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Intermediary-Based Loan Pricing

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We study how shocks to banks transmit to both the price and non-price terms of loans, in a model of multidimensional contracting between heterogeneous risky borrowers and intermediaries with limited lending capacity. The elasticities of loan demand and default rates to interest rates are sufficient statistics which predict how the cross-section of loan terms and banks' portfolio risk react to changes in lenders' capital, funding costs and regulation, and borrowers' risk and liquidity. Our results explain differences in the pass-through of shocks between markets and risk categories. These elasticities also drive the dynamic incidence of credit crises through impact and persistence.

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1 Introduction

The terms of loans offered by banks determine borrowers' access to credit and financial institutions' exposure to credit risk, making them crucial for the economy and financial stability. They make up rich contracts which typically consist of prices (interest rates), quantity limits (e.g., collateral), and other non-price terms (e.g., covenants, maturity), whose variations across borrowers and over time have attracted considerable attention. Recently, non-price terms have come under renewed scrutiny for households during the housing cycle (e.g., Acharya et al. 2020) and for corporate borrowers during the Covid-19 recession (e.g., Chodorow-Reich et al. 2021).

However, it is still largely unclear how loan terms are set in the cross-section of borrowers and how they are affected by the financial health of the very lenders which offer them. Especially puzzling is that, while credit supply tends to go hand in hand with bank health (e.g., Chodorow-Reich and Falato 2021), credit contractions and expansions often affect various types of borrowers differently and through various loan terms. The early 2000s offer a striking illustration, when thriving U.S. banks expanded credit to risky borrowers in the mortgage market (Mian and Sufi 2009), but to safe borrowers in the credit card market (Agarwal et al. 2018). While the mortgage expansion hinged on both relaxed leverage constraints and lower rates, the increase in credit card debt was mostly driven by heightened credit limits. Despite the outsized role of credit conditions in macro-finance (e.g., Stiglitz and Weiss 1981, Holmstrom and Tirole 1997, Bernanke et al. 1999), what is missing is a model that can jointly explain how shocks to banks are transmitted to the loan terms faced by different groups of borrowers.

We fill this gap in an equilibrium model of multidimensional loan contracting between heterogeneous risky borrowers and financial intermediaries with limited lending capacity. Intermediaries' capacity constraints and costs of funds are subject to shocks which arise from changes in asset prices or financial regulation and monetary policy. We show that two sufficient statistics, the interest rate elasticities of borrowers' loan demand and default probability, can explain several features of credit markets: (i) Banks simultaneously use interest rates and quantity limits or other non-price terms to control the supply of credit. (ii) The pass-through of bank shocks to loan terms varies across markets (e.g., mortgages vs. credit cards), borrowers within a market (e.g., safe vs. risky), and bank health (e.g., well vs. poorly capitalized). (iii) The responses of the different loan terms govern the persistence of credit contractions and expansions. We emphasize the role of intermediaries' balance sheets for credit supply in specialized asset markets as in intermediary asset pricing (He and Krishnamurthy 2013, Brunnermeier and Sannikov 2014). But we highlight a specificity of credit markets: banks use non-price terms to control borrowers' default probabilities when the latter are endogenous due to informational frictions.

We motivate our analysis by documenting heterogeneity in loan terms and the pass-through of shocks for the main classes of bank loans. First, loan growth is strongly negatively associated with bank charge-off rates at the aggregate level across mortgage, credit card, and commercial and industrial (C&I) loans since 1995. The more losses on their assets banks make, the less they lend. Second, interest rates and non-price terms act as substitute channels through which mortgage expansions and contractions take place, but as complement channels for credit card and C&I loans. While loan to value (LTV) and debt to income (DTI) constraints tend to be relaxed when mortgage spreads increase, the percentage of banks tightening credit standards tends to increase with credit card and C&I spreads. Third, the pass-through of bank shocks resulting from changes in asset prices or regulation and monetary policy to various borrowers differs across markets. Mortgage sizes grew for both high-risk and low-risk borrowers in the 2000s, but then fell deeply and persistently for high-risk ones despite similar changes in interest rates. In contrast, credit card expansions and contractions tend to be mostly transmitted to low-risk borrowers. Similarly, C&I loan sizes are more volatile for low-risk firms. Despite widespread interest in these markets taken in isolation, there is a limited understanding of what explains their differences and commonalities.

We set up and solve a model of multidimensional loan contracting. We make three contributions. Our first contribution is to jointly endogenize the price and non-price terms of loan contracts. While common in the data, they have been studied separately in theoretical settings so far: either in stylized economies with endogenous credit rationing but fixed loan rates (e.g., Stiglitz and Weiss 1981); or in Walrasian economies where rates adjust to clear credit markets but borrowing constraints are exogenous (e.g., Holmstrom and Tirole 1997).[1] In our model, realistic price and non-price terms of loans are the optimal outcomes of a contracting problem between heterogeneous borrowers and banks. Banks compete for borrowers subject to capacity constraints on lending. Therefore their balance sheets affect loan prices in a way similar to intermediary asset pricing. We add two features which distinguish loan markets from generic asset markets (such as stock markets). First, asset payoffs are endogenous to asset prices. Interest rates affect default probabilities and losses given default because of microeconomic frictions which induce borrowers to default when the loan repayment value is too high – our model accommodates moral hazard, adverse selection, and a "double trigger" default motive. The interest rate elasticity of borrowers' repayment probability $\epsilon_{1-\mu}$ captures this channel. Second, we depart from a Walrasian framework where the loan repayment value is linear in a single interest rate clearing credit markets. Instead, banks can offer non-linear and multidimensional contracts with quantity limits and other non-price terms which capture lender screening and monitoring. Non-price

[1] Borrowing constraints and their variations play a crucial role for household, firm, and asset prices dynamics in most of the recent macro-finance literature. However, they are usually assumed to be exogenous, leaving unclear the ultimate drivers of these fluctuations. Examples include Bernanke et al. (1999), Jermann and Quadrini (2012), Favilukis et al. (2017), and Guerrieri and Lorenzoni (2017).

terms arise from the feedback between loan prices and loan payoffs for banks. Quantity limits and screening or monitoring allow banks to manage borrowers' default risk while holding rates fixed. The interest rate on a given loan compensates banks for their cost of funds (risk-free rate), the borrower credit risk (credit risk premium), and their capacity constraint tightness (excess loan premium). This problem generates a multidimensional contract curve, which relates both borrower and lender characteristics to loan terms in a similar way to reduced-form credit surfaces which focus on borrowers (e.g., Geanakoplos 2010). The contract curve provides a comprehensive measure of credit tightness based on all the terms of loan contracts. It is described by its interest rate elasticity ϵ_ℓ, which captures how demand changes in response to rates when borrowers are endogenously constrained by multiple loan terms. In turn, ϵ_ℓ depends on two objects. The first is the unconstrained interest rate elasticity of loan demand ϵ_ℓ^u when borrowers borrow as much as they want at the going interest rate, which depends on preferences and technology. The second is the elasticity of borrowers' repayment probability $\epsilon_{1-\mu}$, which determines whether banks adjust interest rates or non-price terms to control borrowers' default probability and depends on preferences, technology, and microeconomic frictions.

We use our framework to understand how shocks to banks' lending capacity and cost of funds affect the price and non-price terms of loan in equilibrium. When banks' balance sheets deteriorate, they trade off tightening the various terms. Our second contribution is a set of pass-through formulas which highlight the role of the interest rate elasticities of borrowers' loan demand and repayment probabilities, ϵ_ℓ and $\epsilon_{1-\mu}$, as two sufficient statistics which determine how bank shocks are transmitted to the cross-section of loan terms. Our approach has the benefit of relying on two objects which are identified in borrower- and loan-level data.

We begin by studying the impact of credit supply shocks, modeled as changes in banks' lending capacity. Our results explain why credit contractions and expansions affect safe and risky borrowers differently and through various loan terms across markets (e.g., credit cards vs. mortgages). The higher the elasticity of borrowers' loan demands ϵ_ℓ on a given market, the more their loan sizes vary in response to a given interest rate change. Positive credit supply shocks lower interest rates across markets, but markets with more elastic borrowers see larger increases in sizes and smaller decreases in rates. As a result, loan repayment values increase (decreases) in markets with more (less) elastic borrowers. The repayment elasticity $\epsilon_{1-\mu}$ determines how credit risk reacts to these changes. On more (less) elastic markets, the increase (decrease) in loan repayment values generates a small increase (decrease) in credit risk for safe borrowers, but a large increase (decrease) for risky ones. To maximize their total profits, banks adjust loan terms to equate the returns from lending to safe and risky borrowers – if it were not the case, they could increase their total profits by lending more to borrowers with a higher return. This leads them to lending more to safe borrowers on markets with more elastic borrowers (e.g., credit cards), and more to risky borrowers on markets with less elastic borrowers (e.g., mortgages).[2]

We next study how shocks to banks' cost of funds which arise from monetary policy shocks transmit to the cross-section of loan terms. Our results explain why the transmission of monetary policy is weakened when bank balance sheets are impaired, and why the strength of the bank lending channel is heterogeneous across loan markets. The pass-through of the policy rate to loan rates and sizes depends on the elasticity ϵ_ℓ and on banks' lending capacity. Unconstrained banks transmit changes in the policy rate more than one-for-one to low-elasticity borrowers (e.g., mortgages), but this transmission is dampened for higher-elasticity ones, resulting in stickier rates (e.g., credit cards). When banks are capacity-constrained but lending capacity is not sensitive to the policy rate, the transmission of monetary policy is further dampened. This feature tends to insulate borrowers from the negative effects on credit supply of a policy rate hike, but also from the positive effects of a rate cut.

We explore the implications for the credit risk of the entire portfolio of bank loans. A monetary policy shock has two effects. First, a change in both price and non-price terms within a given pool of borrowers; second, a reallocation of bank loans towards specific borrowers. We find that policy rate cuts tend to increase the credit risk of borrowers with a high loan demand elasticity ϵ_ℓ and to decrease it for others. If high-elasticity borrowers are also riskier (high $\epsilon_{1-\mu}$), then policy rate cuts increase credit risk for the entire portfolio of bank loans. We also show how a low policy rate environment gives rise to covenant-lite lending for low elasticity borrowers but to tighter covenants for high elasticity ones, a novel finding in line with the data.

We conclude by showing that the two elasticities ϵ_ℓ and $\epsilon_{1-\mu}$ also drive the dynamics of banking and credit supply crises. We build a dynamic version of the model where bank shocks and borrower credit risk affect loan spreads which feed back into bank capital, which affects the tightness of lending capacity constraints over time. Our third contribution is to show that the adjustment of non-price terms in response to bank shocks increase the persistence of financial crises – a finding which speaks to the post-2008 experience. Negative bank shocks result in higher spreads and tighter non-price terms. The more they are tightened, the less spreads need to increase to control credit risk. The lesser increase in spreads in turn lowers excess returns earned by constrained banks. It slows down their recapitalization and makes the credit crunch more persistent.

We illustrate the quantitative implications of our results in a version of the model calibrated to the post-2008 U.S. mortgage market. The model matches empirical estimates for ϵ_ℓ and $\epsilon_{1-\mu}$.

[2] Estimates for the interest rate elasticity of mortgage debt tend to be lower than one and range between 0.07 and 0.5 (e.g., Best et al. 2019, Fuster and Zafar 2021, Benetton 2021), with the exception of DeFusco and Paciorek (2017). In contrast, estimates for the elasticity of credit card debt tend to be greater than one (e.g., Gross and Souleles 2002 estimate an elasticity of 1.3).

In response to a contraction in bank lending capacity, the excess loan premium on mortgages increases, reflecting the tightness of banks' constraints. Loan sizes fall and mortgage spreads increase for all households, lowering LTV ratios. As in the data, the responses of loan terms differ across borrowers. Loan size falls twice more for borrowers with high ϵ_ℓ and $\epsilon_{1-\mu}$ while their mortgage spread increases by less. As a result, credit risk falls sharply for high-elasticity borrowers and increases for low-elasticity ones. Two complementary policies partly mitigate the credit crunch. First, direct household debt relief reduces default risk by making them effectively richer, and allows relatively more borrowing during the credit crunch, mitigating the decrease in LTV ratios. Second, a recapitalization of banks relaxes their lending capacity constraint, and achieves similar results by increasing the total volume of credit available.

Related literature Our results contribute to the microeconomic literature on credit markets and to the macro-finance literature on the dynamics of credit crises.

First, our model helps explain the transmission mechanism from losses in banks' portfolios to reductions in credit documented in empirical analyses (e.g., Peek and Rosengren 1997, Murfin 2012, Chodorow-Reich 2014, Huber 2018, Greenstone et al. 2020). Reductions in new credit take the form of tighter lending standards and higher loan spreads. We endogenize this mechanism and study its implications. Our model explains why banks use non-price terms to control the volume of credit for a given rate, such as debt covenants for firms (Chodorow-Reich and Falato 2021) and credit card limits for households (Agarwal et al. 2018). It generates heterogeneous responses of loan terms in the cross-section of borrowers and credit markets, as in the data when banks' balance sheets change (Khwaja and Mian 2008, Chakraborty et al. 2018, Ivashina et al. 2020) and when their cost of fund changes because of monetary policy (Jimenez et al. 2012, Chakraborty et al. 2020) or access to external finance (Paravisini 2008, Ivashina and Scharfstein 2010). We trace these differences back to two sufficient statistics with well-identified empirical estimates: the interest rate elasticities of borrowers' loan demand and repayment probability. Estimates for these elasticities come from survey data (Fuster and Zafar 2021), regression discontinuity analyses (Best et al. 2019, Fuster and Willen 2017), and structural models (Buchak et al. 2020, Benetton 2021, Robles-Garcia 2020). We show how they depend on structural parameters in our model which affect borrowers' loan demand and default. Their variation across markets may arise from moral hazard, adverse selection, or borrowers' liquidity constraints (Adams, Einav and Levin 2009, Einav, Jenkins and Levin 2012).

Second, we add to canonical macro-finance models by introducing multidimensional credit contracts with both price and non-price terms. We build on the theoretical literature on credit rationing (Stiglitz and Weiss 1981, Jaffee and Russell 1976) surveyed in Jaffee and Stiglitz (1990), and focus on rationing at the intensive margin: risky borrowers face tighter non-price terms rather than being excluded from credit markets. Our analysis complements models of the credit surface (e.g., Geanakoplos 2010) where an increase in borrower credit risk in bad times leads more pessimistic lenders to requiring more collateral, and heterogeneous household models where the price of unsecured consumer loans depends on borrower characteristics (e.g., Chatterjee et al. 2007, Livshits et al. 2007). We show how lenders' financial conditions also affect the terms of lending, as in recent work by Diamond and Landvoigt (2021). We extend the intermediary asset pricing framework (He and Krishnamurthy 2013, Brunnermeier and Sannikov 2014) to credit markets by modeling the joint effect of banks' capacity constraints and borrower default risk on interest rates, quantity limits, and other non-price terms of loans. Endogenizing these terms generates the excess bond premium found in the data (Gilchrist and Zakrajsek 2012). It also generates a new transmission mechanism of shocks as non-price adjustments affect the dynamics of credit crises. As in canonical macro-finance models (e.g., Gertler and Kiyotaki 2010, Rampini and Viswanathan 2019), credit crises occur when banks' net worth falls. However, bank losses need not generate a sharp increase in spreads which would quickly recapitalize banks and make the credit crunch short-lived. Instead, the endogenous tightening of non-price terms by banks results in spreads increasing by less, which slows down banks' recapitalization and makes the credit crisis more persistent. This finding helps explain the persistence of the post-2008 U.S. mortgage crisis (e.g., Justiniano et al. 2019).

The rest of the paper is organized as follows. Section 2 documents stylized facts on heterogeneity in loan terms. Section 3 describes our model of multidimensional loan contracting and the credit market equilibrium. Section 4 studies the transmission of credit supply shocks and monetary policy shocks to the cross-section of loan terms. Section 5 analyzes the dynamics of credit crises, which depends on the interaction between the cross-section of loan terms and banks' balance sheets. We illustrate our findings in a version of the model calibrated to the U.S. mortgage market, where we compare the effectiveness of debt relief and bank recapitalization policies. Section 6 concludes.

2 Evidence on Heterogeneity in Loan Terms

This section documents motivating facts on heterogeneity in loan terms and the pass-through of shocks for the main classes of bank loans: mortgages, credit cards, and C&I loans.

Data sources. We combine data on bank losses and the volume and characteristics of mortage, credit card, and commercial and industrial (C&I) loans.

Data on charge-off rates comes from the Consolidated Reports of Condition and Income of the Federal Financial Institutions Examination Council at the U.S. Federal Reserve Board.

For mortgages, data on loan sizes, loan rates, and maximum LTV and DTI ratios comes from Fannie Mae and Freddie Mac. For credit cards and C&I loans, data on the net percentage of banks tightening lending standards and spreads comes from the Senior Loan Officer Opinion Survey (SLOOS) of the U.S. Federal Reserve Board.

2.1 Bank Health and Loan Growth

The more losses on their assets banks make, the less they lend. The upper panel of Appendix Figure 12 plots year-to-year percentage changes in the quantity of mortgage (solid line, left axis) and the associated charge-off rates by banks (dashed line, right axis). There is a strongly negative relationship at the aggregate level between loan growth and bank charge-off rates. As the upper panels of Appendix Figures 13 and 14 show, this relationship holds for credit card and C&I loans too.

2.2 Interest Rates and Non-Price Terms

Interest rates and non-price terms act as substitute channels through which mortgage expansions and contractions take place, but as complement channels for credit card and C&I loans. The lower panel of Appendix Figure 12 plots the maximum LTV ratios on mortgages issued by banks (blue line, left axis), which reflects the looseness of borrowing constraints, and the associated interest rate spread over the Federal Funds Rate (red line, right axis). The lower panels of Appendix Figures 13 and 14 respectively plot the net percentages of banks tightening credit standards on credit card and C&I loans (blue line, left axis), which reflects the tightness of borrowing constraints, and the associated interest rate spreads (red line, right axis). While LTV constraints tend to decrease in lockstep with mortgage spreads, the percentage of banks tightening credit standards tends to increase with credit card and C&I spreads.

2.3 Pass-Through of Bank Shocks

The pass-through of bank shocks resulting from changes in asset prices or monetary policy to various borrower types differs across markets. The upper left panel of Appendix Figure 15 plots the average loan sizes in dollars of high-FICO score, low-risk mortgage borrowers (blue line) and low-FICO score, high-risk mortgage borrowers (red line). The upper right panel plots the associated loan rates, the lower left panel the associated maximum DTI ratios, and the lower right panel the associated maximum LTV ratios. Mortgage sizes grew quickly for both high-risk and low-risk borrowers in the 2000s, but fell more deeply and persistently for high-risk ones despite similar interest rate changes. This finding is consistent with narratives of the Great Recession which attribute a prominent role to the relaxation and subsequent tightening of mortgage standards to subprime, high-risk borrowers (Mian and Sufi 2009, Justiniano et al. 2019).

In contrast, credit card expansions and contractions tend to be mostly transmitted to low-risk borrowers. Appendix Figure 16, from Agarwal et al. 2018, plots changes in borrower credit card limits by FICO score bins in response to a decrease in banks' cost of funds. It shows that credit supply expansions are mostly transmitted to low-risk borrowers. High-risk consumers with a higher marginal propensity to borrow do not face large changes in these limits. Comparing Appendix Figure 12 and Appendix Figure 13 also suggests that interest rates on credit cards are stickier than on mortgages, and therefore that credit card expansions and contractions mostly happen through changes in credit limits.

Finally, Appendix Figure 17 plots the average sizes in dollars of C&I loans (blue line, left axis) and the corresponding interest rates (red line, right axis), for low-risk firms in the left panel and high-risk firms in the right panel. Similar to credit cards, C&I loan sizes are more volatile for low-risk borrowers, despite similar changes in interest rates across risk categories. This finding is consistent with a recent literature which suggests that much of the adjustment in the volume of corporate loans happens through non-price terms. For instance, covenants are stricter after banks suffer defaults (Murfin, 2012), while easy liquidity spurs covenant-lite lending (Diamond, Hu and Rajan, 2020).

3 A Model of Multidimensional Loan Contracting

This section presents a general model of multidimensional loan contracting with financial frictions affecting lenders. We add two features to the intermediary asset pricing framework. First, a distinctive feature of credit markets is that asset prices, i.e., loan interest rates, affect default probabilities and losses given default. Thus asset payoffs are endogenous to asset prices. Second, we depart from the standard Walrasian framework in that endogenous credit risk makes it optimal for banks to impose quantity limits to borrowers, i.e. to offer non-linear contracts.

3.1 Environment

We consider a unit continuum of identical lenders indexed by b, “banks”, and a unit mass of heterogeneous borrowers indexed by i.

Banks. Banks have a funding cost R^f. The expected profit on a loan contract to borrower i, which consists of an interest rate R^i (price term), a loan amount l^i (quantity limit), and a vector of non-price terms $\boldsymbol{z}^i = \left(z^i_k\right)_k$, is

$$\pi^i\left(R^i, l^i, \boldsymbol{z}^i\right) = \left(R^i - R^f\right) l^i - \lambda^i\left(R^i l^i, \boldsymbol{z}^i\right) - c(\boldsymbol{z}^i). \tag{1}$$

Absent default, bank profits would be $\left(R^i - R^f\right) l^i$. λ^i captures the expected loss, and can be thought of as a put option in the Merton model (Merton 1974). In most settings, R and l only affect the expected loss through face value of the debt Rl, as is implicit in (1). We discuss below in which cases R and l matter independently and how to extend our framework to this case. $c(z^i)$ is the cost of control for lenders due to monitoring and screening efforts when borrowers can default. $c(\cdot)$ is an increasing and convex function of the non-price terms $z^i \geq 0$ such that $c(0) = 0$.

The benefit of this reduced-form formulation is to nest rich environments with ex-ante or ex-post asymmetric information which makes the expected loss endogenous to the terms of the contract through the default probability, the loss given default, or both. The particular setting determines how to measure the dependence of λ in Rl. In Appendix B, we detail several examples to show how to map microeconomic frictions to λ: strategic default, liquidity default, collateralized loans, and adverse selection.

Our results derive from the fact that we can always write an isomorphic model with zero recovery rate and an "effective default probability" μ defined as

Definition 1. $\mu^i\left(Rl, \boldsymbol{z}\right) = \frac{\lambda^i(Rl,\boldsymbol{z})}{Rl}$ *is the effective default probability.*

Expected profit can then be written as $\pi = \left[R\left(1 - \mu^i\right) - R^f\right] l$. With a positive recovery rate, μ will be higher than the actual default probability. We work with μ as a primitive, and define the elasticity of the repayment rate, one of the two sufficient statistics from which our results derive.

Definition 2. *The elasticity of the loan repayment probability* $1 - \mu$ *to its face value* Rl *is*

$$\epsilon_{1-\mu}\left(Rl, z\right) = \frac{Rl\mu'\left(Rl, z\right)}{1 - \mu\left(Rl, z\right)}$$

We make the following assumption.

Assumption 1. *The elasticity of the repayment probability satisfies* $0 \leq \epsilon_{1-\mu} < 1$ *everywhere.*

Assumption 1 ensures that the zero profit curve $\pi\left(R, l, z\right) = 0$, or more generally iso-return curves (defined as constant $\frac{\pi(R,l,z)}{l}$), are always upward sloping, since they have a slope $\frac{dR/R}{dl/l} = \frac{\epsilon_{1-\mu}}{1-\epsilon_{1-\mu}}$. Therefore we rule out standard credit rationing of the type studied by Williamson (1987), which takes place when borrowers' loan demand curve is always above lenders' backward-bending supply curve. Since the implications of this "pure credit rationing" are well-known, we restrict our attention to settings where borrowers have access to credit, albeit at a smaller scale than they would like, because banks use binding non-price terms to limit their credit risk.

To study the propagation of credit supply shocks, we study banks with a capacity constraint on lending

$$\int \rho^i l^i di \leq \overline{L} \tag{2}$$

where l^i is the dollar amount lent to borrower i and $\rho^i \in [0,1]$ is a risk-weight that measures how much balance sheet space a loan to borrower i requires. Heterogeneity in ρ can arise from regulatory risk weights. It can also arise from banks' ability to securitize a given loan and take it off their balance sheets. An example is conforming mortgages with a low weight ρ, versus non-conforming mortgages with a high weight ρ, which can further depend on liquidity conditions in the private label securitization market.

Constraint (2) can arise from regulatory constraints (e.g., Basel regulation) or market-based constraints imposed by bank creditors due to informational issues such as those affecting the bank-borrower relationship. A large literature (e.g., Holmstrom and Tirole 1997, Gertler and Kiyotaki 2010) provides microfoundations for the moral hazard or limited commitment problems that lead banks themselves to be credit constrained. Our focus is on the transmission of such a constraint on bank lending to different types of borrowers through the multiple terms of loans.

Borrowers. Borrowers are characterized by their indirect utility over loan contracts $V^i(l, R, z)$. We make the following standard assumption:

Assumption 2. *For each i, $V_R^i < 0$, and the marginal utility of additional borrowing is lower at higher interest rates: $V_{lR}^i < 0$.*

Assumption 2 holds in most settings, such as those described in Appendix B. The second part of the assumption implies that the unconstrained loan demand curve, defined as the solution l to $V_l^i(l, R, z) = 0$, is decreasing in the interest rate R.

The second sufficient statistics from which our results derive, the interest rate elasticity of borrowers' loan demand in the constrained problem, depends on the unconstrained elasticity of loan demand. We define the latter as follows.

Definition 3. *The elasticity of the unconstrained loan demand to the interest rate R is*

$$\epsilon_\ell^u = -\frac{R}{l}\frac{dl}{dR}\Big|_{V_l=0}$$

Finally, tighter non-price terms $z^i \geq 0$ leading borrowers to renouncing control is costly for them, i.e. $V_z^i \leq 0$.

3.2 Equilibrium: Bertrand-Nash with Capacity Constraints

Banks are perfectly competitive and subject to capacity constraints on lending. Banks post contracts $C^i = \left(l^i, R^i, z^i\right)$ with commitment, and borrowers optimally choose which bank they apply to. We assume exclusive contracts, i.e. borrowers cannot borrow from multiple banks. A bank with lending capacity $\bar{L}$ choose contracts and a number x_b^i of loans to borrowers i to solve

$$\begin{aligned} \max_{\{x^i,R^i,l^i,z^i\}} & \int x^i \pi^i \left(l^i, R^i, z^i\right) di \\ \text{s.t.} & \int x^i \rho^i l^i di \leq \bar{L} \\ & V^i \left(l^i, R^i, z^i\right) \geq \overline{V}^i \end{aligned}$$

where $\pi^i \left(l^i, R^i, z^i\right)$ is the profit per loan and banks are subject to a participation constraint from borrowers.

Definition 4. *An equilibrium is an optimal strategy* $i \mapsto \left\{x_b^i, C_b^i\right\}$ *for each bank such that borrowers optimize:*

$$\overline{V}^i = \max_{b'} V^i \left(C_{b'}^i\right)$$

and markets clear:

$$1 = \int_{\arg\max_{b'} V^i\left(C_{b'}^i\right)} x_b^i db.$$

With symmetric banks, we focus on symmetric equilibria such that all banks offer the same contract and choose $x^i = 1$ for all borrowers.

Proposition 1. *In a symmetric equilibrium,*

i. l^i, R^i, *and* z^i *satisfy for each* i

$$\tau^i \left(l^i, R^i, z^i\right) = \frac{\epsilon_{1-\mu}^i \left(R^i l^i, z^i\right)}{1 - \epsilon_{1-\mu}^i \left(R^i l^i, z^i\right)} \tag{3}$$

where $\tau^i \left(l, R, z\right) = -\frac{l V_l^i(l,R,z)}{R V_R^i(l,R,z)}$.

ii. Banks make the same profit per risk-weighted dollar for each borrower, i.e.,

$$\frac{\pi^i \left(l^i, R^i, z^i\right)}{\rho^i l^i} = \frac{\pi^j \left(l^j, R^j, z^j\right)}{\rho^j l^j} \equiv \nu \tag{4}$$

iii. *The marginal cost of tightening non-price terms for banks is weakly lower than their marginal benefit, i.e.,*

$$\frac{\partial c}{\partial z_k}\left(z^i\right) = -R^i l^i \frac{\partial \mu^i}{\partial z_k} - \frac{V^i_{z_k}}{V^i_R} l^i \left(1-\mu^i\right)\left(1-\epsilon^i_{1-\mu}\right) \tag{5}$$

Proof. See Appendix C.1. □

We show below that τ can be interpreted as an intertemporal wedge measuring how constrained borrowers are. If $\tau^i = 0$, then borrowers i are on their unconstrained demand curve $V^i_l = 0$.

Part (i) of Proposition 1 states that banks will optimally constrain borrowers according to the elasticity of their repayment rate to debt $\epsilon_{1-\mu}$. The riskier borrowers are, i.e. the more likely they are to default when the face value of their debt increases, the more banks will restrict the size of their loans for a given interest rate.

Part (ii) describes the optimal capital allocation across different classes of borrowers. Banks use both price (R) and non-price terms (l and z) to equalize the profit per risk-weighted dollar ν across borrowers. If it were not the case, banks could increse profits by lending more to borrowers with a higher profit per dollar.

Part (iii) describes how banks set non-price terms z. If borrowers have no preferences over z, i.e., $V^i_z = 0$, then banks equalize the marginal cost of non-price terms to their marginal benefit. This defines an optimal tightness $\hat{z}^i$, where R^i and l^i are endogenously determined by (3) and (4). In general, non-price terms are costly for borrowers due to for instance a loss of control rights ($V^i_z < 0$); this lowers the marginal benefit from tightening z because banks need to keep attracting borrowers, hence banks optimally relax non-price terms to $z^i < \hat{z}^i$.

Unconstrained vs. constrained banks. For each i and taking $\boldsymbol{z}^i$ as given for now, the unconstrained loan quantity $l^{i,*}$ is defined as solving (3) together with the zero-profit conditions $\pi^i\left(l^i, R^i, \boldsymbol{z}^i\right) = 0$. Banks are said to be *unconstrained* if given unconstrained loan quantities $\left\{l^{i^*}\right\}$, their constraint (2) is satisfied, that is

$$\int \rho^i l^{i^*} \leq \overline{L}.$$

Otherwise, banks are *constrained*, and must make a positive profit per dollar $\nu > 0$.

3.3 Implications

Our framework has three implications which affect the pass-through of credit supply and monetary shocks in the cross-section of borrowers. To illustrate them, we assume that borrower income is i.i.d. and follows a Pareto distribution with shape parameter α, where a higher value corresponds to a riskier distribution of income. The cumulative distribution function of income y is $F_y(y) = 1 - (\frac{y_{min}}{y})^\alpha$ if $y_{min} \leq y \leq y_{max}$ and $F_y(y) = 0$ if $y < y_{min}$ or $y > y_{max}$, where $\alpha, y_{min}, y_{max} > 0$. The remainder of the paper presents our results for a generic income process that satisfies our main assumptions, but for simplicity we maintain the assumption of Pareto income risk in all examples and figures.

Excess loan premium. To first-order, equilibrium interest rates satisfy

$$\begin{aligned} & \log R_i = \log\left(R^f + \rho^i \nu\right) - \log\left(1 - \mu_i\right) \\ \Leftrightarrow \qquad & r^i \approx r^f + \mu^i + \rho^i \nu \end{aligned}$$

Suppose for simplicity that $\rho^i = 1$ for all i. Then interest rates net of the common premium $r^i - \nu$ are actuarially fair. Importantly, this can only be achieved thanks to the endogenous non-price terms l^i, which give banks another instrument to control credit (repayment) risk and thereby offset the common increase in interest rates that arises from credit supply shocks or monetary shocks. Banks optimally tighten l_i by more for riskier borrowers i, as measured by their repayment elasticity $\epsilon^i_{1-\mu}$.

When do banks constrain borrowers? Equation (3) implies that banks only impose a binding borrowing constraint when credit risk is endogenous:

Corollary 1. *Suppose that μ^i is independent of C^i. Then borrower i's allocation can be implemented with a price-posting mechanism where banks only quote an interest rate R^i and borrowers borrow as much as they want given R^i.*

In the constant μ case, a credit supply shock translates into a higher rate, as in other asset markets. It is enough for banks to charge a higher rate to compensate for higher default risk. Menus of loan contracts or non-price terms like covenants, two empirical features of credit markets, are not needed for banks to allocate the supply of credit. Loans can still have time-varying risk through changes in the effective default probability μ (similar to equity with risky dividends). But there is no feedback loop between asset prices and asset payoffs. Such feedback is absent from most asset markets but it is a central feature of credit markets.

Interest rates do not fully capture credit conditions. In particular, the measured equilibrium loan rate is a non-monotonic function of the borrower's income risk. Riskier borrowers may be charged a lower interest rate and still be more credit-constrained than safer borrowers, because banks tighten their quantity limits by more. On Figure 1, the black curve depicts the contractual rate charged to borrowers by banks, the red curve depicts the shadow rate, which accounts for the credit rationing wedge. The contractual rate is increasing for low risk levels, and then becomes decreasing at the point where an increase in the rate would decrease bank's expected profits (because it would increase the borrower's default probability). At that point, the associated loan size decreases more to compensate for the lesser increase, or decrease in the rate. It translates into a shadow loan rate that is increasing in borrower's risk, because it accounts for the wedge τ. The wedge acts like a tax imposed by banks on borrowers, such that those borrow less for a given rate.

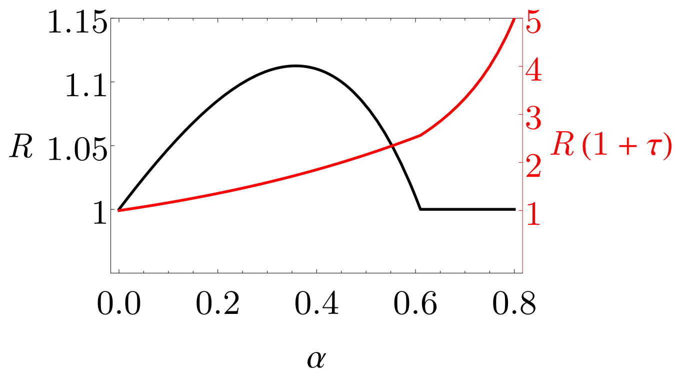

Figure 1: Equilibrium interest rate as a function of borrower risk α, where α is the shape parameter of a Pareto distribution (a higher α corresponds to a riskier distribution). The black curve depicts the contractual rate charged to borrowers by banks. The red curve depicts the shadow rate, which accounts for the rationing wedge.

3.4 Properties of the Contract Curve ℓ

The main determinant of loan terms is the shape of the multidimensional loan contract curve defined by (3). We show below that it helps explain the variation of loan terms in the cross-section of borrowers and their response to bank shocks.

Definition 5. *The contract curve $\ell^i\left(R^i, z^i\right)$ is the solution to (3).*

Definition 6. *The interest rate elasticity of the contract curve is*

$$\epsilon_\ell = -\frac{R}{\ell}\frac{d\ell}{dR},$$

where ℓ satisfies (3) and (4).

The elasticity of the contract curve ℓ^i depends on the unconstrained loan demand elasticity ϵ_ℓ^u which reflects borrower preferences V^i, and on the repayment elasticity $\epsilon_{1-\mu}^i$ which captures the strength of microeconomic frictions affecting repayment risk. The elasticity of ℓ can be decomposed as shown in the following proposition (we omit the superscript i for clarity).

Proposition 2. *The interest rate elasticity of the contract curve can be decomposed as*

$$\epsilon_\ell = \frac{-R\tau_R + \frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}}{-l\tau_l + \frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}} \tag{6}$$

If $\epsilon'_{1-\mu} > 0$ *then*

$$\epsilon_\ell > \frac{R\tau_R}{l\tau_l} \Leftrightarrow \frac{R\tau_R}{l\tau_l} < 1$$

Hence if $\frac{\tau_R}{\tau_l} = \frac{l}{R}$ *then* $\epsilon_\ell = 1$ *and the equilibrium effective default probability* μ *is constant.*

Proof. See Appendix C.1.

□

In response to a small deviation $dx = \frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}$, ϵ_ℓ varies according to

$$\frac{d}{dx}\left(\frac{-R\tau_R + x}{-l\tau_l + x}\right) = \frac{R\tau_R - l\tau_l}{(l\tau_l + x)^2}$$

All else equal, an increase in $\frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}$ makes the contract curve more elastic than the unconstrained demand if the unconstrained elasticity $\frac{R\tau_R}{l\tau_l}$ is lower than 1. That is, the quantity limits of riskier borrowers (with high $\frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}$) vary more on loan markets where borrowers' demand for credit is less sensitive to changes in interest rates ($\frac{R\tau_R}{l\tau_l} < 1$).

The intuition for this result is as follows. Suppose that banks did not impose binding borrowing constraints. Then, for a given reduction in loan size l (e.g. due to a negative credit supply shock or a monetary contraction), the interest rate faced by less elastic borrowers would have to increase by more than one-for-one to induce them to reduce their loan demand. This would result in an increase in the total face value of the loan Rl. Hence it would increase default risk μ and lower bank's expected profits. Instead, the optimal response of banks when quantity limits are endogenous, is to offer a contract with a lower interest rate but with a binding borrowing constraint. Because the latter forces borrowers to adjust the loan quantity demanded, this eventually translates into a more elastic contract curve.

Conversely, an increase in $\frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}$ makes the contract curve less elastic if the unconstrained elasticity $\frac{R\tau_R}{l\tau_l}$ is higher than 1. Hence a positive term $\frac{Rl\epsilon'_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}$ serves as an "elasticity dampener", bringing back final elasticities towards 1.

Elasticity and default rate μ. The elasticity of borrowers' demand for credit to changes in interest rates determines how changes in total loan volume affect the default risk borne by banks. When credit supply $\overline{L}$ falls, loan rates increase because of the higher excess loan premium ν associated with tighter capacity constraints for banks. However, despite the decrease in loan volume, the resulting change in default risk μ if ambiguous. The higher loan rate R leads to more default holding loan size fixed, but the reduction in loan size also reduces the likelihood of default. The balance between these two forces depends once again on the elasticity ϵ_ℓ, since

$$\frac{d}{d\overline{L}}\mu\left(R\left(l\right)l\right) = R\mu'\left(1-\frac{1}{\epsilon_\ell}\right)$$

Thus a reduction in total lending due to a credit supply shock increases default risk μ if and only if the elasticity ϵ_ℓ is below 1. Following our earlier discussion, this is the case if and only if the unconstrained loan demand elasticity $\frac{R\tau_R}{l\tau_l}$ is itself lower than 1.

Determinants of the contract curve elasticity ϵ_ℓ. Figure 2 studies the determinants of ϵ_ℓ in our workhorse model. In particular, the model generates key features of mortgage data. First, the contract curve elasticity increases in the elasticity of intertemporal substitution, but it increases less for riskier borrower types (Best, Cloyne, Ilzetzki and Kleven 2019). Second, ϵ_ℓ increases in borrowers' cash-on-hand, and increases by less at higher levels. Third, ϵ_ℓ increases with borrower income. These last two results are in line with Buchak, Matvos, Piskorski and Seru 2020, who structurally estimate elasticities for different groups of households.

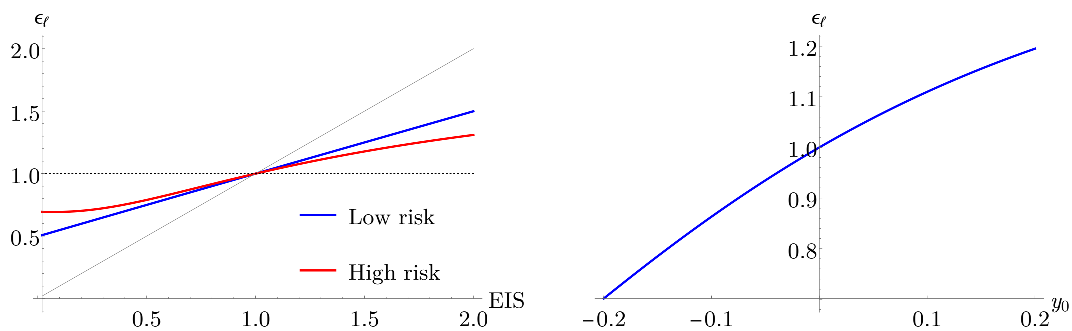

Figure 2: Sensitivity of the interest rate-elasticity of the contract curve curve to borrowers' elasticity of intertemporal substitution and initial endowment y_0.

3.5 Interpretation: State-Dependent Credit Surface

Credit surfaces map loan sizes and borrower characteristics to equilibrium interest rates. They can be estimated using loan- and borrower-level data in multiple settings, including household portfolio choice, mortgage pricing, and sovereign default. We now show that loan contracts with multiple dimensions, including non-price terms, can be interpreted as a credit surface. Instead of the traditional nonparametric estimations of credit surfaces, our settings relies on sufficient statistics based on borrowers' loan and repayment elasticities.

Borrowers are constrained at R in that that they would like to borrow more at the prevailing interest rate if

$$V_l\left(\ell\left(R\right),R\right)>0.$$

This may be the case even if $\ell\left(R\right)=L^{*}$, that is, *lenders* are unconstrained. Our equilibrium condition (3) implies that borrowers are unconstrained only if $\epsilon_{1-\mu}\left(\ell\left(R\right)R\right)=0$. If V is separable, $V\left(l,R\right)=u\left(l\right)-w\left(Rl\right)$, then we can interpret the unconstrained condition $V_l=0$ as a standard Euler equation

$$\frac{u'\left(l\right)}{w'\left(Rl\right)}=R$$

When borrowers are constrained, our equilibrium with endogenous borrowing constraints—which arises from the optimal contract between banks and borrowers—can also be implemented as a competitive equilibrium in which borrowers choose l subject to a non-linear *interest rate schedule* $R\left(l|\bar{L}\right)$, as in Livshits, MacGee and Tertilt 2007 and Chatterjee, Corbae, Nakajima and Rios-Rull 2007. The collection of schedules faced by different borrower types corresponds to the credit surface described by Geanakoplos 2010. Bank lending capacity $\bar{L}$ then acts as a credit supply shifter of this surface.[3]

Given the function $R\left(\cdot|\bar{L}\right)$ associated with her type, a borrower solves

$$\max_{l} u\left(l\right)-w\left(R\left(l|\bar{L}\right)l\right)$$

hence

$$\frac{u'\left(l\right)}{w'\left(R\left(l|\bar{L}\right)l\right)}=R\left(l|\bar{L}\right)\left[1+\frac{lR'\left(l|\bar{L}\right)}{R\left(l|\bar{L}\right)}\right]$$

Combining with (3), the function $R\left(l|\bar{L}\right)$ solves the differential equation (for each type)

$$\frac{d\log R\left(l|\bar{L}\right)}{d\log l}=\frac{\epsilon_{1-\mu}\left(lR\left(l|\bar{L}\right)\right)}{1-\epsilon_{1-\mu}\left(lR\left(l|\bar{L}\right)\right)}\tag{7}$$

Equivalently, $R\left(l|\bar{L}\right)$ gives rise to the locus $R\left(1-\mu\left(Rl\right)\right)=R^{f}+\nu\left(\bar{L}\right)$ where $\nu\left(\bar{L}\right)$ is the excess loan premium. Note that the function $l\mapsto R\left(l|\bar{L}\right)$ is conceptually different from the contract curve or inverse contract curve that we have described earlier, which is the equilibrium outcome

[3] We only make the dependence on $\bar{L}$ explicit below, but other shocks can also shift the credit surface (e.g., changes in R^{f} and to the distribution of default risk).

as we vary $\bar{L}$ instead. To pin down the exact level of $R\left(l|\bar{L}\right)$, we use as boundary condition the fact that $R\left(l|\bar{L}\right)$ must go through the actual equilibrium contract $\left(\bar{R}, \bar{L}\right)$.

The interest rate schedules capture the supply side of credit. Borrower preferences V then determine which contract pair is chosen. Figure 3 shows the interest rate schedules for two types of households indexed by the Pareto parameter α of their income process. α determines the repayment elasticity $\epsilon_{1-\mu}$, with our measure of credit risk being increasing in α. We find that a shock to total lending capacity $\bar{L}$ corresponding to an excess loan premium of $\nu = 5\%$ shifts the interest rate schedule upwards for both borrowers.

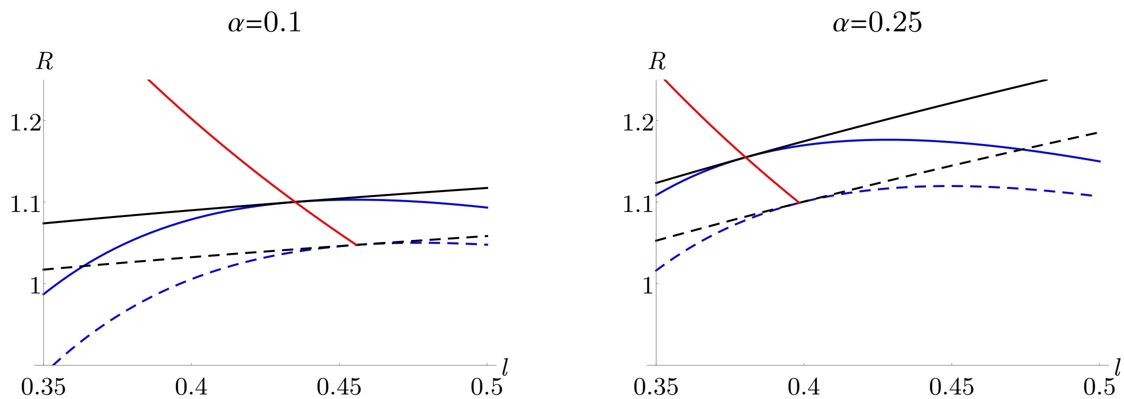

Figure 3: Each borrower α faces an increasing interest rate schedule $R_\alpha(l)$ (black lines). Borrowers choose the point at which their indifference curve (blue lines) is tangent to the rate schedule. The red line is the contract curve $\ell_\alpha(R)$ obtained by varying total lending capacity $\bar{L}$ or equivalently the excess loan premium ν.

Example. In the case of a constant repayment elasticity $\epsilon_{1-\mu}$, we can solve the differential equation (7) in closed form:

$$R(l) = \bar{R} \times \left(\frac{l}{\bar{L}}\right)^{\frac{\epsilon_{1-\mu}}{1-\epsilon_{1-\mu}}} \tag{8}$$

where $\bar{R}$ is the equilibrium rate given lending capacity $\bar{L}$, that is $\ell\left(\bar{R}\right) = \bar{L}$. With several borrower types (e.g. indexed by their FICO score in the case of households, or by their credit rating in the case of firms), we obtain one curve (8) per borrower. These curves together give rise to a credit surface as in Geanakoplos and Rappoport 2019.

The effect of various shocks on the credit surface is captured by the sufficient statistics

$$\left\{\frac{R_i}{\ell_i(R_i)^{\frac{\epsilon^i_{1-\mu}}{1-\epsilon^i_{1-\mu}}}}\right\}_i,$$

where $\{R_i\}_i$ are the equilibrium loan rates. In particular, a shock to total lending capacity $\bar{L}$ induces a general upward shift in the credit surface. The interest rate schedules faced by both more elastic borrowers (for whom R_i is sticky while $\ell_i(R_i)$ drops) and less elastic ones (for whom R_i jumps while $\ell_i(R_i)$ is less responsive) increase. However, the shift is heterogeneous across types. Differences in $\epsilon^i_{1-\mu}$ determine which parts of the surface increase by more, according to our formulas from section 4.

After analyzing the determinants of the cross-section of loan terms, we turn to their responses to bank shocks. We start with credit supply shocks, then turn to monetary policy. The next two sections analyze the steady state response of the cross-section of loan terms to these shocks, and the last section studies the full dynamics of a multidimensional credit crisis.

4 Transmission of Shocks to Loan Terms

This section studies how credit supply shocks, which affect banks' lending capacity, and monetary policy shocks, which affect banks' funding costs, transmit to the cross-section of loan terms. We start by focusing on two terms: loan rates and quantities, holding other non-price terms z fixed. In Section 4.3 we incorporate the endogenous response of other non-price terms z.

4.1 Credit Supply Shocks

There is important heterogeneity in the data in how different borrowers are affected by credit supply shocks on different loan markets. In particular, in the credit card market credit supply expansions benefit low-risk (high FICO score) borrowers, but are not passed through to higher risk borrowers who would be the ones with a higher propensity to borrow (Agarwal et al. 2018). Effects are more ambiguous in mortgage markets: the leading post-crisis narrative was that the credit boom of the early 2000s mostly benefitted subprime borrowers ((Mian and Sufi, 2009)), while there is also evidence that the bulk of new credit was directed towards households in the top half of the income and wealth distribution (Adelino, Schoar and Severino 2016).

Our main result is a formula which governs how loan quantities and interest rates vary across borrowers in response to a shock to bank lending capacity $\overline{L}$.

Definition 7. *Let the risk-adjusted elasticity of loan demand be*

$$\widetilde{\epsilon}^i_\ell = \frac{\epsilon^i_\ell}{\left(1 - \epsilon^i_{1-\mu}\right) + \epsilon^i_\ell \epsilon^i_{1-\mu}},$$

which lies between 1 and ϵ^i_ℓ.

The next proposition shows that the risk-adjusted elasticity of loan demand governs the responses of loan terms to changes in bank lending capacity. This elasticity depends on two sufficient statistics, the repayment elasticity and the elasticity of loan demand.

Proposition 3. *Denote the risk-weighted loan share of borrower i as $\omega^i = \frac{\rho^i \ell^i(R^i)}{\sum \rho^i \ell^i(R^i)}$.*
A change in $\overline{L}$ affect borrowers i's loan quantities and rates as follows:

$$\frac{d\log l^i}{d\log \overline{L}} = \frac{\widetilde{\epsilon}^i_\ell \frac{\rho^i}{R^i(1-\mu^i)}}{\sum \omega^j \widetilde{\epsilon}^j_\ell \frac{\rho^j}{R^j(1-\mu^j)}} \approx \frac{\widetilde{\epsilon}^i_\ell \rho^i}{\sum \omega^j \widetilde{\epsilon}^j_\ell \rho^j}$$

$$\frac{d\log R^i}{d\log \overline{L}} = -\frac{1}{\widetilde{\epsilon}^i_\ell} \times \frac{d\log l^i}{d\log \overline{L}} \approx -\frac{1}{\left(1-\epsilon^i_{1-\mu}\right) + \widetilde{\epsilon}^i_\ell \epsilon^i_{1-\mu}} \times \frac{\rho^i}{\sum \omega^j \widetilde{\epsilon}^j_\ell \rho^j}$$

Proof. See Appendix C.1. □

Proposition 3 provides closed-form formulas for the endogenous contractual response across the full spectrum of borrowers, in terms of simple sufficient statistics. The risk-adjusted elasticities $\widetilde{\epsilon}^i_\ell$ can be constructed from contract curve elasticities $\widetilde{\epsilon}^i_\ell$ and repayment elasticities $\epsilon^i_{1-\mu}$. $\widetilde{\epsilon}^i_\ell$ are in turn obtained from $\epsilon^i_{1-\mu}$ and unconstrained loan demand elasticities $\epsilon^{u,i}_\ell$, using equation (6). All else equal, the pass-through to the loan quantity of borrower i is high if the risk-weight ρ^i or the elasticity $\widetilde{\epsilon}^i_\ell$ is high. However, there is an important asymmetry. Loan contracts with low risk-weights are insulated from credit supply shocks on both the quantity margin $\frac{d\log l^i}{d\log \overline{L}}$ and interest rate margin $\frac{d\log R^i}{d\log \overline{L}}$. On the other hand, while inelastic borrowers are also insulated on the quantity margin, they can experience a sharp increase in their interest rate R^i.

The effect of risk. A central question is whether interest rates rise more or less for riskier borrowers in response to the shock. To illustrate our results, consider a simplified economy with two types a and b, in equal mass, where type b borrowers are riskier. Both types of borrowers have the same preferences or technology, and only differ in their risk. Then:

- In an elastic market, such as corporate loans, riskier borrowers are less elastic ($1 < \widetilde{\epsilon}^b_\ell < \widetilde{\epsilon}^a_\ell$), hence their loan quantity contracts by less and their loan rate rises by more in response to an aggregate tightening $\Delta\overline{L} < 0$, as depicted in Figure 4.

- In an inelastic market, such as mortgages, riskier borrowers are more elastic ($\widetilde{\epsilon}^a_\ell < \widetilde{\epsilon}^b_\ell < 1$), hence their loan quantity contracts by more and their loan rate rises by less in response to an aggregate tightening $\Delta\overline{L} < 0$, as depicted in Figure 5.

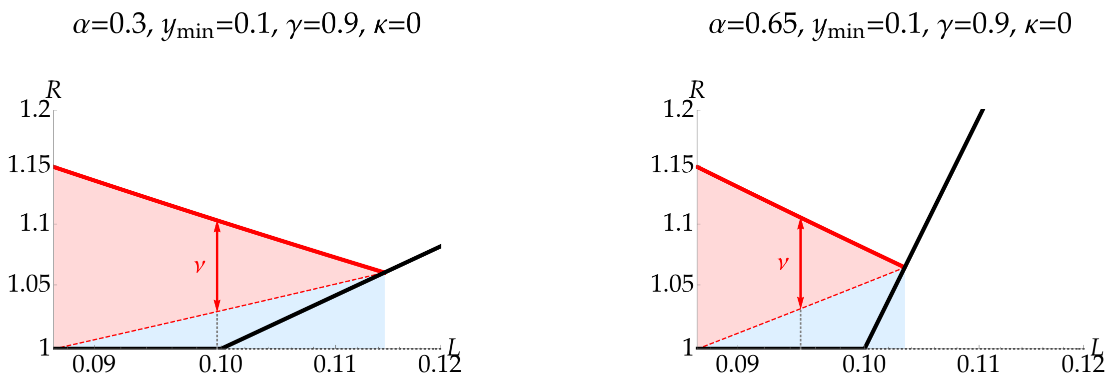

Figure 4: Firms with high $\gamma = 0.9$, log-log scale. Left: safer borrowers (low α), right: riskier borrowers (high α). The vertical red segment has length $\nu \approx 7\%$: the bank equalizes ν across types, which tells us how quantities and rates react for each type. Both types have the same actual loan demand elasticity (the orange line with slope $-(1-\gamma)$) but the contract curve is less elastic for riskier borrowers ($1 < \epsilon_\ell^b < \epsilon_\ell^a$) hence credit contracts more for safer borrowers.

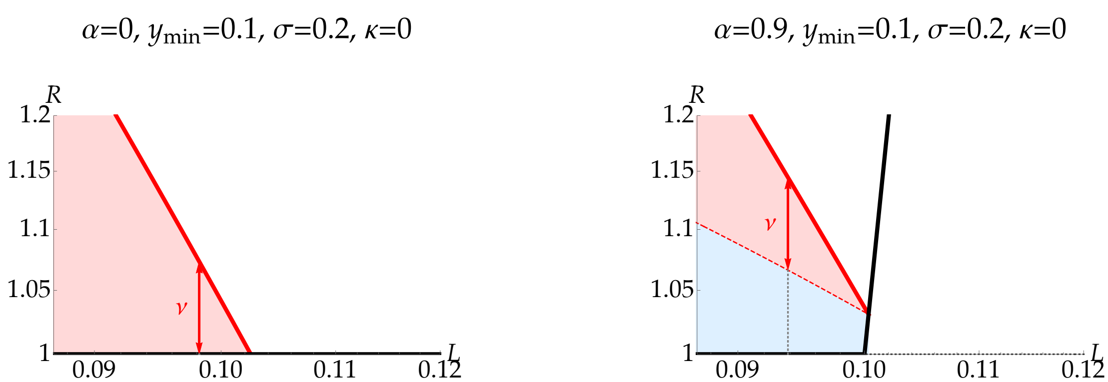

Figure 5: Households with low EIS $\sigma = 0.2$, log-log scale. Left: safe borrowers ($\alpha = 0$), right: risky borrowers (high α). The vertical red segment has length $\nu \approx 7\%$: the bank equalizes ν across types, which tells us how quantities and rates react for each type. The contract curve is more elastic for risky borrowers ($\epsilon_\ell^a < \epsilon_\ell^b < 1$) hence credit contracts more for them.

4.2 Monetary Policy Shocks

This section studies the transmission of monetary policy shocks, which affect banks' cost of funds, to the cross-section of loan terms. The transmission of monetary policy is weakened when bank balance sheets are impaired (e.g., Jimenez et al. 2012, Acharya et al. 2019). Furthermore, there is significant variation in the extent to which changes in the policy rate are passed through to borrowers. In particular, interest rates on new loans are sticky for credit cards, but vary significantly over time for mortgages. In this section, we explain why the bank lending channel is borrower-and product-dependent. The transmission of policy rates to loan terms depends on borrowers' elasticities of loan demand and repayment probability to loan rates. These elasticities depend on borrower characteristics that affect credit risk, as we have shown in section 3.4. They depend on loan types because different borrowers select into different loans, and because the features of loans themselves affect the credit risk of a given borrower.

4.2.1 Pass-Through Formulas

The pass-through of shocks to banks' funding cost R^f into loan rates and quantities for different borrowers depends on whether total bank lending capacity is binding or not.

Unconstrained banks. If banks' loan supply $\bar{L}$ is high enough (i.e. if it is larger than the aggregate unconstrained loan demand L^*), then the interest rate pass-through works through banks' zero profit condition for each borrower $R_i(1-\mu_i) = R^f$. This is true even if banks exert deposit market power. Total pass-through after accounting for changes in loan quantities is

$$\frac{d\log R_i}{d\log R^f} = \frac{1}{1-\epsilon^i_{1-\mu}+\epsilon^i_{1-\mu}\epsilon^i_\ell}$$

Pass-through to borrower type i is imperfect – the risky loan rate R^i increases less than one for one with banks' funding cost R^f – if and only if $\epsilon^i_{1-\mu}\left(\epsilon^i_\ell - 1\right) > 0$. Therefore, pass-through can be weak for some borrowers (with high elasticity ϵ^i_ℓ) and strong for others.

Constrained banks. The reaction of credit markets depends on how lending capacity reacts to monetary policy shocks, $-\frac{d\log\bar{L}}{d\log R^f}$. If $\bar{L}$ does not react to R^f, then steady state loan rates and quantities do not change and R^f only affects banks' static profit per dollar. As we show in the next section, there can still be a dynamic effect of monetary policy on $\bar{L}_t$. We interpret $\bar{L}$ as stemming from a financial constraint; then, monetary shocks affect $\bar{L}$ in a way that depends on the duration of equity. Another possibility is that $\bar{L}$ is determined by banks' market power on deposits, together with constraints on wholesale funding. While our loan market is perfectly competitive, we can interpret the "deposit channel of monetary policy" of Drechsler, Savov and Schnabl 2017 as an effect of R^f on banks' total loan supply $\bar{L}$, inclusive of how banks use their market power on deposits.

Therefore (assuming no risk-weights for simplicity), we obtain the following proposition.

Proposition 4. *Suppose $\rho^i = 1$. The pass-through of banks' funding cost R^f to loan rate R^i is*

- *When banks are unconstrained:*

$$\frac{d\log R^i}{d\log R^f} = \frac{1}{1-\epsilon^i_{1-\mu}+\epsilon^i_{1-\mu}\epsilon^i_\ell} = \frac{\widetilde{\epsilon}^i_\ell}{\epsilon^i_\ell}$$
$$\frac{d\log l^i}{d\log R^f} = -\widetilde{\epsilon}^i_\ell$$

- *When banks are constrained*

$$\frac{d\log R^i}{d\log R^f} = -\frac{1}{\epsilon^i_\ell}\times\frac{\widetilde{\epsilon}^i_\ell}{\widetilde{\overline{\epsilon}}_\ell}\frac{d\log\bar{L}}{d\log R^f},$$
$$\frac{d\log l^i}{d\log R^f} = -\widetilde{\epsilon}^i_\ell\times\left(-\frac{1}{\widetilde{\overline{\epsilon}}_\ell}\frac{d\log\bar{L}}{d\log R^f}\right)$$

If $\frac{d\log\bar{L}}{d\log R^f} = -\widetilde{\overline{\epsilon}}_\ell$, monetary policy transmission to loan terms does not depend on the bank's capacity constraint.

Proof. See Appendix C.1. □

In a model with deposit market power from a monopolist bank and no wholesale funding, $-\frac{d\log\bar{L}}{d\log R^f}$ is the deposit demand elasticity. In a model with equity constraints, $-\frac{d\log\bar{L}}{d\log R^f}$ depends on the duration of equity (either its book or market value, depending on which one determines bank lending capacity).

Figure 6 describes the percentage changes in loan rates and sizes in response to changes in deposit rates, as a function of borrowers' elasticity of loan demand and banks' capacity constraint. When faced with a tightening of monetary policy, unconstrained banks (left panels) pass through the increase in the policy rate more than one-for-one to low elasticity borrowers, while the pass-through to high-elasticity is dampened. However, the latter face a steeper decrease in loan size, therefore they are eventually relatively more credit-rationed. When banks are constrained but their capacity constraints are somewhat inelastic to policy rates (middle panels), the transmission of monetary policy is further dampened. Loan terms are largely determined by bank's lending capacity, so the relative insensitivity of the latter to the policy rate partly insulates borrowers from a credit tightening. Conversely, an insensitive bank lending capacity reduces the transmission of policy cuts to the cross-section of loan terms. Finally, a high sensitivity of banks' capacity constraints to policy rates (right panels) amplifies the transmission of monetary policy to loan terms, as rates reacting more than one-for-one for all types of borrowers.

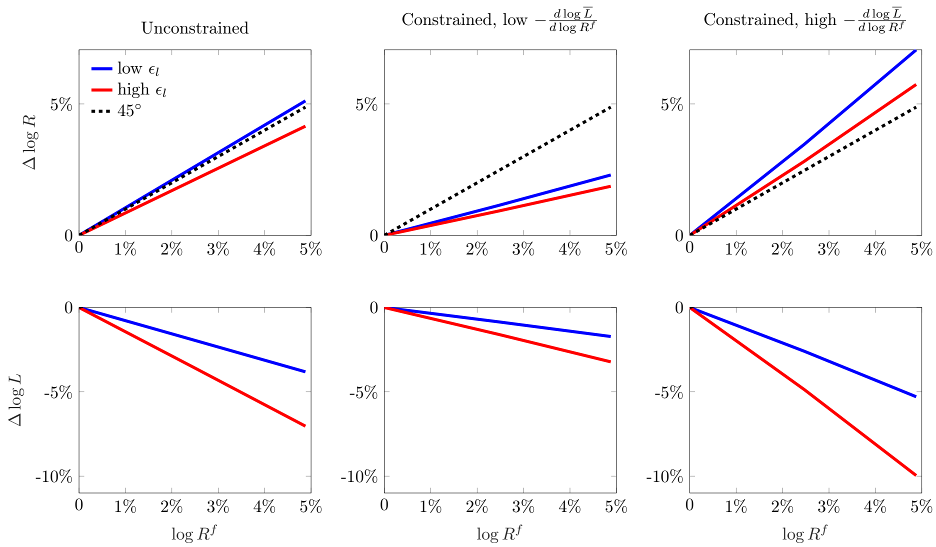

Figure 6: Percentage changes in loan rates and sizes as a function of banks' funding cost. Changes are plotted in cases where banks are unconstrained (left panels), constrained with inelastic lending capacity (middle panels), and constrained with elastic lending capacity (right). For each case, they are plotted for borrowers with a low (blue) vs. high (red) elasticity of loan demand.

4.2.2 Elasticity and Pass-Through

Proposition 4 implies that the main two household loan classes, credit cards and mortgages, react differently to monetary policy shocks. Their responses depend on the elasticity of borrowers' loan demand to loan rates.

Elastic market: credit cards. On the credit card market, banks do not pass-through lower funding costs to borrowers who would like to borrow (Agarwal et al. 2018). In our model, the optimal contract leads banks to pass-through R^f shocks to heterogeneous borrowers according to their elasticity $\tilde{\epsilon}^i_\ell$. As we saw earlier, the comovement of $\tilde{\epsilon}^i_\ell$ with risk is subtle, and depends on the unconstrained loan demand elasticity $\epsilon^{u,i}_\ell$. In the case of a unit elasticity, $\tilde{\epsilon}^i_\ell$ is constant across borrowers and equal to 1. Credit card rates display little variation; suppose this reflects a highly elastic loan demand.[4] Then $\tilde{\epsilon}^i_\ell$ is lower for high risk borrowers, and so is $\tilde{\epsilon}^i_\ell$. Consider a market with $\frac{R\tau_R}{l\tau_l} > 1$ for all types, and the repayment elasticity is non-decreasing $\epsilon'_{1-\mu} \geq 0$. Then $\tilde{\epsilon}^i_\ell$ is lower for high-risk borrowers, hence banks pass through reductions in R^f relatively more to low-risk borrowers, both when they are constrained and unconstrained.

Inelastic market: mortgages. Policy rates affect mortgage terms through several channels, including adjustable-rate mortgage payments, rates on newly originated fixed-rate mortgages,

[4]This may be an equilibrium outcome or due to regulation.

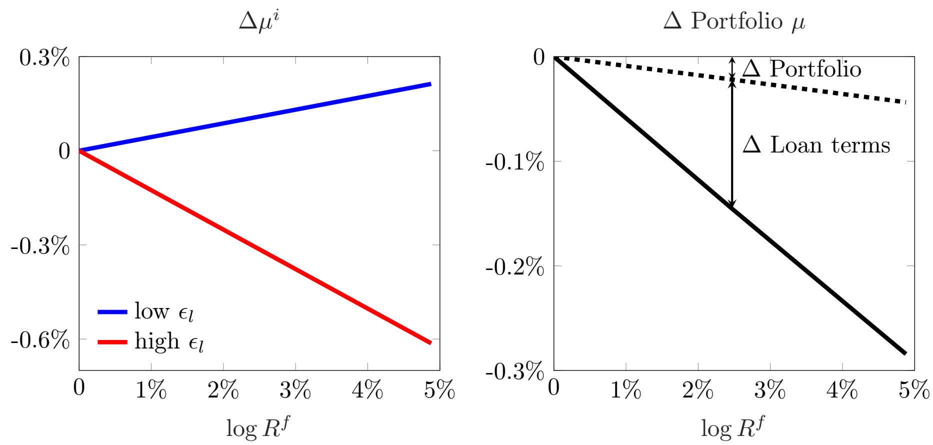

Figure 7: Effect of the risk-free rate on individual default probabilities for borrowers with low vs. high loan demand elasticity (left panel), and on the total default probability of the bank's loan portfolio (right panel).

payment-to-income constraints, and refinancing rates. Appendix B shows how monetary policy affects *all* the dimensions of mortgage contracts, including non-price terms. We extend our baseline model to include mortgages, and compute the loan-to-value (LTV) and payment-to-income ratios associated with a given loan. Figure 18 depicts their response to changes in banks' cost of fund. They are characterized by two main features. First, borrowers with a high interest rate-elasticity of loan demand have higher LTV and PTI ratios than borrowers with a high elasticity. Second, while their LTV decreases more than for low elasticity borrowers when banks' cost of funds increases, their PTI increases less. For a given change in loan rates, they adjust their optimal loan size by more than low elasticity borrowers, resulting in strongly heterogeneous responses in loan term changes across borrower types. Finally, as implied by Proposition 4, these changes are amplified by a greater sensitivity of bank lending capacity to their cost of fund.

4.2.3 Banks' Portfolio Risk: Intensive vs. Extensive Margin

After having studied how changes in banks' cost of fund affect loan terms, we now analyze how banks' portfolio risk react to a decrease in R^f. The total effect can be decomposed into two components: first, a change in both price and non-price loan terms for a given pool of borrowers; second, a reallocation of banks' loans towards specific borrowers.

Within borrower (for a given borrower type), lower rates lead to lower risk for inelastic loan products and higher risk for the elastic ones. In addition, a composition effect towards high ϵ_ℓ products and borrowers arises. Suppose as an illustration that default risk μ is positively correlated with ϵ_ℓ. Then, within relatively inelastic products such as mortgages, riskier borrowers are more elastic. Hence a relaxation of credit supply will increase the weight of risky mortgages in banks' portfolios. The opposite holds within relatively elastic categories such as credit cards. The total effect is

$$d \log \left(\mathbf{E} \left[1 - \mu^i \right] \right) \approx \mathbf{Cov} \left(\mu^i, \widetilde{\epsilon}^i_\ell \right) d \log R^f - \frac{\mathbf{E} \left[d\mu^i \right]}{\mathbf{E} \left[1 - \mu^i \right]}$$

Figure 7 shows how individual default probabilities react to changes in banks' cost of fund, and the resulting change in the total default risk of the bank's portfolio of loans. First, as shown earlier, the increase in banks' cost of fund results in a smaller increase in the loan rate of high elasticity borrowers, who face a larger decrease in loan size. As a result, their credit risk falls significantly when rates increase. In contrast, low elasticity borrowers' credit risk increases, even though their interest rate increases more than one-for-one with banks' cost of fund. Therefore, the fact that payoffs are endogenous to interest rates on credit markets leads to opposite results for borrowers with different elasticities. Second, even though borrowers have identical weights in bank's loan portfolio, the total effect is a decrease in total credit risk. Quantitatively, most of it comes from changes in loan terms, which induce a stark decrease in credit risk for high elasticity borrowers. However, the lower the banks' cost of fund is, the stronger the composition effect is, by which banks reallocate lending towards high elasticity, safer borrowers.

4.3 Other non-price terms

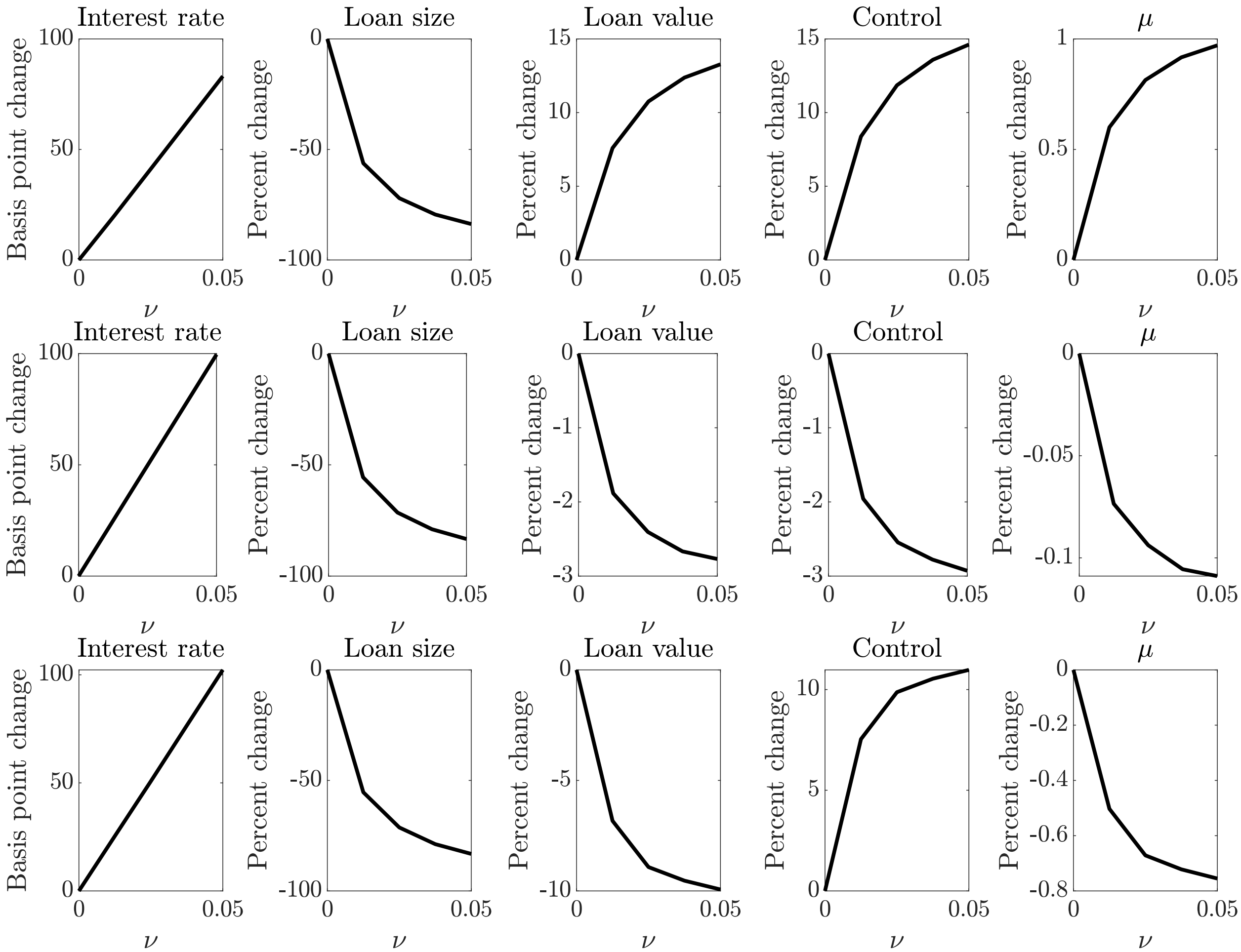

Figure 8: Equilibrium loan terms as a function of the tightness of banks' lending capacity constraint. Change relative to the unconstrained case. "Control" stands for the non-price terms z. In the top panels, income follows a Pareto distribution with parameter $\alpha = 1.5, y_{min} = 0.05, y_{max} = 0.25$, and the cost of non-price terms for lender is zero. In the middle panels, income follows a Pareto distribution with parameter $\alpha = 1.5, y_{min} = 0.05, y_{max} = 10$, and the cost of non-price terms for lender is zero. In the bottom panels, income follows a Pareto distribution with parameter $\alpha = 1.5, y_{min} = 0.05, y_{max} = 10$, and the cost of non-price terms for lender is linear and additively separable such that $V_z = -0.5$.

Reaching for yield: covenant-lite loans at low interest rates. The rising issuance of loans with weak covenants has been linked to the historically low risk-free rates (e.g., Roberts and Schwert 2020). We conclude this section by describing how lenders trade off price and non-price terms as interest rates fall. As Figure 9 illustrates, this trade off depends on the elasticity ϵ_ℓ. Low interest rates are associated with looser covenants z^* for low elasticity borrowers, whose default risk falls when rates are low. However, they are associated with *tighter* covenants for high elasticity borrowers, whose default risk increases because their loan size increases relatively more than their interest rate.

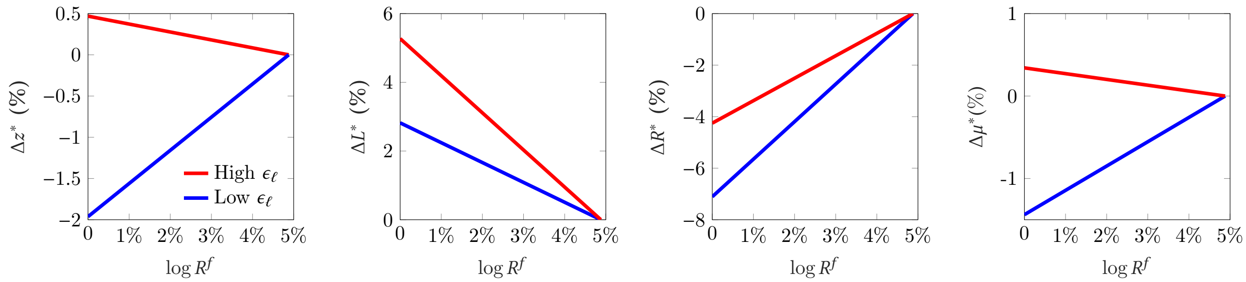

Figure 9: Covenant-lite loans: effect of bank's cost of fund on loan terms starting from a baseline $\log R^f = 5\%$, contrasting low elasticity (blue) and high elasticity (red) borrowers.

5 Dynamics of Credit Crises: Impact vs. Persistence

After analyzing how the cross-section of loan terms reacts to changes in credit supply and monetary policy in the steady state of the model, we turn to the transition dynamics of credit crises and the associated policy responses. We first study the impact and the persistence of a deterioration in banks' balance sheets. Then we apply our results to a calibrated model of the U.S. mortgage market. We conclude by discussing how policy interventions can improve the slow recovery from a credit crisis.

5.1 Dynamic Model

For simplicity, we start by assuming that there is a single type of borrower. Starting from an unconstrained steady state loan demand l^* with $\nu^* = 0$, we consider a credit supply shock that lowers banks' capacity constraint $\overline{L}$ to $\overline{L_0}$ such that an excess loan premium $\nu_0 > 0$ arises. Aggregate loan supply evolves according to

$$l_{t+1} = (1 + \phi \nu_t)\, l_t$$

where ϕ captures the earnings retention ratio times the leverage.

At each date, we have

$$(1 + \phi \nu_t)\, \ell\left(R\left(l\left(\nu_t\right), \nu_t\right)\right) = \ell\left(R\left(l\left(\nu_{t+1}\right), \nu_{t+1}\right)\right),$$

where as earlier $R(l, \nu)$ solves (holding R^f and z fixed) $R\left(1 - \mu\left(Rl\right)\right) = R^f + \nu$, and where $l(\nu)$ solves the static loan market clearing condition $l = \ell\left(R\left(l, \nu\right)\right)$. Linearizing around the steady state l^* and using our previous expressions for $\frac{\partial R}{\partial v}, \frac{\partial R}{\partial l}$, we obtain the law of motion for the excess loan premium v_t:

$$v_{t+1} = \left(1 - \frac{\phi R^f}{\widetilde{\epsilon}_\ell}\right) v_t$$

where as earlier $\widetilde{\epsilon}_\ell = \frac{1}{\left(1-\epsilon_{1-\mu}\right)\frac{1}{\epsilon_\ell}+\epsilon_{1-\mu}}$. The initial jump is $v_0 = \frac{R^f}{\widetilde{\epsilon}_\ell} \times \frac{l^*-\bar{L}_0}{l^*}$. Therefore:

Proposition 5. *Let $\varphi = \frac{\phi R^f}{\widetilde{\epsilon}_\ell}$. To first-order in the size of the initial credit supply shock $\delta = \frac{l^*-\bar{L}_0}{l^*}$, the excess loan premium v_t and bank lending L_t follow*

$$v_t = \frac{R^f}{\widetilde{\epsilon}_\ell}\delta\left(1-\varphi\right)^t, \tag{9}$$

$$l_t = l^*\left[1-\delta\left(1-\varphi\right)^t\right]. \tag{10}$$

We can measure the persistence of the crisis through the half-life of v, defined as the time T such that the excess loan premium has reverted to half its initial value $v_T = v_0/2$:

$$T = \frac{\log 2}{-\log\left(1-\varphi\right)}.$$

A special case of Proposition 5 holds in standard macro-finance models (e.g. Kiyotaki and Moore 1997, Gertler and Kiyotaki 2010): credit risk μ is exogenous, hence $\widetilde{\epsilon}_\ell$ and ϵ_ℓ are both equal to the unconstrained elasticity ϵ_ℓ^u. Accounting for non-linear contracts and endogenous default risk brings the elasticity $\widetilde{\epsilon}_\ell$ closer to 1, hence the half-life gets closer to $\frac{\log 2}{-\log\left(1-\phi R^f\right)}$, relative to the exogenous default (constant μ) case. In particular, in the limit of infinitely elastic unconstrained loan demand, the half-life increases without bounds when μ is constant, while it remains bounded as long as $\epsilon_{1-\mu} > 0$. This is because, as shown earlier, default risk decreases endogenously during credit crises if $\widetilde{\epsilon}_\ell > 1$. In very elastic markets, interest rates do not decrease to offset the lower default risk. As a result, banks earn excess returns v_t that recapitalize them back to the steady state in finite time.

Impact vs. persistence: the role of elasticities. For a given initial shock, the crisis is more persistent if loan demand is very elastic since the loan rate R and hence profits cannot jump enough. How are borrowers' loan payments affected? At a higher elasticity ϵ_l, the initial jump v_0 is also smaller. Overall, this gives rise to an intertemporal trade-off between borrowers. Present borrowers are hurt more (as measured by the spread v they face) with a lower elasticity $\widetilde{\epsilon}_\ell$, as the crisis is sharp and short-lived. Future borrowers are hurt more with a higher elasticity $\widetilde{\epsilon}_\ell$.

Accounting for non-linear contracts and endogenous credit risk slows down or speeds up bank recapitalization relative to the benchmark with linear contracts, depending on the elasticity:

- if $\epsilon_l < 1$, then $\frac{\log 2}{-\log\left(1-\frac{\phi R^f}{\tilde{\epsilon}_l}\right)} \leq T \leq \frac{\log 2}{-\log\left(1-\phi R^f\right)}$: crises are milder on impact, but more prolonged;
- if $\epsilon_l > 1$, then $\frac{\log 2}{-\log\left(1-\phi R^f\right)} \leq T \leq \frac{\log 2}{-\log\left(1-\frac{\phi R^f}{\tilde{\epsilon}_l}\right)}$: crises are sharper on impact, but shorter-lived.

We find that impact and persistence balance each other exactly in the following sense:

Proposition 6. *For an initial credit supply shock $\delta = \frac{l^*-\bar{L}_0}{l^*}$, the cumulative excess loan premium is given by*

$$\sum_{t=0}^{\infty} \nu_t = \frac{\delta}{\phi}$$

and is therefore independent of $\tilde{\epsilon}_\ell$.

This result highlights the benefits from using sufficient statistics in our analysis. In the relatively wide range of settings that we consider, the cumulative impact of the shock, as measured by the cumulative excess loan premium, does not depend on the details of the environment, such as borrower preferences, information asymmetries that affect $\epsilon_{1-\mu}$, and the feasible contract space (linear vs. non-linear contracts). Proposition 6 provides a testable prediction for different crises across time and space: as long as the parameter ϕ remains the same (or can be controlled for), the cumulative spread relative to the percent impact effect on quantities δ should be unchanged.

Dynamics with heterogeneous borrowers. We can combine our dynamic results with our previous results on the incidence of credit supply shocks in the cross-section of borrowers. Assume equal risk-weights $\rho^i = 1$ for simplicity. To first order in each period, aggregate loan supply

$$\begin{aligned}\sum_i l_{t+1}^i\left(\nu_{t+1}\right) &= \left(1+\phi\nu_t\right)\sum_i l_t^i\left(\nu_t\right) \\ &= \left(1+\phi\nu_t\right)\sum_i l^{i,*}\left(1-\epsilon_\ell^i\nu_t\right)\end{aligned}$$

hence the excess loan premium ν_t follows the same dynamics (9) as with homogeneous borrowers, except that $\tilde{\epsilon}_\ell = \sum \omega^i \tilde{\epsilon}_\ell^i$ is now a weighted average of individual elasticities with weights equal to the steady state loan shares $\omega^i = \frac{l^{i,*}}{\sum l^{i,*}}$. Thus the speed of recapitalization is now governed by the average (risk-adjusted) elasticity: credit crunches will be more persistent if banks lend to more elastic borrowers.

How are different borrowers affected over time? Each loan quantity evolves as

$$l_t^i = l^{i,*} - \tilde{\epsilon}_\ell^i \nu_t$$

All loan quantities recover at the same speed, since they all depend on the common excess loan premium ν_t. High $\widetilde{\epsilon}^i_\ell$ borrowers suffer a larger initial tightening, and for two types i, j the relative tightening is constant over time:

$$\frac{l^{j,*} - l^j_t}{l^{i,*} - l^i_t} = \frac{\widetilde{\epsilon}^j_\ell}{\widetilde{\epsilon}^i_\ell} \quad \forall t$$

Finally, the bank earns expected profits $\nu_t l^i_t$ from type i borrowers, and profit per dollar ν_t is common to all borrowers. As a result, inelastic borrowers are hurt less by the credit crunch in terms of credit growth, but they are also the ones paying for the bank recapitalization.

5.2 Example: Mortgage Market Crisis

To illustrate our findings, we conclude by analyzing a credit crisis in a simple calibrated model of the U.S. mortgage market, where loan contracts with many price and non-price terms are traded. We study the transition dynamics of mortgage markets in response to a contraction of banks' balance sheets. We introduce overlapping generations of households in the two-period model of Appendix B, and let banks allocate credit to multiple borrowers with different loan supply elasticities. We calibrate the model to recent U.S. data and discuss estimates for our two sufficient statistics: the interest-rate elasticities of loan supply, and of borrowers' repayment probability. We then present impulse response functions, and study two policy interventions to mitigate the shortgage of credit: direct household debt relief and bank recapitalization. In addition to mortgage spreads, we focus on two non-price dimensions of loans: maximum loan-to-value (LTV) and payment-to-income (PTI) ratios, which limit mortgage sizes for a given interest rate.

5.2.1 Calibration

There are two equal measures of types of borrowers, with respectively high and low elasticities of loan demand. We calibrate the model pre-crisis steady state to match six key moments for the U.S. mortgage market in 2000-2007, shown in the table below.

We target a low interval for the interest rate-elasticity of borrowers' loan demand, which reflects the range of estimates available in the literature. We obtain 0.6 for low-elasticity (low EIS) borrowers, and 1.4 for high-elasticity (high EIS) borrowers. In survey data, Fuster and Zafar 2021 estimate an elasticity of 0.11. Relying on the discontinuity of interest rates at various loan sizes, Best et al. 2019 estimate the elasticity of LTV to be 0.5 for the U.K. nortgage market. In a structural model of the U.S. banking system, Benetton 2021 estimates an elasticity of 0.07.[5]

[5] The interest rate elasticity of mortgage demand can be decomposed into a within-loan elasticity for households who borrow using the same loan, and a between-loan elasticity for households who switch between loan products altogether. those who change loan altogether. We focus on the within-loan elasticity because our model does not feature a discrete loan product choice.

To calibrate the interest-rate elasticity of borrowers' repayment probability, we compute the elasticity of the complement event: the elasticity of borrowers' default probability, which can be more easily estimated. We also target an interval which reflects the range of estimates available in the literature. We obtain a default rate elasticity of 1.45. Fuster and Willen 2017 estimate that the elasticity of mortgage default to monthly payment size is 1.1, and Di Maggio et al. 2017 that it is 2.

Finally, we target values for mortgage rates, maximum LTV and PTI ratios to reflect averages for those variables over the period 2000-2007. We obtain an average mortgage rate of 15%, a LTV ratio of 0.82, and a PTI ratio of 0.15, with the first two moments in the corresponding intervals of [3%, 18%] and [0.8, 1] for new originated mortgages (source: Black Knight, eMBS, HMDA, SIFMA, CoreLogic and Urban Institute), and the third moment below the [0.3, 0.5] interval.

Variable	Description	Value	Target	Source
ϵ_l	Int. rate elasticity of loan demand	0.6, 1.4	[0.11, 5]	See text
ϵ_μ	Int. rate elasticity of default prob.	1.45	[0.15, 2]	See text
R	Mortgage rate	1.15	[1.03, 1.18]	Primary Mortgage Survey (30-Year FRM)
LTV	Max. loan-to-value	0.82	[0.8, 1]	Urban Institute
PTI	Max. payment-to-income	0.15	[0.3, 0.5]	Urban Institute

5.2.2 Results

Figure 10 describes the dynamics of a credit crisis with multidimensional mortgage contracts. At $t = 0$, the mortgage market is in steady state. At $t = 0^+$, banks' lending capacity $\overline{L}$ unexpectedly contracts, resulting in an increase in the excess loan premium ν, which reflects the tightness of the banks' constraint (black line). Loan sizes fall and mortgage spreads increase for both types of households, as a result of the negative shock to credit supply. Because the dynamics of house prices remains unchanged, LTV ratios fall in response to the decrease in loan sizes. Interestingly, the response of PTI ratios depends on the relative changes in interest rates and loan sizes. Because the increase in rates dominates the decrease in loan sizes, PTI ratios increase for both borrower types.

The responses of loan terms are heterogeneous when disaggregated across borrowers. Loan size falls twice as much for high elasticity borrowers as for low elasticity ones, while the mortgage spread increases by less. As a result, default risk falls sharply for high elasticity borrowers, while it increases by about the same amount for low elasticity ones.

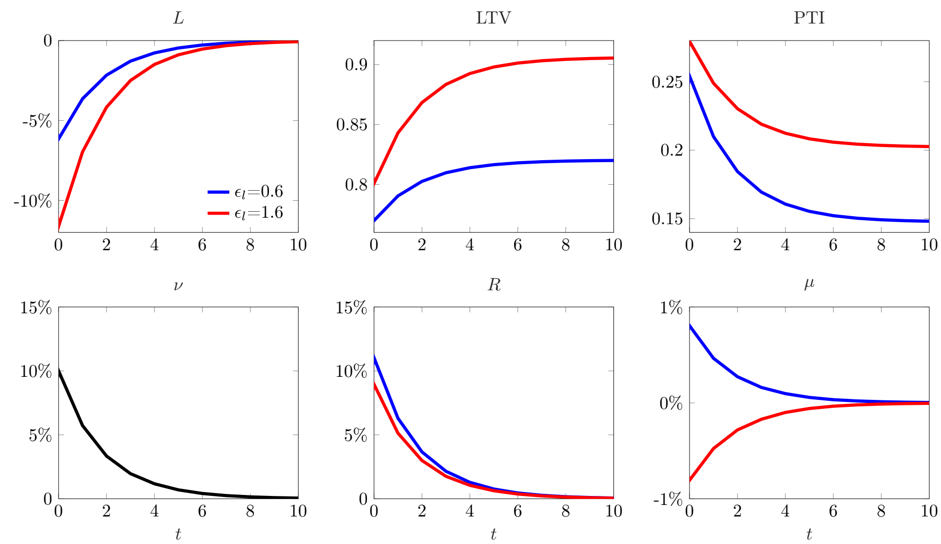

Figure 10: Dynamics of the cross-section of loans terms in the U.S. mortgage market in response to a tightening in banks' lending capacity. Impulse response functions for loan sizes, loan-to-value and payment-to-income ratios, excess loan premium, mortgage spreads, and default risk are plotted for low (blue lines) and high interest-rate elasticity borrowers (red lines).

5.2.3 Credit-Ameliorating Policies: Debt Relief and Bank Recapitalization

We conclude by studying the effectiveness of two policies designed to ameliorate the shortgage of credit in response to a credit supply shock. Direct borrower debt relief is modeled as a lump-sum transfer when households initially borrow from banks, which effectively reduces their indebtedness level. Bank recapitalization is modeled as a relaxation of banks' lending constraints, which can be implemented by direct equity injections. Both policies have been advocated during the U.S. mortgage market crisis, yet without clear guidance on the best way to direct credit to the borrowers who need it most.

Figure 11 illustrates how these policies work. Thick lines show laissez-faire outcomes under the same credit crisis as Figure 10, and dotted lines show the equilibrium path under the two policies, for the two types of borrowers. In our calibration, the two policies have very similar effects in mitigating the fall in LTV ratios due to a decrease in loan sizes. In the case of debt relief, households internalize that they are effectively richer when the loan is originated, and can borrow relatively more without excessively increasing their credit risk. In the case of bank recapitalization, banks' lending capacity does not fall as much, and therefore more credit is available to households, who decide to use it because they are credit-constrained. While the two policies have similar effects across borrowers, debt relief increases the speed at which loan size recovers for high elasticity borrowers slightly more than bank recapitalization.

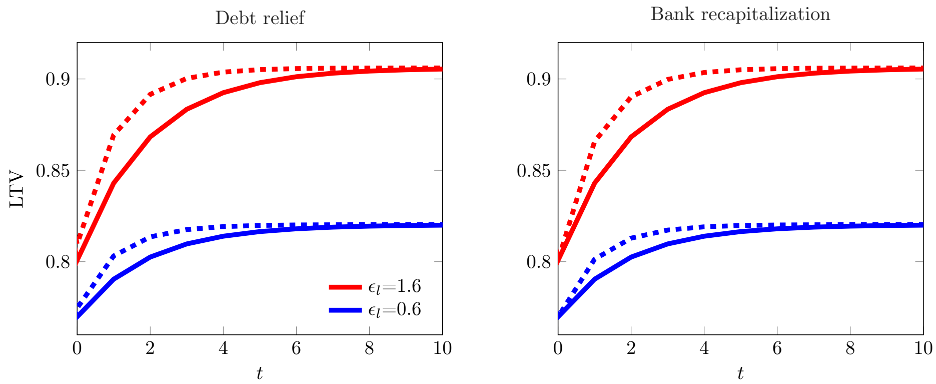

Figure 11: Impulse responses under two credit-ameliorating policies: debt relief and bank recapitalization. Thick lines show laissez-faire outcomes, and dotted lines show the equilibrium path under the two policies. Red (resp. blue): high (resp. low) elasticity borrowers.

6 Conclusion

We propose a model of multidimensional contracting between heterogeneous borrowers and intermediaries with limited lending capacity. We show that two sufficient statistics, the interest rate elasticities of borrowers' loan demand and default rates, predict how the cross-section of loan terms and banks' portfolio risk react to changes in bank capital and funding costs. Our results help explain key and puzzling features of loan markets: in particular, the heterogeneous pass-through of shocks across across borrowers and loan products, and the rise of covenant-lite lending in low risk-free rate environments. They also have normative implications. Our sufficient statistics drive the dynamic incidence of credit crises through the combination of impact and persistence. They provide guidance on how policies can best direct credit to borrowers who need it the most during downturns.

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Appendix

A Figures

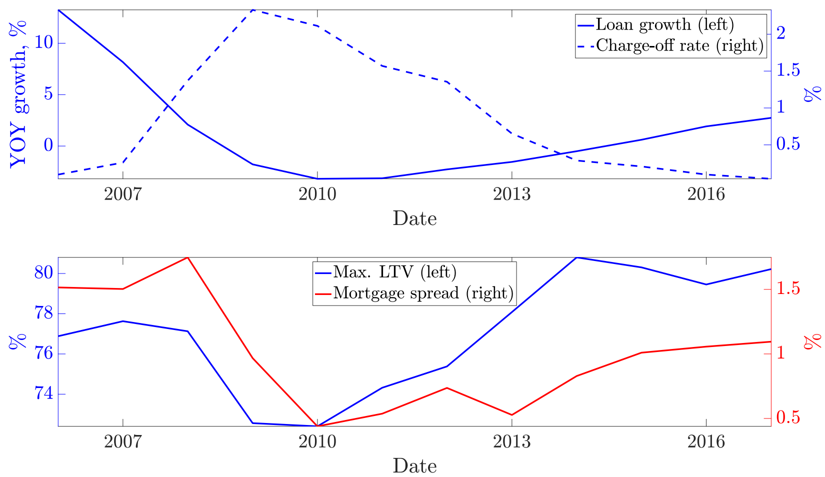

Figure 12: Bank balance sheets and mortgage loans. Sources: Federal Reserve Board, Fannie Mae, Freddie Mac.

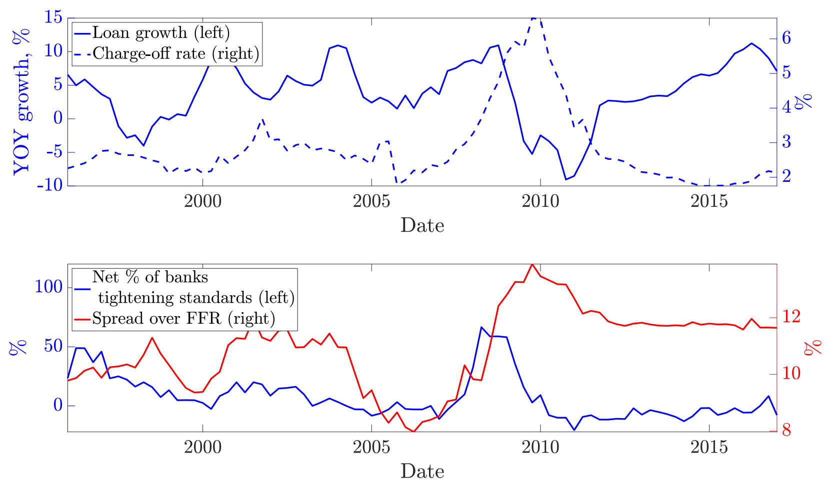

Figure 13: Bank balance sheets and credit card loans. Sources: Federal Reserve Board, SLOOS.

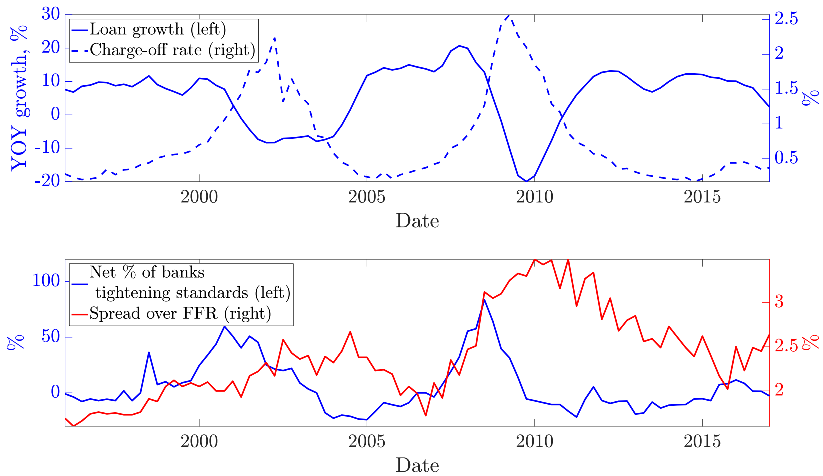

Figure 14: Bank balance sheets and commercial and industrial loans. Sources: Federal Reserve Board, SLOOS.

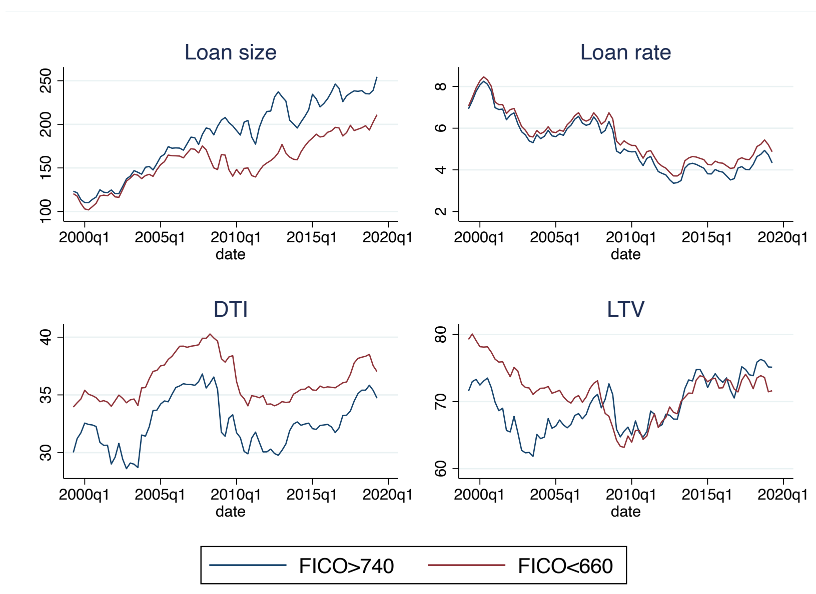

Figure 15: Conforming mortgage loan terms for low-risk (FICO above 750) and high-risk (FICO below 650) borrowers. Sources: Fannie Mac, Freddie Mae.

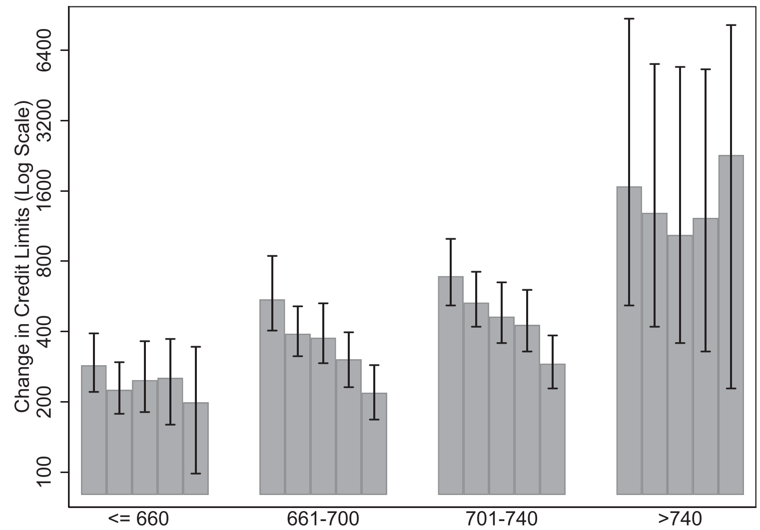

Figure 16: Source: Agarwal et al. 2018.

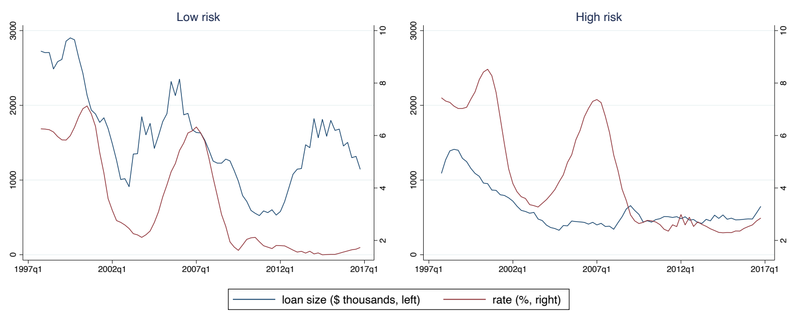

Figure 17: Loan size and loan rates for low and high risk short-term commercial and industrial loans. Source: Board of Governors of the Federal Reserve System (US).

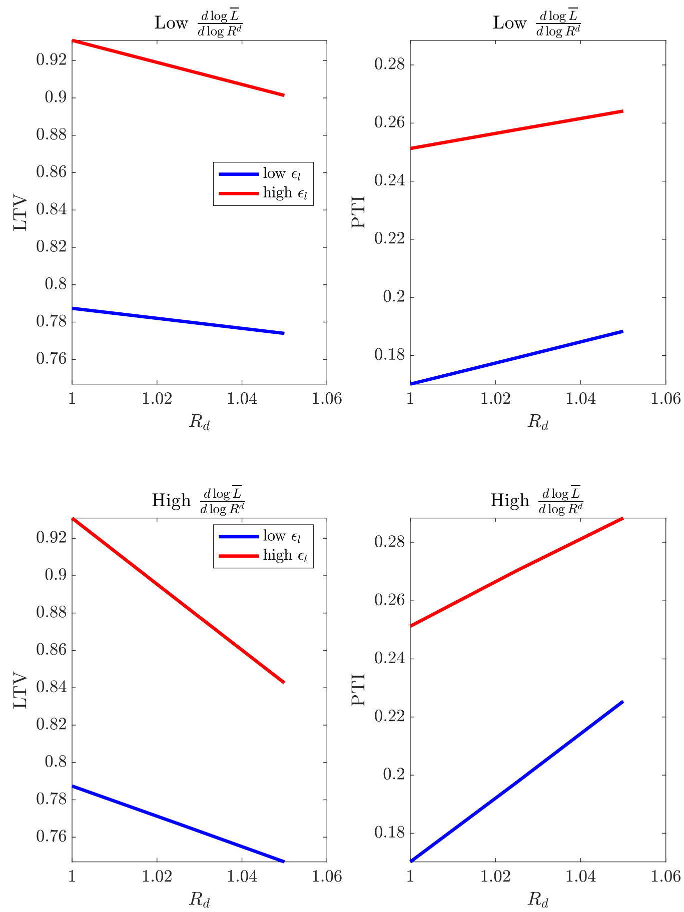

Figure 18: Effect of the bank deposit rate on mortgage terms, for low (blue) and high (red) elasticity borrowers, and as a function of the elasticity of banks' lending capacity to banks' cost of fund (low in upper panels, high in lower panels).

B Examples of Effective Default Probabilities μ

Strategic or liquidity default. A higher repayment Rl makes it harder to repay, hence

$$\lambda = \mu\left(Rl, z\right) \times Rl$$

where $\mu\left(\cdot\right)$ is increasing. For instance, suppose that borrowers have stochastic income y at the date of repayment and default if and only if $Rl \geq y$ (liquidity default): then $\mu\left(Rl\right) = \mathbf{P}\left(y \leq Rl\right)$. More generally, many of our examples will feature a standard debt contract such that the bank recovers $\rho\left(y, z\right)$ in case of default, which happens when y falls below a threshold $\hat{y}\left(Rl, z\right)$. In that case,

$$\underbrace{\mu\left(Rl, z\right)}_{\text{effective default prob.}} = \underbrace{F\left(\hat{y}\left(Rl, z\right)\right)}_{\text{actual default prob.}} \left(1 - \mathbf{E}\left[\frac{\rho\left(y, z\right)}{Rl} | y \leq \hat{y}\left(Rl, z\right)\right]\right)$$

Collateralized loans. We describe a simple way to capture mortgages within our framework. Households can buy a house worth P_0 by borrowing l from the bank and contributing a down-payment d such that $P_0 = d + l$. They then consume

$$c_0 = y_0 - d = y_0 - P_0 + l$$

At date-1, income y_1 and house price P_1 are realized and households have utility

$$\begin{cases} u\left(y_1 - Rl + \chi P_1\right) & \text{if they repay the mortgage} \\ u\left(\kappa y_1 + \underline{c}\right) & \text{if they default} \end{cases}$$

$\left(1 - \kappa\right) y_1$ captures the disutility of renting as well as the costs of exclusion from financial markets. χP_1 captures the pecuniary value of owning. Hence households default if and only if

$$z_1 \equiv y_1 + \frac{\chi}{1 - \kappa} P_1 \leq \frac{\underline{c} + Rl}{1 - \kappa}$$

We can thus define

$$V\left(l, R\right) = u\left(y_0 - P_0 + l\right) + \beta \left[\int_{z_1 \leq \frac{\underline{c}+Rl}{1-\kappa}} u\left(\kappa y_1 + \underline{c}\right) dF\left(y_1, P_1\right) + \int_{z_1 > \frac{\underline{c}+Rl}{1-\kappa}} u\left(y_1 - Rl + \chi P_1\right) dF\left(y_1, P_1\right)\right]$$

If upon default the bank recovers ζP_1, we have

$$\mu\left(Rl\right) = \int \int_{z_1 \leq \frac{\underline{c}+Rl}{1-\kappa}} \left(1 - \frac{\zeta P_1}{Rl}\right) dF\left(y_1, P_1\right)$$

The loan-to-value ratio (LTV) is then LTV $= \frac{l}{P_0}$ or just l if we normalize P_0 to 1. The debt to income ratio is l/y_0 while the payment to income ratio is $(R-1)l/y_0$.

Models of mortgage markets typically assume exogenous LTV (k) and PTI (θ) constraints, such that households maximize $V(l, R)$ subject to

$$l \leq \min\left\{kP_0, \frac{\theta y_0}{R-1}\right\}$$

Given a mortgage rate R, define $\ell^u(R)$ as solving $V_l(\ell^u(R), R) = 0$. The actual amount borrowed by the household is then

$$l = \min\left\{kP_0, \frac{\theta y_0}{R-1}, \ell^u(R)\right\} \tag{11}$$

Once we allow for endogenous constraints, the implementation of the contractual loan amount is indeterminate in the case of a single borrower type, assuming we are in the case $V_l(l, R) > 0$ hence $l < l^u(R)$, there are two unknowns (k, θ) for a single equation (11). Given the contractual rate R, the equilibrium loan amount l could arise from a binding LTV constraint with

$$k = \frac{l}{P_0}$$

coupled with any PTI constraint with $\theta > \frac{l(R-1)}{y_0}$; or alternatively from a binding PTI constraint with

$$\theta = \frac{l(R-1)}{y_0}$$

together with any LTV constraint with $k > \frac{l}{P_0}$.

Once we have several borrower types, say $i = a, b$, with different incomes y_0^i and house prices P_0^i, we can, under plausible conditions, interpret the equilibrium contracts $\left(l^i, R^i\right)$ by requiring that both borrowers face the same LTV and PTI constraints. Denote $r^i = R^i - 1$.

Proposition 7. *Suppose (without loss of generality) that* $\frac{P_0^a}{P_0^b} < \frac{y_0^a/r^a}{y_0^b/r^b}$ *and*

$$\frac{P_0^a}{P_0^b} < \frac{l^a}{l^b} < \frac{y_0^a/r^a}{y_0^b/r^b} \tag{12}$$

Then we can implement the contracts $\left(l^i, R^i\right)$ *with common LTV and PTI constraints*

$$k = \frac{l^a}{P_0^a}, \quad \theta = \frac{r^b l^b}{y_0^b}$$

LTV binds for a *and PTI binds for* b*.*

If (12) fails, the equilibrium cannot arise from common constraints, hence we need type-specific constraints $\left(k^i, \theta^i\right)$.[6] For each type, we are then back in the indeterminate case described earlier.

Adverse selection. A higher repayment Rl deters good borrowers, hence the average default probability goes up:

$$\lambda = \mu\left(Rl, z\right) \times Rl$$

where $\mu\left(\cdot\right)$ is increasing. For instance, suppose borrowers have an unobservable propensity to default u_i as well as an unobservable component in the utility from loans v_i. Holding rates fixed, if $\mathbf{Cov}\left(u_i, v_i\right) > 0$ then borrowers who demand a higher loan size (e.g., LTV) are more likely to default.

[6]We can still allow for a common LTV constraint k and type-specific PTI constraints θ^i, or conversely a single PTI constraint θ and type-specific LTV constraints k^i.

C Proofs and derivations

C.1 Main Propositions

Proof of Proposition 1. Each bank solves

$$\max_{\{x^i,R^i,l^i,z^i\}} \int x^i \pi^i \left(l^i, R^i, z^i\right) di$$

$$\text{s.t.} \int x^i \rho^i l^i di \leq \bar{L} \tag{13}$$

$$V^i \left(l^i, R^i, z^i\right) \geq \overline{V}^i \tag{14}$$

Denote ν the multiplier on the bank lending constraint (13) and λ_i the one on borrower i's participation constraint (14). The first-order conditions with respect to l^i, R^i and x^i are respectively

$$x^i \pi^i_R + \lambda_i V^i_R = 0$$

$$x^i \pi^i_l + \lambda_i V^i_l - \nu \rho^i = 0$$

$$\pi^i - \nu \rho^i l^i = 0$$

Therefore banks must equalize the profit per risk-weighted dollar across loans

$$\frac{\pi^i}{\rho^i l_i} = \nu$$

Note that this nests the case in which the lending constraint is not binding and thus $\nu = 0$ and banks make zero profits.

In a symmetric equilibrium with $x^i = 1$ for all i, the price and quantity of each loan must solve

$$-\frac{V^i_l}{V^i_R} = \frac{\frac{\pi^i}{l^i} - \pi^i_l}{\pi^i_R}.$$

Using

$$\pi^i = \left[R^i \left(1 - \mu^i\right) - R^d\right] l^i$$

we have

$$\frac{\frac{\pi^i}{l^i} - \pi^i_l}{\pi^i_R} = \frac{R^i}{l^i} \frac{l^i \mu^i_l / \left(1 - \mu^i\right)}{1 - R^i \mu^i_R / (1 - \mu^i)}$$

$$= \frac{R^i}{l^i} \frac{\epsilon^i_{1-\mu}}{1 - \epsilon^i_{1-\mu}}$$

where the second line uses $\mu^i = \mu^i \left(R^i l^i, z^i\right)$.

Proof of Proposition 2. We fix one borrower type i and omit the superscripts i. Differentiating (3) yields

$$-\frac{l\tau_l}{\tau}\frac{dl}{l}-\frac{R\tau_R}{\tau}\frac{dR}{R}=\theta\left(\frac{dl}{l}+\frac{dR}{R}\right)(1+\tau)$$

where $\theta=\frac{Rl\epsilon_{1-\mu}'}{\epsilon_{1-\mu}}$. Using $1+\tau=\frac{1}{\epsilon_{1-\mu}-1}$ hence $\tau\left(1+\tau\right)=\frac{-\epsilon_{1-\mu}}{\left(1-\epsilon_{1-\mu}\right)^2}$ we get

$$\epsilon_l=\frac{-R\tau_R+\frac{Rl\epsilon_{1-\mu}'}{\left(1-\epsilon_{1-\mu}\right)^2}}{-l\tau_l+\frac{Rl\epsilon_{1-\mu}'}{\left(1-\epsilon_{1-\mu}\right)^2}}.$$

Letting $x=\frac{Rl\epsilon_{1-\mu}'}{\left(1-\epsilon_{1-\mu}\right)^2}$, we have $\frac{d}{dx}\left(\frac{-R\tau_R+x}{-l\tau_l+x}\right)=\frac{l\tau_l-R\tau_R}{(b+x)^2}$ hence if $\theta>0$ then $\epsilon_l>\frac{R\tau_R}{l\tau_l}$ if and only if $l\tau_l-R\tau_R>0$.

Proof of Proposition 3 and Proposition 4. We detail the case where R^f is fixed and $\overline{L}$ is shocked; the converse case follows exactly the same steps. First, the bank lending constraint implies

$$\sum_i dl^i=d\bar{L}$$

$$-\sum_i\frac{l^i}{\bar{L}}\epsilon_l^i d\log R^i=d\log\bar{L}$$

To obtain $d\log R^i$, rewrite (4) as

$$\frac{R^i\left[1-\mu^i\left(R^i\ell^i\left(R^i\right),z^i\right)\right]-R^f}{\rho^i}=\nu$$

and differentiate to get for i,j

$$\frac{d\log R^i}{\rho^i}R^i\left(1-\mu^i\right)\left[1-\epsilon_{1-\mu}^i\left(1-\epsilon_l^i\right)\right]=\frac{d\log R^j}{\rho^j}R^j\left(1-\mu^j\right)\left[1-\epsilon_{1-\mu}^j\left(1-\epsilon_l^j\right)\right]$$

Therefore

$$
\begin{aligned}
-1 &= \sum_i \frac{l^i}{\bar{L}} \epsilon_l^i \frac{d\log R^i}{d\log \bar{L}} \\
&= \frac{l^i}{\bar{L}} \epsilon_l^i \frac{d\log R^i}{d\log \bar{L}} + \sum_{j\neq i} \frac{l^j}{\bar{L}} \epsilon_l^j \frac{d\log R^j}{d\log \bar{L}} \\
&= \frac{d\log R^i}{d\log \bar{L}} \frac{R^i\left(1-\mu^i\right)\left[1-\epsilon_{1-\mu}^i\left(1-\epsilon_l^i\right)\right]}{\rho^i} \left\{\sum_j \omega^j \epsilon_l^j \frac{\rho^j}{R^j\left(1-\mu^j\right)}\right\}
\end{aligned}
$$

where $\omega^j = \frac{l^j}{\bar{L}}$ are loan weights and $\epsilon_l^j = \frac{\epsilon_l^j}{1-\epsilon_{1-\mu}^j\left(1-\epsilon_l^j\right)} = \frac{\epsilon_l^j}{\left(1-\epsilon_{1-\mu}^j\right)+\epsilon_l^j\epsilon_{1-\mu}^j}$ is the risk-adjusted elasticity. This rewrites

$$
\frac{d\log R^i}{d\log \bar{L}} = -\frac{\rho^i}{R^i\left(1-\mu^i\right)} \frac{\epsilon_\ell^i}{\epsilon_l^i} \times \frac{1}{\sum_j \omega^j \epsilon_l^j \frac{\rho^j}{R^j\left(1-\mu^j\right)}}
$$

which implies

$$
\begin{aligned}
\frac{d\log l^i}{d\log \bar{L}} &= -\epsilon_l^i \frac{d\log R^i}{d\log \bar{L}} \\
&= \frac{\frac{\rho^i}{R^i\left(1-\mu^i\right)}\epsilon_\ell^i}{\sum_j \omega^j \epsilon_l^j \frac{\rho^j}{R^j\left(1-\mu^j\right)}}
\end{aligned}
$$

and $\frac{d\log R^i}{d\log \bar{L}} = -\frac{1}{\epsilon_l^i}\frac{d\log l^i}{d\log \bar{L}}$.

Since $R^i\left(1-\mu^i\right) = R^f + \rho^i \nu$, for small $\left(\rho^i - \rho^j\right)\nu$ we have $R^i\left(1-\mu^i\right) \approx R^j\left(1-\mu^j\right)$ hence

$$
\frac{d\log l^i}{d\log \bar{L}} \approx \frac{\rho^i \epsilon_\ell^i}{\sum_j \omega^j \rho^j \epsilon_l^j}.
$$

C.2 Other Calculations

The effective default probability is lower than the actual one thanks to the positive recovery rate. Then

$$
\mu'\left(Rl\right) = \kappa f\left(\hat{y}\left(Rl\right)\right) + \frac{1-\kappa}{\left(Rl\right)^2} \int_{y_{\min}}^{\hat{y}(Rl)} y dF\left(y\right)
$$

while

$$
\begin{aligned}
\epsilon_{1-\mu}\left(Rl\right) &= \frac{Rl\mu'\left(Rl\right)}{1-\mu\left(Rl\right)} \\
&= \frac{Rl\kappa\frac{f(Rl)}{F(Rl)} + \left(1-\kappa\right)\mathbf{E}\left[\frac{y}{Rl}|y \leq Rl\right]}{\frac{1-F(Rl)}{F(Rl)} + \left(1-\kappa\right)\mathbf{E}\left[\frac{y}{Rl}|y \leq Rl\right]}
\end{aligned}
$$

If $\kappa = 0$ then $\epsilon_{1-\mu} \in [0, 1]$.

Pareto distribution.

- Suppose $\kappa = 0$ and
$$F(y) = 1 - \left(\frac{y_{\min}}{y}\right)^{\alpha}$$
for $\alpha > 0$ and $y > y_{\min}$,, then $f(y) = \alpha \frac{1-F(y)}{y}$ and
$$\epsilon_{1-\mu}(Rl) = \alpha \times \frac{1 - \left(\frac{y_{\min}}{Rl}\right)^{1-\alpha}}{1 - \alpha \left(\frac{y_{\min}}{Rl}\right)^{1-\alpha}} \in [0, 1]$$
When $y_{\min}$ is very small this is approximately α. When $\alpha = 1$ this is 0. More generally if $\kappa \leq \frac{1}{\alpha}$ then $\epsilon_{1-\mu} \in [0, 1]$. The general formula is
$$\epsilon_{1-\mu} = \alpha \times \frac{1 - \alpha\kappa - (1-\kappa)\left(\frac{y_{\min}}{Rl}\right)^{1-\alpha}}{1 - \alpha\kappa - \alpha(1-\kappa)\left(\frac{y_{\min}}{Rl}\right)^{1-\alpha}}$$
$$\epsilon'_{1-\mu} = \frac{(1-\alpha)^2 \alpha(1-\kappa) y_{\min}(1-\alpha\kappa)\left(\frac{y_{\min}}{Rl}\right)^{\alpha}}{\left(\alpha(1-\kappa)y_{\min} - Rl(1-\alpha\kappa)\left(\frac{y_{\min}}{Rl}\right)^{\alpha}\right)^2}$$
$$\theta_{1-\mu} = \frac{(1-\alpha)^2 (1-\kappa) y_{\min}}{\left(\alpha \frac{1-\kappa}{1-\alpha\kappa}\left(\frac{y_{\min}}{Rl}\right)^{1-\alpha} - 1\right)\left(\frac{1-\kappa}{1-\alpha\kappa}\left(\frac{y_{\min}}{Rl}\right)^{1-\alpha} - 1\right)}$$

- With the power law example,
$$\theta_{1-\mu} = \frac{(1-\alpha)^2 Rl y_{\min}\left(\frac{y_{\min}}{Rl}\right)^{\alpha}}{\left(y_{\min} - Rl\left(\frac{y_{\min}}{Rl}\right)^{\alpha}\right)\left(\alpha y_{\min} - Rl\left(\frac{y_{\min}}{Rl}\right)^{\alpha}\right)}$$
is always positive. If $\alpha > 1$ the denominator is the product of two positive terms. If $\alpha < 1$ it's the product of two negative terms.

Examples of borrower utility $V(l, R)$.

- Starting with no risk hence no default:
  - Households
  $$V(l, R) = u(y_0 + l) + \beta u(y_1 - Rl)$$
  we see that
  $$\tau(l, R) = \frac{u'(y_0 + l)}{\beta R u'(y_1 - Rl)} - 1$$

Consistent with the intertemporal wedge interpretation, $\tau \geq 0$ measures how constrained the household ends up since $u_0' = \beta R (1+\tau) u_1'$. Suppose CRRA utility with EIS σ, $u(c) = c^{1-1/\sigma}$. Then

$$\frac{R\tau_R}{l\tau_l} = \frac{(y_0 + l)(\sigma(y_1 - Rl) + Rl)}{l(Ry_0 + y_1)}$$

If $y_0 = 0$ then this simplifies to

$$\frac{R\tau_R}{l\tau_l} = \sigma \times \left(1 - \frac{Rl}{y_1}\right) + 1 \times \frac{Rl}{y_1}$$

This is a weighted average of σ and 1, so it's above 1 if and only if $\sigma \geq 1$.

- Firms

  $$V(l, R) = f(k_0 + l) - Rl$$

  then

  $$\tau(l, R) = \frac{f'(k_0 + l)}{R} - 1$$

  we have the same wedge interpretation: $f'(k_0 + l) = (1+\tau) R$. Then

  $$\frac{R\tau_R}{l\tau_l} = \frac{f'(k_0 + l)}{-lf''(k_0 + l)}$$

  the inverse curvature of the production function. So for $f(k) = Ak^\gamma$ we have $\frac{R\tau_R}{l\tau_l} = \frac{(k_0+l)}{l(1-\gamma)}$. With $k_0 = 0$,

  $$\frac{R\tau_R}{l\tau_l} = \frac{1}{1-\gamma} \geq 1$$

  A higher γ leads to a higher interest rate elasticity of the unconstrained loan demand.

- Once we add risk and default we need to compute V numerically:

  - Households with income shocks y_1:

    $$V(l, R) = u(y_0 + l) + \beta \left[\int_{Rl}^{\infty} u(y_1 - Rl)\, dF(y_1) + \int_0^{Rl} u(\kappa y_1)\, dF(y_1) \right]$$

  - Firms with stochastic TFP shocks A, so that firm repays if and only if $Af(k_0 + l) \geq Rl$:

    $$V(l, R) = \int_{\frac{Rl}{(y_0+l)^\gamma}}^{\infty} \left[A(y_0 + l)^\gamma - Rl\right] dF(A) + \kappa (y_0 + l)^\gamma \int_0^{\frac{Rl}{(y_0+l)^\gamma}} A dF(A)$$