernoulli equation in $v_{\text{pri}}(\zeta_1)$ whose implicit solution is
\[
v_{\text{pri}}^{-1}(v_{\text{pri}}) = C e^\frac{-a}{2\kappa} v_{\text{pri}} - a \left(\kappa + \frac{1}{4}\right) - \frac{1}{2a\kappa} e^\frac{-a}{2\kappa} v_{\text{pri}} e^\frac{-a}{2\kappa} v_{\text{pri}}^{-1}(v_{\text{pri}} + a) d v_{\text{pri}}.
\]

(III) In a similar vein, we can show that for $\zeta_1 \in [\zeta_o, a/2]$, the implicit solution to (14) is given by
\[
v_{\text{pri}}^{-1}(v_{\text{pri}}) = C' e^\frac{-a}{2\kappa} v_{\text{pri}} - a \left(\kappa - \frac{1}{4}\right) - \frac{1}{2a\kappa} e^\frac{-a}{2\kappa} v_{\text{pri}} e^\frac{-a}{2\kappa} v_{\text{pri}}^{-1}(v_{\text{pri}} - a) d v_{\text{pri}}.
\]

Combining (17) and (18) allows us to derive closed-form solutions. With (18), (17) becomes $v_{\text{pri}}^{-1}(v_{\text{pri}}) = C e^\frac{-a}{2\kappa} v_{\text{pri}} - a \left(\frac{3}{2}\kappa + \frac{1}{8}\right) - C' e^\frac{-a}{2\kappa} v_{\text{pri}} + \left(\frac{1}{2a\kappa}\right)^2 e^\frac{-a}{2\kappa} v_{\text{pri}} e^\frac{-a}{2\kappa} v_{\text{pri}}^{-1}(v_{\text{pri}} + a) d v_{\text{pri}}$. Note that $v_{\text{pri}}^{-1}(v_{\text{pri}}) - 2a\kappa (v_{\text{pri}}^{-1}(v_{\text{pri}}))' + (a\kappa)^2 (v_{\text{pri}}^{-1}(v_{\text{pri}}))^n = v_{\text{pri}}^{-1}(v_{\text{pri}})/4 - a(3\kappa/2 + 1/8)$. This is an equation of damped vibrations with canonical solution $v_{\text{pri}}^{-1}(v_{\text{pri}}) = \gamma_4 e^\frac{-a}{2\kappa} v_{\text{pri}} + \gamma_5 e^\frac{-a}{2\kappa} v_{\text{pri}} - a \left(\frac{1}{6} - 2\kappa\right)$. In an identical way, we can derive the canonical solution for $\zeta_1 \in [\zeta_o, a/2]: v_{\text{pri}}^{-1}(v_{\text{pri}}) = \gamma_4 e^\frac{-a}{2\kappa} v_{\text{pri}} + \gamma_5 e^\frac{-a}{2\kappa} v_{\text{pri}} + a \left(\frac{1}{6} - 2\kappa\right)$. Thus, the canonical solution to (14) is given by
\[
v_{\text{pri}}^{-1}(v_{\text{pri}}) = \begin{cases} 
\gamma_4 e^\frac{-a}{2\kappa} v_{\text{pri}} + \gamma_5 e^\frac{-a}{2\kappa} v_{\text{pri}} - a \left(\frac{1}{6} + 2\kappa\right) & \text{if } v_o - a \leq v_{\text{pri}} < v_o \\
\gamma_5 e^\frac{-a}{2\kappa} v_{\text{pri}} - a & \text{if } v_o \leq v_{\text{pri}} \leq v_o \\
\gamma_4 e^\frac{-a}{2\kappa} v_{\text{pri}} + \gamma_5 e^\frac{-a}{2\kappa} v_{\text{pri}} + a \left(\frac{1}{6} - 2\kappa\right) & \text{if } v_o < v_{\text{pri}} \leq v_o + a,
\end{cases}
\]
where $v_o = v_{\text{pri}}(\zeta_u)$, $v_o = v_{\text{pri}}(\zeta_o)$, $v_o - a = v_{\text{pri}}(-a/2)$, and $v_o + a = v_{\text{pri}}(a/2)$. With the substitution $u = \exp(v/(2a\kappa))$ we can represent $v_{\text{pri}}^{-1}(v_{\text{pri}})$ as a set of cubic equations, which we can solve for $v_{\text{pri}}(\zeta_1)$ with standard mathematical tools (Oliver et al. 2010, p. 131) to gain
\[
v_{\text{pri}}(\zeta_1) = \begin{cases} 
2a\kappa \cdot \ln \left(\sqrt{\frac{4a\kappa}{\gamma_3}} \cdot \sin \left(\frac{1}{4} \cdot \sin^{-1} \left(\sqrt{\frac{-3+24\kappa}{4\gamma_3} + a(1+12\kappa)/2\kappa}\right)\right)\right) & \text{if } -\frac{\alpha}{2} \leq \zeta_1 < \zeta_u \\
\alpha \kappa \cdot \ln \left(\frac{\zeta_1 + 2\kappa}{\gamma_3}\right) & \text{if } \zeta_u \leq \zeta_1 \leq \zeta_o \\
2a\kappa \cdot \ln \left(\frac{1}{6} \cdot \sqrt{2(\zeta_1 + 2\kappa)} - 2 \cdot \frac{a}{12\gamma_3} \cdot \frac{1}{\sqrt[3]{(\zeta_1 + 2\kappa)}}\right) & \text{if } \zeta_o < \zeta_1 \leq \frac{\alpha}{2}.
\end{cases}
\]
It remains to determine the integration constants. From (II), it follows readily that $\gamma_4 = -\gamma_1 a^3$ and $\gamma_5 = \gamma_2 a$. Moreover, (14) satisfies all requirements of the Implicit Function Theorem. Therefore, $v_{\text{pri}}(\zeta_1)$ is continuously differentiable. From the continuity of $(v_{\text{pri}}(\zeta_1))'$, it follows that $\gamma_2 a = 2\gamma_3 xy(n^3 x + y)/(n^2 x^2 + y^2)$, and $3\gamma_1 a = 2\gamma_3 (ny - y)/(n x^2 + y^2)$. Additionally, the continuity of $v_{\text{pri}}(\zeta_1)$ implies that $\zeta_o = \gamma_3 x - a\kappa$, $\zeta_o = \gamma_3 y^2 - a\kappa$, $\gamma_3 = 3a(\kappa + 1/6)(n x^2 + y^2)/(x^2(3y^2 - n x^2 + 4n^3 x y))$, and $\gamma_3 = 3a(\kappa - 1/6) n(x^2 + y^2)/(y^2(3n^2 x^2 - ny^2 + 4xy))$. Equating these two expressions leads to (6). Finally, the integral equation (14) becomes $a/(6\kappa) - a(\kappa - 1/6) \ln(y) - a(\kappa + 1/6) \ln(x) - a\kappa + 2\gamma_1 ((y/n)^3 - x^3)/3 + 3\gamma_3 (y^2 + x^2)/2 = 0$, which is the same as (7).
As a last step, we need to verify that our initial conjecture that $v^{pri}(\zeta_{11})$ increases in $\zeta_{11}$ is true. Note that because $0 < x < y$, and $n > 1$, we have $\gamma_1, \gamma_2, \gamma_3 > 0$. Thus, it is obvious that $v^{pri}(\zeta_{11})$ increases in $\zeta_{11}$ for $\zeta_{11} \geq \zeta_u$. For $\zeta_{11} < \zeta_u$, we have $(v^{-1}(v^{pri}))' > 0$ if $3\gamma_4 x^2 + \gamma_5 = 2\gamma_3 x > 0$, which is true. Therefore, $v^{pri}(\zeta_{11})$ increases in $\zeta_{11}$, which concludes the proof.

**Note on our solution methodology:** The crucial step is to transform the integral equation (14) into an ordinary differential equation (ODE) by differentiating both sides of the equality with respect to $\zeta$. Clearly, the unique solution to (14) also solves the ODE. However, the ODE may have solutions that do not solve (14). To circumvent this problem, we identify the ODE’s canonical solution (19), which defines the solution to (14) up to certain constants. These constants are, in turn, uniquely defined by the properties of (14).

**Proof of Corollary 1.** Let $\tilde{x} = e^{-(\kappa-1)/(4\kappa^2)} = n^{-(\kappa-1)/(2\kappa)}$ and $\tilde{y} = e^{(\kappa+1)/(4\kappa^2)} = n^{(\kappa+1)/(2\kappa)}$. We now show that $(\tilde{x}, \tilde{y})$ is the solution to the system of equations (6)-(7) as $\kappa \to \infty$. Inserting $\tilde{x}$ and $\tilde{y}$ in (6) reveals that the left-hand side is equal to $-2n^{2/\kappa} \cdot (2 + n^2 + m(1 + 2n^2))$, which converges to zero as $\kappa \to \infty$ because $\lim_{\kappa \to \infty} n = 1$, and $\lim_{\kappa \to \infty} m = -1$. Similarly, the left-hand side of (7) is given by $(1 - 6\kappa^2)/(\kappa(1 + 6\kappa)) + (m + 1)/(4\kappa) + (m - 1)/(4\kappa^2) + 3(1 + n^2)/(2(1 + 2n^2))$, which clearly converges to zero as $\kappa \to \infty$. Moreover, $\lim_{\kappa \to \infty} v_u = \lim_{\kappa \to \infty} 2\kappa \ln(\tilde{x}) = -a/2$, and $\lim_{\kappa \to \infty} v_e = \lim_{\kappa \to \infty} 2\kappa \ln(\tilde{y}) = a/2$. From (5), it follows that only the middle sector persists as $\kappa \to \infty$. Also, by inserting $\tilde{x}$ and $\tilde{y}$ in the formula for $\gamma_3$ in Proposition 3, $\lim_{\kappa \to \infty} \tilde{\gamma}_3 = \lim_{\kappa \to \infty} \gamma_3$. Taken together, this implies that $\lim_{\kappa \to \infty} \tilde{e}_2(\zeta_{11}) = \lim_{\kappa \to \infty} e^{pri}_2(\zeta_{11})$ for all $\zeta_{11}$.

**Proof of Proposition 4A.** (i) From Propositions 1 and 2, it follows readily that $e^{no}_1 = e^{pub}_1$. It remains to show that $e^{no}_1 > e^{pri}_1$. Note that (12) reveals that $e^{pri}_2(\zeta_{11}) < Ak_\kappa/(2ac)$ for all $\zeta_{11}$. Thus, $0 \leq e^{pri}_1 = E_{\zeta_{11}}[e^{pri}_2(\zeta_{11})] < Ak_\kappa/(2ac)$. It follows that $\lim_{a \to \infty} e^{no}_1 = \lim_{a \to \infty} e^{pri}_1 = 0$. Furthermore, $\partial e^{no}_1/\partial a = -e^{no}_1/a$, and $\partial e^{pri}_1/\partial a = -e^{pri}_1/a + \partial(e^{pri}_1)/\partial a > -e^{pri}_1/a$. As a result, $e^{no}_1 = e^{pri}_1 = 0$ for $a \to \infty$, but $\partial e^{no}/\partial a > \partial e^{pri}/\partial a$; i.e., $e^{no}_1$ decreases less steeply than $e^{pri}_1$. This implies that $e^{no}_1 > e^{pri}_1$ if $a$ becomes an $\epsilon > 0$ smaller. But if $e^{no}_1 > e^{pri}_1$, then $e^{pri}_1$ decreases even less steeply compared to $e^{no}_1$. By an inductive argument, it follows that $e^{no}_1 - e^{pri}_1 > 0$, and this difference decreases in $a$.

(ii) The result is a direct consequence of (i) in combination with Propositions 1 - 3.

(iii) By (i) and (ii), it follows that $\Pi_{avg}^{pub} = E[(e^{pub}_{x_2} + v^{pub}_{x_2})/2 - A] = (e^{pub}_{x_1} + E_{\Delta_{x_1}}(e^{pub}_{x_2}(\Delta_{x_1}))) - A = (e^{no}_1 + e^{pri}_2) - A = \Pi_{avg}^{no} > \Pi_{avg}^{pri} = (e^{pri}_{x_1} + E_{\zeta_{11}}[e^{pri}_{x_2}(\zeta_{11}))] - A$.

**Proof of Proposition 4B.** (i) The result follows directly from comparing the firm’s expected profits under the two different feedback policies: $\Pi_{best}^{pub} = E[\max\{\zeta_{11} + \zeta_{x_2} + 2k_e e^{no}_{x_2}\}] - A = 2k_e e^{no}_{x_1} + E[\max\{\zeta_{x_1} + \zeta_{x_2}\}] - A = a(2/(3\kappa) + 7/30) - A$; and $\Pi_{best}^{pub} = E[\max\{\zeta_{x_1} + \zeta_{x_2} + k_e e^{pub}_{x_1} + k_e e^{pub}_{x_2}(\zeta_{11})\}] - A = 2k_e e^{pub}_{x_1} + E[\max\{\zeta_{x_1} + \zeta_{x_2}\}] - A = \Pi_{best}^{no}$, where we made use of the well-known fact that $\max\{a,b\} = (a+b+|a-b|)/2$.

(ii) Let $\kappa = 1$. Then, $\Pi_{best}^{no} \approx 0.889a - A < 0.9a - A = \Pi_{best}^{pub}$, and by the continuity of $\Pi_{best}^{pub}$ and $\Pi_{best}^{no}$, it follows that there exists a $\kappa > 1$, such that $\Pi_{best}^{pub} > \Pi_{best}^{no}$ for all $\kappa < \kappa$.

(iii) The proof proceeds in two steps. First, we establish a lower bound for the firm’s expected profits under a private feedback policy, $\Pi_{best}^{pri} < \Pi_{best}^{no}$, and show that $\Pi_{best}^{pri} > \Pi_{best}^{pub}$ if $\gamma_3$ is sufficiently low. Last, we verify that there exists a $\pi$ such that $\gamma_3$ becomes sufficiently low for all $\kappa > \pi$. 
Lower bound. The firm’s expected profit is $\Pi_{\text{pri}}^{\text{best}} = k_e c_{1}^{\text{pri}} + E \left[ \max \{ \zeta_1 + \zeta_2 + k_e c_2^{\text{pri}}(\zeta_1) \} \right] - A$. Clearly, for any effort function $\epsilon_2(\zeta_1)$ with $\epsilon_2(\zeta_1) \leq \epsilon_2^{\text{pri}}(\zeta_1)$ for all $\zeta_1$, we have $\Pi_{\text{best}}^{\text{pri}} = k_e c_1^{\text{pri}} + E \left[ \max \{ \zeta_1 + \zeta_2 + k_e \epsilon_2(\zeta_1) \} \right] - A \leq \Pi_{\text{best}}^{\text{pri}}$. In the remainder, we set $\epsilon_2(\zeta_1) \equiv -\frac{\zeta_1}{k_e} + \frac{a_k}{k_e} \ln(\zeta_1 + \alpha k) - \frac{e_\gamma}{k_e} \ln(\gamma_3)$. To see that this is indeed a lower bound on $\epsilon_2^{\text{pri}}(\zeta_1)$, note that $\epsilon_2(\zeta_1)$ solves the integral equation (15) for all $\zeta_1$. By doing so, however, we ignore the fact that for some $\zeta_1$ and $\zeta_{j,1}$, we have $\zeta_1 + k_e \epsilon_2(\zeta_1) - \zeta_{j,1} + k_e \epsilon_2(\zeta_1) \notin [-a, a]$. This implies that the left-hand side of (15) is extended by negative terms compared to the correct solution outlined in Proposition 3. Now, since the left-hand side is smaller, it follows by equality that the right-hand side is smaller as well, thereby implying $\epsilon_2(\zeta_1) \leq \epsilon_2^{\text{pri}}(\zeta_1)$.

With $\epsilon_2(\zeta_1)$, $\Pi_{\text{best}}^{\text{pri}} = a \cdot (-\kappa^3(2^3 + \frac{1}{4})(e^{\frac{1}{\kappa}} - e^{-\frac{1}{\kappa}}) + \kappa^4(e^{\frac{1}{\kappa}} + e^{-\frac{1}{\kappa}}) + \kappa(2\kappa - 1)(2\kappa + 1)^3 + \frac{1}{4} \kappa \ln(a^4(2\kappa - 1)(2\kappa + 1)^3) + \frac{4}{3} \kappa^2 - 2\kappa(1 + \ln(2) + \ln(\gamma_3)) + \frac{4}{3} \gamma_3)$, and $\Pi_{\text{best}}^{\text{pri}} > \Pi_{\text{best}}^{\text{pub}}$ if and only if $\gamma_3 < \gamma_3$, with

$$\gamma_3 = \frac{a e^{\frac{1}{4}(-2\kappa + 1)}(e^{\frac{1}{\kappa}} - e^{-\frac{1}{\kappa}})}{e^{\frac{1}{4}(-2\kappa + 1)} + \frac{1}{4} \kappa \ln(\gamma_3) + 2 \kappa + 1} \sqrt{(2\kappa - 1)(2\kappa + 1)} e^{\frac{1}{4}(-2\kappa + 1)} - \frac{4}{3} \kappa^2. \quad (21)$$

Taking the limit. To test whether $\gamma_3 < \gamma_3$, we will first derive an upper bound on $\gamma_3$, and then show that this upper bound is smaller than $\gamma_3$. As a preliminary step, define the function $\Gamma(x, y) = 2\gamma_3/(a(1 + 6\kappa)) = (n^2 x^2 + y^2)/(x^2(3y^2 - n^2 x^2 + 4n^3 xy))$, which decreases in $x$ and $y$. Since $x \in \left[ e^{-1/(4\kappa)}, e^{-1/(4\kappa)}(1 - 1/\kappa) \right]$ and $y \in \left[ e^{1/(4\kappa)}, e^{1/(4\kappa)}(1 + 1/\kappa) \right]$, it follows that $\Gamma(x, y) \in \left[ e^{1/(2\kappa)}, (1 + 2e^{1/\kappa})^2(2e^{1/2\kappa}), e^{1/(2\kappa)}(1 + 2e^{1/\kappa}) \right]$. Given the monotonicity of $\Gamma(x, y)$, we can build the inverse function of $\Gamma(x, y)$ with respect to $y$.

$$y(x, \Gamma) = \frac{n x}{1 - 3 \Gamma x^2} \cdot \left( 2n^2 \Gamma x^2 - \sqrt{4(1 + n^4) \Gamma^2 x^4 - (\Gamma x^2 - 1)^2} \right). \quad (22)$$

Inserting (22) in (6) and (7) allows us to eliminate $y$ from the system of equations, and to represent it in variables $x$ and $\Gamma$. Now, $\Pi_{\text{best}}^{\text{pri}} \leq \Pi_{\text{best}}^{\text{pub}}$ if and only if the transformed system of equations has a solution for $\Gamma$ in the interval $[\Gamma_1, \Gamma]$, and $x$, arbitrary, where $\Gamma_1 = 2\gamma_3/(a(1 + 6\kappa))$. We proceed to show that for sufficiently large $\kappa$, such a solution does not exist; but before doing so, we derive some important properties.

Let $l_1(x, y)$ be the left-hand side of (6), and $l_2(x, y)$ be the left-hand side of (7). Straightforward differentiation verifies that there exists a $\pi$ such that for all $\kappa > \pi$, $l_1(x, y)$ increases in $x$ and decreases in $y$, whereas $l_2(x, y)$ decreases in $x$ and $y$. Furthermore, denote by $x_1(y)$ (resp. $x_2(y)$) the solution to $l_1(x_1(y), y) = 0$ (resp. $l_2(x_2(y), y) = 0$) for any $y$. Applying the Implicit Function Theorem reveals that $x_1(y)$ increases in $y$, while $x_2(y)$ decreases in $y$ for $\kappa > \pi$. In a next step, we transfer these results to the transformed system of equations, which we denote by $l_1'(x, \Gamma) = 0$ and $l_2'(x, \Gamma) = 0$. Analogously to above, let $x_1'(\Gamma)$ (resp. $x_2'(\Gamma)$) be the solution to $l_1'(x_1'(\Gamma), \Gamma) = 0$ (resp. $l_2'(x_2'(\Gamma), \Gamma) = 0$) for any $\Gamma$. Moreover, note that by the Inverse Function Theorem, $y(x, \Gamma)$ decreases in $\Gamma$, because $\Gamma(x, y)$ decreases in $y$. Therefore, by total differentiation, it follows that $\partial x_1'(\Gamma)/\partial \Gamma = \partial x_1(y)/\partial y \cdot \partial y(x, \Gamma)/\partial \Gamma < 0$, and $\partial x_2'(\Gamma)/\partial \Gamma = \partial x_2(y)/\partial y \cdot \partial y(x, \Gamma)/\partial \Gamma > 0$ for $\kappa > \pi$.

We are now well-equipped to complete the proof. We want to show that there exists a $\pi$ such that for all $\kappa > \pi$, the transformed system of equations $l_1'(x^*, \Gamma^*) = 0$ and $l_2'(x^*, \Gamma^*) = 0$ admits no solution with $\Gamma^* \in [\Gamma_1, \Gamma]$. We do so by verifying that for all $\kappa > \pi$, $l_2'(x, \Gamma) > 0$ for any $x$ and $\Gamma \in [\Gamma_1, \Gamma]$. Note that for $\kappa > \pi$, $l_2'(x, \Gamma)$ decreases in $x$, and increases in $\Gamma$. This is true because $\partial l_2'(x, \Gamma)/\partial \Gamma = \partial l_2(x, y)/\partial y \cdot \partial y \cdot \partial \Gamma > 0$, and,
by the Implicit Function Theorem, $\partial l_2(x, \Gamma)/\partial x = - (\partial l_2(x, \Gamma)/\partial \Gamma)/(\partial x_2(\Gamma)/\partial \Gamma) < 0$. Therefore, for $\kappa > \pi$, $l'_2(x, \Gamma) \geq l'_2(\pi, \Gamma_0)$, where $\pi = e^{-1/(4\kappa)} (1-1/\kappa)$. It remains to demonstrate that $l'_2(\pi, \Gamma_0) > 0$, or equivalently, $l'_2(\pi, y(\pi, \Gamma_0)) > 0$ for $\kappa > \pi$. We will conclude this final step with the help of a two-step Taylor series expansion. As a starting point, we substitute $1/\kappa$ by $z$. This substitution allows us to develop the Taylor series at $z = 0$. Now, as a first step, the Taylor series of $y(\pi, \Gamma_0)$ at $z = 0$ is given by $y^{Taylor}(z) = 1 + z/4 - 3z^2/32 - 1177z^3/4480 - 611z^4/14336 + O(z^5)$. In a second step, we can now derive the Taylor series of $l'_2(\pi, y(\pi, \Gamma_0)) = 11/(3360\kappa^3) - 23/(960\kappa^3) + O(1/\kappa^4)$. Since the first term is positive, we can conclude that there exists a $\kappa < \infty$ such that $l'_2^{Taylor}(\pi, y(\pi, \Gamma_0)) > 0$ for all $\kappa > \pi$.

**Proof of Proposition 5.** (i) Suppose that both solvers have invested arbitrary first-round efforts $e_1$. Each solver will make his submission decision so as to maximize his expected continuation utility.

Case (a): If both solvers submit their intermediate solutions, they perfectly learn $v_1$. According to (10), each solver will invest a second-round effort of $e_2^{ss}(v_1) = Ak_e/2c \cdot g_{\Delta \zeta_2}(v_1 - v_{j1})$. As a result, each solver’s expected continuation utility before submission is $u_2^{ss}(v_1) = A \cdot E_{v_1}[G_{\Delta \zeta_2}(v_1 - v_{j1})] - E_{v_1}[e_2^{ss}(v_1)^2]$. Case (b): If only solver $i$ submits his intermediate solution, both solvers only learn $v_i$. The solvers’ equilibrium second-round effort is $e_2^{sn}(v_1) = Ak_e/2c \cdot E_{v_j}[g_{\Delta \zeta_2}(v_1 - v_{j1})|v_{i1}]$, and each solver’s expected continuation utility is $u_2^{sn}(v_1) = A \cdot E_{v_1}[G_{\Delta \zeta_2}(v_1 - v_{j1})] - E_{v_1}[e_2^{sn}(v_1)^2]$. Case (c): If no solver submits his solution, then no feedback is transmitted and according to (9), the solvers’ equilibrium second-round effort is $e_2^{nn}(v_1) = Ak_e/2c \cdot E_{v_{j1}}[g_{\Delta \zeta_2}(v_1 - v_{j1})]|v_{i1}]$, which yields the following expected continuation utility for solver $i$: $u_2^{nn}(v_1) = A \cdot E_{v_1}[G_{\Delta \zeta_2}(v_1 - v_{j1})] - E_{v_1}[e_2^{nn}(v_1)^2]$. By symmetry, case (c) is an equilibrium if and only if $u_2^{nn}(v_1) > u_2^{ss}(v_1)$, or equivalently, $E_{v_{i1}}[e_2^{nn}(v_1)^2] > (e_2^{nn})^2$. This is true because Jensen’s inequality implies that $E_{v_{i1}}[e_2^{nn}(v_1)^2] > (E_{v_{i1}}[e_2^{nn}(v_1)])^2 = (e_2^{nn})^2$. To show that case (c) is the unique equilibrium we verify that case (a) is not an equilibrium: $u_2^{nn}(v_1) > u_2^{ss}(v_1)$ follows from Jensen’s inequality because $E_{v_{i1}}[e_2^{nn}(v_1)^2] = E_{v_{i1}}[E_{v_{j1}}[e_2^{nn}(v_1)^2]|v_{i1}] < E_{v_{i1}}[E_{v_{j1}}[e_2^{nn}(v_1)^2]|v_{i1}] = E_{v_{i1}}[e_2^{nn}(v_1)^2]$.

(ii) In the case of private feedback, a solver’s submission decision is unobservable to the other solver. Hence it is sufficient to verify that a solver’s expected utility from submitting is larger than from not submitting for any mixed strategy of solver $j$ and arbitrary $e_1$. Let $q_j$ be solver $j$’s probability of submitting, and let $e_j^{ss}$ and $e_j^{nn}$ be his second-round efforts if he submits or not, respectively (Lemma A1 guarantees uniqueness).

Solver $i$’s expected utility when submitting his intermediate solution and receiving feedback $v_{i1}$ is $u_2^{ss}(e_{i2}, v_{i1}) = q_i(A \cdot E_{v_j}[G_{\Delta \zeta_2}(v_{i1} + k_e e_{i2} - v_{j1} - k_e e_{j2})|v_{i1}] - e_2^{ss}) + (1 - q_i)(A \cdot E_{v_j}[G_{\Delta \zeta_2}(v_{i1} + k_e e_{i2} - v_{j1} - k_e e_{j2})|v_{i1}] - e_2^{ss})$. Let $u_2^{ss}(e_{i2}) = E_{v_{i1}}[u_2^{ss}(e_{i2}, v_{i1})]$ be his corresponding expected continuation utility, and note that his expected continuation utility from not submitting is $u_2^{nn}(e_{i2}) = q_i(A \cdot E_{v_j}[G_{\Delta \zeta_2}(v_{i1} + k_e e_{i2} - v_{j1} - k_e e_{j2})|v_{i1}] - e_2^{ss}) + (1 - q_i)(A \cdot E_{v_j}[G_{\Delta \zeta_2}(v_{i1} + k_e e_{i2} - v_{j1} - k_e e_{j2})|v_{i1}] - e_2^{ss})$. Furthermore, let $e_{i2}^{ss}(v_{i1})$ and $e_{i2}^{nn}$ be solver $i$’s optimal effort choices if he submits or not, respectively, his intermediate solution. Then it is true that $u_2^{ss}(e_{i2}^{ss}(v_{i1})) = E_{v_{i1}}[u_2^{ss}(e_{i2}^{ss}(v_{i1}), v_{i1})] > E_{v_{i1}}[u_2^{ss}(e_{i2}^{nn}, v_{i1})] = u_2^{nn}(e_{i2}^{nn})$, which proves the claim.
Proof of Proposition 6. Let $A_1 = \alpha A$ and $A_2 = (1 - \alpha)A$, $\alpha \in [0, 1]$, be the awards for the first- and second-round winner, respectively, and note that Proposition 5 implies that $\alpha$ must be sufficiently large to incentivize solvers to submit their intermediate solutions. Clearly, if $\alpha$ is not large enough then a milestone award only reduces the overall contest incentives, which cannot be optimal. In contrast, if $\alpha$ is large enough, then solvers submit their intermediate solutions. In this case, following the same steps as in the proof of Proposition 2, it is straightforward to show that the unique PBE is symmetric and that the solvers’ equilibrium efforts are $e_1^{\text{int}} = (1 + \alpha/2)e^{\text{pub}}_1$ and $e_2^{\text{int}}(\Delta \zeta_1) = (1 - \alpha)e^{\text{pub}}_2(\Delta \zeta_1)$. It follows that $e_1^{\text{int}} + \mathbb{E}[e_2^{\text{int}}(\Delta \zeta_1)] = (2 - \alpha/2)e^{\text{pub}}_1$, implying that the firm’s expected profits decrease in $\alpha$. Thus the firm always chooses $\alpha = 0$ in optimum.

Proof of Proposition 7. Suppose that after round one, the firm truthfully reveals the solvers’ ranking, but not $v_1$; and wlog suppose that solver $i$ is currently the leader. Hence both solvers know that $\Delta v_1 = v_i - v_j > 0$. Solver $i'$’s second-round equilibrium effort solves $e_{i'}^{\text{eq}} \in \arg\max_{e_{i'} \in A} A \cdot \mathbb{E}_{e_1}[G_{\Delta \zeta_2}(v_i + k_i e_{i'} - v_j - k_i e_{i'}^2)] = ce_{i'}^{\text{eq}}$, and the corresponding necessary and sufficient first-order optimality condition is $Ak_i \cdot \mathbb{E}_{e_1}[g_{\Delta \zeta_2}(v_i + k_i e_{i'} - v_j - k_i e_{i'}^2)] | \Delta v_1 > 0 = 2ce_{i'}^{\text{eq}}$. By the symmetry of $g_{\Delta \zeta_2}$ around zero, the unique second-round equilibrium is symmetric, $e_{i'}^{\text{eq}} = e_{i'}^{\text{eq}}$. In the first round, solver $i$’s effort maximizes $\Pi_e[i, k_i, \zeta_i, \kappa]$ subject to the constraint $v_i - v_j > 0$. By the same argument as in the proof of Proposition 2, the first-round equilibrium is also unique and symmetric; that is $e_1^{\text{eq}}(\Delta \zeta_1) = \Delta v_i - \Delta v_j$.

Proof of Proposition 8. (i) Upon learning $v_1$ under a noisy public-feedback policy, solver $i$ chooses his second-round equilibrium effort by solving $e_{i'}^{\text{eq}} \in \arg\max_{e_{i'} \in A} A \cdot \mathbb{E}_{e_1}[G_{\Delta \zeta_2}(v_i + k_i e_{i'} - v_j - k_i e_{i'}^2)] = ce_{i'}^{\text{eq}}$, and the corresponding necessary and sufficient first-order optimality condition is $Ak_i \cdot \mathbb{E}_{e_1}[g_{\Delta \zeta_2}(v_i + k_i e_{i'} - v_j - k_i e_{i'}^2)] | \Delta v_1 > 0 = 2ce_{i'}^{\text{eq}}$. By the same argument as in the proof of Proposition 2, the first-round equilibrium is also unique and symmetric. Hence, by the same argument as in the proof of Proposition 2, the first-round equilibrium is also unique and symmetric. In particular, equilibrium efforts are given by $e_1^{\text{noi}} = E_{\Delta \zeta_1}[e_2^{\text{noi}}(\Delta \zeta_1)] = Ak_i/(3ac)$, and $e_2^{\text{noi}}(\Delta \zeta_1) = Ak_i/(2c) \cdot (g_{\Delta \zeta_2}(\Delta \zeta_1) = (1 - q)E_{\Delta \zeta_1}[g_{\Delta \zeta_2}(\Delta \zeta_1)]).$ Finally, using the same methodology as in the proofs of Proposition 4A(ii) and 4B(i) shows that $\Pi_2^{\text{noi}}$ and $\Pi_2^{\text{noi}}$ are invariant in $q$.

(ii) In a way identical to the proof of Proposition 3, it can be shown that the equilibrium under a noisy private-feedback policy is unique and symmetric. Moreover, $e_1^{\text{noi}} = E_{\zeta_1}[e_2^{\text{noi}}(\zeta_1)]$, and $e_2^{\text{noi}}(\zeta_1)$ solves $qE_{\zeta_1}[g_{\Delta \zeta_2}(\zeta_1 + k_i e_{\text{eq}}^{\text{noi}}(\zeta_1) - \zeta_1 - k_i e_{\text{eq}}^{\text{noi}}(\zeta_1))] + (1 - q)E_{\zeta_1}[g_{\Delta \zeta_2}(\zeta_1 + k_i e_{\text{eq}}^{\text{noi}}(\zeta_1) - \zeta_1 - k_i e_{\text{eq}}^{\text{noi}}(\zeta_1))] = 2ce_{\text{eq}}^{\text{noi}}(\zeta_1)/(Ak_i)$. Let $e_{\text{eq}}^{\text{noi}}(\zeta_1) = e_{\text{eq}}^{\text{noi}}(\zeta_1, \kappa)$ be the effort function defined in (5) for given $\kappa$. Then, $e_2^{\text{noi}}(\zeta_1) = e_2^{\text{noi}}(\zeta_1, \kappa/q) + (1 - q)Ak_i/(2c) \cdot \mathbb{E}_{e_1}[g_{\Delta \zeta_2}(\zeta_1 + k_i e_{\text{eq}}^{\text{noi}}(\zeta_1, \kappa/q) - \zeta_1 - k_i e_{\text{eq}}^{\text{noi}}(\zeta_1, \kappa/q))$. Having derived the equilibrium efforts we can follow exactly the same procedure as in the proof of Proposition 4B(iii) to gain the required Taylor series $I_2^{\text{Taylor}}(\pi, y(\pi, \lambda_2)) = 11q/(3360\kappa^2) - 23/(960\kappa^3) + O(1/\kappa^4)$. Since the first term is positive: for any fixed $q > 0$, there exists a $\pi < \infty$ such that $I_2^{\text{Taylor}}(\pi, y(\pi, \lambda_2)) > 0$ for all $\kappa > \pi$.

Acknowledgments
The authors are grateful to Pascale Crama and Nektarios Oraiopoulos for helpful comments. The authors are also grateful for the constructive suggestions made by three anonymous reviewers, the associate editor, and the department editor.

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