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Bundling in a Symmetric Bertrand Duopoly

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Competitive bundling may lead to such different outcomes as preempting entry, intensifying price competition, or softening it. These different outcomes have been shown to emerge under different industry structures when firms have restricted ranges of action. But how general are these results? In this paper, we investigate whether they still hold under the most generic model of competition, namely: Two symmetric firms competing on price with regard to two homogeneous zero-cost components, without restrictions on their product offering. We show that all three outcomes emerge in equilibrium, respectively as a full mixed-bundling monopoly, a full mixed-bundling competitive duopoly, and a pure or partial-mixed bundling differentiated duopoly. Furthermore, we establish that there are first-mover advantages to bundling and that, unlike in a monopoly, firms may be better off limiting their product offering. Our stylized approach highlights the importance of market operating rules for equilibrium selection: Bundling is not anticompetitive per se, unless firms attempt to coordinate or preempt entry by fully covering the market.

Keywords: Industrial Organization; Bundling; Bertrand Competition; Non-cooperative Game Theory

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"There are only two ways to make money in business: One is to bundle; the other is unbundle." —Jim Barksdale, former CEO of Netscape

1. Introduction

Bundling is a fundamental business decision that affects not only pricing and revenue management, but also product design and horizontal expansion. Since bundling opportunities are greater with lower marginal costs (Bakos and Brynjolfsson 1999), bundling is especially prevalent on online platforms, which leverage economies of scope (driven by digital convergence) to laterally expand across verticals. For example, Netflix's subscription gives access to a bundle of movies, second-run shows, independent shows, purchased shows, produced shows, and—soon—games; Amazon Prime membership gives access to no less than 28 services (e.g., delivery, music, movies, video games, books, magazines, and cloud services); and Apple, as part of its strategy shifts toward services, recently launched the Apple One bundle (covering Apple Music, Apple TV+, Apple Arcade, and iCloud storage).

This increased bundling practice tends to comfort large online platforms (e.g., Google, Apple, Facebook, Amazon, and Microsoft—the GAFAM) in their dominant position and raises numerous antitrust concerns. A classical case study was the charge against Microsoft's bundling of its Internet Explorer browser with its Windows operating system. In a curious repeat of history, Slack, a team communication software, recently filed a lawsuit against Microsoft for its bundling of Teams communication software with its Office 365 (now Microsoft 365) suite of office productivity applications.[1] Is bundling always anticompetitive?

In a departure from its historical focus on monopoly settings, which "is obviously very restrictive to understand bundling as used in practice" (Jeuland 1984, p. S234), the academic literature on bundling has more recently studied competitive settings. It has established that bundling could lead to different outcomes such as (i) preempting entry, (ii) intensifying price competition, or (iii) softening price competition. The divergence in outcomes stems from differences in market structures (duopoly vs. oligopoly), type of firm differentiation (horizontal vs. vertical), levels of differentiation, firm asymmetry (leader vs. follower), distribution of customer valuations, and set of feasible actions (pure vs. partial-mixed vs. full-mixed bundling). Are these three outcomes specific to these industry structures or more general?

In order to assess the generalizability of these results, we consider the most generic model of competition: Two symmetric firms competing on price with regard to two homogeneous zero-cost components, without restrictions on their product offering. Thus, firms can offer any combination of the single-component products and the bundle. A firm adopts a *pure component* strategy if it offers only both single-component products; a *pure bundling* strategy if it offers only the bundle; a *partial-mixed bundling* strategy if it offers the bundle and a single-component product; and a *full-mixed bundling* strategy if it offers all products.

We model the firms' bundling and pricing decisions as a two-stage non-cooperative game: First, firms choose their product offering and then, they compete on price in a Bertrand-Nash pricing game. Consistent with our motivation of shedding light on online platforms' bundling practices, we consider zero costs of production and distribution, as is common for many information goods. Also, we consider uncapacitated Bertrand competition, which is the most prevalent form of competition in online channels of information goods. Although firms are *ex-ante* identical, they may choose to differentiate in equilibrium; thus, firm

[1] `https://stratechery.com/2020/the-slack-social-network/`, accessed on August 6, 2021.

differentiation is an integral part of the model and not assumed exogenously. We focus on the duopoly case, which appears to be the dominant market structure in practice (e.g., Microsoft and Amazon in public cloud, Microsoft and Sony in gaming, Amazon and Google in shopping searches, Google and Facebook in digital advertising). We also consider a generic (symmetric) distribution of customer valuations. Within that setting, we investigate the following research questions:

- Are the three aforementioned competitive bundling equilibrium outcomes idiosyncratic to specific market conditions? Or can they emerge in a symmetric market with unrestricted action sets?
- How should a firm's bundling (or unbundling) strategy change depending on whether it operates in a monopoly or in a duopoly? In particular, does it always pay off to offer more products even if there is no direct competition?
- Can competition alone explain the online platforms' urges to quickly expand their offering horizontally?

We obtain the following results:

- Three outcomes always emerge in equilibrium, namely: (i) a full-mixed bundling monopoly, thus preempting entry from competitors; (ii) a full-mixed bundling competitive duopoly, leading to a price war in which, at most, one firm (namely, the bundler if it is the only one) can earn positive profit; and (iii) a pure or partial-mixed bundling differentiated duopoly, in which both firms earn positive profits. Hence, the three equilibrium outcomes identified in the literature under specific market conditions still emerge in the most generic form of competition. In almost all outcomes, all products are offered and firms have differentiated offerings.

What determines the equilibrium that will be played are the equilibrium selection—or market operating—rules. Firms might coordinate to avoid the payoff-dominated outcome of competitive duopoly. With sequential entry and (infinitesimal) fixed costs of product offering, a first-mover firm will always attempt to expand its product offering as quickly as possible to preempt entry. In such cases, bundling is anticompetitive. However, if firms are unsure about their competitors' decisions, any outcome that involves full-mixed bundling is trembling-hand imperfect, and the differentiated and competitive duopoly outcomes become more plausible—thereby benefiting customers.

- Unlike in a monopoly, firms are not necessarily better off when they offer more products even if there is no direct competition. Specifically, a firm that competes against a (pure or partial-mixed) bundler is always better off when it offers only one single-component product to differentiate its offering from the bundler and avoid head-to-head price competition. Conversely, a bundler competing against a single-component product firm may not benefit from offering anything else; this happens, in particular, when customer valuations are perfectly correlated. In fact, the bundler may see its profit drop by adding the other single-component product to its offering: Even though the new product may enable the bundler to capture unserved market segments, it might redirect the bundle towards competing head-to-head against the competitor's single-component product, thereby leading to a price war and loss of value.
- Across all three equilibrium outcomes, the firm that offers the bundle—if it is the only one—always earns more profit than its competitor. Thus, there are first-mover advantages to bundling. The online platforms' urges to expand their offerings horizontally can therefore be simply explained on purely competitive grounds.

2. Literature Review

The economics and management literature has explored the numerous benefits and pitfalls of bundling—first, from a monopolist's perspective and, more recently, in oligopolistic settings. For surveys of the literature, see Stremersch and Tellis (2002), Kobayashi (2005), and Venkatesh and Mahajan (2009).

For a monopolist, bundling offers several benefits in terms of product performance (Ulrich and Eppinger 2003), in case of component complementarity (Venkatesh and Kamakura 2003), and economies of scope in production, distribution, and promotional activities (Eppen et al. 1991, Evans and Salinger 2005). More subtly, bundling is a form of price discrimination: A monopolist can extract more surplus from its customers by offering a bundle, for which there is little heterogeneity in valuations, than by offering the components separately (Stigler 1963, Adams and Yellen 1976, Schmalensee 1984, McAfee et al. 1989, Salinger 1995). This opportunity is greater for goods that have lower marginal costs, such as information goods (Bakos and Brynjolfsson 1999, Raghunathan and Sarkar 2016). Pure bundling may or may not dominate a pure component strategy, depending on the distribution of valuations, the number of components, and their marginal costs (Fang and

Norman 2006, Ibragimov and Walden 2010). When firms are free to choose any product offering, as we consider here, full-mixed bundling is a dominant strategy for a monopolist (Bhargava 2013). So, the comparison between the pure bundling and pure component strategies is irrelevant, and this enables us to consider general distributions of valuations.

Bundling also gives a monopolist leverage in other markets. Thus, a firm that is a monopolist on one component, but competes with other firms on another component, can leverage its monopolist position by bundling the two components together and foreclosing rivals' sales, thereby increasing its market power in a competitive market (Whinston 1990). In fact, bundling can even *pre-empt the entry* of potential competitors or force the exit of current ones (Carlton and Waldman 2002, Nalebuff 2004, Peitz 2008). Bakos and Brynjolfsson (2000) find that bundling allows large bundlers of information goods to outbid smaller ones in securing upstream content and discourages competitors' entries in the bundler's market while favoring entry of the bundler in adjacent markets.

In oligopoly markets, the effect of bundling is ambiguous. On the one hand, bundling can *intensify competition* because a pure component firm that competes against a bundler finds it "doubly profitable" to lower the price of one of its products: By doing so, it increases the sales of its two products as customers defect from the bundle (Zhou 2021). See Matutes and Regibeau (1988, 1992), Economides et al. (1989), Anderson and Leruth (1993), Nalebuff (2004), Reisinger (2004), Thanassoulis (2007), Armstrong and Vickers (2010), Ahn and Yoon (2012), and Zhou (2021). On the other hand, bundling can *soften competition* because it provides an opportunity for firms to differentiate their offerings and price less aggressively. See Carbajo et al. (1990), Chen (1997), Nalebuff (2004), Zhou et al. (2020), Zhou (2017), and Hurkens et al. (2019).

Most of the literature on competitive bundling assumes some form of horizontal differentiation using spatial models (Matutes and Regibeau 1988, 1992, Economides et al. 1989, Nalebuff 2000, Ghosh and Balachander 2007, Thanassoulis 2007, Armstrong and Vickers 2010, Vamosiu 2018), and more recently—and with great progress—random utility models (Anderson and Leruth 1993, Zhou 2017, 2021). Yet, the most generic model of competition assumes firm symmetry (Bertrand 1883). One might argue that there is no meaningful scope for bundling when firms are identical (Zhou 2017), but this turns out to be not necessarily true since differentiation may emerge as an equilibrium outcome. While modeling horizontal differentiation is useful to capture the reality of many settings, it is thus not necessary to describe the different outcomes of competitive bundling.

In sum, competitive bundling has been shown to (i) help a monopolist preempt entry or force exit, (ii) intensify price competition, or (iii) soften price competition. The divergence in outcomes depends on the market structure (monopoly vs. duopoly vs. oligopoly), the distribution of customer valuations, the firms' degrees of differentiation (horizontal, vertical), asymmetry (leader vs. follower, presence in different markets), and ability to practice pure vs. mixed bundling. In contrast to most of the literature, which makes specific assumptions regarding these different dimensions to identify particular outcomes, we show that all three outcomes emerge in equilibrium *within* the same model, for any distribution of valuations and when firms are free to choose any product offering. Our model is parsimonious and yet, comprehensive. It takes a stylized approach to demonstrate the robustness of the outcomes derived in different settings in the literature

3. Model

Two identical firms, indexed by $i \in \{1,2\}$, compete in the market of an information good (or service) that consists of two homogenous components (e.g., a suite of office productivity applications and a team collaboration software). Each firm can offer any combination of the following three products, indexed by $k \in \{1,2,b\}$: stand-alone component 1; stand-alone component 2; and bundle b of both components.

There is no firm-specific component differentiation and no capacity constraints; accordingly, firms compete on price *à la* Bertrand. As is typical of many information goods, we assume zero marginal production and distribution costs. We also assume that any product can be included in a firm's offering at no cost. Usually, once a firm decides on a product offering, it must remain committed to that decision for a substantial amount of time (Stremersch and Tellis 2002). We accordingly model competition in two stages: a bundling game and a pricing game.

Bundling Game. In the first stage, firms choose their product offering (or bundling) strategy to maximize their individual payoffs. For any $i \in \{1,2\}$, let $\mathbf{z}_i = (z_{ik})_{k=1,2,b}$ be firm i's offering decision; here $z_{ik} = 1$ if firm i offers product k and $z_{ik} = 0$ otherwise. Let $\mathbf{z} = (\mathbf{z}_1, \mathbf{z}_2)$. In contrast to the extant literature, we assume that firms can offer any subset of products. Accordingly, the set of feasible offerings is defined as $\mathcal{Z} = \{0,1\} \times \{0,1\} \times \{0,1\}$. In particular, firm i adopts a *pure component* strategy if $\mathbf{z}_i = (1,1,0)$, a *pure bundling* strategy if $\mathbf{z}_i = (0,0,1)$, a *partial-mixed bundling* strategy if either $\mathbf{z}_i = (1,0,1)$ or $\mathbf{z}_i = (0,1,1)$, and a *full-mixed bundling* strategy if $\mathbf{z}_i = (1,1,1)$. We assume that a firm that is indifferent between offering a bundle or not will opt to offer it. Let $\pi_i(\mathbf{z}_i;\mathbf{z}_{-i})$ be firm i's profit when it offers $\mathbf{z}_i$, and its competitor offer $\mathbf{z}_{-i}$. (Throughout, we denote with $-i \doteq 3-i$ the firm other than firm i, for any $i \in \{1,2\}$.)

When firms choose their offerings simultaneously, a pure-strategy Nash equilibrium of the bundling game is the solution to:

$$\mathbf{z}_i^* \in \arg\max_{\mathbf{z}_i \in \mathcal{Z}} \pi_i(\mathbf{z}_i;\mathbf{z}_{-i}^*) \quad i \in \{1,2\}. \tag{1}$$

The bundling game turns out to always have a pure-strategy Nash equilibrium in the cases we consider (and so we ignore any mixed-strategy equilibria). Yet it might have *multiple* pure-strategy equilibria, in which case we employ the following selection rules:

- Equilibrium $\mathbf{z}^*$ is *payoff-dominant* if $\pi_i(\mathbf{z}_i^*;\mathbf{z}_{-i}^*) \geq \pi_i(\mathbf{z}_i;\mathbf{z}_{-i})$ for all $i=1,2$ for all $\mathbf{z}$. If the inequality is reversed for all $\mathbf{z}$, $\mathbf{z}^*$ is *payoff-dominated.*
- Equilibrium $\mathbf{z}^*$ survives *sequential entry with infinitesimal fixed costs of product offering* if for some small $\gamma > 0$ and some $i \in \{1,2\}$, $\mathbf{z}_i^* = \arg\max_{\mathbf{z}_i \in \mathcal{Z}} \pi_i(\mathbf{z}_i;\mathbf{z}_{-i}^*(\mathbf{z}_i)) - \gamma \sum_{k\in\{1,2,b\}} z_{ik}$, in which $\mathbf{z}_{-i}^*(\mathbf{z}_i) = \arg\max_{\mathbf{z}_{-i}\in\mathcal{Z}} \pi_{-i}(\mathbf{z}_{-i};\mathbf{z}_i) - \gamma\sum_{k\in\{1,2,b\}} z_{-i,k}$.
- Equilibrium $\mathbf{z}^*$ is *trembling-hand perfect* if, for $i=1,2$, there exists a sequence of probabilities $\lambda_{-i}^n(\mathbf{z}_{-i}) > 0$ for all $\mathbf{z}_{-i} \in \mathcal{Z}$ with $\lambda_{-i}^n(\mathbf{z}_{-i}^*) \to 1$ such that $\sum_{\mathbf{z}_{-i}\in\mathcal{Z}} \lambda_{-i}^n(\mathbf{z}_{-i})\pi_i(\mathbf{z}_i^*;\mathbf{z}_{-i}) \geq \sum_{\mathbf{z}_{-i}\in\mathcal{Z}} \lambda_{-i}^n(\mathbf{z}_{-i})\pi_i(\mathbf{z}_i;\mathbf{z}_{-i})$ for any $\mathbf{z}_i \in \mathcal{Z}$ (Selten 1975). If the inequality fails to hold for all sequences of probabilities, $\mathbf{z}^*$ is *trembling-hand imperfect.*

Pricing Game. In the second stage, firms choose their pricing decisions simultaneously and non-cooperatively. Customers have heterogeneous valuations for each component $\mathbf{v} = (v_1, v_2)$. Let $F(\mathbf{v})$ denote the joint cumulative distribution function of customer valuations. Since the components are homogenous, $F(\mathbf{v})$ is assumed to be symmetric, i.e., $F(v_1,v_2) = F(v_2,v_1)$ for any v_1, v_2. We also assume that valuations are positive and finite, i.e., $F(0,v) = 0$ for any v and $F(\overline{v},\overline{v}) = 1$ for some $\overline{v} < \infty$.

The components may have complementary value, which is unleashed through bundling. This happens, in particular, when bundling components is associated with a higher-performing integral design, whereas their separate purchase is associated with a lower-performing modular design (Ulrich and Eppinger 2003). Accordingly, customer valuations for the bundle may be higher than the sum of their valuations for the individual components

(Venkatesh and Kamakura 2003). We assume that this complementarity effect is additive and independent of the valuations; accordingly, a customer who values the components at $\mathbf{v}$ will value the bundle at $v_1 + v_2 + \Delta$, with $\Delta \geq 0$.

Let $\mathbf{p}_i = (p_{ik})_{k=1,2,b}$ denote firm i's vector of pricing decisions. Since the component valuations are positive and bounded from above, the firms' prices can be restricted to the nonempty compact sets: $p_{ik} \in [0, \overline{v}_k]$ with $\overline{v}_k = \overline{v}$ for $k \in \{1,2\}$ and $\overline{v}_b = 2\overline{v} + \Delta$. For any $i \in \{1,2\}$ and $k \in \{1,2,b\}$, if firm i does not offer product k, we set the corresponding price to its upper bound, i.e., $z_{ik} = 0 \Rightarrow p_{ik} = \overline{v}_k$.

Customers always purchase a product at its lowest available price; accordingly, p_k, the market price of product $k \in \{1,2,b\}$, is equal to $\min_{i\in\{1,2\}} p_{ik}$. Customers optimize their purchasing decision to maximize their net surplus with ties being broken in favor of the bundle. For any $k \in \{1,2,b\}$, let $\zeta_k(\mathbf{v},\mathbf{p}) \in \{0,1\}$ indicate whether a customer with component valuations $\mathbf{v}$ buys product k given market prices $\mathbf{p} = (p_k)_{k\in\{1,2,b\}}$. Therefore,

$$\zeta_b(\mathbf{v},\mathbf{p}) = 1 \Leftrightarrow p_b \leq \min\{v_1, p_1\} + \min\{v_2, p_2\} + \Delta \tag{2}$$

$$\text{for } k \in \{1,2\}: \ \zeta_k(\mathbf{v},\mathbf{p}) = 1 \Leftrightarrow v_k \geq p_k \text{ and } p_b > \min\{v_1, p_1\} + \min\{v_2, p_2\} + \Delta. \tag{3}$$

We consider a unit market size and assume that customers buy, at most, one unit of each of component, i.e., that there is full satiation in consumption. Resale by customers is unfeasible. Accordingly, the demand for product k at price $\mathbf{p}$ equals $D_k(\mathbf{p}) = \int \zeta_k(\mathbf{v},\mathbf{p}) dF(\mathbf{v})$.

For any $i \in \{1,2\}$ and $k \in \{1,2,b\}$, let $x_{ik}(p_{ik}; p_{-i,k})$ denote firm i's market share on product k when firm i's price is p_{ik}, and its competitor's price is $p_{-i,k}$. We assume that in the case of a tie, both firms generate sales. Without loss of generality, we consider an even market share split. Accordingly, firm i's market share on product k is equal to

$$x_{ik}(p_{ik}; p_{-i,k}) = \mathbb{1}_{[p_{ik} < p_{-i,k}]} + \frac{1}{2}\mathbb{1}_{[p_{ik} = p_{-i,k}]}; \tag{4}$$

here, $\mathbb{1}_{[y>0]}$ is the indicator function, i.e., $\mathbb{1}_{[y>0]} \doteq 1$ if $y > 0$ and 0 if $y \leq 0$. Accordingly, firm i's profit given its prices $\mathbf{p}_i$ and offering $\mathbf{z}_i$, and its competitor's prices $\mathbf{p}_{-i}$ and offering $\mathbf{z}_{-i}$ equals:

$$\pi_i(\mathbf{p}_i; \mathbf{p}_{-i}, \mathbf{z}_i, \mathbf{z}_{-i}) = \sum_{k\in\{1,2,b\}} p_{ik} x_{ik}(p_{ik}; p_{-i,k}) D_k(\min\{\mathbf{p}_i, \mathbf{p}_{-i}\}).$$

We consider pricing mixed strategies. Even if $\pi_i(\mathbf{p}_i; \mathbf{p}_{-i}, \mathbf{z}_i, \mathbf{z}_{-i})$ is discontinuous whenever a probability mass of customers is indifferent between two options (e.g., buying the bundle or buying the two single-component products), their sum turns out to be upper semicontinuous. Moreover, as is typical in Bertrand games, a firm that lowers the price of one of its products may see its profit drop only slightly, as long as the other firm's mixed strategy does not change dramatically; hence, the extension of the game to mixed strategies is payoff-secure. As a result, for any $\mathbf{z}$, there exists a mixed-strategy Nash equilibrium in the pricing game (Reny 1999, Corollary 5.2).

Let $\Phi_i(\mathbf{p}_i)$ be firm i's price distribution restricted to belong to $\mathcal{F}(\mathbf{z}_i)$, the set of (Borel) probability measures on $[0,1]$ with support on $[\mathbf{0},\overline{\mathbf{v}}]$ such that, if $z_{ik}=0$, $\Phi_i(\mathbf{p})=0$ for all $\mathbf{p}$ such that $p_{ik}<\overline{v}_k$. The pair $(\Phi_1^*(\mathbf{p}_1),\Phi_1^*(\mathbf{p}_2))$ is a mixed-strategy Nash equilibrium if

$$\Phi_i^*(\mathbf{p}_i)\in\arg\max_{\Phi_i\in\mathcal{F}(\mathbf{z}_i)}\int\pi_i(\mathbf{p}_i;\mathbf{p}_{-i},\mathbf{z}_i,\mathbf{z}_{-i})d\Phi_{-i}^*(\mathbf{p}_{-i})d\Phi_i(\mathbf{p}_i)\quad i\in\{1,2\}. \tag{5}$$

For any $i\in\{1,2\}$, let $\pi_i(\mathbf{z}_i;\mathbf{z}_{-i})=\int\pi_i(\mathbf{p}_i;\mathbf{p}_{-i},\mathbf{z}_i,\mathbf{z}_{-i})d\Phi_{-i}^*(\mathbf{p}_{-i})d\Phi_i^*(\mathbf{p}_i)$ denote the equilibrium profit in the pricing game, which will be an input into the bundling game (1).

We solve the game by backward induction, by first solving the pricing game (5) in §4, and then solving the bundling game (1) in §5.

4. Price Equilibria

In this section, we characterize the Nash equilibria of the pricing game for each of the $2^3\times2^3=64$ possible combination of offerings (since $\mathcal{Z}=\{0,1\}^3$). We start with two standard results. First, we show that, in the context of our three-product Bertrand market, if two firms offer the same product, they engage in a price war, which drives the product's equilibrium price to zero. (All proofs appear in an electronic companion.)

Lemma 1 **(Bertrand Price Collapse)**. *For any $k\in\{1,2,b\}$, if $z_{1k}=z_{2k}=1$ and $D_k(\mathbf{p}^*)>0$, then $p_k^*=0$.*

Second, we show that, similar to a monopolist, a competitive bundler is always weakly better off generating sales from its bundle, in the same vein as Anderson and Leruth (1993).

Lemma 2 **(Selling Bundle Always Pays Off)**. *For any $i\in\{1,2\}$, suppose that $z_{ib}=1$. Then firm i is always (weakly) better-off selling the bundle; that is $x_i(p_{ib}^*,p_{-i,b}^*)D_b(\mathbf{p}^*)>0$.*

Next, we consider two specific industry structures involving a firm that pursues a pure component strategy and a firm that adopts either a partial-mixed or a pure bundling strategy. In both cases, the prices of both individual components are driven to zero, the price of the bundle settles to Δ, and the bundling firm is better off than its competitor.

Lemma 3 **(Partial-Mixed Bundling vs. Pure Components)**. *For any* $i \in \{1,2\}$, *suppose that* $\mathbf{z}_i = (1,0,1)$ *or* $(0,1,1)$ *and* $\mathbf{z}_{-i} = (1,1,0)$. *Then,* $\pi_i(\mathbf{p}_i^*; \mathbf{p}_{-i}^*) = \Delta$ *and* $\pi_{-i}(\mathbf{p}_{-i}^*; \mathbf{p}_i^*) = 0$.

Lemma 4 **(Pure Bundling vs. Pure Components)**. *For any* $i \in \{1,2\}$, *suppose that* $\mathbf{z}_i = (0,0,1)$ *and* $\mathbf{z}_{-i} = (1,1,0)$. *Then,* $\pi_i(\mathbf{p}_i^*; \mathbf{p}_{-i}^*) = \Delta$ *and* $\pi_{-i}(\mathbf{p}_{-i}^*; \mathbf{p}_i^*) = 0$.

The proof of Lemma 3 is straightforward. By Lemma 1, the price of the common single-component product is driven down to zero. As a result, the other single-component product competes directly against the bundle, leading to a price war on these two products.

In contrast, the proof of Lemma 4 is more subtle. While the existence of a perfectly competitive equilibrium is easy to establish, showing that it is unique is nontrivial. When facing a bundle price p_b with no complementarity effect ($\Delta = 0$), the profit of the firm that adopts a pure component strategy when the price of each of its products p lies between $p_b/2$ and p_b, is equal to $2p \int_p^\infty \int_0^{p_b - p} dF(\mathbf{v})$. When valuations are independent and uniformly distributed, $p^* = p_b/2$; hence, the single-component product prices add up exactly to the bundle price, leading to a price war. Although this might seem specific to uniform distributions (since in general the maximizer of $2p \int_p^\infty \int_0^{p_b - p} dF(\mathbf{v})$ is not half the bundle price), the price war turns out to hold, in general.

Combining these results leads to the following characterization of the equilibrium profits when both firms pursue either a pure component strategy or a (pure, partial-mixed, full-mixed) bundling strategy. In all cases, perfect competition follows, either leading both firms to earn zero profit or, in cases where only one firm offers the bundle, conferring the bundler an advantage due to the component complementarity.

Corollary 1. *Suppose that* $\mathbf{z}_i \in \{(1,1,0),(0,0,1),(1,0,1),(0,1,1),(1,1,1)\}$ *for* $i = 1,2$. *Then* $\pi_i(\mathbf{z}_i; \mathbf{z}_{-i}) = \Delta$ *if* $z_{ib} = 1$ *and* $z_{-i,b} = 0$ *and* $\pi_i(\mathbf{z}_i; \mathbf{z}_{-i}) = 0$ *otherwise.*

Besides these competitive equilibria, it is also straightforward to characterize the equilibria where one or both firms operate as monopolists. Let $\pi_{1:\emptyset}$, $\pi_{b:\emptyset}$, $\pi_{1b:\emptyset}$, $\pi_{12b:\emptyset}$ denote a monopolist's profit when it offers, respectively, only product 1, only the bundle, the bundle and product 1, and all three products. When $\mathbf{z}_i = (1,0,0)$ and $\mathbf{z}_{-i} = (0,1,0)$, both firms operate as local monopolists on their respective single-component products, yielding each of them a profit of $\pi_{1:\emptyset}$. When $\mathbf{z}_i = (1,1,0)$ and $\mathbf{z}_{-i} = (0,0,0)$, firm i is a monopolist on both single-component products and earns $2\pi_{1:\emptyset}$.

Table 1 Firms' Equilibrium Payoffs

$(\mathbf{z}_i, \mathbf{z}_{-i})$	(0,0,0)	(1,0,0)	(0,1,0)	(1,1,0)	(0,0,1)	(1,0,1)	(0,1,1)	(1,1,1)
$(0,0,0)$	$0,0$	$0,\pi_{1:\emptyset}$	$0,\pi_{1:\emptyset}$	$0,2\pi_{1:\emptyset}$	$0,\pi_{b:\emptyset}$	$0,\pi_{1b:\emptyset}$	$0,\pi_{1b:\emptyset}$	$0,\pi_{12b:\emptyset}$
$(1,0,0)$	$\pi_{1:\emptyset},0$	$0,0$	$\pi_{1:\emptyset},\pi_{1:\emptyset}$	$0,\pi_{1:\emptyset}$	$\pi_{1:b},\pi_{b:1}$	$0,\pi^{\Delta}_{1:\emptyset}$	$\pi_{1:2b},\pi_{2b:1}$	$0,\pi^{\Delta}_{1:\emptyset}$
$(0,1,0)$	$\pi_{1:\emptyset},0$	$\pi_{1:\emptyset},\pi_{1:\emptyset}$	$0,0$	$0,\pi_{1:\emptyset}$	$\pi_{1:b},\pi_{b:1}$	$\pi_{1:2b},\pi_{2b:1}$	$0,\pi^{\Delta}_{1:\emptyset}$	$0,\pi^{\Delta}_{1:\emptyset}$
$(1,1,0)$	$2\pi_{1:\emptyset},0$	$\pi_{1:\emptyset},0$	$\pi_{1:\emptyset},0$	$0,0$	$0,\Delta$	$0,\Delta$	$0,\Delta$	$0,\Delta$
$(0,0,1)$	$\pi_{b:\emptyset},0$	$\pi_{b:1},\pi_{1:b}$	$\pi_{b:1},\pi_{1:b}$	$\Delta,0$	$0,0$	$0,0$	$0,0$	$0,0$
$(1,0,1)$	$\pi_{1b:\emptyset},0$	$\pi^{\Delta}_{1:\emptyset},0$	$\pi_{2b:1},\pi_{1:2b}$	$\Delta,0$	$0,0$	$0,0$	$0,0$	$0,0$
$(0,1,1)$	$\pi_{1b:\emptyset},0$	$\pi_{2b:1},\pi_{1:2b}$	$\pi^{\Delta}_{1:\emptyset},0$	$\Delta,0$	$0,0$	$0,0$	$0,0$	$0,0$
$(1,1,1)$	$\pi_{12b:\emptyset},0$	$\pi^{\Delta}_{1:\emptyset},0$	$\pi^{\Delta}_{1:\emptyset},0$	$\Delta,0$	$0,0$	$0,0$	$0,0$	$0,0$

Note. The shaded cells correspond to the potential set of weakly-dominating bundling Nash equilibria under simultaneous choices of offering. Whether the outcomes in the lighter gray cells are equilibria depends on whether $\pi_{b:1} \geq \pi_{2b:1}$.

The most interesting situation—which has received the most attention in the literature (Carbajo et al. 1990, Nalebuff 2004)—occurs when one firm offers only a single-component product and the other firm offers at least the bundle. If the bundler also offers the same single-component product as its competitor, the price of that product is driven down to zero by Lemma 1 and the bundler operates as a local monopoly on the other single component. Because of the complementarity effect associated with the bundle, a customer with valuation $\mathbf{v}$ who is offered single-component product 2 for free will value the bundle at $v_1 + \Delta$. In that case, the bundler's profit is the same as that of a monopolist that offers product 1 when the marginal distribution of valuations is shifted positively by Δ, which we denote by $\pi^{\Delta}_{1:\emptyset} \doteq \max_p p\mathbb{P}[v_1 + \Delta \geq p]$. Obviously, $\pi^{\Delta}_{1:\emptyset} \geq \pi_{1:\emptyset}$.

In case the bundling firm does not offer the same single-component product as its competitor because it offers nothing other than the bundle (resp., because it offers a single-component product different from the one offered by its competitor), the bundling firm's profit is denoted as $\pi_{b:1}$ (resp., $\pi_{2b:1}$) and its competitor's as $\pi_{1:b}$ (resp., $\pi_{1:2b}$). Using these notations and combining the above results leads to the following proposition.

Proposition 1 **(Price Equilibrium Profits)**. *For any combination of offerings* $(\mathbf{z}_i, \mathbf{z}_{-i})$*, the equilibrium profits in the pricing game (5) are as given in Table 1.*

To illustrate Proposition 1, Table 2 shows the profit values corresponding to no complementarity effects, i.e., $\Delta = 0$, and independent uniform valuations over the unit interval, i.e., $F(\mathbf{v}) = v_1 v_2$ for any $v_k \in [0,1]$.[2]

[2] When $\mathbf{z}_i = (0,0,1)$ and $\mathbf{z}_{-i} = (1,0,0)$, Nalebuff (2004, p.180) argues that the solution should be $(p_1^*, p_b^*) \approx (0.24, 0.59)$ yielding profits $\pi_i^* \approx 0.366$ and $\pi_{-i}^* \approx 0.064$, but that conclusion is based on an erroneous statement of the first-order conditions, which should read instead $p_1^* = (1 + p_b - p_b^2/2)/2$.

Table 2 Firms' Equilibrium Payoffs under No Complementarity Effects and Independent Uniform Valuations

$(\mathbf{z}_i, \mathbf{z}_{-i})$	(0,0,0)	(1,0,0)	(0,1,0)	(1,1,0)	(0,0,1)	(1,0,1)	(0,1,1)	(1,1,1)
(0,0,0)	0, 0	0, 0.25	0, 0.25	0, 0.5	0, 0.544	0, 0.546	0, 0.546	0, 0.549
(1,0,0)	0.25, 0	0, 0	0.25, 0.25	0, 0.25	0.067, 0.369	0, 0.25	0.067, 0.369	0, 0.25
(0,1,0)	0.25, 0	0.25, 0.25	0, 0	0, 0.25	0.067, 0.369	0.067, 0.369	0, 0.25	0, 0.25
(1,1,0)	0.5, 0	0.25, 0	0.25, 0	0, 0	0, 0	0, 0	0, 0	0, 0
(0,0,1)	0.544, 0	0.369, 0.067	0.369, 0.067	0, 0	0, 0	0, 0	0, 0	0, 0
(1,0,1)	0.546, 0	0.25, 0	0.369, 0.067	0, 0	0, 0	0, 0	0, 0	0, 0
(0,1,1)	0.546, 0	0.369, 0.067	0.25, 0	0, 0	0, 0	0, 0	0, 0	0, 0
(1,1,1)	0.549, 0	0.25, 0	0.25, 0	0, 0	0, 0	0, 0	0, 0	0, 0

Note. The shaded cells correspond to bundling Nash equilibria under simultaneous choices of offering.

5. Bundling Equilibria

In this section, we identify the equilibrium strategies in the bundling game (1) while incorporating the equilibrium outcomes of the pricing subgames. To do so, we need to compare profits across conditions. First, a monopolist is trivially better off when it offers more products, i.e., $2\pi_{1:\emptyset} \leq \pi_{12b:\emptyset}$ and $\pi_{b:\emptyset} \leq \pi_{1b:\emptyset} \leq \pi_{12b:\emptyset}$. In contrast, in a duopoly involving a firm that offers only a single-component product, its bundling competitor is better off *not* offering the same single-component product because, as we show next, it would only earn $\pi^{\Delta}_{1:\emptyset}$. (This latter result relies on the assumption of zero marginal costs.)

Lemma 5 **('Are More Products Better?': Monopoly vs. Duopoly)**. *$2\pi_{1:\emptyset} \leq \pi_{12b:\emptyset}$, $\pi_{b:\emptyset} \leq \pi_{1b:\emptyset} \leq \pi_{12b:\emptyset}$, and $\pi^{\Delta}_{1:\emptyset} \leq \min\{\pi_{b:1}, \pi_{2b:1}\}$.*

Hence, the optimal bundling strategies in a duopoly may differ from those in a monopoly. Even if firms may freely add products to their offering, they should not necessarily do so in a duopoly, as it might limit their ability to differentiate. To provide insight into the second result, consider a firm that offers only the bundle and its competitor that offers only product 1. The bundling firm can always set its price equal to $p_2^M = \arg\max_p p\mathbb{P}[v_2 + \Delta \geq p]$—the price of a monopolistic firm that would only offer product 2 and face a demand shifted by Δ. Because the demand for the bundle at p_2^M is higher than the demand for product 2 at p_2^M, the bundling firm setting its price at p_2^M faces larger sales than if it offered only product 2 at the same price. Hence, $\pi_{b:1} \geq \pi^{\Delta}_{1:\emptyset}$.

In the same vein, Table 2 suggests, in the particular case of independent uniform valuations, no real advantage for a pure bundler ($\mathbf{z}_i = (0,0,1)$) that competes against a single-component product firm (e.g., $\mathbf{z}_{-i} = (1,0,0)$) to offer the other single-component product (i.e., $\mathbf{z}_i = (0,1,1)$) since the bundler's profit remains equal to 0.369. Is that specific to independent uniform distributions or more general? The next lemma shows that independence is not the driving factor. Specifically, for perfectly correlated valuations, adding the other single-component product to the bundler's offering provides no benefit.

LEMMA 6 **(Partial-Mixed Bundling May Be the Same as Pure Bundling).** *Suppose that the distributions of valuations are perfectly (negatively or positively) correlated. Then,* $\pi_{2b:1} = \pi_{b:1}$.

However, it is easy to construct instances where the bundler strictly benefits from adding the single-component product to its offering, as shown in the next example.

EXAMPLE 1. Consider a discrete distribution of valuations where $\mathbf{v}$ equals $(1/2, 1/2)$ with probability 3/4, $(2/3, 0)$ with probability 1/8, and $(0, 2/3)$ with probability 1/8. Suppose $\Delta = 0$. When $\mathbf{z}_1 = (0,0,1)$ and $\mathbf{z}_2 = (1,0,0)$, the price equilibrium is $(p_1^*, p_b^*) = (2/3, 1)$, yielding $\pi_1(p_b^*; p_1^*) = 3/4$ and $\pi_2(p_1^*; p_b^*) = 1/12$. When $\mathbf{z}_1 = (0,1,1)$ and $\mathbf{z}_2 = (1,0,0)$, the price equilibrium is $(p_1^*, p_2^*, p_b^*) = (2/3, 2/3, 1)$ yielding $\pi_1(p_2^*, p_b^*; p_1^*) = 5/6$ and $\pi_2(p_1^*; p_2^*, p_b^*) = 1/12$. Hence, $\pi_{2b:1} > \pi_{b:1}$. □

In fact, this benefit tends to materialize for most (continuous) distributions when valuations are independent, as shown in the next lemma. The lemma requires that the distribution has an increasing failure rate, which is a common assumption in pricing and revenue management (Zhou 2017), but also that it has a decreasing reverse hazard rate. Although this assumption is less common in the literature, it is satisfied by most common distributions, such as Gaussian, uniform, exponential, gamma, logistic, Gumbel, Weibull, and unimodal beta (Chechile 2011). Hence, unlike the specific case of uniform distributions, one might expect, in general, that $\pi_{2b:1} \geq \pi_{b:1}$ under independent valuations.

LEMMA 7 **(Partial-Mixed Bundling May Weakly Dominate Pure Bundling).** *Suppose that the distributions of valuations for the two components are independent, absolutely continuous, and their marginal distribution* $F(x)$ *is such that (i) its failure rate,* $\frac{f(x)}{\bar{F}(x)}$, *is increasing, with* $f(x) \doteq F'(x)$ *and* $\bar{F}(x) \doteq 1 - F(x)$, *and (ii) its reverse hazard rate,* $\frac{f(x)}{F(x)}$, *is decreasing. Then,* $\pi_{2b:1} \geq \pi_{b:1}$.

The intuition behind this result is that, for these distributions of valuations, the game, when $\mathbf{z}_i = (0,1,1)$ and $\mathbf{z}_{-i} = (1,0,0)$, is supermodular in $(p_1, -p_2, p_b)$ within the region of interest with positive sales of the three products. Introducing product 2 in the bundler's offering is equivalent to dropping product 2's price, which then leads to an increase in both the price of the bundle and the price of product 1. Because the price of product 1 increases, the bundler effectively faces softer price competition and is thus able to increase its profit.

Still, supermodularity is not guaranteed and it is possible that the introduction of the single-component product into the bundler's offering initiates a price war on the other single component, potentially resulting in lower profits for both firms, as shown in the next example.

Example 2. Consider a uniform discrete distribution of valuations where $\mathbf{v}$ equals $(0,1)$, $(0,1/2)$, $(3/5,2/5)$, and $(3/5,0)$ with equal probability $1/4$. Suppose $\Delta=0$. When $\mathbf{z}_1=(0,0,1)$ and $\mathbf{z}_2=(1,0,0)$, a price equilibrium is $(p_1^*,p_b^*)=(3/5,1)$, yielding $\pi_1(p_b^*;p_1^*)=1/2$ and $\pi_2(p_1^*;p_b^*)=3/20$. (There exists other price equilibria with $p_b^*=1/2$, yielding $\pi_1(p_b^*;p_1^*)=1/2$ and $\pi_2(p_1^*;p_b^*)=0$.) When $\mathbf{z}_1=(0,1,1)$ and $\mathbf{z}_2=(1,0,0)$, there exists no pure-strategy price equilibrium. The following distributions constitute a mixed-strategy price equilibrium:

$$\Phi_1^*(p_2,p_b)=\begin{cases}0 & \text{if } p_b<\frac{2}{5},\\ \mathbb{1}_{[p_2\geq 1/2]}\left(1-\frac{2}{5p_b}\right) & \text{if } \frac{2}{5}\leq p_1<\frac{3}{5},\\ \mathbb{1}_{[p_2\geq 1]} & \text{if } \frac{3}{5}\leq p_b;\end{cases}\quad\text{and}\quad \Phi_2^*(p_1)=\begin{cases}0 & \text{if } p_1<\frac{2}{5},\\ 4-\frac{8}{5p_1} & \text{if } \frac{2}{5}\leq p_1<\frac{1}{2},\\ 2-\frac{3}{5p_1} & \text{if } \frac{1}{2}\leq p_1<\frac{3}{5},\\ 1 & \text{if } \frac{3}{5}\leq p_1.\end{cases}$$

It can be checked that $\pi_1(\Phi_1^*(p_2,p_b);\Phi_2^*(p_1))=2/5$ and $\pi_2(\Phi_2^*(p_1);\Phi_1^*(p_2,p_b))=1/10$. Hence, $\pi_{2b:1}<\pi_{b:1}$. □

Unlike in a monopoly, a bundler may not always benefit—and at times, may be hurt—from adding a single-component product to its offering, even if there is no direct competition on that product. On the one hand, adding a single-component product may enable the bundler to capture unserved customers. On the other hand, it may lead to a price war between the bundle and the competitor's single-component product for the following reason: In addition to catering to unserved customers, the new single-component product might also make some customers switch from the bundle to the new product. Since the bundle is no longer needed to capture these customers, it can be used to capture other customers—such as those targeted by the single-product competitor. A price war might ensue, destroying value for potentially both firms.

Using Lemma 5 leads to the identification of the bundling equilibria, highlighted in the shaded cells of Tables 1 and 2. (Note that Table 2 shows one more equilibrium, namely $\mathbf{z}_i=$

$\mathbf{z}_{-i} = (1,1,0)$, which arises because $\Delta = 0$, but is weakly dominated otherwise.) Although there are numerous product offering combinations that result in bundling equilibria, we note that (i) by firm symmetry, only half of them needs to be considered and (ii) several combinations are payoff-equivalent, really leading to three types of equilibria.

THEOREM 1 **(Bundling Equilibria)**. *In the bundling game (1), the weakly-dominating bundling equilibria are:*

- *Full-Mixed Bundling Monopoly: One firm offers all three products, i.e.,* $\mathbf{z}_i = (1,1,1)$*, and the other firm offers nothing, i.e.,* $\mathbf{z}_{-i} = (0,0,0)$*;*
- *Full-Mixed Bundling Competitive Duopoly: Both firms offer both components and at least one firm offers the bundle, i.e., either* $\mathbf{z}_i = (1,1,0)$ *or* $\mathbf{z}_i = (1,1,1)$ *for* $i = 1,2$ *with* $z_{ib} + z_{-i,b} \geq 1$.
- *Pure or Partial-Mixed Bundling Differentiated Duopoly:*
  - *If* $\pi_{b:1} \geq \pi_{2b:1}$*, one firm offers only the bundle, i.e.,* $\mathbf{z}_i = (0,0,1)$*, and the other firm offers a single component, i.e., either* $\mathbf{z}_{-i} = (1,0,0)$ *or* $\mathbf{z}_{-i} = (0,1,0)$*;*
  - *If* $\pi_{b:1} \leq \pi_{2b:1}$*, one firm offers the bundle and a single-component product, i.e., either* $\mathbf{z}_i = (1,0,1)$ *or* $\mathbf{z}_i = (0,1,1)$*, and the other offers a distinct single-component product, i.e., either* $\mathbf{z}_{-i} = (0,1,0)$ *or* $\mathbf{z}_{-i} = (1,0,0)$*, respectively;*

As reviewed in §2, the literature on competitive bundling has demonstrated, using various models, that bundling could (i) preempt entry from a potential competitor or force exit of an existing one, (ii) intensify price competition, or (iii) soften price competition. Theorem 1 shows that all three outcomes are indeed possible equilibria *within* the same model. Moreover, the models in the literature typically assume some form of firm asymmetry or differentiation and some specific distribution of valuations and restrict firms' offering decisions. In contrast, our model makes no such assumption or restriction. Hence, the competitive bundling equilibrium outcomes identified in the literature are robust, in the sense that they still emerge under the most generic form of competition.

Since the bundle and at least one-single component product are offered in equilibrium, there is no room for potential entry. Hence, in a market with more than two competitors, at most, one of them is making profit; and if there is one, it is a firm that offers the bundle. The restriction to a duopoly is without loss of generality (in our two-component market). With more that two components, we expect multiple forms of the differentiated duopoly equilibrium outcome, reflecting the numerous ways a partial bundle can be created, and potential profitability for more than two competitors (e.g., with n components, $n-1$ firms can offer a single-component product and the nth can could offer the bundle).

Even though games with multiple equilibria lack predictability, it is actually reassuring in this context (given the generic nature of the Bertrand competition model) to see that different outcomes—all plausible—emerge in equilibrium; otherwise, the model lacks validity. To resolve the indeterminacy, one might consider different equilibrium selection rules. In contrast to the literature, which associates different outcomes with different market conditions (e.g., the number of competitors, the distribution of valuations, and whether firms are able to practice mixed or pure bundling), here, the different outcomes are associated with different market operating rules, such as the firms' ability to coordinate their actions, to move sequentially, or to anticipate their competitor's decisions with some degree of certainty. It turns out that different selection rules lead to different predictions. Specifically,

- Equilibria $\mathbf{z} = ((1,1,1),(1,1,1))$ and $\mathbf{z} = ((1,1,0),(1,1,1))$ are Pareto-dominated by $\mathbf{z} = ((0,0,0),(1,1,1))$ since $\pi_{12b:\emptyset} \geq \Delta \geq 0$;
- Equilibrium $\mathbf{z} = ((0,0,0),(1,1,1))$ is the only one that survives sequential entry in the presence of infinitesimal fixed costs of product offering, since $\pi_{12b:\emptyset} \geq \max\{\pi_{b:1}, \pi_{2b:1}\}$.
- Equilibria $\mathbf{z} = ((0,0,0),(1,1,1))$ and $\mathbf{z} = ((1,1,0),(1,1,1))$ are trembling-hand imperfect since strategies $(0,0,0)$ and $(1,1,0)$ are weakly-dominated by $(1,1,1)$.

The payoff-dominance and sequential entry equilibrium selection rules lead to anticompetitive behavior. If firms can coordinate, they will always avoid perfect competition. Worse, with sequential entry, the first mover will preempt entry from any competitor, resulting in low customer welfare. Bundling is, indeed, a terrible "competitive weapon" (Nalebuff 2000), as exemplified by the urge of large online platforms to horizontally expand across industries as an attempt to lock in customers. It has indeed been argued that the extensive benefits covered by the Amazon Prime membership enable Amazon to prevent customer churn. Similarly, Apple's strategy to offer the Apple One bundle has been argued to be a way to extract greater customer surplus.[3]

Fortunately for customers, both the full-mixed bundling competitive duopoly and the pure and partial-mixed bundling differentiated duopoly are trembling-hand perfect, so

[3] `https://stratechery.com/2020/2020-bundles/`, accessed July 16, 2021.

these equilibria tend to be robust to mistakes. If entry is stimulated and coordination banned, some form of competition will ensue, benefiting customers. Customers benefit the most in the competitive duopoly outcome, but this obviously requires that both firms develop both components, which, in practice, may not always be feasible. For instance, customers would certainly benefit from having Slack develop an office suite of productivity software applications to compete with Microsoft 365, but Slack may have little incentive to do so given the high fixed development cost and prospects of intense competition.

A more likely duopoly outcome might be the differentiated one, in which one firm offers a bundle and the other one offers a single-component product. Even though firms are ex-ante symmetric, they choose to differentiate in equilibrium. In this industry structure, the firm that offers the bundle earns the most profit (since $\min\{\pi_{b:1}, \pi_{2b:1}\} \geq \pi_{1:0}$ for otherwise a best response to $\mathbf{z}_i = (1,0,0)$ would be $\mathbf{z}_{-i} = (0,1,0)$). Nalebuff (2004) shows that with independent uniform valuations, the bundling benefit may be substantial.

In fact, there are first-mover advantages to bundling across all three types of equilibria: Whenever one firm bundles and the other does not, the bundler always earns more than its competitor. This explains the drive of some online content platforms to expand their content within their bundle to differentiate themselves from competitors. For instance, Netflix started producing its own content (with such shows as "House of Cards" and "The Crown") to differentiate its content from that available on other movie streaming platforms (e.g., Amazon Prime, Hulu), and it is now expanding its entertainment offering into games. Similarly, Spotify is expanding its offering to podcasts, since its music library is undifferentiated from other music streaming platforms (e.g., Deezer).

6. Conclusions

Although bundling is a fundamental product design decision (every product is a bundle of attributes), it has grown in importance in services with its extensive use by large online platforms (e.g., GAFAM). Because of digitalization, these platforms operate across numerous verticals (e.g., movies, music, news, shopping) and sometimes overlap, which provides an opportunity to package different bundles and compete through bundling.

To test the robustness of the different outcomes that have been shown to emerge under different market conditions (e.g., industry structure, distribution of valuations, firms' asymmetry, differentiation, range of offering), we consider a stylized model of competitive bundling in a symmetric Bertrand duopoly. Even though firms are *ex-ante* identical, they can choose to differentiate in equilibrium.

Our model is comprehensive in the sense that it leads to the three equilibrium outcomes identified in the literature, namely: (i) a full-mixed bundling monopoly, preempting competition, (ii) a full-mixed bundling competitive duopoly, leading to intense price competition, and (iii) a pure or partial-mixed bundling differentiated duopoly, leading to softer price competition. These outcomes are robust, since they emerge in the most generic form of competition.

In contrast to a monopoly, offering more products may not always be beneficial. In particular, a bundler that is competing against a single-component product competitor may be better off if it offers only the bundle to avoid any form of (direct or indirect) competition on the single component offered by its competitor. Still, the firm that offers the bundle—if it is the only one to do so—earns greater profit, suggesting a first-mover advantage to bundling, and providing a rationale for the online platforms' urges to quickly expand the scope of their bundle.

The multiplicity of equilibrium outcomes draws attention to the sensitivity of the competitive nature of bundling to market operating rules (as opposed to the market conditions). In particular, to enhance customer welfare, firms should be banned from coordinating escape from price competition or from preempting entry through full-mixed bundling.

Our model is more stylized than most in the literature, but this is on purpose: to demonstrate the robustness of these outcomes. The literature has already pursued several extensions, bringing greater realism, such as oligopolistic competition (Zhou 2021), different marginal costs (Carbajo et al. 1990), asymmetric valuations (Bhargava 2013), and externalities (Prasad et al. 2010). To better understand online platforms' bundling strategies, one might further investigate the effect of goods with negative valuation (e.g., advertising), supply chain intermediaries (e.g., app stores), externalities on other product markets (e.g., Disney's flywheel effect, Microsoft's Xbox), partially overlapping component sets (e.g., Netflix and Disney+), and user base effects (e.g., Netflix). Operationally, bundling information goods presents many challenges such as pricing (Letham et al. 2014, Abdallah et al. 2021) and digital piracy (Wu et al. 2020). As online platforms keep expanding their offering across different verticals, more research is needed to identify the strategic and operational implications of competitive bundling.

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Appendix: Proofs and Supplementary Results

Proof of Lemma 1. To obtain a contradiction, suppose that there exists an equilibrium $\mathbf{p}^*$ such that $D_k(\mathbf{p}^*) > 0$ and $p_k^* > 0$. We consider three cases:

1. **$p_{ik}^* > p_{-i,k}^* = p_k^*$ for some $i \in \{1,2\}$.** In that case, firm i could lower its price p_{ik} to $p_{-i,k}^*$ without changing anything to $\mathbf{D}(\mathbf{p}^*)$ and increase its profit by $p_k^* D_k(\mathbf{p}^*)/2 > 0$ by (4), a contradiction.

2. **$p_{1k}^* = p_{2k}^* = p_k^*$ and, for any $\kappa \in \{1,2,b\} \setminus \{k\}$, $D_\kappa(\mathbf{p})$ is either decreasing or left-continuous in p_k at $\mathbf{p}^*$.** In that case, for any $i \in \{1,2\}$, firm i could strictly increase its sales of product k by at least $D_k(\mathbf{p}^*)/2$ by setting p_{ik} infinitesimally below $p_{-i,k}^*$, which is feasible given that $p_k^* > 0$. Its sales of any other products $\kappa \in \{1,2,b\} \setminus \{k\}$ might decrease, but only infinitesimally by left-continuity of $D_\kappa(\mathbf{p})$ and by (4). Hence, firm i's profit could strictly increase, a contradiction.

3. **$p_{1k}^* = p_{2k}^* = p_k^*$ and, for some $\kappa \in \{1,2,b\} \setminus \{k\}$, $D_\kappa(\mathbf{p})$ is increasing and right-continuous in p_k at $\mathbf{p}^*$.** This may happen only when $k \in \{1,2\}$, $p_1^* + p_2^* = p_b^* - \Delta$, and $\mathbb{P}[v_1 \geq p_1^*, v_2 \geq p_2^*] > 0$ since, by (2)-(3) and by denoting $-k \doteq 3-k$ the single-component product other than single-component product k,

$$\text{for } k \in \{1,2\}: \; D_k(\mathbf{p}) = \int_{p_k}^{\infty} \int_0^{\min\{p_{-k}, p_b - \Delta - p_k\}} dF(\mathbf{v}) + \int_{p_1}^{\infty} \int_{p_2}^{\infty} dF(\mathbf{v}) \mathbb{1}_{[p_b - \Delta > p_1 + p_2]}$$
$$D_b(\mathbf{p}) = \int_0^{p_1} \int_{\max\{0, p_b - \Delta - v_1\}}^{p_2} dF(\mathbf{v}) + \int_{\max\{0, p_b - \Delta - p_2\}}^{p_1} \int_{p_2}^{\infty} dF(\mathbf{v})$$
$$+ \int_{p_1}^{\infty} \int_{\max\{0, p_b - \Delta - p_1\}}^{p_2} dF(\mathbf{v}) + \int_{p_1}^{\infty} \int_{p_2}^{\infty} dF(\mathbf{v}) \mathbb{1}_{[p_b - \Delta \leq p_1 + p_2]}.$$

Thus, $D(\mathbf{p})$ is left-continuous in p_b, and the only point where $D(\mathbf{p})$ is right-continuous in p_k, $k \in \{1,2\}$, is when $p_1 + p_2 = p_b - \Delta$, and $\mathbb{P}[v_1 \geq p_1, v_2 \geq p_2] > 0$. Without loss of generality, assume that $z_{ib} = 0$ for some firm i. In that case, firm i could drop its price p_{ik} infinitesimally below p_k^*, increase its sales of product k by $\mathbb{P}[v_1 \geq p_1^*, v_2 \geq p_2^*] > 0$, and strictly increase its profit, a contradiction. □

Proof of Lemma 2. Fix i and suppose that $z_{ib} = 1$. We consider two cases:

1. **$D_b(\mathbf{p}^*) > 0$.** If $z_{-i,b} = 0$, then $x_i(\mathbf{p}^*) = 1$ by (4), which establishes the result. Henceforth, we consider the case where $z_{ib} = z_{-i,b} = 1$. By Lemma 1 since $D_b(\mathbf{p}^*) > 0$, $p_b^* = 0$. Then, by (3), $D_k(\mathbf{p}^*) = 0$ for $k = 1,2$. Hence, $\pi_\iota(\mathbf{p}^*) = 0$ for $\iota = 1,2$. If $p_{ib}^* > p_b^*$, $\pi_i(\mathbf{p}^*) = 0$ and firm i might as well set p_{ib} equal to $p_{-i,b}^*$. Then, $p_{ib}^* = p_{-i,b}^*$ and $x_i(p_{ib}^*, p_{-i,b}^*) > 0$ by (4) and therefore, $x_i(p_{ib}^*, p_{-i,b}^*) D_b(\mathbf{p}^*) > 0$.

2. **$D_b(\mathbf{p}^*) = 0$.** Without loss of generality, we set $p_b^* = \overline{v}_b$. Denote with $-k \doteq 3-k$ the single-component product other than single-component product k ($k \in \{1,2\}$). Then,

$$\pi_i(\mathbf{p}_i^*; \mathbf{p}_{-i}^*) = \sum_{k=1}^{2} p_k^* x_k(p_{ik}^*, p_{-i,k}^*) D_k(\mathbf{p}^*)$$
$$= \sum_{k=1}^{2} x_k(p_{ik}^*, p_{-i,k}^*) p_k^* \int \mathbb{1}_{\left[v_k \geq p_k^*, v_{-k} < p_{-k}^*\right]} dF(\mathbf{v})$$
$$+ \left(p_1^* x_1(p_{i1}^*, p_{-i,1}^*) + p_2^* x_2(p_{i2}^*, p_{-i,2}^*)\right) \int \mathbb{1}_{\left[v_1 \geq p_1^*, v_2 \geq p_2^*\right]} dF(\mathbf{v})$$

$$
\begin{aligned}
&= \sum_{k=1}^{2} x_k(p^*_{ik}, p^*_{-i,k}) p^*_k \int \mathbb{1}_{\left[v_k \geq p^*_k, p^*_k + \min\{v_{-k}, p^*_{-k}\} < p^*_1 + p^*_2 - \Delta\right]} dF(\mathbf{v}) \\
&\quad + \left(p^*_1 x_1(p^*_{i1}, p^*_{-i,1}) + p^*_2 x_2(p^*_{i2}, p^*_{-i,2})\right) \int \mathbb{1}_{\left[\min\{v_1, p^*_1\} + \min\{v_2, p^*_2\} \geq p^*_1 + p^*_2 - \Delta\right]} dF(\mathbf{v}) \\
&\leq \sum_{k=1}^{2} x_k(p^*_{ik}, p^*_{-i,k}) p^*_k \int \mathbb{1}_{\left[v_k \geq p^*_k, p^*_k + \min\{v_{-k}, p^*_{-k}\} < p^*_1 + p^*_2 - \Delta\right]} dF(\mathbf{v}) \\
&\quad + (p^*_1 + p^*_2) \int \mathbb{1}_{\left[\min\{v_1, p^*_1\} + \min\{v_2, p^*_2\} \geq p^*_1 + p^*_2 - \Delta\right]} dF(\mathbf{v}) \\
&\leq \max_{p_b} \sum_{k=1}^{2} x_k(p^*_{ik}, p^*_{-i,k}) p^*_k \int \mathbb{1}_{\left[v_k \geq p^*_k, p^*_k + \min\{v_{-k}, p^*_{-k}\} < p_b - \Delta\right]} dF(\mathbf{v}) \\
&\qquad + p_b \int \mathbb{1}_{\left[\min\{v_1, p^*_1\} + \min\{v_2, p^*_2\} \geq p_b - \Delta\right]} dF(\mathbf{v}) \\
&= \max_{p_{ib}} \pi_i(p^*_{i1}, p^*_{i2}, p_{ib}; \mathbf{p}^*_{-i}).
\end{aligned}
$$

Here the second equality is by (3), the first inequality is because $x_k(\mathbf{p}) \leq 1$ by (4), and the second inequality is because setting $p_b = p^*_1 + p^*_2$ is a feasible solution. Hence, firm i is always (weakly) better off setting its bundle price $p_b < \overline{v}_b$, i.e., generating sales from the bundle. □

Proof of Lemma 3. Let $k \in \{1,2\}$ such that $z_{1k} = z_{2k} = 1$ and let $-k \doteq 3 - k$ the index of the single-component product other than single-component product k. By Lemma 1, either $D_k(\mathbf{p}^*) = 0$ or $p^*_k = 0$. Then, since $z_{i,-k} = 0$, $\pi_i(\mathbf{p}^*_i; \mathbf{p}^*_{-i}) = p^*_b D_b(\mathbf{p}^*)$. Since $D_b(\mathbf{p}) = 1$ for all $p_b \in [0, \Delta]$ by (2), firm i can earn at least Δ by setting p_b equal to Δ; hence, $\pi_i(\mathbf{p}^*_i; \mathbf{p}^*_{-i}) \geq \Delta$ and $D_b(\mathbf{p}^*) > 0$. The rest of the proof is by contradiction and proceeds in two steps.

1. **Suppose that** $\pi_{-i}(\mathbf{p}^*_{-i}; \mathbf{p}^*_i) > 0$**.** Then, $p^*_{-k} D_{-k}(\mathbf{p}^*) > 0$ since $p^*_k = 0$ and $z_{-i,b} = 0$. This implies, by (3), that $p^*_b > p^*_{-k} + \Delta$. However, this would yield that $D_b(\mathbf{p}^*) = 0$ by (2), a contradiction. Thus, $\pi_{-i}(\mathbf{p}^*_{-i}; \mathbf{p}^*_i) = 0$.
2. **Suppose that** $\pi_i(\mathbf{p}^*_i; \mathbf{p}^*_{-i}) > \Delta$**.** Then, $p^*_b > \Delta$ and, by (2), $\mathbb{P}[v_{-k} \geq p^*_b - \Delta] > 0$. However, this cannot be an equilibrium since firm $-i$ could set its price p_{-k} to $p^*_b - \Delta - \epsilon$ for some small $\epsilon > 0$ (given that $p^*_b > \Delta$) and earn a positive profit $(p^*_b - \Delta - \epsilon)\mathbb{P}[v_{-k} \geq p^*_b - \Delta - \epsilon] > 0$. Thus, $\pi_i(\mathbf{p}^*_i; \mathbf{p}^*_{-i}) > \Delta$. □

Proof of Lemma 4. Because $F(\mathbf{v})$ is symmetric, $p^*_1 = p^*_2 = p^*$.

First, we show that $\mathbf{p}^* = (0, 0, \Delta)$ is an equilibrium. When $p_b = \Delta$, $D_k(\mathbf{p}) = 0$ for $k = 1, 2$ for all (p_1, p_2) by (3), so setting $p_k = 0$, $k \in \{1, 2\}$, is a best response to $p^*_b = \Delta$. When $p_1 = p_2 = 0$, $D_b(\mathbf{p}) = 1$ if $p_b \leq \Delta$ and zero otherwise by (2), so setting $p_b = \Delta$ is the best response to $p^*_1 = p^*_2 = 0$.

Next, we show that there exists no other equilibrium that generates different payoffs. On the one hand, no equilibrium exists, such that $\pi_i(\mathbf{p}^*_i; \mathbf{p}^*_{-i}) < \Delta$, since firm i can always capture the entire market by setting p_b equal to Δ. On the other hand, we show that there exists an equilibrium such that $\pi_i(\mathbf{p}^*_i; \mathbf{p}^*_{-i}) > \Delta$. The proof proceeds in four steps.

Step 1: $(p_b^* - \Delta)/2 \le p^* < p_b^* - \Delta$. When $\pi_i(\mathbf{p}_i^*;\mathbf{p}_{-i}^*) > \Delta$, $p_b^* > \Delta$ and, by (2), $2p^* \ge p_b^* - \Delta$. Thus,

$$p^* \ge \frac{1}{2}(p_b^* - \Delta). \tag{EC.1}$$

Since firm $-i$ can set p slightly below to $(p_b^* - \Delta)/2$, which is feasible since $p_b^* > \Delta$, and capture market $\int_{\frac{p_b^*-\Delta}{2}}^{\infty}\int_0^{\infty} dF(\mathbf{v})$ for each product by (3),

$$\pi_{-i}(\mathbf{p}_{-i}^*;\mathbf{p}_i^*) \ge (p_b^* - \Delta)\int_{\frac{p_b^*-\Delta}{2}}^{\infty}\int_0^{\infty} dF(\mathbf{v}). \tag{EC.2}$$

In particular, $\pi_{-i}(\mathbf{p}_{-i}^*;\mathbf{p}_i^*) > 0$, since $p_b^* > \Delta$. Thus, $p^* > 0$ and $D_k(\mathbf{p}^*) > 0$ for $k = 1, 2$. By (3),

$$p^* < p_b^* - \Delta. \tag{EC.3}$$

Combining (EC.1) and (EC.3), we obtain that $(p_b^* - \Delta)/2 \le p^* < p_b^* - \Delta$. Hence, both firms sell their products and

$$\pi_i(\mathbf{p}_i^*;\mathbf{p}_{-i}^*) = p_b^* \int_{p_b^*-\Delta-p^*}^{\infty}\int_{p_b^*-\Delta-\min\{v_1,p^*\}}^{\infty} dF(\mathbf{v}) \text{ and} \tag{EC.4}$$

$$\pi_{-i}(\mathbf{p}_{-i}^*;\mathbf{p}_i^*) = 2p^* \int_{p^*}^{\infty}\int_0^{p_b^*-\Delta-p^*} dF(\mathbf{v}). \tag{EC.5}$$

Step 2: Lower bound on $\pi_i(\mathbf{p}_i^*;\mathbf{p}_{-i}^*)$. Because firm i can set $p_b = p^* + \Delta$ and capture market $1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})$ by (2) Moreover,

$$\pi_i(\mathbf{p}_i^*;\mathbf{p}_{-i}^*) \ge (p^* + \Delta)\left(1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})\right).$$

Moreover, $1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v}) \ge 2\int_{p^*}^{\infty}\int_0^{p^*} dF(\mathbf{v})$. Hence,

$$\begin{aligned}
\pi_i(\mathbf{p}_i^*;\mathbf{p}_{-i}^*) &\ge 2p^* \int_{p^*}^{\infty}\int_0^{p^*} dF(\mathbf{v}) + \Delta\left(1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})\right) \\
&\ge 2p^* \int_{p^*}^{\infty}\int_0^{p_b^*-\Delta-p^*} dF(\mathbf{v}) + \Delta\left(1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})\right);
\end{aligned} \tag{EC.6}$$

where the third inequality is by (EC.1).

Step 3: Lower bound on $\int_{\frac{p_b^*-\Delta}{2}}^{\infty}\int_{p_b^*-\Delta-\min\{v_1,p^*\}}^{\frac{p_b^*-\Delta}{2}} dF(\mathbf{v})$. Using Steps 1 and 2 above, we obtain:

$$\begin{aligned}
& p_b^* \int_{p_b^*-\Delta-p^*}^{\infty}\int_{p_b^*-\Delta-\min\{v_1,p^*\}}^{\infty} dF(\mathbf{v}) \\
&= \pi_i(\mathbf{p}_i^*;\mathbf{p}_{-i}^*) && \text{by (EC.4)} \\
&\ge 2p^* \int_{p^*}^{\infty}\int_0^{p_b^*-\Delta-p^*} dF(\mathbf{v}) + \Delta\left(1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})\right) && \text{by (EC.6)} \\
&= \pi_{-i}(\mathbf{p}_{-i}^*;\mathbf{p}_i^*) + \Delta\left(1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})\right) && \text{by (EC.5)} \\
&\ge (p_b^* - \Delta)\int_{\frac{p_b^*-\Delta}{2}}^{\infty}\int_0^{\infty} dF(\mathbf{v}) + \Delta\left(1 - \int_0^{p^*}\int_0^{p^*-v_1} dF(\mathbf{v})\right) && \text{by (EC.2).}
\end{aligned}$$

Equivalently, after dividing both sides by p_b^* and rearranging the terms, we obtain

$$\int_{p_b^*-\Delta-p^*}^{\frac{p_b^*-\Delta}{2}} \int_{p_b^*-\Delta-v_1}^{\infty} dF(\mathbf{v}) \tag{EC.7}$$

$$\geq \int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_0^{p_b^*-\Delta-\min\{v_1,p^*\}} dF(\mathbf{v}) + \frac{\Delta}{p_b^*}\left(\int_0^{\frac{p_b^*-\Delta}{2}} \int_{p^*-v_1}^{\infty} dF(\mathbf{v}) - \int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p^*-v_1} dF(\mathbf{v})\right). \tag{EC.8}$$

Since $F(\mathbf{v})$ is symmetric,

$$\int_{p_b^*-\Delta-p^*}^{\frac{p_b^*-\Delta}{2}} \int_{p_b^*-\Delta-v_1}^{\infty} dF(\mathbf{v}) = \int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_{p_b^*-\Delta-\min\{v_1,p^*\}}^{\frac{p_b^*-\Delta}{2}} dF(\mathbf{v}). \tag{EC.9}$$

Therefore, plugging (EC.9) into (EC.8) yields

$$\int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_{p_b^*-\Delta-\min\{v_1,p^*\}}^{\frac{p_b^*-\Delta}{2}} dF(\mathbf{v})$$

$$\geq \int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_0^{p_b^*-\Delta-\min\{v_1,p^*\}} dF(\mathbf{v}) + \frac{\Delta}{p_b^*}\left(\int_0^{\frac{p_b^*-\Delta}{2}} \int_{p^*-v_1}^{\infty} dF(\mathbf{v}) - \int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p^*-v_1} dF(\mathbf{v})\right). \tag{EC.10}$$

Step 4: p^* is not a best response to p_b^*. As a result,

$$\begin{aligned}
&\pi_{-i}\left(\frac{p_b^*-\Delta}{2}, \frac{p_b^*-\Delta}{2}; p_b^*\right)\\
&= (p_b^*-\Delta)\int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_0^{\frac{p_b^*-\Delta}{2}} dF(\mathbf{v}) && \text{by (EC.5)}\\
&= (p_b^*-\Delta)\left(\int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_0^{p_b^*-\Delta-\min\{v_1,p^*\}} dF(\mathbf{v}) + \int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_{p_b^*-\Delta-\min\{v_1,p^*\}}^{\frac{p_b^*-\Delta}{2}} dF(\mathbf{v})\right)\\
&\geq 2(p_b^*-\Delta)\int_{\frac{p_b^*-\Delta}{2}}^{\infty} \int_0^{p_b^*-\Delta-\min\{v_1,p^*\}} dF(\mathbf{v})\\
&\quad + (p_b^*-\Delta)\frac{\Delta}{p_b^*}\left(\int_0^{\frac{p_b^*-\Delta}{2}} \int_{p^*-v_1}^{\infty} dF(\mathbf{v}) - \int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p^*-v_1} dF(\mathbf{v})\right) && \text{by (EC.10)}\\
&> 2p^*\int_{p^*}^{\infty} \int_0^{p_b^*-\Delta-p^*} dF(\mathbf{v}) + 2(p_b^*-\Delta)\int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p_b^*-\Delta-v_1} dF(\mathbf{v})\\
&\quad + (p_b^*-\Delta)\frac{\Delta}{p_b^*}\left(\int_0^{\frac{p_b^*-\Delta}{2}} \int_{p^*-v_1}^{\infty} dF(\mathbf{v}) - \int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p^*-v_1} dF(\mathbf{v})\right) && \text{by (EC.3)}\\
&> 2p^*\int_{p^*}^{\infty} \int_0^{p_b^*-\Delta-p^*} dF(\mathbf{v})\\
&\quad + (p_b^*-\Delta)\frac{\Delta}{p_b^*}\left(2\int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p_b^*-\Delta-v_1} dF(\mathbf{v}) + \int_0^{\frac{p_b^*-\Delta}{2}} \int_{p^*-v_1}^{\infty} dF(\mathbf{v}) - \int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p^*-v_1} dF(\mathbf{v})\right) && \text{by } p_b^* > \Delta\\
&= 2p^*\int_{p^*}^{\infty} \int_0^{p_b^*-\Delta-p^*} dF(\mathbf{v})\\
&\quad + (p_b^*-\Delta)\frac{\Delta}{p_b^*}\left(\int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_0^{p_b^*-\Delta-v_1} dF(\mathbf{v}) + \int_0^{\frac{p_b^*-\Delta}{2}} \int_{p^*-v_1}^{\infty} dF(\mathbf{v}) + \int_{\frac{p_b^*-\Delta}{2}}^{p^*} \int_{p^*-v_1}^{p_b^*-\Delta-v_1} dF(\mathbf{v})\right)
\end{aligned}$$

$$> 2p^* \int_{p^*}^{\infty} \int_0^{p_b^* - \Delta - p^*} dF(\mathbf{v}) \qquad \text{by } p_b^* > \Delta$$
$$= \pi_{-i}(p^*, p^*; p_b^*) \qquad \text{by (EC.5).}$$

Hence, p^* is not a best response to p_b^* and no equilibrium exists, such that $\pi_i(\mathbf{p}_i^*; \mathbf{p}_{-i}^*) > \Delta$. □

Proof of Corollary 1. We consider the following cases:

1. $\mathbf{z}_{-i} = (1,1,0)$**.** If $\mathbf{z}_i = (1,1,0)$, the result follows by Lemma 1. If $\mathbf{z}_i = (0,0,1)$, the result follows by Lemma 4. If $\mathbf{z}_i = (1,0,1)$ or $(0,1,1)$, the result follows by Lemma 3. If $\mathbf{z}_i = (1,1,1)$, then by Lemma 1, $p_1^* = p_2^* = 0$, and, therefore, $D_b(\mathbf{p}^*) = 1$ if $p_b \le \Delta$ and zero otherwise; hence, $p_b^* = \Delta$.
2. $\mathbf{z}_{-i} = (0,0,1)$**.** If $\mathbf{z}_i = (0,0,1)$, $(1,0,1)$, $(0,1,1)$, or $(1,1,1)$, $p_b^* = 0$ by Lemma 1 and $D_k(\mathbf{p}^*) = 0$ for $k \in \{1,2\}$.
3. $\mathbf{z}_{-i} = (1,0,1)$ **or** $(0,1,1)$**.** If $\mathbf{z}_i = (1,0,1)$, $(0,1,1)$, or $(1,1,1)$, $p_b^* = 0$ by Lemma 1 and $D_k(\mathbf{p}^*) = 0$ for $k \in \{1,2\}$.
4. $\mathbf{z}_{-i} = (1,1,1)$**.** If $\mathbf{z}_i = (1,1,1)$, $p_b^* = 0$ by Lemma 1 and $D_k(\mathbf{p}^*) = 0$ for $k \in \{1,2\}$. □

Proof of Proposition 1. We establish the equilibrium profits by considering the following cases:

- When $\mathbf{z}_1 = \mathbf{z}_2$, the result follows by Lemma 1.
- When $\mathbf{z}_1, \mathbf{z}_2 \in \{(1,1,0),(0,0,1),(1,0,1),(0,1,1),(1,1,1)\}$, the result follows by Corollary 1.
- When $\mathbf{z}_1, \mathbf{z}_2 \in \{(0,0,0),(1,0,0),(0,1,0),(1,1,0)\}$, either firms are directly competing head to head with the same product, in which case their profit from that product is zero by Lemma 1, or they have a monopoly, in which case their profit from that product—given that valuations are symmetric—is equal to $\pi_{\{1\}:\emptyset}$.
- When for any $k \in \{1,2\}$, $z_{ik} = 1$, $z_{ib} = 1$, $z_{-i,k} = 1$, and $z_{-i,-k} = z_{-i,b} = 0$ with $-k \doteq 3 - k$, then $p_k^* = 0$ by Lemma 1. Firm i's equilibrium profit simplifies to $\max_{p_{ib}, p_{i,-k}} p_{ib} D_b(0, p_{i,-k}, p_{ib}) + p_{i,-k} z_{i,-k} D_{-k}(0, p_{i,-k}, p_{ib})$. If $z_{i,-k} = 1$, it is optimal for firm i to set $p_{i,-k} = \infty$ because product $-k$ is directly competing with the bundle by (2)-(3): either $D_b(0, p_{-k}, p_b) > 0$, in which case $p_b \le p_{-k} + \Delta$ or $D_{-k}(0, p_{-k}, p_b) > 0$, in which case $p_b > p_{-k} + \Delta$. By Lemma 2, firm i prefers to sell the bundle. Therefore, firm i's equilibrium profit simplifies to $\max_{p_{ib}} p_{ib} D_b(0, \infty, p_{ib}) = \max_{p_{ib}} p_{ib} \int \mathbb{1}_{[p_{ib} \le v_{-k} + \Delta]} dF(\mathbf{v}) = \pi^{\Delta}_{\{1\}:\emptyset}$. □

Proof of Lemma 5. • The inequalities $2\pi_{1:\emptyset} \le \pi_{12b:\emptyset}$ and $\pi_{b:\emptyset} \le \pi_{1b:\emptyset} \le \pi_{12b:\emptyset}$ follow from the fact that, with more products, a monopolist can only do better as it can always price the incremental product to infinity.

- To show that $\pi_{b:1} \ge \pi^{\Delta}_{1:\emptyset}$, consider the pricing game associated with $(\mathbf{z}_i, \mathbf{z}_{-i}) = ((0,0,1),(1,0,0))$. Denote by $(\Phi_i^*(p_b), \Phi_{-i}^*(p_1))$ the corresponding mixed-strategy price equilibrium solving (5) and by p_2^M the optimal price of a monopolist facing a shifted demand $D_2^{\Delta}(\overline{v}_1, p_2, \overline{v}_b)$, i.e., $p_2^M = \arg\max_{p_2} p_2 D_2^{\Delta}(\overline{v}_1, p_2, \overline{v}_b)$, which exists by Weierstrass theorem because the monopolist's action set can be constrained to belong to $[0, \overline{v}_2]$ and its profit is upper semi-continuous. For any p and p_1, we have

$$D_b(p_1, \overline{v}_2, p) = \int \mathbb{1}_{[\min\{v_1, p_1\} + v_2 \ge p - \Delta]} dF(\mathbf{v}) \ge \int \mathbb{1}_{[v_2 + \Delta \ge p_2]} dF(\mathbf{v}) \doteq D_2^{\Delta}(p_1, p, \overline{v}_b). \tag{EC.11}$$

Then,

$$
\begin{aligned}
&\pi_{b:1} \\
&= \int p_b D_b(p_1, \overline{v}_2, p_b) d\Phi^*_{-i}(p_1) d\Phi^*_i(p_b) \\
&\geq p_2^M \int D_b(p_1, \overline{v}_2, p_2^M) d\Phi^*_{-i}(p_1) && \text{because setting } p_2^M \text{ with probability 1 is feasible} \\
&\geq p_2^M \int D_2^\Delta(p_1, p_2^M, \overline{v}_b) d\Phi^*_{-i}(p_1) && \text{by (EC.11)} \\
&= p_2^M D_2^\Delta(\overline{v}_1, p_2^M, \overline{v}_b) && \text{because } D_2^\Delta(p_1, p_2, \overline{v}_b) \text{ is independent of } p_1 \\
&= \pi^\Delta_{1:\emptyset}.
\end{aligned}
$$

- To show $\pi_{2b:1} \geq \pi^\Delta_{1:\emptyset}$, consider the pricing game associated with $(\mathbf{z}_i, \mathbf{z}_{-i}) = ((0,1,1),(1,0,0))$ and denote by $(\Phi^*_i(p_2, p_b), \Phi^*_{-i}(p_1))$ the corresponding mixed-strategy price equilibrium solving (5). Let $p_2^M = \arg\max_{p_2} p_2 D_2^\Delta(\overline{v}_1, p_2, \overline{v}_b)$. Accordingly,

$$
\begin{aligned}
&\pi_{2b:1} \\
&= \int \left(p_2 D_2(p_1, p_2, p_b) + p_{ib} D_b(p_1, p_2, p_b)\right) d\Phi^*_{-i}(p_1) d\Phi^*_i(p_2, p_b) \\
&\geq p_2^M \int D_b(p_1, \overline{v}_2, p_2^M) d\Phi^*_{-i}(p_1) && \text{as setting } (p_2, p_b) = (\overline{v}_2, p_2^M) \text{ w.p. 1 is feasible} \\
&\geq p_2^M \int D_2^\Delta(p_1, p_2^M, \overline{v}_b) d\Phi^*_{-i}(p_1) && \text{by (EC.11)} \\
&= p_2^M D_2^\Delta(\overline{v}_1, p_2^M, \overline{v}_b) && \text{because } D_2^\Delta(p_1, p_2, \overline{v}_b) \text{ is independent of } p_1 \\
&= \pi^\Delta_{1:\emptyset}.
\end{aligned}
$$

□

Proof of Lemma 6. Suppose that $\mathbf{z}_i = (0,1,1)$ and $\mathbf{z}_{-i} = (1,0,0)$. We separately consider the cases of negatively and positively correlated valuations.

Perfectly negatively correlated valuations. Let V be customers' (identical) valuations for both products; i.e., $V \doteq \min\{v_1 | F(v_1, 0) = 1\}$. Suppose, to obtain a contradiction, that all three products are sold in equilibrium. By (2), customers with valuation $\mathbf{v}$ purchase the bundle if $p_b \leq \min\{v_1, p_1\} + \min\{V - v_1, p_2\} + \Delta \leq p_1 + p_2 + \Delta$. By (3), customers with valuation $\mathbf{v}$ purchase product 2 if $v_2 \geq p_2$ and $p_b > p_2 + \min\{V - v_2, p_1\} + \Delta$. When $p_b \leq p_1 + p_2 + \Delta$, the latter condition implies that $p_1 > \min\{V - v_2, p_1\}$, i.e., $p_1 > V - v_2$. Thus, $p_b > p_2 + \min\{V - v_2, p_1\} + \Delta$ is equivalent to $p_b > p_2 + V - v_2 + \Delta$, i.e., $v_2 > p_2 + V - p_b + \Delta$. Using a symmetric argument, customers with valuation $\mathbf{v}$ purchase product 1 if $v_1 > p_1 + V - p_b + \Delta$, i.e., given that $v_1 + v_2 = V$, if $v_2 < p_b - \Delta - p_1$. Defining $F(v_2)$ the marginal distribution of v_2, firm i's profit can be expressed as $p_2 \bar{F}(p_2 + V - p_b + \Delta) + p_b\left(F(p_2 + V - p_b + \Delta) - F(p_b - \Delta - p_1)\right)$. Because $p_b \geq p_2 + \Delta$, firm i's is increasing in p_2; it is optimal for firm i to set p_2 as large as possible until $D_2(\mathbf{p}) = 0$, a contradiction.

Perfectly positively correlated valuations. Here, $F(\mathbf{v}) = F(\min\{v_1, v_2\}, \min\{v_1, v_2\})$. Suppose, to obtain a contradiction, that in equilibrium, $D_2(\mathbf{p}^*) > 0$ and $D_b(\mathbf{p}^*) > 0$. If $D_1(\mathbf{p}^*) = 0$, firm $-i$ could increase its profit by setting $p_1^* = p_2^*$, a contradiction, so we also have that $D_1(\mathbf{p}^*) > 0$. For any $k \in \{1,2\}$ and $-k \doteq 3-k$, suppose that $p_k^* \geq p_{-k}^*$. By (3), for any customer $\mathbf{v}$ who buys product k, $v_k \geq p_k^*$. Because $v_1 = v_2$, $v_{-k} \geq p_k^*$, which implies, since $p_k^* \geq p_{-k}^*$, that $v_{-k} \geq p_{-k}^*$. Moreover, (3) yields that, for such customer $\mathbf{v}$, $p_b^* > \min\{v_1, p_1^*\} + \min\{v_2, p_2^*\} + \Delta \geq p_1^* + p_2^* + \Delta$. By (2), for any customer $\mathbf{v}$ who purchases the bundle, $p_b^* \leq \min\{v_1, p_1^*\} + \min\{v_2, p_2^*\} + \Delta \leq p_1^* + p_2^* + \Delta$. Since these two conditions are exclusive, we obtain a contradiction. $\square$

LEMMA EC.1. *Suppose that the distributions of valuations for the two components are independent, absolutely continuous, and their marginal distribution $F(x)$ is such that its failure rate, $\frac{f(x)}{\bar{F}(x)}$, is increasing, with $f(x) \doteq F'(x)$ and $\bar{F}(x) \doteq 1 - F(x)$. For any p_1, p_2, $\underline{p}_b$, and $\overline{p}_b$ with $\underline{p}_b < \overline{p}_b$, let $G(p_1) \doteq \overline{p}_b \bar{F}(\overline{p}_b - \Delta - p_1) - \underline{p}_b \bar{F}(\underline{p}_b - \Delta - p_1)$. Then, $G(p_1) \geq 0 \Rightarrow G'(p_1) > 0$.*

Proof. Suppose that $G(p_1) \geq 0$. Then,

$$\begin{aligned} G'(p_1)\Big|_{G(p_1)\geq 0} &= \Big(\overline{p}_b f(\overline{p}_b - \Delta - p_1) - \underline{p}_b f(\underline{p}_b - \Delta - p_1)\Big)_{G(p_1)\geq 0} \\ &\geq \underline{p}_b \left(f(\overline{p}_b - \Delta - p_1) \frac{\bar{F}(\underline{p}_b - \Delta - p_1)}{\bar{F}(\overline{p}_b - \Delta - p_1)} - f(\underline{p}_b - \Delta - p_1) \right) \\ &> 0, \end{aligned}$$

in which the second inequality follows because $\frac{f(x)}{\bar{F}(x)}$ is increasing and $\overline{p}_b > \underline{p}_b$. $\square$

LEMMA EC.2. *Suppose that the distributions of valuations for the two components are independent, absolutely continuous, and their marginal distribution $F(x)$ is such that (i) its failure rate, $\frac{f(x)}{\bar{F}(x)}$, is increasing, with $f(x) \doteq F'(x)$ and $\bar{F}(x) \doteq 1 - F(x)$, and (ii) its reverse hazard rate, $\frac{f(x)}{F(x)}$, is decreasing. If $\mathbf{z}_i = (0,1,1)$ and $\mathbf{z}_{-i} = (1,0,0)$, $\pi_i(p_2, p_b; p_1, \mathbf{z})$ and $\pi_{-i}(p_1; p_2, p_b, \mathbf{z})$ are supermodular in $(p_1, -p_2, p_b)$ when $p_1, p_2 \leq p_b - \Delta \leq p_1 + p_2$.*

Proof. When $p_1, p_2 \leq p_b - \Delta \leq p_1 + p_2$,

$$\pi_i(p_2, p_b; p_1) = p_b \int_{p_b - \Delta - p_2}^{\infty} \int_{p_b - \Delta - \min\{v_1, p_1\}}^{\infty} dF(\mathbf{v}) + p_2 \int_0^{p_b - \Delta - p_2} \int_{p_2}^{\infty} dF(\mathbf{v}) \tag{EC.12}$$

$$\pi_{-i}(p_1; p_2, p_b) = p_1 \int_{p_1}^{\infty} \int_0^{p_b - \Delta - p_1} dF(\mathbf{v}). \tag{EC.13}$$

We consider in turn firm $-i$'s and firm i's profits.

Firm $-i$'s profit. For any $\underline{p}_1, \overline{p}_1, p_2$, and $\underline{p}_b$, such that $\underline{p}_1 < \overline{p}_1$ and $\overline{p}_1, p_2 \le \underline{p}_b - \Delta \le \underline{p}_1 + p_2$, suppose that $\pi_{-i}(\overline{p}_1; p_2, \underline{p}_b) \ge \pi_{-i}(\underline{p}_1; p_2, \underline{p}_b)$. Given that valuations are independent, $\pi_{-i}(p_1; p_2, p_b) = p_1 \bar{F}(p_1) F(p_b - \Delta - p_1)$ by (EC.13). Therefore,

$$\frac{\overline{p}_1 \bar{F}(\overline{p}_1)}{\underline{p}_1 \bar{F}(\underline{p}_1)} \ge \frac{F(\underline{p}_b - \Delta - \underline{p}_1)}{F(\underline{p}_b - \Delta - \overline{p}_1)}.$$

Since $f(x)/F(x)$ is decreasing and $\underline{p}_1 < \overline{p}_1$, for any $p_b \ge \Delta + \overline{p}_1$, $\frac{f(p_b - \Delta - \underline{p}_1)}{F(p_b - \Delta - \underline{p}_1)} \le \frac{f(p_b - \Delta - \overline{p}_1)}{F(p_b - \Delta - \overline{p}_1)}$, i.e., $\frac{F(p_b - \Delta - \underline{p}_1)}{F(p_b - \Delta - \overline{p}_1)}$ is decreasing in p_b. As a result, for any $\overline{p}_b \ge \underline{p}_b$, such that $\overline{p}_b - \Delta \le \underline{p}_1 + p_2$

$$\frac{\overline{p}_1 \bar{F}(\overline{p}_1)}{\underline{p}_1 \bar{F}(\underline{p}_1)} \ge \frac{F(\overline{p}_b - \Delta - \underline{p}_1)}{F(\overline{p}_b - \Delta - \overline{p}_1)},$$

i.e., $\pi_{-i}(\overline{p}_1; p_2, \overline{p}_b) \ge \pi_{-i}(\underline{p}_1; p_2, \overline{p}_b)$. Moreover, $\pi_{-i}(p_1; p_2, p_b)$ is independent of p_2 when $p_1, p_2 \le p_b - \Delta \le p_1 + p_2$. Thus, $\pi_{-i}(p_1; p_2, p_b)$ is supermodular in $(p_1, -p_2, p_b)$ when $p_1, p_2 < p_b - \Delta \le p_1 + p_2$.

Firm i's profit. For any $\overline{p}_1$, $\underline{p}_2$, $\overline{p}_2$, and p_b, such that $\underline{p}_2 < \overline{p}_2$ and $\overline{p}_1, \overline{p}_2 \le p_b - \Delta \le \overline{p}_1 + \underline{p}_2$, suppose that $\pi_i(\overline{p}_2, p_b; \overline{p}_1) \ge \pi_i(\underline{p}_2, p_b; \overline{p}_1)$, i.e., using (EC.12),

$$p_b \int_{p_b - \Delta - \overline{p}_2}^{p_b - \Delta - \underline{p}_2} \int_{p_b - \Delta - v_1}^{\infty} dF(\mathbf{v}) + \overline{p}_2 \int_0^{p_b - \Delta - \overline{p}_2} \int_{\overline{p}_2}^{\infty} dF(\mathbf{v}) - \underline{p}_2 \int_0^{p_b - \Delta - \underline{p}_2} \int_{\underline{p}_2}^{\infty} dF(\mathbf{v}) \ge 0.$$

Since the left-hand side does not depend on $\underline{p}_1$, the same inequality must also hold at any $\underline{p}_1 < \overline{p}_1$, such that $\underline{p}_1, \overline{p}_2 \le p_b - \Delta \le \underline{p}_1 + \underline{p}_2$. Hence, $\pi_i(\overline{p}_2, p_b; \underline{p}_1) \ge \pi_i(\underline{p}_2, p_b; \underline{p}_1)$.

For any $\underline{p}_1$, p_2, $\underline{p}_b$, and $\overline{p}_b$, such that $\underline{p}_b < \overline{p}_b$ and $\underline{p}_1, p_2 \le \underline{p}_b - \Delta < \overline{p}_b - \Delta \le \underline{p}_1 + p_2$, suppose that $\pi_i(p_2, \overline{p}_b; \underline{p}_1) \ge \pi_i(p_2, \underline{p}_b; \underline{p}_1)$. Using (EC.12) and given that valuations are independent, this is equivalent to:

$$\begin{aligned}
&\overline{p}_b \int_{\overline{p}_b - \Delta - p_2}^{\infty} \bar{F}(\overline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) f(v_1) dv_1 - \underline{p}_b \int_{\underline{p}_b - \Delta - p_2}^{\infty} \bar{F}(\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) f(v_1) dv_1 \\
&\ge -p_2 \bar{F}(p_2) \int_{\underline{p}_b - \Delta - p_2}^{\overline{p}_b - \Delta - p_2} f(v_1) dv_1 \\
\Leftrightarrow \quad & \int_{\overline{p}_b - \Delta - p_2}^{\infty} \left(\overline{p}_b \bar{F}(\overline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) - \underline{p}_b \int_{\underline{p}_b - \Delta - p_2}^{\infty} \bar{F}(\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) \right) f(v_1) dv_1 \\
&\ge \int_{\underline{p}_b - \Delta - p_2}^{\overline{p}_b - \Delta - p_2} \left(\underline{p}_b \bar{F}(\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) - p_2 \bar{F}(p_2) \right) f(v_1) dv_1. \qquad \text{(EC.14)}
\end{aligned}$$

When $v_1 \ge \underline{p}_b - \Delta - p_2$ and $\underline{p}_b - \Delta \le \underline{p}_1 + p_2$, $\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\} \le p_2$, i.e., $\bar{F}(\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) \ge \bar{F}(p_2)$. Therefore, the right-hand side of (EC.14) is nonnegative. Thus (EC.14) implies that:

$$\int_{\overline{p}_b - \Delta - p_2}^{\infty} \left(\overline{p}_b \bar{F}(\overline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) - \underline{p}_b \int_{\underline{p}_b - \Delta - p_2}^{\infty} \bar{F}(\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) \right) f(v_1) dv_1 \ge 0. \qquad \text{(EC.15)}$$

Next, we show when (EC.15) holds, $\overline{p}_b f(\overline{p}_b - \Delta - p_1) \ge \underline{p}_b f(\underline{p}_b - \Delta - p_1)$ for all $p_1 \ge \underline{p}_1$. The proof is by contradiction. To obtain a contradiction, suppose the contrary, i.e., there exists some $\tilde{p}_1 \ge \underline{p}_1$ such that $\overline{p}_b f(\overline{p}_b - \Delta - \tilde{p}_1) < \underline{p}_b f(\underline{p}_b - \Delta - \tilde{p}_1)$. By Lemma EC.1, this implies that $\overline{p}_b \bar{F}(\overline{p}_b - \Delta - p_1) < \underline{p}_b \bar{F}(\underline{p}_b - \Delta - p_1)$ for all $p_1 \le \tilde{p}_1$. Therefore, $\overline{p}_b \bar{F}(\overline{p}_b - \Delta - \min\{v_1, \underline{p}_1\}) < \underline{p}_b \bar{F}(\underline{p}_b - \Delta - \min\{v_1, \underline{p}_1\})$ for all v_1, contradicting (EC.15).

Since $\overline{p}_b f(\overline{p}_b-\Delta-p_1)\geq \underline{p}_b f(\underline{p}_b-\Delta-p_1)$ for all $p_1\geq \underline{p}_1$, $\overline{p}_b\bar{F}(\overline{p}_b-\Delta-p_1)-\underline{p}_b\bar{F}(\underline{p}_b-\Delta-p_1)$ is increasing in p_1 for all $p_1\geq \underline{p}_1$. Therefore,

$$\begin{aligned}&\overline{p}_b\int_{\overline{p}_b-\Delta-p_2}^{\infty}\bar{F}(\overline{p}_b-\Delta-\min\{v_1,\overline{p}_1\})f(v_1)dv_1-\underline{p}_b\int_{\underline{p}_b-\Delta-p_2}^{\infty}\bar{F}(\underline{p}_b-\Delta-\min\{v_1,\overline{p}_1\})f(v_1)dv_1\\ \geq\ &\overline{p}_b\int_{\overline{p}_b-\Delta-p_2}^{\infty}\bar{F}(\overline{p}_b-\Delta-\min\{v_1,\underline{p}_1\})f(v_1)dv_1-\underline{p}_b\int_{\underline{p}_b-\Delta-p_2}^{\infty}\bar{F}(\underline{p}_b-\Delta-\min\{v_1,\underline{p}_1\})f(v_1)dv_1.\end{aligned}$$

Combining this last inequality with (EC.14), we obtain that

$$\begin{aligned}&\overline{p}_b\int_{\overline{p}_b-\Delta-p_2}^{\infty}\bar{F}(\overline{p}_b-\Delta-\min\{v_1,\overline{p}_1\})f(v_1)dv_1-\underline{p}_b\int_{\underline{p}_b-\Delta-p_2}^{\infty}\bar{F}(\underline{p}_b-\Delta-\min\{v_1,\overline{p}_1\})f(v_1)dv_1\\ \geq\ &\int_{\underline{p}_b-\Delta-p_2}^{\overline{p}_b-\Delta-p_2}\left(\underline{p}_b\bar{F}(\underline{p}_b-\Delta-\min\{v_1,\underline{p}_1\})-p_2\bar{F}(p_2)\right)f(v_1)dv_1,\end{aligned}$$

i.e., $\pi_i(p_2,\overline{p}_b;\overline{p}_1)\geq \pi_i(p_2,\underline{p}_b;\overline{p}_1)$. □

Proof of Lemma 7. Suppose that $\mathbf{z}_i=(0,1,1)$ and $\mathbf{z}_{-i}=(1,0,0)$. By Lemma 2, $D_b(\mathbf{p})>0$ in equilibrium. Hence, in equilibrium, $p_b\leq p_1+p_2+\Delta$ by (2). Without loss of generality, we can constrain p_2 to be smaller than $p_b-\Delta$, since when $p_2\geq p_b-\Delta$, $D_2(\mathbf{p})=0$ by (3). Similarly, we can constrain p_1 to be smaller than $p_b-\Delta$. Hence in equilibrium, $p_1,p_2\leq p_b-\Delta\leq p_1+p_2$. By Lemma EC.2, the firms' profit functions, $\pi_i(p_2,p_b;p_1)$ and $\pi_{-i}(p_1;p_2,p_b)$, are supermodular in $(p_1,-p_2,p_b)$, and they can be checked to be upper semicontinuous, so there exists a Nash equilibrium in pure strategies (Vives 1999, Theorem 4.2). Let $\hat{\mathbf{p}}$ be one such equilibrium and suppose that $D_2(\hat{\mathbf{p}})>0$ for otherwise $\pi_{\{2,b\}:\{1\}}=\pi_{\{b\}:\{1\}}$. Consider a game where firm i chooses only p_b; p_2 is an external parameter, which we set sufficiently large that $D_2(\mathbf{p})=0$. Again, there exists a pure-strategy Nash equilibrium in that game, denoted by $\check{\mathbf{p}}$. Without loss of generality, we assume that $\check{p}_2\geq \max\{\hat{p}_b,\check{p}_b\}-\Delta$. Hence, $\hat{p}_2\leq\check{p}_2$. Because $\pi_i(p_2,p_b;p_1)$ and $\pi_{-i}(p_1;p_2,p_b)$ are supermodular in $(p_1,-p_2,p_b)$ when $p_1,p_2\leq p_b-\Delta\leq p_1+p_2$, $\hat{p}_1\geq\check{p}_1$ by (Vives 1999, Theorem 3.1). Therefore, $\pi_i(\hat{p}_2,\hat{p}_b;\hat{p}_1)\geq\pi_i(\check{p}_2,\check{p}_b;\hat{p}_1)\geq \pi_i(\check{p}_2,\check{p}_b;\check{p}_1)$, i.e., $\pi_{\{2,b\}:\{1\}}\geq\pi_{\{b\}:\{1\}}$. □

Proof of Theorem 1. The equilibria follow from Table 2. □