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Fast or Slow? Competing on Publication Frequency

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For many information goods, longer publication cycles are more economical, but often result in less timely - and, therefore, less valuable - information. While the digitalization of publication processes reduced fixed publication costs, making shorter publication cycles (or batches of information) more economically viable, competing firms adapted their publication cycle differently: Whereas some of them publish more frequently, others publish less frequently. In this paper, we build a game-theoretic model to determine how, in a duopoly, information providers should choose their publication cycles and prices under competition. We find that when the firms are *ex-ante* identical, they choose different publication frequencies in equilibrium. While a reduction in the fixed cost of publication yields shorter publication cycles, it may also intensify the competitive dynamics, which lead firms to differentiate their publication cycles further, especially when information is either very ephemeral or timeless. When publishers have access to sufficiently differentiated content, digitalization additionally creates an incentive for publishers to move toward independent publishing. Given the first-mover advantages of publishing at high frequency, our analysis informs publishers to take a proactive approach to digitalization and adapt their publication frequency accordingly.

Keywords: Publishing; Batching; Information Goods; Competition; Vertical Differentiation; Digitalization

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1. Introduction

How frequently an information good should be published is a key operational decision faced by many publishers across industries (e.g., news, entertainment, financial, weather). At the core of that decision lies a fundamental trade-off between economies of scale in publication and information obsolescence. As an example, consider a travel guidebook, such as *Lonely Planet* or *Fodor's*. Between two successive editions, some information about the listed attractions (e.g., addresses, opening hours) may change, making the guidebook less relevant. To counteract that obsolescence, the guidebook could be published more frequently, but at the expense of a higher publication and distribution cost.

This trade-off is resolved differently by different publishers. Guidebooks often coexist with travel websites; financial institutions offer services both in real time and with a delay (Shapiro and Varian 1998); *The Economist* publishes news weekly, while *The New York Times* does so daily; editions of encyclopedia *Britannica* appear every decade, unlike Wikipedia, which is updated almost continuously; Disney+ releases new content once a week, whereas YouTube publishes 500 hours of new content every minute (Thompson 2021); and web novels are typically updated daily (Hong 2017), unlike printed novels, which are released in bulk. Hence, within the same industry, different publishers adopt different publication cycles, or "clockspeeds," which contribute to differentiating their offerings. In this paper, we study how competing information providers should batch their information content and pace their publication cycles.

An information provider's publication cycle—or batching—decision is naturally dependent on the magnitude of the publication cost, the time sensitivity of the information, and the degree of content differentiation. Over the last decades, publication costs significantly decreased with the digitalization of the publication and distribution processes (e.g., website, streaming platforms; see Doyle 2013), making more frequent publishing economically viable (Meyer 2010). For instance, since 2008, Google Finance has offered real-time NYSE ticker updates in place of quotes that were delayed by 15 minutes, rendering its service more valuable to time-sensitive trades (Lowensohn 2018).

Yet, not every publisher chooses to operate more frequently. For instance, *Les Inrockuptibles*, a cultural magazine, recently switched from a weekly to monthly release. According to the magazine's CEO, "out-of-series and special issues, which remain in press shops for longer periods of time, are selling better. This motivated the switch to a monthly frequency" (Bernard and Patri 2021). Similarly, *China Business News Weekly* (*CBN*) opted to switch from weekly to monthly frequency and changed its focus from purely transmitting information to carrying in-depth content (Jiang 2018).

Digitalization also opened up new distribution channels and led to the development of independent publishing (e.g., Substack, Reddit, TikTok, YouTube). While the development of independent publishing is typically attributed to the wider geographical reach of digital channels, which makes it economically viable to offer niche content to heterogeneous customers (Evans and Wurster 1997, Doyle 2013), can it also be explained by the publishing competitive dynamics, even when customers have homogeneous tastes?

To be sure, there is a large literature on competition with batching—mostly based on the Economic Order Quantity (EOQ) model (Cachon and Harker 2002, Bernstein and Federgruen 2003)—but its focus is on physical, and not information, goods. Information goods differ from physical goods along two dimensions: First, there is no inventory holding cost borne by the firm; second, the value of information decays over time, affecting demand (Caro and Martínez-de-Albéniz 2020). As a result, the current literature offers little guidance to information providers for setting their publication cycles under competition.

As a first step toward that objective, in this paper, we study the competitive dynamics that arise between two *ex-ante* identical information publishers.[1] We focus on their publishing (or batching) decision, which usually comes on top of content generation, capture, and curation (Karmarkar and Apte 2007). We assume that firms first simultaneously choose their publication cycles and then compete on price.

To isolate the effect of publishing, we assume that the providers have access to the same information content, which has a constant inflow and decays linearly over time.[2] We consider a representative set of customers who, given the publication cycles and prices of the information products, optimize their consumption portfolios to maximize their surplus.

As is common to information goods, every publication comes with a fixed setup cost per publication (including the packaging, marketing, and distribution) and zero marginal cost of distribution (Jones and Mendelson 2011). We conceptualize the process of digitalization as a reduction in the fixed (setup) publication costs, ignoring other aspects (e.g., easier access to information or wider market reach).

Then, we extend the base model to consider situations where firms have partially differentiated content, different fixed publication costs, or different information decay rates. Still considering homogeneous customers, we model content differentiation by assuming that each firm's product contains some fraction of exclusive content.

Within this context, we investigate the following questions:

- How frequently should information providers publish content under competition? Are there first-mover advantages in choosing publication cycles?
- How should information providers adapt their batching strategy to a reduction in the fixed (setup) publication cost, e.g., due to the digitalization of publication and distribution processes? Does digitalization lift all boats, i.e., make everybody win?
- How does partial content differentiation affect the nature of the publication cycle equilibrium and its dynamics following a reduction in the fixed publication cost?
- How does asymmetry in fixed publication costs or information timeliness affect the equilibrium cycles and profits?

[1] Although the duopoly setting is considered here for tractability, it is relevant in many markets; e.g., local news were traditionally offered by one or two newspapers (George and Waldfogel 2006).

[2] Since firms offer the same content, there is no room for content-based horizontal differentiation (Hotelling 1990). This is obviously a simplification of reality—for instance, newspapers are often differentiated along the political spectrum—but this allows us to isolate the effect of publication cycle on competition.

We obtain the following results. First, when firms are *ex-ante* identical, a pair of asymmetric equilibria emerges, with firms choosing different publication frequencies. Moreover, they publish at a lower frequency than monopolists. Maximum cycle differentiation happens when information is either very ephemeral or timeless, but not in the intermediate range. This explains why we observe greater cycle differentiation in entertainment (e.g., YouTube vs. Disney+) than in news (e.g., *The New York Times* vs. *The Economist*), for instance. Moreover, the fastest publisher earns more profit than the slowest, so there are first-mover advantages of choosing a fast publication cycle.

Second, a reduction in the fixed cost of publication (e.g., driven by digitalization) creates two opposing forces. On the one hand, it induces shorter publication batches, due to diminishing economies of scale (as in the EOQ model). On the other hand, it may intensify the competitive dynamics, forcing firms to further differentiate their publication cycles. In particular, when the fixed cost is zero, the level of publication cycle differentiation is (locally) the highest. These two forces lead to non-monotone effects when the fixed publication cost decreases: While the fastest publisher always publishes more frequently, the slowest one may publish more frequently if the fixed cost is large and less frequently if it is small, which may explain the decision by *Les Inrockuptibles* or *CBN* to publish at a lower frequency. Furthermore, digitalization leads to some winners, but also, potentially, to some losers: While the fastest publisher always benefits from a reduction in the fixed publication cost, the slowest publisher might suffer from it.

Third, a cycle-based differentiated duopoly is more prevalent with lower levels of content differentiation. If content is sufficiently differentiated, publishers choose the same pace of publication and operate as local monopolies on their fraction of differentiated content—some form of monopolistic competition (Chamberlin 1933). Moreover, a reduction in the fixed setup cost may lead to an equilibrium shift from a cycle-based differentiated duopoly to content-based monopolistic competition, and this shift is moderated by the degree of time-sensitivity of information. This result supports the fast growth of independent publishers on online platforms offering differentiated content (e.g., Substack, YouTube), relative to channels that offer more similar content (e.g., news, financial information), which differ primarily on their publication cycles.

Fourth, we find that a publisher always benefits from a reduction in its own fixed cost of production (e.g., by migrating its content online) or in its information decay rate (e.g., by making its content more timeless), unlike the other publisher, who suffers from it. The levels of product quality (measured as information timeliness) turn out to be strategic substitutes; so, if one firm improves the quality of its product (either by publishing it more frequently, due to a reduction in its fixed cost, or by making its content less subject to decay), the other firm degrades its quality by publishing at a lower frequency.

The structure of the paper is as follows. In §2, we review the related literature and delineate our contribution. In §3, we model the choice of publication cycle and pricing as a two-stage game and present the analysis for publishers that are *ex-ante* identical. In §4, we characterize the equilibrium for duopolies differentiated in content, fixed cost of publication, and time sensitivity. In §5, we discuss the results and the limitations of our analysis. All proofs are provided in an electronic companion.

2. Literature Review

Our study contributes to three streams of literature: time-based competition, quality choice and vertical differentiation, and the management of information provision and publishing.

Because batching affects the product value, our work relates to the growing research on time-based competition (Blackburn 1991). The early literature on that topic focused on throughput time in a manufacturing (Just In Time) or service context (De Vany and Saving 1983, Lederer and Li 1997, Allon and Federgruen 2007). More recently, this literature has considered other time dimensions, such as time between the releases of successive generations of a product (Lobel et al. 2016, Barriola and Martínez-de-Albéniz 2021), assortment rotation cycle time (Caro and Martínez-de-Albéniz 2012, Bernstein and Martínez-de-Albéniz 2017), and, for information providers, time between successive updates of a database (Anant and Karmarkar 2007) and release time of new content (Choi 2015). In most of this literature, customers batch their purchasing decisions because search costs prevent them from continuously visiting stores (Bernstein and Martínez-de-Albéniz 2017) or because their utility from consuming a product gradually declines over time (Barriola and Martínez-de-Albéniz 2021), e.g., due to satiation (Caro and Martínez-de-Albéniz 2012, Ferreira and Goh 2021) or technological obsolescence (Lobel et al. 2016). In contrast, here we study a situation where batching is driven by supply considerations, as in the traditional EOQ model (Cachon and Harker 2002, Bernstein and Federgruen 2003), with a focus on information, and not physical goods, which are not associated with holding costs—but which value decays over time, as in Anant and Karmarkar (2007) and Choi (2015).

Since more frequent publications lead to the release of more timely information, the publication cycle decision relates to quality choice in product design and vertical differentiation. In a seminal paper, Moorthy (1988, p. 164) highlights that "a firm's equilibrium product strategy (quality) is the result of two opposing forces, one bringing the firms closer, the other moving them apart." Extending Moorthy (1988), Motta (1993) and Lehmann-Grube (1997) consider different competition formats, and Jones and Mendelson (2011) consider information goods. One common assumption in these papers is that customers buy, at most, one product and the choice of quality results in market segmentation. In contrast, we operationalize quality as publication frequency, which makes it possible for customers to purchase a portfolio of products. When customers are allowed to buy more than one product, it might be optimal for a monopolist to introduce multiple versions of a product (Calzada and Valletti 2012), and a market with homogeneous customers, which we consider here, may be profitable. Much like Moorthy (1988), we uncover two opposing forces in the equilibrium choice of product cycles. However, we find that the quality levels chosen by the duopolists are both lower than in a monopoly, in contrast to this literature, which finds that, under uniform customer valuations, quality levels are driven apart from the monopoly benchmark. As another extension of Moorthy (1988), Netessine and Taylor (2007) show how a monopolist's optimal product line design is affected by production setup costs and inventory accumulation. Our paper adds to their analysis by considering the effect of competition with a specific focus on information goods.

Finally, our work relates the research on the management of information provision and publishing. A large body of literature in economics characterizes the network effects between newspaper readers and advertisers in a "one newspaper-town" setup. They show that the economies of scale (Rosse 1967) and the positive-feedback loop between advertisers and customers (Bucklin et al. 1989, Blair and Romano 1993, and Dertouzos and Trautman 1990) result in an inevitable shift to local monopolies in the newspaper industry. A handful of papers study the effect of competition on advertising (Gabszewicz et al. 2001, Armstrong 2006, Rochet and Tirole 2003) but do not consider publication cycles as a dimension of differentiation. Besides advertising, information providers' operational decisions include fundraising (Kind et al. 2009), content creation (Sun and Zhu 2013), dynamic content allocation (Bernstein et al. 2021), content sizing (Anant and Karmarkar 2007), and content batching and release (Choi 2015, Caro and Martínez-de-Albéniz 2020).

Our paper enriches this growing stream of literature by exploring the role of publication cycles as another operational lever and its effect on competitive dynamics.

3. Base Case: Symmetric Firms

We consider a market with two *ex-ante* identical information providers who have access to the same content and periodically publish an information good over an infinite time horizon. The market, which size is normalized to one, consists of homogeneous customers who optimize their consumption to maximize their surplus.

Publication frequency decisions are often more strategic than pricing decisions. Accordingly, we assume that firms choose, first, their publication cycles simultaneously and non-cooperatively and then, their prices (again, simultaneously and non-cooperatively).

Section 3.1 introduces the model. As a benchmark, we consider the case of a monopoly in §3.2. We then characterize the competitive equilibrium in §3.3. We characterize how the equilibrium varies as a function of the timeliness of information provided in §3.4 and a reduction in the fixed publication cost in §3.5.

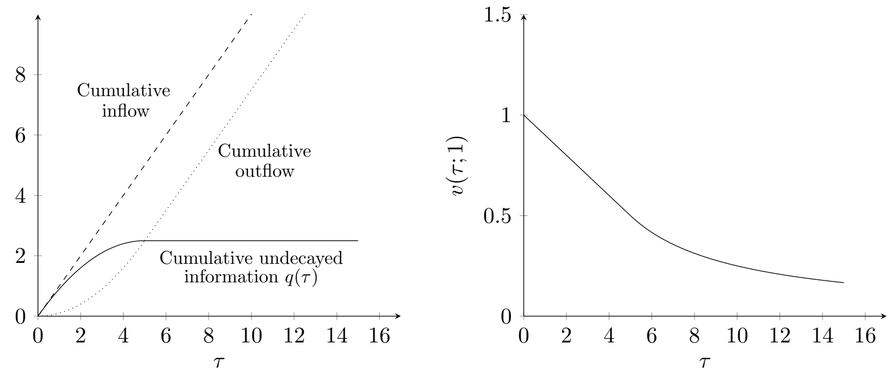
3.1. Model

We first introduce the supply and the demand sides, and then formulate the pricing and the publishing games.

Supply. Publishers obtain information at a constant rate, normalized to 1 bit/period. As is commonly assumed for perishable goods and information goods (Anant and Karmarkar 2007, Choi 2015, Caro and Martínez-de-Albéniz 2020), information continuously decays over time. Here, we assume a linear decay at a rate $\alpha > 0$ (expressed in bits/period^2), which measures the information time sensitivity. For instance, financial news is associated with a high decay rate, whereas cultural magazines and movies are associated with a low decay rate. At any point in time, the amount of information that decays cannot exceed the information inflow. Thus, the cumulative amount of undecayed information initially grows over time as the inflow exceeds the outflow and then reaches a steady state; see Figure 1. Specifically, the net amount of information accumulated over τ periods, denoted by $q(\tau)$, equals:

$$q(\tau) = \int_0^\tau 1 - \min\{1, \alpha t\} dt = \begin{cases} \tau\left(1 - \frac{\alpha}{2}\tau\right) & \tau \leq \frac{1}{\alpha} \\ \frac{1}{2\alpha} & \tau > \frac{1}{\alpha} \end{cases}. \tag{1}$$

Figure 1 Cumulative amount of information inflow, outflow, undecayed information $q(\tau)$ in absolute units (left); and undecayed information averaged over the time $v(\tau, 1)$ (right).



Note. In both plots, $\alpha = 0.2$.

We focus on stationary policies. Let u denote a firm's publication cycle (or batch of information) duration and ϕ be the fraction of the publication cycle during which customers have not received information from other sources. For instance, if a firm publishes weekly ($u = 7$ days) every Sunday, but the customers purchase a daily newspaper on Mondays, Tuesdays, and Wednesdays,[3] the fraction of new content associated with the weekly periodical is $\phi = 4/7$. Accordingly, the net amount of information collected over ϕu periods, averaged over the publication cycle u, equals

$$v(u,\phi) := q(\phi u)/u = \begin{cases} \phi\left(1 - \frac{\alpha}{2}\phi u\right) & \phi u \le \frac{1}{\alpha} \\ \frac{1}{2\alpha u} & \phi u > \frac{1}{\alpha} \end{cases}. \tag{2}$$

Each issuance incurs a fixed cost $k > 0$ (in \$), associated with its publishing, marketing, and distribution, but zero marginal cost, as is commonly assumed for information goods (Jones and Mendelson 2011)—an assumption that has also been empirically validated by Rosse (1967) and Wagner (1981). Therefore, the publisher's batching decision needs to trade off the fixed (setup) publication cost against information decay. Throughout our analysis, we assume that $\alpha \cdot k < 0.5$, which guarantees that a monopolist makes positive profit (see §3.2). Let r denote the time-average publication price; for instance, if a weekly publication costs \$10, $r = \$10/7$ per day.

Let $\mathbf{u} = (u_1, u_2)$ and $\mathbf{r} = (r_1, r_2)$, with u_i and r_i denoting Firm i's publication cycle and time-average price, for $i \in \{1, 2\}$. Similar to Barriola and Martínez-de-Albéniz (2021), and

[3] If customers purchase both the weekly and a fraction of dailies, it is optimal for them to purchase the dailies in sequence, published just after the release of the weekly. We assume that optimal purchasing pattern throughout.

in the same spirit as multi-echelon inventory policies (Roundy 1985), we consider cyclic policies; that is, we restrict the firms' publication cycles to be integer multiple of each other, i.e., $\frac{\max\{u_1,u_2\}}{\min\{u_1,u_2\}} \in \mathbb{N}$, and assume that publications are synchronized every $\max\{u_1, u_2\}$ periods; for instance, both firms publish at time 0. Without loss of generality, we refer to Firm 1 as the fastest publisher, i.e., $u_1 \le u_2$, in the consumers' consumption problem and the pricing game. To simplify the exposition, we sometimes refer to the fastest product as a "daily" and the slowest one as a "weekly," though u_2/u_1 is not necessarily equal to 7.

Demand. Since the contents are undifferentiated, the market can be aggregated into a set of homogeneous customers who optimize their consumption over an undiscounted infinite time horizon (Chamberlin 1933, Caro and Martínez-de-Albéniz 2012). The information good is consumed immediately after it is purchased and is proportionally valued to the net amount of information it contains. We normalize customer valuations, so that one bit of information is worth \$1. The customers have no capacity constraint limiting their consumption. Thus, they can opt to buy daily news every day or can wait for a weekly to come out to consume an entire week's news altogether, although some of it would have been obsolete by the time of consumption. Alternatively, the customers can use any combination of dailies and the weekly; indeed, European newspaper readers often buy more than one product (Statistica 2020b) and make several purchases a week (Statistica 2020a). Accordingly, the customers' time-average utility from consuming a fraction $d \in D := \{i \cdot \frac{u_1}{u_2} | i = 0, 1, \ldots, \frac{u_2}{u_1}\}$ of dailies and $w \in \{0, 1\}$ weekly per weekly cycle is

$$U(d, w; \mathbf{r}, \mathbf{u}) = d \cdot (v(u_1, 1) - r_1) + w \cdot (v(u_2, 1 - d) - r_2). \tag{3}$$

Given publication cycles $\mathbf{u}$ and prices $\mathbf{r}$, the customers' optimal consumption portfolios solve the following problem:

$$(d^*(\mathbf{r}, \mathbf{u}), w^*(\mathbf{r}, \mathbf{u})) = \underset{d \in D, w \in \{0,1\}}{\arg\max} \; U(d, w; \mathbf{r}, \mathbf{u}). \tag{4}$$

In case two product portfolios achieve the same utility, we select, as a tie-breaking rule, the one that contains the weekly, and, among those that achieve the same utility with the weekly, the one that contains the largest number of dailies.

3.2. Monopoly

Before analyzing the competitive equilibrium, we first consider, as a benchmark, a single-product monopolist who operates with publication cycle u and price r. (Throughout this section, we use superscript 'm' to refer to this case.) The customers' time-average utility from consuming the product is

$$U^m(u,r) = v(u,1) - r. \tag{5}$$

Hence, customers buy the product if and only if $r \le v(u,1)$. Accordingly, given a publication cycle u, the monopolist's time-average profit is

$$\Pi^m(r;u) = \begin{cases} r - \frac{k}{u} & r \le v(u,1) \\ 0 - \frac{k}{u} & r > v(u,1). \end{cases} \tag{6}$$

Given u, the monopolist prices at $r^m(u) = \arg\max_r \Pi^m(r;u) = v(u,1) - \frac{k}{u}$ to maximize its profit. Accordingly, the monopolist chooses its optimal publication cycle by solving $u^m = \arg\max_u v(u,1) - \frac{k}{u} = \sqrt{\frac{2k}{\alpha}}$. Let $\pi^m := \Pi^m(r^m, u^m)$ denote the optimal profit.

THEOREM 1. *A monopolist publishes its good with cycle $u^m = \sqrt{\frac{2k}{\alpha}}$ and sets its price $r^m = 1 - \sqrt{\frac{\alpha k}{2}}$, yielding time-average profit $\pi^m = 1 - \sqrt{2\alpha k}$. Even if the monopolist had the option to offer multiple products, it would offer only one product.*

Like the EOQ model, where one trades off a fixed cost against an inventory holding cost, the monopolist chooses its publication cycle by trading off a fixed setup cost against information decay. Hence, the information decay rate α plays a similar role to the inventory holding cost. Naturally, the monopolist's optimal publication cycle is smaller when the fixed cost decreases or when information decays faster. Also, the monopolist's optimal profit is completely determined by the product $\alpha \cdot k$. In particular, assuming that $\alpha \cdot k < 0.5$ is a necessary and sufficient condition for the monopolist to make a positive profit (under the normalization that information is generated at a rate of 1 bit/period and that customers value information at \$1/bit). Finally, Theorem 1 shows that the monopolist could not achieve a higher profit by offering multiple products.

3.3. Competitive Equilibrium

To characterize the competitive equilibrium, we solve the game backward, by first considering the customers' consumption problem, then the firms' pricing problem, and finally the firms' publication frequency choices.

Customer Choice. We sequentially solve the customers' choice problem (4) by separating the consumption choice of weekly from that of dailies. Let $d^*(w;\mathbf{r},\mathbf{u}) := \arg\max_{d\in D} U(d,w;\mathbf{r},\mathbf{u})$. See Lemma EC.1 in the electronic companion.

For tractability, in our solution of the pricing game, we approximate $d^*(w;\mathbf{r},\mathbf{u})$ as a continuous function whenever the customers buy some, but not all, dailies together with the weekly. In these cases, which happen when $w=1$, $\frac{\partial U(0,1)}{\partial d}\geq 0$ and $U(1,1)<U\left(1-\frac{u_1}{u_2},1\right)$,[4] the customers' optimal quantities turn out to be identifiable by the first-order optimality conditions: $d^*(1;\mathbf{r},\mathbf{u}) = \arg\min_{j\in\left\{0,\ldots,\frac{u_2}{u_1}\right\}}\left|\frac{\partial U\left(j\frac{u_1}{u_2},1;\mathbf{r},\mathbf{u}\right)}{\partial d}\right|$. Accordingly, we use the following approximation:

$$\tilde{d}^*(w;\mathbf{r},\mathbf{u}) := \begin{cases} \arg\min_{j\in\left[0,\frac{u_2}{u_1}\right]}\left|\frac{\partial U\left(j\frac{u_1}{u_2},1;\mathbf{r},\mathbf{u}\right)}{\partial d}\right| & \text{if } w=1,\ \frac{\partial U(0,1)}{\partial d}\geq 0 \text{ and } U(1,1)<U\left(1-\frac{u_1}{u_2},1\right), \\ d^*(w;\mathbf{r},\mathbf{u}) & \text{otherwise.} \end{cases} \tag{7}$$

Let $\tilde{w}^*(\mathbf{r},\mathbf{u}) := \arg\max_{w\in\{0,1\}} U(\tilde{d}^*(w;\mathbf{r},\mathbf{u}),w;\mathbf{r},\mathbf{u})$ and $\tilde{d}^*(\mathbf{r},\mathbf{u}) := \tilde{d}^*(\tilde{w}^*(\mathbf{r},\mathbf{u});\mathbf{r},\mathbf{u})$.

Pricing Game. Taking into account the customers' choices, the competing publishers simultaneously choose their prices. Given publication cycles $\mathbf{u}$, and still denoting Firm 1 as the fastest publisher, the publishers' time-average undiscounted profits are

$$\Pi_1(r_1;r_2,\mathbf{u}) = r_1\cdot\tilde{d}^*(\mathbf{r},\mathbf{u}) - \frac{k}{u_1}; \quad \Pi_2(r_2;r_1,\mathbf{u}) = r_2\cdot\tilde{w}^*(\mathbf{r},\mathbf{u}) - \frac{k}{u_2}. \tag{8}$$

It turns out that the firms' best responses do not cross. Intuitively, firms will compete on price *à la* Bertrand in order to capture the entire market (so that customers purchase only all dailies or only the weekly). When the market price falls below a certain threshold, the fast publisher will give up on the idea of capturing the entire market, and instead target partial market coverage (so that customers purchase a fraction of dailies), enabling it to raise its price. No longer facing price pressure, the slowest publisher will then also raise to the highest profitable price, leading to another round of price war. Thus, there exists no pure-strategy Nash equilibrium in the pricing game.

However, there exists a mixed-strategy Nash equilibrium, which can be interpreted as inter-temporal price dispersion, e.g., through random promotions or sales (Varian 1980). See Lemma EC.2 in the electronic companion. For any $i\in\{1,2\}$, we denote by $F_i(\cdot)$ Firm

[4] Note that we preserve the discontinuous character of $d^*(w;\mathbf{r},\mathbf{u})$ near the upper end of D. Otherwise, Firm 1 would be given a disadvantage to capture the whole market, distorting the nature of the equilibrium in the case with differentiated content studied in §4.

i's pricing cumulative distribution function defined on support S_i. A pair of supports (S_1, S_2) together with the corresponding cumulative distributions $(F_1(\cdot), F_2(\cdot))$ constitute a mixed-strategy Nash equilibrium (Mas-Colell et al. 1995) if $\forall i \in \{1,2\}$,

- $\forall r_i \in S_i$, $\Pi_i(r_i; F_{-i}(\cdot), \mathbf{u}) = \Pi_i^*(\mathbf{u})$ for some $\Pi_i^*(\mathbf{u})$;
- $\forall r_i \notin S_i$, $\Pi_i(r_i; F_{-i}(\cdot), \mathbf{u}) \leq \Pi_i^*(\mathbf{u})$;

where $-i := 3 - i$ refers to the firm other than i. In other words, Firm i solves the following problem:

$$S_i = \arg\max_{r_i} \Pi_i\left(r_i; F_{-i}(\cdot), \mathbf{u}\right) \quad \forall i \in \{1,2\}. \tag{9}$$

Let $\pi_i(u_i; u_{-i}) := \Pi_i^*(\mathbf{u})$, $\forall i \in \{1,2\}$, denote Firm i's equilibrium profit in the pricing game.

Publishing Game. Anticipating their equilibrium profits in the pricing game $\pi_i(u_i; u_{-i})$, the firms simultaneously choose their publication cycles to maximize their profits. Given $\frac{\max\{u_1,u_2\}}{\min\{u_1,u_2\}} \in \mathbb{N}$, either $u_1 = u_2$ or $\frac{\max\{u_1^*,u_2^*\}}{\min\{u_1^*,u_2^*\}} \geq 2$. To obtain closed-form expressions of the equilibrium publication cycle best responses, we relax the constraint that $\frac{\max\{u_1,u_2\}}{\min\{u_1,u_2\}} \in \mathbb{N}$ and consider only the latter implications, resulting in the following game:

$$u_i^* = \underset{u_i > 0 : \frac{\max\{u_i, u_{-i}^*\}}{\min\{u_i, u_{-i}^*\}} \geq 2 \text{ if } u_i \neq u_{-i}^*}{\arg\max} \pi_i(u_i; u_{-i}^*) \quad \forall i \in \{1,2\}. \tag{10}$$

This game turns out to have two asymmetric pure-strategy Nash equilibria.

THEOREM 2. *There exists a unique pair of asymmetric equilibria in the publishing game (10). Denoting Firm 1 as the fastest publisher in equilibrium, the equilibrium publication cycles solve:*

$$u_1^* = \sqrt{\frac{2ku_2^*}{\alpha u_2^* - 1}}, \quad u_2^* = \frac{\left(\sqrt{6\alpha k + 1} + 2\right)(2 - \alpha u_1^*)}{2\alpha(1 - 2\alpha k)}. \tag{11}$$

Moreover, $\pi_1^* > \pi_2^*$.

Even though (or because) firms are *ex-ante* identical, they choose to differentiate their publication cycles to avoid a price war. Since $\alpha u_1^* \leq 1$, Product 1's information accumulation process (1) has not reached a steady state before its release, unlike Product 2 $(\alpha u_2^* > 1)$.

Since the firms' best responses are downward-sloping, their publication cycles, i.e., their product quality levels (measured as information timeliness), are strategic substitutes. We will demonstrate the robustness of this insight when firms have asymmetric fixed costs (§4.2) or asymmetric decay rates (§4.3).

Using the monopoly benchmark enables us to disentangle the forces underlying the duopolists' equilibrium publication cycles (11). Their best-response functions can be decomposed into two components: one reflecting the fixed-cost effect and the other reflecting the competition effect. For u_1^*, we define the fixed-cost effect at the monopolist's optimal cycle. For u_2^*, we factor the fixed-cost effect, so that it tends to one when $k \to 0$. Specifically,

$$u_1^* = \underbrace{\sqrt{\frac{u_2^*}{u_2^* - 1/\alpha}}}_{\text{competition effect}} \cdot \underbrace{\sqrt{\frac{2k}{\alpha}}}_{\text{fixed-cost effect}} \quad ; \quad u_2^* = \underbrace{\frac{3(2 - \alpha u_1^*)}{2\alpha}}_{\text{competition effect}} \cdot \underbrace{\frac{\sqrt{6\alpha k + 1} + 2}{3(1 - 2\alpha k)}}_{\text{fixed-cost effect}}. \tag{12}$$

Accordingly, the equilibrium choices of publication cycles are the result of two opposing forces: (i) the desire to set the publication frequency at the monopoly level, and (ii) the need to be apart from each other to ease the competition. This is akin to quality differentiation models (Moorthy 1988).

Next, we compare the equilibrium publication cycles (11) to their monopoly benchmark.

Proposition 1. *The publication cycles are longer in a duopoly than in a monopoly:*

$$u_2^* > u_1^* > u^m.$$

Proposition 1 reveals that competition leads to less frequent publications, i.e., the provision of less timely information. This is because competition dilutes market shares, which creates stronger economies of scale, engendering larger batches of information than in a monopoly. This kind of distortion is reminiscent of Moorthy (1988), who shows that competition pushes firms to choose different quality levels. But here they are both lower (lower publication frequency) than in the monopoly case, unlike generic models of vertical differentiation with heterogeneous customers (Lehmann-Grube 1997).

Finally, because $\pi_1^* > \pi_2^*$, firms have an incentive to signal that they would publish more frequently than their rivals, i.e., there are first-mover advantages of publishing at high frequency.

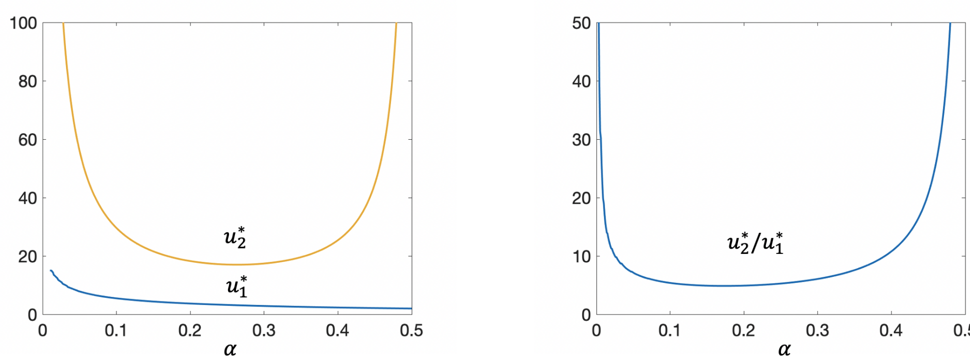
3.4. Information Timeliness

Is publication cycle differentiation more salient in industries that provide more or less timely information? The next proposition shows that maximum cycle differentiation happens when the information is either very ephemeral (i.e., very large α) or timeless (very small α). When the information decays quicker, the fastest publisher operates under shorter cycles (similar to a monopolist, see Theorem 1), but the slowest publisher may actually operate under either shorter or longer cycles. The characterization of the slowest publisher's equilibrium frequency involves two thresholds, $\underline{\alpha}$ and $\overline{\alpha}$, which turn out to be very close to each other (specifically, we can show that $\overline{\alpha} - \underline{\alpha} \leq \frac{0.014}{k}$).

PROPOSITION 2 **(Information Timeliness).**

- *The ratio of publication cycles (u_2^*/u_1^*) decreases in α when $\alpha < \hat{\alpha}$ and increases when $\alpha > \tilde{\alpha}$ for some $\tilde{\alpha} \geq \hat{\alpha} > 0$.*
- *Firm 1 publishes more frequently as α increases; in particular, $u_1^* \to \infty$ when $\alpha \to 0$ and $u_1^* \to 2k$ when $\alpha \to 0.5/k$;*
- *Firm 2 publishes more frequently as α increases when $\alpha < \underline{\alpha}$, and less frequently when $\alpha > \overline{\alpha}$, for some thresholds $0 < \underline{\alpha} \leq \overline{\alpha} < 0.5/k$; in particular, $u_2^* \to \infty$ when either $\alpha \to 0$ or $\alpha \to 0.5/k$.*

Figure 2 Equilibrium publication cycles and the corresponding ratios as a function of α



Note. In both plots, $k = 1$.

Consider the case where $\alpha < \min\{\hat{\alpha}, \underline{\alpha}\}$.[5] Proposition 2 indicates that industries that provide more ephemeral information (i.e., greater α) are associated with more frequent publications and lower cycle differentiation. To illustrate this point, consider the following four industries, ranked from the least to the most timely: encyclopedias, entertainment, news, and stock market information. Table 1 presents examples of the fastest and slowest information providers in each industry. Consistent with Proposition 2, we observe that, as

[5] This case is the most relevant in competitive settings. In the other case, i.e., when the information decay rate α is high, the profit potential is low (Theorem 1); as a result, providers of very ephemeral information (e.g., weather, disruption report) often operate as natural monopolies or need to be publicly subsidized (Byrne 2012, Frijters and Velamuri 2010).

Table 1 Example of the fastest and slowest publishers in select industries.

Industry	Fastest Publisher		Slowest Publisher	
	Example	Publishing Frequency	Example	Publishing Frequency
Encyclopedias	Wikipedia	Months	*Britannica*	Decades
Entertainment	YouTube	Days	Disney	Months
News	CNN.com	Hours	*The New York Times*	Days
Stock Market	Bloomberg Terminal	Seconds	Bloomberg TV	Minutes

Note: The publishing frequency is meant to be representative, not exact. For instance, in 2020, the time between every 10 million edits on the English Wikipedia was 54 days (https://en.wikipedia.org/wiki/Wikipedia:Time_Between_Edits, accessed on December 6, 2021).

information becomes more timely, the fastest publisher operates more frequently, and there is smaller cycle differentiation between the fastest and the slowest publishers. There are of course other factors that can explain these different industries' publishing frequencies and degrees of differentiation (such as the rate of information accumulation and the level of content differentiation), but our base model, which assumes *ex-ante* identical publishers, is consistent with these observed patterns.

3.5. Digitalization

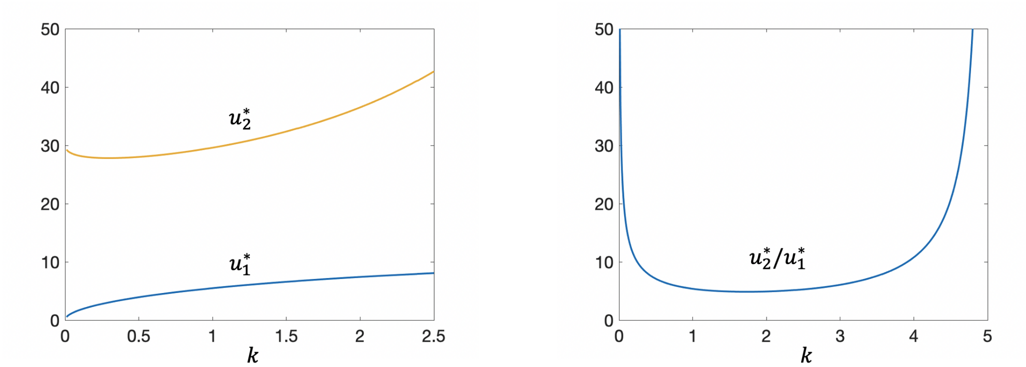
Next, we study how the equilibrium publication cycles and profits are affected by changes in the fixed publication cost, e.g., driven by the digitalization of their processes. Our first result establishes that, following a reduction in the fixed publication cost k, the fastest publisher will publish more frequently (similar to the EOQ model), unlike the slowest publisher. The proposition introduces two thresholds, $\underline{k}$ and $\overline{k}$, which turn out to be very close to each other (specifically, we can show that $\overline{k} - \underline{k} \leq \frac{0.014}{\alpha}$).

PROPOSITION 3 **(Fixed Publication Cost)**. *When the fixed publication cost k decreases, in equilibrium,*

- *Firm 1 publishes more frequently;*
- *Firm 2 publishes less frequently when $k < \underline{k}$ and more frequently when $k > \overline{k}$, for some thresholds $0 < \underline{k} \leq \overline{k} < 0.5/\alpha$.*

The left panel of Figure 3 illustrates Proposition 3. Recall from (12) that the firm's best responses are the results of two forces: a competition effect and a fixed-cost effect. For Firm 1, the fixed-cost effect dominates the competition-effect coefficient, which remains bounded, lying between 1 and $\sqrt{2}$ (since $u_2^* > 2/\alpha$ by Lemma EC.6). As a result, Firm 1's equilibrium cycle is monotonically increasing in k, similar to a monopolist (Theorem 1).

Figure 3 **Equilibrium publication cycles as a function of k**



Note. In both plots, $\alpha = 0.1$. For readability, we consider different ranges of values for k since $u_2^* \to \infty$ when $\alpha k \to 0.5$.

In contrast, Firm 2's equilibrium cycle needs to balance the competition and fixed-cost effects, which relative strengths vary with k, resulting in a non-monotone behavior. Specifically, the fixed-cost effect dominates, making u_2^* increase in k, when k is large; whereas the competitive effect dominates, making u_2^* decrease in k, when k is small. As a result, the cycle differentiation u_2^*/u_1^*, illustrated in the right panel of Figure 3, reaches a local maximum when k is very small. Therefore, a reduction in fixed publication costs (e.g., driven by digitalization) may not necessarily yield more frequent publications across an entire industry, as illustrated by the cases of *Les Inrockuptibles* and *CBN* discussed in §1.

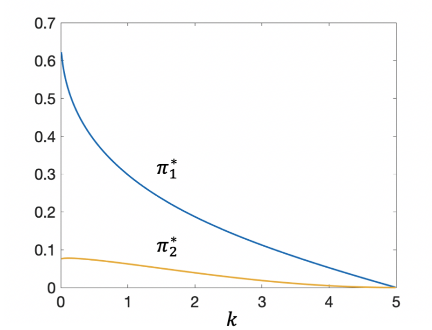
We know that a monopolist always benefits from a setup cost reduction (Theorem 1), but is that the case in a competitive equilibrium? The next proposition shows that again the fastest publisher behaves like a monopolist, in the sense that its equilibrium profit decreases in k. In contrast, the slowest publisher's profit is, in general, non-monotone. See Figure 4. The proposition expresses the profit comparative statics in terms of $\alpha \cdot k$.

PROPOSITION 4 **(Profits)**. *When αk decreases,*

- *Firm 1's profit π_1^* monotonically increases from $\pi_1^* \to 0$ when $\alpha k \to 0.5$ to $\pi_1^* \to \frac{2}{3}$ when $\alpha k \to 0$;*
- *Firm 2's profit π_1^* monotonically decreases when $\alpha k < \bar{c}$, for some $\bar{c} > 0$; moreover, $\pi_2^* \to 0$ when $\alpha k \to 0.5$ and $\pi_2^* \to \frac{2}{27}$ when $\alpha k \to 0$.*

Hence, a reduction in fixed publication costs creates some winners and, potentially, some losers: While the fastest publisher always benefits from an industry-wide setup cost reduction, the slowest publisher prefers operating at intermediate levels of the setup cost

Figure 4 Equilibrium profits as a function of k



Note. Here, $\alpha = 0.1$.

since its profit is increasing in k when $k < \bar{c}/\alpha$, for some $\bar{c} > 0$ and tends to zero when $\alpha k \to 0.5$. Digitalization does not make all boats rise: In the U.S., while the total sales of the top four newspaper publishers increased by 1.3% from 2012 to 2017, those of the top four magazine publishers decreased by 9.0% (U.S. Census Bureau 2012, 2017).

4. Extensions: Asymmetric Firms

In this section, we extend our base model of *ex-ante* identical firms by sequentially considering publishers with partially differentiated contents, different fixed publication costs, and different time sensitivity.

4.1. Publishers with Partially Differentiated Contents

So far, we have assumed that firms provided the same information to isolate the effect of publication cycles as a source of differentiation. Although this is relevant to some industries where information is standard (e.g., finance, weather), other industries are prone to content differentiation. To capture that dimension, we assume the products offered by the duopoly contain a fraction β of shared content and $1-\beta$ of exclusive content. Since customers are assumed to be homogeneous, the type of exclusive content that is offered is irrelevant to our discussion. When $\beta = 1$, the model reduces to our base case. When $\beta = 0$, both firms offer only exclusive content and, thus, can act as monopolists as characterized in §3.2. In that case, they will not differentiate their publication cycles and will publish at the same frequency u^m.

In this section, we investigate the intermediate cases of partial differentiation $0 < \beta < 1$. As a we will soon show, and consistent with the extreme cases $\beta \in \{0, 1\}$, two types of outcomes emerge in equilibrium: (i) *content-based monopolistic competition* (referred to with a superscript $\beta,1$), in which both firms operate under the same publication cycle, competing *à la* Bertrand on the common content and acting as monopolists on their exclusive content and (ii) a *cycle-based differentiated duopoly* (referred to with a superscript $\beta,2$), in which firms differentiate their publication cycles, capturing value on both their common and the exclusive content.

We first extend (3) to the case with a β-fraction of common content. In that case, denoting Firm 1 as the fastest publisher, the customers' time-average utility with purchase portfolios (d,w) for any given publication cycles $\mathbf{u}$ and prices $\mathbf{r}$ is:

$$U^{\beta}(d,w;\mathbf{u},\mathbf{r}) = d\left(v(u_1,1)-r_1\right)+w\left(\beta v(u_2,1-d)+(1-\beta)v(u_2,1)-r_2\right). \tag{13}$$

Unlike (3), the customers might derive positive value from buying both products each time they are released, due to the presence of exclusive content.

Similar to the base case, we first solve the customers' consumption optimization problem (4) with $U^{\beta}(d,w;\mathbf{u},\mathbf{r})$ defined as (13); then approximate the number of consumed dailies d as continuous as in (7); solve the pricing game (9), which can now have either a pure or mixed-strategy equilibrium; and finally consider the publishing game (10). This leads to the following publishing equilibria.

Theorem 3. *When the publishers share a fraction β of the same content, the publishing game (10) has the following equilibria:*

- *When $0<\beta<\min\left\{\frac{1}{2},t_2(\alpha k)\right\}$, there exists a unique equilibrium with publication cycles $u_1^{\beta,1}=u_2^{\beta,1}=\sqrt{\frac{2k}{(1-\beta)\alpha}}$.*
- *When $\beta\geq t_1(\alpha k)$, there exists a unique pair of asymmetric equilibria. Denoting Firm 1 as the fastest publisher in equilibrium, the equilibrium cycles solve:*

$$u_1^{\beta,2}=\sqrt{\frac{2ku_2^{\beta,2}}{\alpha u_2^{\beta,2}-1}};\quad u_2^{\beta,2}=\frac{(2-\alpha u_1^{\beta,2})\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)}{2\alpha\beta(1-2\alpha k)}.$$

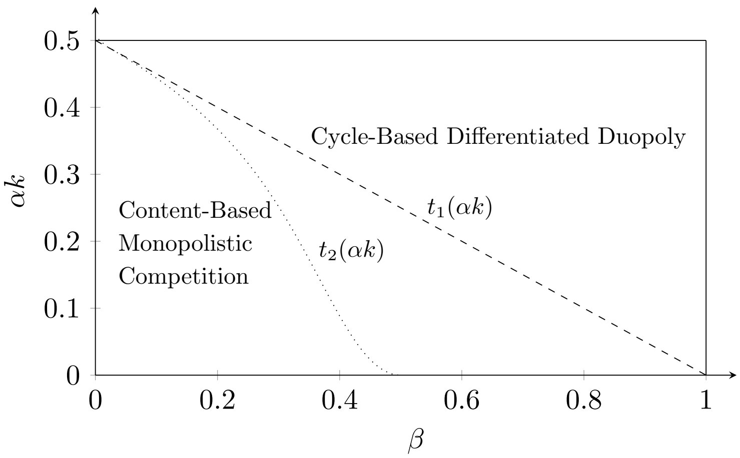
Moreover, $t_1(\alpha k)$ and $t_2(\alpha k)$ are monotonically decreasing in αk.

Theorem 3 formally establishes the existence of the two types of equilibria and characterizes the conditions under which they occur; see Figure 5 for an illustration.[6] On the

[6] The functional forms for $t_1(\alpha k)$ and $t_2(\alpha k)$ are provided in (EC.56)-(EC.57) in the electronic companion. The conditions defining the boundaries, $\beta<\min\{1/2,t_2(\alpha k)\}$ and $\beta\geq t_1(\alpha k)$, are sufficient only. Thus, the regions where the two types of equilibria emerge may be larger than as depicted in Figure 5. In particular, numerical analysis shows that when $\beta>0.92$, only the cycle-based differentiated duopoly equilibrium can sustain for all αk. In the intermediate region, i.e., when $\min\{1/2,t_2(\alpha k)\}\leq\beta<t_1(\alpha k)$, the firms' best response functions cannot be expressed in closed form, which is why we omit the equilibrium characterization.

one hand, monopolistic competition arises when the information products are sufficiently different, i.e., when β is low, or when the industry potential profit is high, i.e., when αk is low. In that equilibrium, firms engage in a head-to-head competition on their shared content by choosing the same publication cycle and the same price in a pure-strategy Nash equilibrium, as if they were monopolists endowed with only $(1-\beta)$ amount of exclusive information. On the other hand, a cycle-based differentiated duopoly emerges when the information products share a large fraction of the same content, i.e., when β is high, or when the industry potential profit is low, i.e., when αk is high. In that case, firms differentiate by choosing different cycles, as in the base case (§3.3). Hence, more monopolistic behavior can be expected in industries with diversified content and high profitability potential (e.g., independent publishing on Substack, Reddit, YouTube, or TikTok) than in those with more common content and low profitability potential (e.g., news reporting).

Figure 5 **Equilibrium outcomes when products share β fraction of common content**



Theorem 3 and Figure 5 demonstrate another implication of the digitalization of publication processes. With partial content differentiation, a decrease in the fixed publication cost k may lead to an equilibrium switch from a cycle-based differentiated duopoly to monopolistic competition. Thus, the development of independent publishing (which is a form of monopolistic competition) can arise even when customers have homogeneous tastes, complementing the traditional argument based on the economic viability of offering niche content to heterogeneous customers (Evans and Wurster 1997, Doyle 2013), because of the changes in the competitive dynamics due to digitalization.

Next, we characterize how, within each equilibrium market structure, the publication cycles change when products contain more exclusive content. We restrict our comparative statics analysis to the set of equilibria that we have characterized.

PROPOSITION 5 **(Exclusive Contents)**. *When the products contain more exclusive content (smaller β), denoting Firm 1 as the fastest publisher,*

- *If $\beta \geq t_1(\alpha k)$,*
  - *— Firm 1 publishes less frequently and Firm 2 publishes more frequently;*
  - *— Firm 1 earns less and Firm 2 earns more.*
- *If $\beta < \min\left\{\frac{1}{2}, t_2(\alpha k)\right\}$,*
  - *— Both firms publish more frequently;*
  - *— Both firms earn more profit.*

First, consider the case of a cycle-based differentiated duopoly, which happens when $\beta \geq t_1(\alpha k)$. When more exclusive content is offered, the firms feel less pressure to differentiate their publication cycles. Accordingly, Firm 1 publishes less frequently, whereas Firm 2 publishes more frequently. Hence, greater content differentiation leads to less cycle differentiation. Because the products' cycles are more comparable, the customers purchase fewer dailies, which hurts Firm 1's profit, while the customers are willing to pay more for the weekly, which benefits Firm 2. Numerically, we observe that, while Firm 2's profit only increases marginally, Firm 1's profit decreases significantly. This suggests a divergence in the quest for content differentiation: While the slowest publisher (Firm 2) seeks to provide more exclusive content, the fastest one (Firm 1) is happy to provide common content.

Next consider the case of monopolistic competition, which happens when $\beta < \min\{\frac{1}{2}, t_2(\alpha k)\}$. When more exclusive content is offered, both players benefit. Hence, in contrast to a cycle-based differentiated duopoly, there is no divergence in the quest for content differentiation, as evidenced by the rise in prominence of some bloggers and the development of fan subscription schemes on Facebook and Twitter for providers of exclusive content (Silberling 2021, Kastrenakes 2021).

4.2. Publishers with Different Fixed Costs

In this section, we relax the assumption that firms have the same fixed publication cost to explore the changes in competitive dynamics when only one publisher manages to reduce its fixed publication cost through digitalization of its publication processes. Throughout this section, we refer to the equilibrium quantities with a superscript 'k'.

Let k_i be Firm i's fixed publication cost for $i \in \{1,2\}$. Without loss of generality, assume that $k_1 \leq k_2$. Unlike in the base case, publishers are no longer *ex-ante* identical, and Firm 1 has a cost advantage. If Firm 1's cost advantage is not significant, there exists a pair of equilibria, similar to the base case (Theorem 2). However, if its cost advantage is significant, only the equilibrium where Firm 1 operates as the fastest publisher sustains.[7] The next proposition characterizes the changes in equilibrium publication cycles and profits as a result of a decrease in Firm i's fixed publication cost.

PROPOSITION 6 **(Publisher-Specific Fixed Publication Costs)**. *Suppose that $k_1 \leq k_2$ and consider the equilibrium where Firm 1 publishes faster than Firm 2. For any $i \in \{1,2\}$, when Firm i's fixed publication cost k_i decreases,*

- *u_i^k decreases, u_{-i}^k increases;*
- *π_i^k increases, π_{-i}^k decreases.*

Because the equilibrium cycles are strategic substitutes (Theorem 2), if a firm publishes more frequently (due to a decrease in its fixed publication cost), the other firm responds by publishing less frequently, and vice versa.

Proposition 6 applies to either publisher. On the one hand, when the fixed publication cost of Firm 1 (the fastest publisher) decreases, u_1^k decreases and u_2^k increases, which creates a higher level of differentiation. On the other hand, when Firm 2's fixed setup cost decreases, u_1^k increases and u_2^k decreases, which results in less differentiation. Therefore, the cycle differentiation is highest when k_1 is very small and k_2 is very large; and it is lowest when k_1 is very large and k_2 is very small.

Naturally, the firm that experiences a reduction in its fixed publication cost benefits from it, whereas the other firm is hurt because it becomes less competitive. Hence, firms have an incentive to be the first to digitalize their publication processes. This reinforces our early observation on the first-mover advantages of operating under short publication cycles.

[7] For brevity, we prove only the existence of the latter equilibrium. The other equilibrium, where Firm 2 publishes faster than Firm 1, can be shown to exist when $\alpha k_2 \leq \frac{(1+3\alpha k_1)-\sqrt{1+6\alpha k_1}}{2\alpha k_1}$.

4.3. Publishers with Different Time Sensitivity

We next extend the base model to account for publishers' different time sensitivities, similar to Caro and Martínez-de-Albéniz (2012), who consider different levels of product satiation. Although the publishers offer the same content, they may provide it in a more or less timeless way by offering additional commentary like *Les Inrockuptibles* or *CBN*.

For any $i \in \{1, 2\}$, we denote by α_i Firm i's product's time sensitivity. Without loss of generality, assume that $\alpha_1 \leq \alpha_2$, so that Firm 1 has a value advantage. If Firm 1's value advantage is not significant, there exists a pair of equilibria, similar to the base case (Theorem 2). However, if the value advantage is significant, only the equilibrium where Firm 1 operates as the fastest publisher sustains.[8] The next proposition characterizes the effect of a single publisher's change in its information decay rate.

Proposition 7 **(Publisher-Specific Information Decay Rates)**. *Suppose that $\alpha_1 \leq \alpha_2$ and consider the equilibrium where Firm 1 publishes faster than Firm 2.*

- *When Firm 1's information decay rate α_1 increases,*
  - *Both Firm 1 and Firm 2 publish more frequently;*
  - *Firm 1 makes less profit, and Firm 2 makes more profit.*
- *When Firm 2's information decay rate α_2 increases,*
  - *Firm 1 publishes more frequently;*
  - *Firm 2 publishes more frequently when $\alpha_2 < \tilde{\alpha}$, for some threshold $\tilde{\alpha} > 0$;*
  - *Firm 1 makes more profit, and Firm 2 makes less profit.*

Similar to a monopolist (Proposition 1), if Firm 1 experiences a reduction in its information decay rate α_1, it publishes less frequently. In contrast, if Firm 2 experiences a reduction in its information decay rate α_2, its response may be non-monotone as a result of two forces: namely, the economics of batching (which should lead to less frequent publications) and the need to differentiate its product, similar to the decomposition presented in (12). However, when Firm 2's product's decay rate is low, the economics of batching dominate the competition effect. This provides another explanation for the decision by *Les Inrockuptibles* and *CBN* to shift to monthly releases—besides digitalization inferred from Proposition 3—by making their content more focused on in-depth stories.

[8] For brevity, we prove only the existence of the latter equilibrium. The other equilibrium, where Firm 2 publishes faster than Firm 1, can be shown to exist when $\alpha_1 < 4/3\alpha_2$ and $\alpha_1 k \leq \frac{1+3\alpha_2 k-\sqrt{6\alpha_2 k+1}}{2\alpha_2 k}$.

A reduction in a product's information decay rate improves the quality (information timeliness) of the product. This quality improvement benefits the firm that offers this product and hurts its competitor. Because quality levels are strategic substitutes (Theorem 2), the competitor responds by publishing less frequently, therefore degrading the quality of its own product.

5. Conclusion

For many providers of information goods, choosing the right publication frequency is a strategic decision. For a monopolist, this batching decision reduces to trading off publication costs with information obsolescence. But for competitors, publication frequency can also be a source of product differentiation: Within the same industry, some publishers operate under short clocks, while others operate under longer ones.

Over the last few decades, the digitalization of publication processes has completely transformed the economics of publication, not only by reducing the publication costs and making economically viable frequent publishing, but also by intensifying the competitive dynamics, which leads firms to further differentiate their offering. As of now, there is little guidance for publishers to follow when choosing their publication frequency during these turbulent times.

In this paper, we take a first step toward understanding the publishing competitive dynamics. We consider a stylized model of *ex-ante* identical publishers competing in a duopoly. Firms simultaneously choose their publication frequency and then compete on price. Content is undifferentiated, and the market consists of a homogeneous set of customers who optimize their consumption portfolios to maximize their surplus. We characterize the equilibrium publication cycles and profits and how they are affected by changes in the fixed publication cost (either at the industry level or at only one publisher), changes in the information time sensitivity (again, either at the industry level or at only one publisher), and, when content is partially differentiated, changes in content exclusivity.

Our analysis highlights that the publication frequencies are strategic substitutes. In equilibrium, a publisher balances the desire to operate at the monopoly-level frequency and the need to differentiate itself from its competitor, resulting in an asymmetric equilibrium (Theorem 2). Because of the overlap in content, competition makes it more expensive to operate at a fast pace and, consequently, leads to less frequent publishing than in a monopoly (Proposition 1). The level of cycle differentiation is the strongest when the information is either very ephemeral or very timeless (Proposition 2). The need to differentiate product quality (measured as information timeliness) also arises when firms are asymmetric, in terms of their fixed costs of publication (Proposition 6) or information decay rates (Proposition 7).

Content differentiation reduces the need to differentiate publishing cycles, especially if the industry profit potential is high (Theorem 3). Still, publishers offering a large fraction of similar content will not uniformly seek to differentiate it, unlike in monopolistic competition: Whereas the slowest publisher benefits from exclusive content, the fastest publisher benefits from sharing common content (Proposition 5).

While a reduction in the fixed cost of publication (e.g., due to the digitalization of its processes) yields shorter publication cycles, it may also intensify the competitive dynamics, which may lead firms to differentiate their publication cycles further (Proposition 3). When publishers have access to sufficiently differentiated content, digitalization may further lead to a change in the nature of competition, from a cycle-based differentiated duopoly equilibrium toward content-based monopolistic competition—especially in industries with diversified content and high profit potential (Theorem 3).

We report the existence of first-mover advantages in choosing to publish at high frequency (Theorem 2)—and this is even more salient if the fixed publication costs decrease, not only at the firm level (Proposition 6), but also at the industry level (Proposition 3). As more distribution channels become digitalized, our analysis recommends to publishers to be proactive (i.e., be first movers), and it informs them to adapt their response as a function of the time sensitivity and exclusivity of their content.

Our stylized model focuses exclusively on the publication frequency decision, putting aside decisions that revolve around content creation, capture, and curation. However, content can be tailored to different tastes, and publishers face a positioning decision (Hotelling 1990). In particular, digitalization widened the reach of some publishers (while we assume a fixed market size), which makes the provision of niche content economically viable (Evans and Wurster 1997). Also, publishers can invest in differentiating their content (e.g., investigative journalism) (Sun and Zhu 2013) or content inflow (which we assume to be identical), along with their publication cycle. We focused on pay-per-content pricing, but, in the presence of customers with time-varying valuations, could also consider alternative pricing models (e.g., subscriptions). We also assume a stationary environment, but in practice, user base effects are important for publishers (Caro and Martínez-de-Albéniz 2020), especially if their main source of revenue comes from advertising (Gabszewicz et al. 2006). Finally, we did not model customer access costs (Bernstein and Martínez-de-Albéniz 2017); releasing large batches of content (like Netflix does) is more customer-friendly, but—consistent with our decay model—may generate less value because it does not give enough time for critics to review it and generate hype (like Disney does); see Thompson (2021). We hope that future research will incorporate these features to bring greater realism and investigate the interaction between content development and its publishing.

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Appendix: Proofs and Supplementary Results

EC.1. Monopolist

Proof of Theorem 1. The monopolist's profit function, $v(u_1,1)-\frac{k}{u_1}=\begin{cases}1-\frac{\alpha u_1}{2}-\frac{k}{u_1} & u_1\le 1/\alpha\\ \frac{1-2\alpha k}{2\alpha u_1} & u_1>1/\alpha\end{cases}$, is continuous and pseudo-concave when $\alpha k<0.5$. Hence, the first-order optimality conditions are necessary and sufficient, yielding u_1^m. Now consider a multi-product monopolist who offers $n\ge 2$ products indexed by $i\in\{1,2,\ldots,n\}$, each incurring a fixed publication cost k when released. Label the products such that $u_1\le u_2\le\cdots\le u_n$. Under the integrality assumption, $\frac{u_i}{u_1}\in\mathbb{N}$, $\forall i$. All products are synchronized every U period, where U is the least common multiple of $u_1,\ldots,u_n$. The customers can buy any subset of the products offered in any period. By (2), the average amount of information provided by Product i in a full cycle, $v(u_i,1)$, is decreasing in u_i. Hence, $q(u_i)\le q(u_1)u_i/u_1$, i.e., the customers obtain more information buying Product 1 all the time than buying any other Product i. Hence, the customers' willingness to pay for information—and therefore the firm's revenue—is bounded from above by $v(u_1,1)$. Accordingly, for any prices $\mathbf{r}=(r_1,r_2,\ldots,r_n)$ and publication cycles $\mathbf{u}=(u_1,u_2,\ldots,u_n)$, the firm's time-average profit is bounded by

$$\pi(\mathbf{u},\mathbf{r})\le v(u_1,1)-\sum_{i=1}^{n}\frac{k}{u_i}<v(u_1,1)-\frac{k}{u_1}\le\pi^m. \tag{EC.1}$$

Thus, there is no benefit from offering multiple products. □

EC.2. Symmetric Publishers

Lemma EC.1. *For any given publication cycles $\mathbf{u}$ and prices $\mathbf{r}$, suppose $\alpha u_1\le 1$ and $\alpha u_2>1$. The customers' optimal consumption portfolios solving (4) is:*

$$(d^*(\mathbf{r},\mathbf{u}),w^*(\mathbf{r},\mathbf{u}))=\begin{cases}(\delta(r_1,\mathbf{u}),1) & r_1\le 1-\frac{\alpha u_1}{2}, r_2\le\Lambda(\delta(r_1,\mathbf{u}),r_1,\mathbf{u})\\ (1,0) & r_1\le 1-\frac{\alpha u_1}{2}, r_2>\Lambda(\delta(r_1,\mathbf{u}),r_1,\mathbf{u})\\ (0,1) & r_1>1-\frac{\alpha u_1}{2}, r_2\le\frac{1}{2\alpha u_2}\\ (0,0) & r_1>1-\frac{\alpha u_1}{2}, r_2>\frac{1}{2\alpha u_2}\end{cases}, \tag{EC.2}$$

where $\Lambda(\delta,r_1,\mathbf{u}):=(1-\delta)\left(r_1+\frac{\alpha u_1}{2}-(1-\delta)\frac{\alpha u_2}{2}\right)$ *and* $\delta(r_1,\mathbf{u}):=\frac{u_1}{u_2}\cdot\arg\min_{j\in\left\{0,\ldots,\frac{u_2}{u_1}\right\}}\left|1-\frac{u_1}{2u_2}-\frac{r_1}{\alpha u_2}-j\frac{u_1}{u_2}\right|$.[9]

Proof. This is a special case of Lemma EC.8 when $\beta=1$. □

Lemma EC.2. *For any given publication cycles $\mathbf{u}$, suppose $\alpha u_1\le 1$ and $\alpha u_2>1$. The pricing game (9) does not have a pure-strategy equilibrium, but it has a mixed-strategy equilibrium yielding the following firms' profits:*

$$\pi_1(u_1;u_2)=\begin{cases}\frac{\alpha(u_1-2u_2)^2}{16u_2}-\frac{k}{u_1} & u_1\le\frac{4-2\alpha u_2}{\alpha}\\ \frac{(2-\alpha u_1)(\alpha u_2-1)}{2\alpha u_2}-\frac{k}{u_1} & u_1>\frac{4-2\alpha u_2}{\alpha}\end{cases};\quad \pi_2(u_1;u_2)=\begin{cases}\frac{\alpha(u_1+2u_2)^4}{512u_2^3}-\frac{k}{u_2} & u_2\le\frac{4-\alpha u_1}{2\alpha}\\ \frac{(\alpha u_1+2\alpha u_2-2)^2}{8\alpha^3u_2^3}-\frac{k}{u_2} & u_2>\frac{4-\alpha u_1}{2\alpha}\end{cases}. \tag{EC.3}$$

[9] In case there are multiple solutions to the inner minimization problem, we select the largest one, consistent with the tie-breaking rule that customers prefer to consume more to less.

Proof. This is a special case of Lemma EC.10 when $\beta = 1$. $\square$

LEMMA EC.3. *Suppose that $u_1^* \le u_2^*$. Then, $\alpha u_1^* \le 1$.*

Proof. This is a special case of Lemma EC.12. In particular, when $\beta = 1$, $\beta u_2 = u_2$, and Firm 1's profit in (EC.48) monotonically decreases. Thus, any strategy $u_1 > 1/\alpha$ is strictly dominated. $\square$

LEMMA EC.4. *For any given publication cycles $\mathbf{u}$ and prices $\mathbf{r}$, suppose $\alpha u_1 < \alpha u_2 \le 1$. The customers' optimal consumption portfolios solving (4) is:*

$$(d^*(\mathbf{r},\mathbf{u}), w^*(\mathbf{r},\mathbf{u})) = \begin{cases} (\delta(r_1,\mathbf{u}),1) & r_1 \le \alpha u_2 - \frac{\alpha u_1}{2}, r_2 \le \Lambda(\delta, r_1, \mathbf{u}) \\ (1,0) & r_1 \le \alpha u_2 - \frac{\alpha u_1}{2}, r_2 > \Lambda(\delta, r_1, \mathbf{u}) \\ & \textit{or } \alpha u_2 - \frac{\alpha u_1}{2} < r_1 \le 1 - \frac{\alpha u_1}{2}, r_2 > r_1 - \frac{\alpha}{2}(u_2 - u_1) \\ (0,1) & \alpha u_2 - \frac{\alpha u_1}{2} < r_1 \le 1 - \frac{\alpha u_1}{2}, r_2 \le r_1 - \frac{\alpha}{2}(u_2 - u_1) \\ & \textit{or } r_1 > 1 - \frac{\alpha u_1}{2}, r_2 \le 1 - \frac{\alpha u_2}{2} \\ (0,0) & r_1 > 1 - \frac{\alpha u_1}{2}, r_2 > 1 - \frac{\alpha u_2}{2} \end{cases}, \tag{EC.4}$$

where $\delta(r_1,\mathbf{u})$ and $\Lambda(\delta, r_1, \mathbf{u})$ are as defined in Lemma EC.1.

Proof. This is a special case of Lemma EC.9 when $\beta = 1$. $\square$

LEMMA EC.5. *For any given publication cycles $\mathbf{u}$, suppose $\alpha u_1 < \alpha u_2 \le 1$. There exists no pure-strategy equilibrium in the pricing game (9), but there exists a mixed-strategy equilibrium, yielding the following firms' profits:*

$$\pi_1(u_1;u_2) = \frac{\alpha(u_1 - 2u_2)^2}{16u_2} - \frac{k}{u_1}, \quad \pi_2(u_2;u_1) = \frac{\alpha(u_1 + 2u_2)^4}{512u_2^3} - \frac{k}{u_2}.$$

Proof. This is a special case of Lemma EC.11 when $\beta = 1$. $\square$

LEMMA EC.6. *Suppose that $u_1 \in \left[0, \frac{1}{\alpha}\right]$ and $u_2 \in [0, +\infty)$. The following set of equations*

$$u_1 = \sqrt{\frac{2ku_2}{\alpha u_2 - 1}}, \quad u_2 = \frac{\left(\sqrt{6\alpha k + 1} + 2\right)(2 - \alpha u_1)}{2\alpha(1 - 2\alpha k)}$$

has a unique pair of solutions (u_1^*, u_2^*). Moreover, $\sqrt{\frac{2k}{\alpha}} < u_1^* < 1.77\sqrt{\frac{k}{\alpha}}$ and $u_2^* > \frac{2}{\alpha}$.

Proof. This is a special case of Lemma EC.13 when $\beta = 1$. Moreover, combining the two equations, the desired u_1 is a root to the following equation

$$g(u_1) := \frac{\alpha^2 u_1^3}{2} - \frac{1}{3}\alpha u_1^2\left(\sqrt{6\alpha k + 1} + 1\right) - \alpha k u_1 + 2k = 0. \tag{EC.5}$$

We obtain that $u_1^* < 1.77\sqrt{\frac{k}{\alpha}}$ since

$$g\left(1.77\sqrt{\frac{k}{\alpha}}\right) = k\left(1.003\sqrt{\alpha k} - 1.044\sqrt{6\alpha k + 1} + 0.956\right) < 0,$$

given that $\alpha k < 0.5$ and that $g\left(1.77\sqrt{\frac{k}{\alpha}}\right)$ is a concave function of αk, maximized at $\alpha k = 0.03025$, and negative at its peak. $\square$

Proof of Theorem 2. This is a special case of Theorem 3 when $\beta=1$. In particular, when $\beta=1$, Region 2 simplifies to $0<\alpha k<0.5$, thus, there exists a unique pair of asymmetric Nash equilibria for all $0<\alpha k<0.5$. In particular, assume $u_1^*\le u_2^*$ without loss of generality. The equilibrium publication cycles solve (11), and the equilibrium profits can be obtained by plugging Firm 2's best response in (11) into (EC.3):

$$\pi_1^*=\frac{1}{6}\left(2\sqrt{6\alpha k+1}-3\alpha u_1^*+2\right)-\frac{k}{u_1^*},\quad \pi_2^*=\frac{2\left(6\alpha k\left(\sqrt{6\alpha k+1}-3\right)+\sqrt{6\alpha k+1}+1\right)}{27(2-\alpha u_1^*)}. \tag{EC.6}$$

Using (EC.6) and the fact that $u_1^*\in\left[\sqrt{\frac{2k}{\alpha}},\frac{1}{\alpha}\right]$ by Lemma EC.6, we obtain that

$$\begin{aligned}
\pi_2^*&=\frac{2\left(6\alpha k\left(\sqrt{6\alpha k+1}-3\right)+\sqrt{6\alpha k+1}+1\right)}{27(2-\alpha u_1^*)}\\
&\le\frac{2\left(6\alpha k\left(\sqrt{6\alpha k+1}-3\right)+\sqrt{6\alpha k+1}+1\right)}{27} && \text{since } \alpha u_1^*\le 1 \text{ by Lemma EC.3}\\
&=\frac{4\left(1-6\alpha k+\sqrt{6\alpha k+1}\right)}{9\sqrt{6\alpha k+1}}\cdot\frac{2\sqrt{6\alpha k+1}-6\alpha k-3+2}{6}\\
&<\frac{1}{6}\left(2\sqrt{6\alpha k+1}-6\alpha k-3+2\right) && \text{since } 0<\alpha k<0.5\\
&\le\frac{1}{6}\left(2\sqrt{6\alpha k+1}-3\alpha u_1^*-\frac{6k}{u_1^*}+2\right) && \text{since } -\frac{6\alpha k}{u_1}-3\alpha u_1 \text{ is decreasing in } u_1\in\left[\sqrt{\frac{2k}{\alpha}},\frac{1}{\alpha}\right]\\
& && \text{and } u_1^*\in\left[\sqrt{\frac{2k}{\alpha}},\frac{1}{\alpha}\right] \text{ by Lemma EC.6}\\
&=\pi_1^*. \quad \square
\end{aligned}$$

Proof of Proposition 1. By Theorem 1, $u^m=\sqrt{\frac{2k}{\alpha}}$. By Theorem 2 and Lemma EC.6, $u_2^*>u_1^*>\sqrt{\frac{2k}{\alpha}}$. $\square$

Lemma EC.7. *When $u_1^*\in(0,1/\alpha]$ and $u_2^*\in(0,\infty]$ solve (11), and $0<\alpha k<0.5$,*

$$\frac{\partial g(u_1^*)}{\partial u_1^*}<0,$$

where $g(u_1^*)$ is as defined in (EC.5).

Proof. Differentiating $g(\cdot)$ with respect to u, we have

$$\frac{\partial g(u_1^*)}{\partial u_1^*}=\frac{1}{6}\alpha\left(9\alpha {u_1^*}^2-4\left(\sqrt{6\alpha k+1}+1\right)u_1^*-6k\right),$$

which is convex with respect to u_1^*. Thus, its maximum is obtained at boundaries. Based on Lemma EC.6, $0<u_1^*<2\sqrt{\frac{k}{\alpha}}$. Therefore,

$$\begin{aligned}
\frac{\partial g(u_1^*)}{\partial u_1^*}&\le\max\left\{\frac{\partial g(u_1^*)}{\partial u_1^*}\Big|_{u_1^*=0},\frac{\partial g(u_1^*)}{\partial u_1^*}\Big|_{u_1^*=2\sqrt{\frac{k}{\alpha}}}\right\}\\
&=\max\left\{\underbrace{-6k}_{<0},\frac{1}{\alpha}\underbrace{\left[30\alpha k-8\sqrt{\alpha k}\left(1+\sqrt{1+6\alpha k}\right)\right]}_{\text{strictly convex in }\alpha k}\right\}\\
&<\max\left\{0,\max\left\{\frac{1}{\alpha}\left[30\alpha k-8\sqrt{\alpha k}\left(1+\sqrt{1+6\alpha k}\right)\right]\Big|_{\alpha k=0},\frac{1}{\alpha}\left[30\alpha k-8\sqrt{\alpha k}\left(1+\sqrt{1+6\alpha k}\right)\right]\Big|_{\alpha k=0.5}\right\}\right\}\\
&=\max\left\{0,\max\left\{0,-\frac{1.97}{\alpha}\right\}\right\}\\
&=0. \quad \square
\end{aligned}$$

Proof of Proposition 2. • Dividing Firm 2's best response (11) by u_1^*, we obtain

$$\frac{u_2^*}{u_1^*} = \frac{\left(\sqrt{6\alpha k+1}+2\right)(2-\alpha u_1^*)}{2\alpha u_1^*(1-2\alpha k)}.$$

Taking the total derivative with respect to α yields:

$$\begin{aligned}
\mathrm{d}\frac{u_2^*}{u_1^*}\Big/\mathrm{d}\alpha &= \frac{\partial \frac{u_2^*}{u_1^*}}{\partial \alpha u_1^*}\frac{\partial(\alpha u_1^*(\alpha))}{\partial \alpha} + \frac{\partial \frac{u_2^*}{u_1^*}}{\partial \alpha} \\
&= -\frac{\sqrt{6\alpha k+1}+2}{(\alpha u_1^*)^2(1-2\alpha k)}\frac{\partial(\alpha u_1^*(\alpha))}{\partial \alpha} + \frac{k\left(6\alpha k+4\sqrt{6\alpha k+1}+5\right)(2-\alpha u_1^*)}{2\alpha u_1^*(1-2\alpha k)^2\sqrt{6\alpha k+1}}.
\end{aligned}$$

By Lemma EC.6 and the squeeze theorem, $\lim_{\alpha\to 0}\alpha u_1^*(k)=0$ since $\sqrt{2\alpha k}<\alpha u_1^*<1.77\sqrt{\alpha k}$ and $\lim_{\alpha\to 0}\sqrt{2\alpha k}=\lim_{\alpha\to 0}1.77\sqrt{\alpha k}=0$. Moreover, $\lim_{\alpha\to 0}\frac{\partial \alpha u_1^*}{\partial \alpha}>0$ as we show next. Replacing αu_1 by x in (EC.5), $g(\cdot)$ can be expressed as a function of x. Therefore, by the implicit function theorem,

$$\begin{aligned}
\lim_{\alpha\to 0}\frac{\partial x^*}{\partial \alpha} &= \lim_{\alpha\to 0} -\frac{\frac{\partial g(x^*)}{\partial \alpha}}{\frac{\partial g(x^*)}{\partial x}} \\
&= \lim_{\alpha\to 0}\frac{x^{*2}\left(\sqrt{6\alpha k+1}+\frac{1}{\sqrt{6\alpha k+1}}-3x^*+2\right)}{\alpha\left(6\alpha k+4x^*\left(\sqrt{6\alpha k+1}+1\right)-9x^{*2}\right)} \\
&= \lim_{\alpha\to 0}\frac{x^{*2}}{\alpha}\cdot\lim_{\alpha\to 0}\frac{\sqrt{6\alpha k+1}+\frac{1}{\sqrt{6\alpha k+1}}-3x^*+2}{6\alpha k+4x^*\left(\sqrt{6\alpha k+1}+1\right)-9x^{*2}} \\
&= +\infty \text{ by Lemma EC.6 and the squeeze theorem.}
\end{aligned}$$

Hence, $\lim_{\alpha\to 0}\mathrm{d}\frac{u_2^*}{u_1^*}\Big/\mathrm{d}\alpha=-\infty$. Therefore, due to the continuity of elementary functions, there exists $\hat{\alpha}>0$ such that when $0<\alpha<\hat{\alpha}$, $\mathrm{d}\frac{u_2^*}{u_1^*}\Big/\mathrm{d}\alpha<0$. On the other hand, when $\alpha>\overline{\alpha}$, u_2^* is increasing in α, whereas u_1^* is decreasing in α, so u_2^*/u_1^* is increasing in α.

• Combining the two best responses (11), we obtain (EC.5). First, we establish that

$$\frac{\partial g(u_1^*)}{\partial \alpha} = \frac{u_1^*}{3\sqrt{6\alpha k+1}}\left(3\alpha u_1^{*2}\sqrt{6\alpha k+1}-u_1^*\left(9\alpha k+\sqrt{6\alpha k+1}+1\right)-3k\sqrt{6\alpha k+1}\right)<0. \tag{EC.7}$$

Let

$$l(u_1^*) := 3\alpha u_1^{*2}\sqrt{6\alpha k+1}-u_1^*\left(9\alpha k+\sqrt{6\alpha k+1}+1\right)-3k\sqrt{6\alpha k+1}. \tag{EC.8}$$

Since $l(u_1^*)$ is strictly convex, applying Lemma EC.6,

$$\begin{aligned}
l(u_1^*) &< \max\left\{l(0), l\left(1.77\sqrt{\frac{k}{\alpha}}\right)\right\} \\
&= \max\left\{-3k\sqrt{6\alpha k+1}, l\left(1.77\sqrt{\frac{k}{\alpha}}\right)\right\} \\
&\le \max\left\{0, \frac{1}{\alpha}\left(6.40\alpha k\sqrt{6\alpha k+1}-1.77\sqrt{\alpha k}\left(9\alpha k+\sqrt{6\alpha k+1}+1\right)\right)\right\} \\
&< \max\left\{0, \frac{1}{\alpha}\left(6.40\alpha k\sqrt{6\times 0.5+1}-1.77\sqrt{\alpha k}\left(9\alpha k+\sqrt{6\times 0+1}+1\right)\right)\right\} \\
&\le 0.
\end{aligned}$$

Moreover, $\frac{\partial g(u_1^*)}{\partial u_1^*}<0$ by Lemma EC.7. Accordingly, by the implicit function theorem,

$$\frac{\partial u_1^*}{\partial \alpha} = -\frac{\frac{\partial g(u_1^*)}{\partial \alpha}}{\frac{\partial g(u_1^*)}{\partial u_1^*}}<0.$$

- Plugging Firm 2's best response (11) into (EC.5), we obtain that Firm 2's equilibrium publication cycle u_2^* must be a root to:

$$h(u_2^*) := 9\alpha^2 k u_2^* - 2(\alpha u_2^* - 1)\left(\alpha u_2^*\left(\sqrt{6\alpha k+1}-2\right)+3\right)^2 = 0. \quad \text{(EC.9)}$$

Differentiating it with respect to α and replacing u_2^* by applying Firm 2's best response in (11) yields:

$$\frac{\partial h(u_2^*)}{\partial \alpha} = \frac{3\left(\sqrt{6\alpha k+1}+2\right)(2-\alpha u_1^*)m(u_1^*)}{4(1-2\alpha k)^2\sqrt{6\alpha k+1}},$$

where

$$\begin{aligned} m(u_1^*) =& 12k(1-2\alpha k)\sqrt{6\alpha k+1} + 4u_1^*\left(\sqrt{6\alpha k+1}+\alpha k\left(-18\alpha k - 5\sqrt{6\alpha k+1}+1\right)+1\right) \\ &+ 3\alpha {u_1^*}^2\left(4k\left(\alpha+2\alpha\sqrt{6\alpha k+1}\right)-3\sqrt{6\alpha k+1}\right) \end{aligned}$$

One can easily check that $\lim_{\alpha\to 0} m(u_1^*) = 12k + 8u_1^* > 0$ and $\lim_{\alpha\to 0.5/k} m(u_1^*) = \lim_{\alpha\to 0.5/k} \frac{3{u_1^*}^2}{k^2} - 12u_1^*/k < 0$. Since $m(u_1^*)$ is continuous, there exist thresholds $0 < \underline{\alpha} \le \overline{\alpha} < \frac{0.5}{k}$ such that when $\alpha < \underline{\alpha}$, $m(u_1^*)$ is positive; and when $\alpha > \overline{\alpha}$, $m(u_1^*)$ is negative.

By the implicit function theorem, $\frac{\partial u_2^*}{\partial \alpha} = -\frac{\frac{\partial h(u_2^*)}{\partial \alpha}}{\frac{\partial h(u_2^*)}{\partial u_2^*}}$. Since $\frac{\partial g(u_1^*)}{\partial u_1^*} < 0$ by (EC.7) and $\frac{\partial u_1^*}{\partial u_2^*} < 0$ by (11),

$$\frac{\partial h(u_2^*)}{\partial u_2^*} = \frac{\partial g(u_1^*)}{\partial u_1^*}\cdot\frac{\partial u_1^*}{\partial u_2^*} > 0. \quad \text{(EC.10)}$$

Thus, when $\alpha < \underline{\alpha}$, u_2^* is decreasing in α; and when $\alpha > \overline{\alpha}$, u_2^* is increasing in α. □

Proof of Proposition 3.

- Combining the two best responses (11), we obtain (EC.5). On the one hand, $\frac{\partial g(u_1^*)}{\partial k}$, equal to $-\frac{\alpha^2 {u_1^*}^2}{\sqrt{6\alpha k+1}} - \alpha u_1^* + 2$, is decreasing in u_1^* since

$$\frac{\partial \frac{\partial g(u_1^*)}{\partial k}}{\partial u_1^*} = -\frac{2\alpha^2 u_1^*}{\sqrt{6\alpha k+1}} - \alpha < 0.$$

Since $u_1^* \le 1/\alpha$ by Lemma EC.3 and

$$\left.\frac{\partial g(u_1^*)}{\partial k}\right|_{u_1^*=\frac{1}{\alpha}} = 1 - \frac{1}{\sqrt{1+6\alpha k}} > 0,$$

we obtain

$$\frac{\partial g(u_1^*)}{\partial k} \ge \left.\frac{\partial g(u_1^*)}{\partial k}\right|_{u_1^*=\frac{1}{\alpha}} > 0. \quad \text{(EC.11)}$$

On the other hand, $\frac{\partial g(u_1^*)}{\partial u_1^*} < 0$ by Lemma EC.7. By the implicit function theorem, using (EC.11), we obtain

$$\frac{\partial u_1^*}{\partial k} = -\frac{\frac{\partial g(u_1^*)}{\partial k}}{\frac{\partial g(u_1^*)}{\partial u_1^*}} > 0.$$

- Combining the two best responses (11), we obtain (EC.9). By the implicit function theorem,

$$\frac{\partial u_2^*}{\partial k} = -\frac{\frac{\partial h(u_2^*)}{\partial k}}{\frac{\partial h(u_2^*)}{\partial u_2^*}},$$

with

$$\frac{\partial h(u_2^*)}{\partial k} = \frac{u_1^*(\sqrt{6\alpha k+1}+4\alpha k) - 4k\left(\sqrt{6\alpha k+1}+2\right)}{{u_1^*}^3(1-2\alpha k)\sqrt{6\alpha k+1}}.$$

Let

$$s(u_1^*(k)) := \alpha u_1^*(k)(\sqrt{6\alpha k+1}+4\alpha k) - 4\alpha k\left(\sqrt{6\alpha k+1}+2\right). \tag{EC.12}$$

Accordingly, since $\frac{\partial h(u_2^*)}{\partial u_2^*} > 0$ by (EC.10), $\frac{\partial u_2^*}{\partial k} > 0$ if and only if $s(u_1^*) < 0$. By the squeeze theorem, $\lim_{k\to 0} u_1^*(k) = 0$ since $\sqrt{\frac{2k}{\alpha}} < u_1^* < 1.77\sqrt{\frac{k}{\alpha}}$ by Lemma EC.6 and $\lim_{k\to 0}\sqrt{\frac{2k}{\alpha}} = \lim_{k\to 0} 1.77\sqrt{\frac{k}{\alpha}} = 0$. Hence, $\lim_{k\to 0} s(u_1^*(k)) = 0$ and $\lim_{k\to \frac{0.5}{\alpha}} s(u_1^*(k)) = 4\alpha u_1^* - 8 < 0$. Additionally, applying the total derivatives,

$$\lim_{k\to 0}\frac{\mathrm{d}s}{\mathrm{d}k} = \lim_{k\to 0}\frac{\partial s}{\partial k} + \frac{\partial s}{\partial u_1^*}\frac{\partial u_1^*}{\partial k} = \lim_{k\to 0}\alpha(7\alpha u_1^* - 12) + \alpha\frac{\partial u_1^*}{\partial k} = +\infty$$

since by (EC.5),

$$\lim_{k\to 0}\frac{\partial u_1^*}{\partial k} = \lim_{k\to 0} -\frac{\frac{\partial g(u_1^*)}{\partial k}}{\frac{\partial g(u_1^*)}{\partial u_1^*}} = \lim_{k\to 0}\frac{6(1-\alpha u_1^*)(\alpha u_1^*+2)}{\alpha u_1^*(8-9\alpha u_1^*)} = +\infty.$$

Due to the continuity of elementary functions, there exist $0 < \underline{k} \le \overline{k} < \frac{0.5}{\alpha}$ such that when $0 < k < \underline{k}$, then $s(u_1^*) > 0$ and u_2^* decreases with respect to k; and when $\overline{k} < k < \frac{0.5}{\alpha}$, $s(u_1^*) < 0$ and u_2^* decreases with respect to k. $\square$

Proof of Proposition 4.

- Differentiating Firm 1's profit (EC.6) with respect to αu_1^* and αk, we obtain

$$\frac{\partial \pi_1^*}{\partial \alpha u_1^*} = \frac{\alpha k}{(\alpha u_1^*)^2} - \frac{1}{2} < 0 \text{ (by Lemma EC.6)}, \quad \frac{\partial \pi_1^*}{\partial \alpha k} = \frac{1}{\sqrt{6\alpha k+1}} - \frac{1}{\alpha u_1^*} < 0 \text{ (by Lemma EC.3)}.$$

Combining the two best responses (11), we obtain (EC.5). By replacing αu_1 by x in (EC.5), $g(\cdot)$ can be expressed as a function of x. Denoting αu_1^* by x^* and applying the implicit function theorem, we obtain

$$\frac{\partial x^*}{\partial \alpha k} = -\frac{\frac{\partial g(x^*)}{\partial \alpha k}}{\frac{\partial g(x^*)}{\partial x^*}} = -\frac{2 - \alpha u_1^*\left(\frac{\alpha u_1^*}{\sqrt{6\alpha k+1}}+1\right)}{\frac{1}{6}\left(\alpha u_1^*\left(9\alpha u_1^* - 4\left(\sqrt{6\alpha k+1}+1\right)\right) - 6\alpha k\right)} > 0 \tag{EC.13}$$

since

$$\begin{aligned}
&\alpha u_1^*\left(9\alpha u_1^* - 4\left(\sqrt{6\alpha k+1}+1\right)\right) - 6\alpha k \\
<&9\times 1.77^2\alpha k - 4\sqrt{2\alpha k}\left(\sqrt{6\alpha k+1}+1\right) - 6\alpha k \text{ by Lemma EC.6} \\
\le&\underbrace{9\times 1.77^2\alpha k - 4\sqrt{2\alpha k}\,(1+2\alpha k+1) - 6\alpha k}_{\text{maximized at }\alpha k=0} \quad \text{since } \sqrt{6\alpha k+1} \ge 1+2\alpha k \\
\le&0.
\end{aligned}$$

By the chain rule of total derivatives,

$$\frac{\mathrm{d}\pi_1^*}{\mathrm{d}\alpha k} = \frac{\partial \pi_1^*}{\partial \alpha u_1^*}\frac{\partial \alpha u_1^*}{\partial \alpha k} + \frac{\partial \pi_1^*}{\partial \alpha k} < 0.$$

- Similarly, consider Firm 2's profit (EC.6). Since $\lim_{\alpha k\to 0}\frac{\partial \alpha u_1^*}{\partial \alpha k} = +\infty$ by (EC.13), we obtain, by taking the total derivative,

$$\lim_{\alpha k\to 0}\frac{\mathrm{d}\pi_2^*}{\mathrm{d}\alpha k} = \lim_{\alpha k\to 0}\frac{\partial \pi_2^*}{\partial \alpha k} + \frac{\partial \pi_2^*}{\partial \alpha u_1^*}\frac{\partial \alpha u_1^*}{\partial \alpha k} = \frac{2}{3(\alpha u_1^*-2)} + \frac{4}{27(\alpha u_1^*-2)^2}\lim_{\alpha k\to 0}\frac{\partial \alpha u_1^*}{\partial \alpha k} = +\infty.$$

Additionally,

$$\lim_{\alpha k\to 0}\pi_2^* = \lim_{\alpha k\to 0}\frac{2\left(6\alpha k\left(\sqrt{6\alpha k+1}-3\right)+\sqrt{6\alpha k+1}+1\right)}{27(2-\alpha u_1^*)} = \frac{2}{27}; \quad \lim_{\alpha k\to 0.5}\pi_2^* = 0.$$

Therefore, π_2^* is non-monotonic in αk. Therefore, there exists a threshold $\bar{c} > 0$ such that π_2^* increases in αk when $\alpha k < \bar{c}$. $\square$

EC.3. Publishers with Partially Differentiated Content

LEMMA EC.8. *When the publishers share a fraction β of the same content, for any given publication cycles $\mathbf{u}$ and prices $\mathbf{r}$, suppose that $\alpha u_1 \le 1 < \alpha u_2$. The customers' optimal consumption portfolios solving (4) with utility (13) is:*

$$(d^\beta(\mathbf{r},\mathbf{u}), w^\beta(\mathbf{r},\mathbf{u})) = \begin{cases} (0,0) & r_1 > 1-\frac{\alpha u_1}{2}, r_2 > \frac{1}{2\alpha u_2} \\ (0,1) & r_1 > 1-\frac{\alpha u_1}{2}, r_2 \le \frac{1}{2\alpha u_2} \\ (1,0) & r_1 \le 1-\frac{\alpha u_1}{2}, r_2 > \Lambda^\beta(\delta^\beta(r_1,\mathbf{u}), r_1, \mathbf{u}) \\ (\delta^\beta(r_1,\mathbf{u}),1) & (1-\beta)(1-\frac{\alpha u_1}{2}) < r_1 \le 1-\frac{\alpha u_1}{2}, r_2 \le \Lambda^\beta(\delta^\beta(r_1,\mathbf{u}), r_1,\mathbf{u}) \\ (1,1) & r_1 \le (1-\beta)(1-\frac{\alpha u_1}{2}), r_2 \le (1-\beta)\frac{1}{2\alpha u_2} \end{cases}, \tag{EC.14}$$

where $\delta^\beta(r_1,\mathbf{u}) := \frac{u_1}{u_2} \cdot \arg\min_{j\in\left\{0,\dots,\frac{u_2}{u_1}\right\}} \left|1-\frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2} - j\frac{u_1}{u_2}\right|$ *and* $\Lambda^\beta(\delta, r_1, \mathbf{u}) := (1-\delta)\left(r_1 + \frac{\alpha u_1}{2} - (1-\beta) - \frac{1}{2}\alpha\beta(1-\delta)u_2\right) + (1-\beta)v_2(u_2,1)$.

Proof. When $\alpha u_1 \le 1 < \alpha u_2$, the customer utility (13) can be expanded as:

$$U^\beta(d,w;\mathbf{u},\mathbf{r}) = \begin{cases} d\left((1-\frac{\alpha u_1}{2}) - r_1\right) + w\left(\frac{1}{2\alpha u_2} - r_2\right) & d < 1-\frac{1}{\alpha u_2} \\ d\left((1-\frac{\alpha u_1}{2}) - r_1\right) + w\left(\beta(1-d)\left(1-(1-d)\frac{\alpha u_2}{2}\right) + (1-\beta)\frac{1}{2\alpha u_2} - r_2\right) & d \ge 1-\frac{1}{\alpha u_2} \end{cases}. \tag{EC.15}$$

For simplicity, we omit the conditioning on $\mathbf{u}$ and $\mathbf{r}$. First, we consider the inner maximization problem in (4): $d^\beta(w) = \arg\max_{d\in D} U^\beta(d,w)$. We consider two cases, depending on whether $w=0$ or $w=1$. By (EC.15),

- If $w=0$,

$$U^\beta(d,0) = d\left(\left(1-\frac{\alpha u_1}{2}\right) - r_1\right) \Rightarrow d^\beta(w) = \begin{cases} 1 & r_1 \le 1-\frac{\alpha u_1}{2} \\ 0 & r_1 > 1-\frac{\alpha u_1}{2} \end{cases}.$$

- If $w=1$,

$$U^\beta(d,1) = \begin{cases} d\left((1-\frac{\alpha u_1}{2}) - r_1\right) + \left(\frac{1}{2\alpha u_2} - r_2\right) & d < 1-\frac{1}{\alpha u_2} \\ d\left((1-\frac{\alpha u_1}{2}) - r_1\right) + \left(\beta(1-d)\left(1-(1-d)\frac{\alpha u_2}{2}\right) + (1-\beta)\frac{1}{2\alpha u_2} - r_2\right) & d \ge 1-\frac{1}{\alpha u_2} \end{cases}.$$

  —If $r_1 > 1-\frac{\alpha u_1}{2}$, $U^\beta(d,1)$ is monotonically decreasing. Thus, $d^\beta(w)=0$.

  —If $r_1 < 1-\frac{\alpha u_1}{2}$, $U^\beta(d,1)$ is continuously differentiable and increasing for $d < 1-\frac{1}{\alpha u_2}$. Given that $U^\beta(d,1)$ is quadratic when $d \ge 1-\frac{1}{\alpha u_2}$, it is sufficient to take the first-order optimality condition to find the maximum. Accordingly, $d^\beta(w) = \delta^\beta$.

  —If $r_1 = 1-\frac{\alpha u_1}{2}$, $U^\beta(d,1) = \frac{1}{2\alpha u_2} - r_2$ for $d < 1-\frac{1}{\alpha u_2}$ and decreases for $d \ge 1-\frac{1}{\alpha u_2}$. As customers prefer buying when indifferent, $d^\beta(w) = \delta^\beta$, which merges with the previous case.

In summary,

$$d^\beta(w) = \begin{cases} 1 & r_1 \le 1-\frac{\alpha u_1}{2}, w=0 \\ \delta^\beta & r_1 \le 1-\frac{\alpha u_1}{2}, w=1 \\ 0 & r_1 > 1-\frac{\alpha u_1}{2} \end{cases}. \tag{EC.16}$$

Next, we consider the outer maximization problem in (4) with (EC.16): $w^\beta = \arg\max_{w\in\{0,1\}} U(d^\beta(w), w)$. We consider three cases, depending on the value of r_1.

- If $r_1 > 1-\frac{\alpha u_1}{2}$: $d^\beta(w) = 0, \forall w$.

$$U^\beta(0,w) = w\left(\frac{1}{2\alpha u_2} - r_2\right) \Rightarrow w^\beta = \begin{cases} 0 & r_2 > \frac{1}{2\alpha u_2} \\ 1 & r_2 \le \frac{1}{2\alpha u_2} \end{cases}. \tag{EC.17}$$

- If $(1-\beta)(1-\frac{\alpha u_1}{2}) < r_1 \le 1-\frac{\alpha u_1}{2}$: $d^\beta(w) = \begin{cases} 1 & w=0 \\ \delta^\beta & w=1 \end{cases}$. Note that $U^\beta(1,0) \le U^\beta(\delta^\beta,1)$ if and only if $r_2 \le \Lambda^\beta(\delta^\beta)$, after replacing $v_2(u_2,1)$ with $\frac{1}{2\alpha u_2}$ by (2) in $\Lambda^\beta(\delta^\beta)$ given that $\alpha u_2 > 1$. Since $\Lambda^\beta(\delta^\beta) \le \frac{(2r_1+\alpha u_1-2)^2+4\beta(2r_1+\alpha u_1-1)}{8\alpha\beta u_2}$ and $\frac{(2r_1+\alpha u_1-2)^2+4\beta(2r_1+\alpha u_1-1)}{8\alpha\beta u_2} \le \frac{1}{2\alpha u_2}$ if and only if $1-\beta-\frac{\alpha u_1}{2} \le r_1 \le 1-\frac{\alpha u_1}{2}$, we obtain that when $1-\beta-\frac{\alpha u_1}{2} \le r_1 \le 1-\frac{\alpha u_1}{2}$, $r_2 \le \Lambda^\beta(\delta^\beta) \Rightarrow r_2 \le \frac{1}{2\alpha u_2}$. Therefore,

$$w^\beta = \begin{cases} 1 & r_2 \le \Lambda^\beta(\delta^\beta) \\ 0 & r_2 > \Lambda^\beta(\delta^\beta) \end{cases}. \tag{EC.18}$$

- If $r_1 \le (1-\beta)(1-\frac{\alpha u_1}{2})$, then $U^\beta(1,1) \ge U^\beta\left(1-\frac{u_1}{u_2},1\right)$, i.e., $d^\beta = 1$ and $\Lambda^\beta(\delta^\beta) = (1-\beta)\frac{1}{2\alpha u_2}$. Hence, (EC.18) simplifies to $w^\beta = \begin{cases} 1 & r_2 \le (1-\beta)\frac{1}{2\alpha u_2} \\ 0 & r_2 > (1-\beta)\frac{1}{2\alpha u_2} \end{cases}$.

Combining (EC.16), (EC.17), (EC.18) and this last case yields (EC.14). □

LEMMA EC.9. *When the publishers share a fraction β of the same content, for any given publication cycles $\mathbf{u}$ and prices $\mathbf{r}$, suppose that $\alpha u_1 < \alpha u_2 \le 1$. The customers' optimal consumption portfolios solving (4) with utility (13) is:*

$$(d^\beta(\mathbf{r},\mathbf{u}), w^\beta(\mathbf{r},\mathbf{u})) = \begin{cases} (0,0) & r_1 > 1-\frac{\alpha u_1}{2}, r_2 > 1-\frac{\alpha u_2}{2} \\ (0,1) & r_1 > 1-\frac{\alpha u_1}{2}, r_2 \le 1-\frac{\alpha u_2}{2} \\ & \textit{or } 1-\frac{\alpha u_1}{2} \ge r_1 \ge 1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2}, r_2 \le r_1 - \frac{\alpha}{2}(u_2-u_1) \\ (1,0) & 1-\frac{\alpha u_1}{2} \ge r_1 \ge 1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2}, r_2 > r_1-\frac{\alpha}{2}(u_2-u_1) \\ & \textit{or } (1-\beta)(1-\frac{\alpha u_1}{2}) < r_1 < 1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2}, r_2 > \Lambda^\beta(\delta^\beta(r_1,\mathbf{u}),r_1,\mathbf{u}) \\ & \textit{or } r_1 \le (1-\beta)(1-\frac{\alpha u_1}{2}), r_2 > (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) \\ (\delta^\beta(r_1,\mathbf{u}),1) & (1-\beta)(1-\frac{\alpha u_1}{2}) < r_1 < 1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2}, r_2 \le \Lambda^\beta(\delta^\beta(r_1,\mathbf{u}),r_1,\mathbf{u}) \\ (1,1) & r_1 \le (1-\beta)(1-\frac{\alpha u_1}{2}), r_2 \le (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) \end{cases}, \tag{EC.19}$$

where $\delta^\beta(r_1,\mathbf{u})$ and $\Lambda^\beta(\delta,r_1,\mathbf{u})$ are as defined in Lemma EC.8.

Proof. When $\alpha u_1 \le 1$ and $\alpha u_2 \le 1$, the customer utility (13) can be expanded as:

$$U^\beta(d,w;\mathbf{u},\mathbf{r}) = d\left(\left(1-\frac{\alpha u_1}{2}\right)-r_1\right) + w\left(\beta(1-d)\left(1-(1-d)\frac{\alpha u_2}{2}\right)+(1-\beta)\left(1-\frac{\alpha u_2}{2}\right)-r_2\right). \tag{EC.20}$$

For simplicity, we omit the conditioning on $\mathbf{u}$ and $\mathbf{r}$.

First, we consider the inner maximization problem in (4), leading to (EC.16). Second, we solve the outer maximization problem for different values of r_1.

- If $r_1 > 1-\frac{\alpha u_1}{2}$: $d^\beta(w)=0, \forall w$. Hence,

$$U^\beta(0,w) = w\left(\left(1-\frac{\alpha u_2}{2}\right)-r_2\right) \Rightarrow w^\beta = \begin{cases} 0 & r_2 > 1-\frac{\alpha u_2}{2} \\ 1 & r_2 \le 1-\frac{\alpha u_2}{2} \end{cases}. \tag{EC.21}$$

- If $1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2} \le r_1 \le 1-\frac{\alpha u_1}{2}$: $d^\beta(w) = \begin{cases} 1 & w=0 \\ 0 & w=1 \end{cases}$. Comparing $U^\beta(1,0)$ with $U^\beta(0,1)$, we have

$$w^\beta = \begin{cases} 1 & r_2 \le r_1 - \frac{\alpha}{2}(u_2-u_1) \\ 0 & r_2 > r_1-\frac{\alpha}{2}(u_2-u_1) \end{cases}. \tag{EC.22}$$

- If $(1-\beta)(1-\frac{\alpha u_1}{2}) < r_1 < 1+\beta(\alpha u_2 - 1) - \frac{\alpha u_1}{2}$: $d^\beta(w) = \begin{cases} 1 & w=0 \\ \delta^\beta & w=1 \end{cases}$. Note that $U^\beta(1,0) \le U^\beta(\delta^\beta,1)$ if and only if $r_2 \le \Lambda^\beta(\delta^\beta)$, after replacing $v_2(u_2,1)$ with $1-\frac{\alpha u_2}{2}$ by (2) in $\Lambda^\beta(\delta^\beta)$ given that $\alpha u_2 \le 1$. Moreover, as in Lemma EC.8, when $(1-\beta)(1-\frac{\alpha u_1}{2}) \le r_1 \le 1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2}$, $r_2 \le \Lambda^\beta(\delta^\beta) \Rightarrow r_2 \le 1-\frac{\alpha u_2}{2}$. Therefore,
$$w^\beta = \begin{cases} 1 & r_2 \le \Lambda^\beta(\delta^\beta) \\ 0 & r_2 > \Lambda^\beta(\delta^\beta) \end{cases}. \tag{EC.23}$$
- If $r_1 \le (1-\beta)(1-\frac{\alpha u_1}{2})$, $d^\beta = 1$ and $\Lambda^\beta(d^\beta) = (1-\beta)\left(1-\frac{\alpha u_2}{2}\right)$. Hence, (EC.23) simplifies to
$$w^\beta = \begin{cases} 1 & r_2 \le (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) \\ 0 & r_2 > (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) \end{cases}.$$

Combining (EC.21), (EC.22), (EC.23), and this last case yields (EC.19). $\square$

Lemma EC.10. *When the publishers share a fraction β of the same content, for any given publication cycles $\mathbf{u}$, suppose that $\alpha u_1 \le 1 < \alpha u_2$. The pricing game (9) does not have a pure-strategy Nash equilibrium, but it has a mixed-strategy Nash equilibrium yielding the following profits:*

$$\pi_1^\beta(u_1;u_2) = \begin{cases} (1-\beta)(1-\frac{\alpha u_1}{2}) - \frac{k}{u_1} & u_2 \le \Phi(u_1) \\ \frac{(\alpha u_1 + \beta(2-2\alpha u_2)-2)^2}{16\alpha\beta u_2} - \frac{k}{u_1} & \Phi(u_1) < u_2 \le \max\{\Phi(u_1), \frac{2\beta-\alpha u_1+2}{2\alpha\beta}\} \\ \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2} - \frac{k}{u_1} & u_2 > \max\{\Phi(u_1), \frac{2\beta-\alpha u_1+2}{2\alpha\beta}\} \end{cases}, \tag{EC.24}$$

$$\pi_2^\beta(u_2;u_1) = \begin{cases} (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2} & u_2 \le \Phi(u_1) \\ \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2} & \Phi(u_1) < u_2 \le \max\{\Phi(u_1), \frac{2\beta-\alpha u_1+2}{2\alpha\beta}\} \\ \frac{(\alpha u_1 + 2\alpha\beta u_2 - 2)^2}{8\alpha^3\beta u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2} & u_2 > \max\{\Phi(u_1), \frac{2\beta-\alpha u_1+2}{2\alpha\beta}\} \end{cases}, \tag{EC.25}$$

in which
$$\Phi(u_1) := \frac{2-2\beta-\alpha u_1}{2\alpha\beta} + u_1 + \sqrt{\frac{(1-\beta)(2-\alpha u_1)u_1}{\alpha\beta}}. \tag{EC.26}$$

Proof. Throughout the proof, we omit the arguments $\mathbf{u}$ in the functions since the cycles are fixed in the pricing game. Since $\alpha u_1 \le 1 < \alpha u_2$, the customers' optimal consumption portfolios are given by Lemma EC.8. Using (EC.15), we obtain that $\frac{\partial U^\beta(0,1)}{\partial d} \ge 0 \Leftrightarrow r_1 \le 1 - \frac{\alpha u_1}{2} + \beta(\alpha u_2 - 1)$ and $U^\beta(1,1) < U^\beta\left(1-\frac{u_1}{u_2}, 1\right)$ if and only if $r_1 > (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$. Thus, (7) consists in approximating $\delta^\beta(r_1)$ in (EC.14) with

$$\tilde{\delta}^\beta(r_1) := \frac{u_1}{u_2} \cdot \underset{j\in\left[0,\frac{u_2}{u_1}\right]}{\arg\min} \left| \frac{\partial U^\beta\left(j\frac{u_1}{u_2}, 1\right)}{\partial d} \right| = 1 - \frac{u_1}{2\beta u_2} - \frac{r_1}{\alpha\beta u_2} + \frac{1-\beta}{\alpha\beta u_2}. \tag{EC.27}$$

Correspondingly,
$$\Lambda^\beta(\tilde{\delta}^\beta(r_1), r_1) = \frac{(2\beta + 2r_1 + \alpha u_1 - 2)^2}{8\alpha\beta u_2} + (1-\beta)v(u_2,1). \tag{EC.28}$$

The relevant domain of $\Lambda^\beta(\tilde{\delta}^\beta(r_1), r_1)$ in (EC.14) is $\left((1-\beta)\left(1-\frac{\alpha u_1}{2}\right), 1-\frac{\alpha u_1}{2}\right]$. Because $\Lambda^\beta(\tilde{\delta}(r_1), r_1)$ is increasing on its domain, it is invertible. Therefore, define

$$(\Lambda^\beta)^{-1}(r_2) := \begin{cases} 1-\beta-\frac{\alpha u_1}{2} + \sqrt{2\alpha\beta u_2\left(r_2 - (1-\beta)v(u_2,1)\right)} & (1-\beta)v(u_2,1) + \frac{\alpha\beta u_1^2}{8u_2} < r_2 \le v(u_2,1) \\ 0 & r_2 \le (1-\beta)v(u_2,1) + \frac{\alpha\beta u_1^2}{8u_2} \end{cases}. \tag{EC.29}$$

Here, because $\alpha u_2 > 1$, $v(u_2, 1) = \frac{1}{2\alpha u_2}$ by (2). First, we characterize the firms' best response functions and then characterize the equilibrium.

Firm 1. Plugging (EC.14) with $\tilde{\delta}^\beta(r_1)$ into (8), we express Firm 1's profit in two cases, depending on the value of r_2:

- If $r_2 > \frac{1}{2\alpha u_2}$: Since, when $1-\beta-\frac{\alpha u_1}{2} \le r_1 \le 1-\frac{\alpha u_1}{2}$, $r_2 > \frac{1}{2\alpha u_2} \Rightarrow r_2 > \Lambda^\beta(\delta^\beta)$, the optimal consumption patterns are either $(0,0)$ or $(1,0)$. Thus,
$$\Pi_1(r_1; r_2) = \begin{cases} r_1 - \frac{k}{u_1} & r_1 \le 1-\frac{\alpha u_1}{2} \\ 0 - \frac{k}{u_1} & r_1 > 1-\frac{\alpha u_1}{2} \end{cases}.$$
Firm 1's profit is maximized at $r_1 = 1-\frac{\alpha u_1}{2}$.
- If $r_2 \le \frac{1}{2\alpha u_2}$, for some infinitesimal $\epsilon > 0$,
$$\Pi_1(r_1; r_2) = \begin{cases} r_1 - \frac{k}{u_1} & r_1 \le \max\left\{(1-\beta)(1-\frac{\alpha u_1}{2}), (\Lambda^\beta)^{-1}(r_2) - \epsilon\right\} \\ r_1\tilde{\delta}^\beta(r_1) - \frac{k}{u_1} & \max\left\{(1-\beta)(1-\frac{\alpha u_1}{2}) + \epsilon, (\Lambda^\beta)^{-1}(r_2)\right\} \le r_1 \le 1-\frac{\alpha u_1}{2} \\ 0 - \frac{k}{u_1} & r_1 > 1-\frac{\alpha u_1}{2} \end{cases}. \tag{EC.30}$$
It can be easily seen that $\Pi_1(r_1; r_2)$ experiences a downward jump at each of its breakpoints. The first piece is increasing; the second piece, $r_1\tilde{\delta}^\beta(r_1)$, is concave quadratic; and the third one is constant. If $u_2 \le \frac{2-2\beta-\alpha u_1}{2\alpha\beta} + u_1$, $r_1\tilde{\delta}^\beta(r_1)$ is decreasing for all $r_1 \ge (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$. Thus, in this case, $\Pi_1(r_1; r_2)$ attains its maximum at the first breakpoint. If $\frac{2-2\beta-\alpha u_1}{2\alpha\beta} + u_1 < u_2 \le \Phi(u_1)$, $r_1\tilde{\delta}^\beta(r_1)$ attains its maximum on the interval $r_1 \ge (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$, but its maximum, equal to $\frac{(2-\alpha u_1-2\beta(1-\alpha u_2))^2}{16\alpha\beta u_2}$, is less than or equal to $(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$; again, $\Pi_1(r_1; r_2)$ attains its maximum at the first breakpoint. Finally, if $u_2 > \Phi(u_1)$, the maximum of $r_1\tilde{\delta}^\beta(r_1)$ is greater than $(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$. Since $r_1\tilde{\delta}^\beta(r_1) \le r_1$ for all $r_1 \le \max\left\{(1-\beta)(1-\frac{\alpha u_1}{2}) + \epsilon, (\Lambda^\beta)^{-1}(r_2)\right\}$, the maximum of $\Pi_1(r_1; r_2)$ can be obtained by simply comparing the revenue at $(\Lambda^\beta)^{-1}(r_2) - \epsilon$ (which is equal to $(\Lambda^\beta)^{-1}(r_2) - \epsilon$) with the maximum of $r_1\tilde{\delta}^\beta(r_1)$ over the interval $\left((1-\beta)(1-\frac{\alpha u_1}{2}), 1-\frac{\alpha u_1}{2}\right]$. Accordingly, we characterize Firm 1's best response in two cases, depending on whether $u_2 \le \Phi(u_1)$.
  - —Case 1: $u_2 \le \Phi(u_1)$. In this case, when $r_2 \le \frac{1}{2\alpha u_2}$, $\Pi_1(r_1; r_2)$ is maximized at the first breakpoint—either at $r_1 = (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$ or near $r_1 = (\Lambda^\beta)^{-1}(r_2)$, whichever is larger. Therefore, Firm 1's best response is
$$r_1^\beta(r_2) = \begin{cases} (1-\beta)(1-\frac{\alpha u_1}{2}) & r_2 \le (1-\beta)\frac{1}{2\alpha u_2} + \frac{\alpha\beta u_1^2}{8u_2} \\ (\Lambda^\beta)^{-1}(r_2) - \epsilon & (1-\beta)\frac{1}{2\alpha u_2} + \frac{\alpha\beta u_1^2}{8u_2} < r_2 \le \frac{1}{2\alpha u_2} \\ 1-\frac{\alpha u_1}{2} & r_2 > \frac{1}{2\alpha u_2} \end{cases},$$
where $\epsilon > 0$ is an arbitrarily small amount.
  - —Case 2: $u_2 > \Phi(u_1)$. When $r_2 \le \frac{1}{2\alpha u_2}$, comparing the maximum of $r_1\tilde{\delta}^\beta(r_1)$ over $\left((1-\beta)(1-\frac{\alpha u_1}{2}), 1-\frac{\alpha u_1}{2}\right]$ to $(\Lambda^\beta)^{-1}(r_2) - \epsilon$ gives the best response. In particular, $r_1\tilde{\delta}^\beta(r_1)$ is maximized over $\left((1-\beta)(1-\frac{\alpha u_1}{2}), 1-\frac{\alpha u_1}{2}\right]$ at $r_1 = \frac{1}{4}(2-\alpha u_1 + 2\beta(\alpha u_2 - 1))$ if $u_2 \le \frac{2\beta-\alpha u_1+2}{2\alpha\beta}$ and at the second breakpoint, $r_1 = 1-\frac{\alpha u_1}{2}$, if $u_2 > \frac{2\beta-\alpha u_1+2}{2\alpha\beta}$. Thus,
    - ∗ when $u_2 \le \frac{2\beta-\alpha u_1+2}{2\alpha\beta}$,
$$r_1^\beta(r_2) = \begin{cases} \frac{1}{4}(2-\alpha u_1 + 2\beta(\alpha u_2 - 1)) & r_2 \le \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} \\ (\Lambda^\beta)^{-1}(r_2) - \epsilon & \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} < r_2 \le \frac{1}{2\alpha u_2} \\ 1-\frac{\alpha u_1}{2} & r_2 > \frac{1}{2\alpha u_2} \end{cases};$$

* when $u_2 > \frac{2\beta-\alpha u_1+2}{2\alpha\beta}$,

$$r_1^\beta(r_2)=\begin{cases}1-\frac{\alpha u_1}{2} & r_2\le \frac{(\alpha u_1+2\alpha\beta u_2-2)^2}{8\alpha^3\beta u_2^3}+(1-\beta)\frac{1}{2\alpha u_2}\\ (\Lambda^\beta)^{-1}(r_2)-\epsilon & \frac{(\alpha u_1+2\alpha\beta u_2-2)^2}{8\alpha^3\beta u_2^3}+(1-\beta)\frac{1}{2\alpha u_2}<r_2\le\frac{1}{2\alpha u_2}\\ 1-\frac{\alpha u_1}{2} & r_2>\frac{1}{2\alpha u_2}\end{cases}.$$

Firm 2. From (EC.14), Firm 2's profit can be expressed as the following, depending on the value of r_1:

- If $r_1 \le (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$, then $\tilde{\delta}^\beta(r_1)=1$, and therefore, $\Lambda^\beta(\tilde{d}^\beta(r_1),r_1))=(1-\beta)\frac{1}{2\alpha u_2}$. Accordingly,

$$\Pi_2(r_2;r_1)=\begin{cases}r_2-\frac{k}{u_2} & r_2\le(1-\beta)\frac{1}{2\alpha u_2}\\ 0-\frac{k}{u_2} & r_2>(1-\beta)\frac{1}{2\alpha u_2}\end{cases}. \tag{EC.31}$$

- If $(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)<r_1\le 1-\frac{\alpha u_1}{2}$,

$$\Pi_2(r_2;r_1)=\begin{cases}r_2-\frac{k}{u_2} & r_2\le\Lambda^\beta(\tilde{d}^\beta(r_1),r_1))\\ 0-\frac{k}{u_2} & r_2>\Lambda^\beta(\tilde{d}^\beta(r_1),r_1))\end{cases}. \tag{EC.32}$$

- If $r_1>1-\frac{\alpha u_1}{2}$,

$$\Pi_2(r_2;r_1)=\begin{cases}r_2-\frac{k}{u_2} & r_2\le\frac{1}{2\alpha u_2}\\ 0-\frac{k}{u_2} & r_2>\frac{1}{2\alpha u_2}\end{cases}. \tag{EC.33}$$

In all three cases, $\Pi_2(r_2;r_1)$ is maximized at the right end of the linear function. Hence, Firm 2's best response is

$$r_2^\beta(r_1)=\begin{cases}(1-\beta)\frac{1}{2\alpha u_2} & r_1\le(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)\\ \frac{(2\beta+2r_1+\alpha u_1-2)^2}{8\alpha\beta u_2}+(1-\beta)\frac{1}{2\alpha u_2} & (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)<r_1\le1-\frac{\alpha u_1}{2}\\ \frac{1}{2\alpha u_2} & r_1>1-\frac{\alpha u_1}{2}\end{cases}.$$

Equilibrium. When $u_2\le\Phi(u_1)$, the best-response functions cross only once at $r_1^\beta=(1-\beta)(1-\frac{\alpha u_1}{2})$, $r_2^\beta=(1-\beta)\frac{1}{2\alpha u_2}$. The equilibrium profits are

$$\pi_1^\beta(u_1;u_2)=(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)-\frac{k}{u_1},\quad \pi_2^\beta(u_2;u_1)=(1-\beta)\frac{1}{2\alpha u_2}-\frac{k}{u_2}. \tag{EC.34}$$

When $u_2>\Phi(u_1)$, the best response functions do not cross. Thus, the pricing game does not have any pure-strategy Nash equilibrium. We construct the mixed-strategy Nash equilibrium with distributions F_i defined on the supports $r_i\in[\underline{r}_i,\overline{r}_i]$ for $i=1,2$.

- If $u_2\le\frac{2\beta-\alpha u_1+2}{2\alpha\beta}$, consider the following distributions

$$F_1(r_1)=\begin{cases}1-\frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3u_2^3h(r_1)}-(1-\beta)\frac{v(u_2,1)}{h(r_1)} & \underline{r}_1\le r_1<\overline{r}_1\\ 1 & r_1=\overline{r}_1\end{cases}; \tag{EC.35}$$

$$F_2(r_2)=\left(1-\frac{(\alpha u_1+\beta(2-2\alpha u_2)-2)^2}{16\alpha\beta u_2 h^{-1}(r_2)}\right)\Big/\left(\frac{h^{-1}(r_2)}{\alpha\beta u_2}+\frac{\alpha u_1-2(1-\beta)}{2\alpha\beta u_2}\right),\ \underline{r}_2\le r_2<\overline{r}_2, \tag{EC.36}$$

where $h(r_1):=\frac{(2\beta+2r_1+\alpha u_1-2)^2}{8\alpha\beta u_2}+(1-\beta)v(u_2,1$, with supports

$$\underline{r}_1=\frac{(\alpha u_1+\beta(2-2\alpha u_2)-2)^2}{16\alpha\beta u_2},\ \overline{r}_1=\frac{1}{4}(2-\alpha u_1+2\beta(\alpha u_2-1)), \tag{EC.37}$$

$$\underline{r}_2=\frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3u_2^3}+(1-\beta)v(u_2,1),\ \overline{r}_2=\frac{(\alpha u_1+2(\beta+\alpha\beta u_2-1))^2}{32\alpha\beta u_2}+(1-\beta)v(u_2,1). \tag{EC.38}$$

Here, because $\alpha u_2>1$, $v(u_2,1)=\frac{1}{2\alpha u_2}$ by (2).

Given $F_1(\cdot)$, by (EC.32), Firm 2's expected profit is:

—If $r_2 \in [\underline{r}_2, \overline{r}_2]$,

$$\begin{aligned}\Pi_2(r_2; F_1(\cdot)) &= r_2 \cdot \mathbb{P}\left(r_2 \le h(r_1)\right) + 0 \cdot \mathbb{P}\left(r_2 > h(r_1)\right) - \frac{k}{u_2} \\ &= r_2 \cdot \mathbb{P}\left(r_1 \ge h^{-1}(r_2)\right) - \frac{k}{u_2} \\ &= r_2 \left(1 - \lim_{r \to h^{-1}(r_2)} F_1(r)\right) - \frac{k}{u_2} \\ &= \frac{(2\beta + \alpha u_1 + 2\alpha\beta u_2 - 2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}.\end{aligned}$$

—If $r_2 > \overline{r}_2$,

$$\Pi_2(r_2; F_1(\cdot)) = 0 - \frac{k}{u_2} < \frac{(2\beta + \alpha u_1 + 2\alpha\beta u_2 - 2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}.$$

—If $r_2 < \underline{r}_2$,

$$\Pi_2(r_2; F_1(\cdot)) = r_2 - \frac{k}{u_2} < \frac{(2\beta + \alpha u_1 + 2\alpha\beta u_2 - 2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}.$$

Given $F_2(\cdot)$, by (EC.30), Firm 1's expected profit is:

—If $r_1 \in [\underline{r}_1, \overline{r}_1]$,

$$\begin{aligned}\Pi_1(r_1; F_2(\cdot)) &= r_1 \cdot \mathbb{P}\left(r_2 > h(r_1)\right) + r_1 \left(1 - \frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) \cdot \mathbb{P}\left(r_2 \le h(r_1)\right) - \frac{k}{u_1} \\ &= r_1 \cdot \left(1 - F_2\left(h(r_1)\right)\right) + r_1 \left(1 - \frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) \cdot F_2\left(h(r_1)\right) - \frac{k}{u_1} \\ &= r_1 - r_1 \left(\frac{r_1}{\alpha\beta u_2} + \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) \cdot F_2\left(h(r_1)\right) - \frac{k}{u_1} \\ &= \frac{(\alpha u_1 + \beta(2 - 2\alpha u_2) - 2)^2}{16\alpha\beta u_2} - \frac{k}{u_1};\end{aligned}$$

—If $\overline{r}_1 < r_1 \le 1 - \frac{\alpha u_1}{2}$, as $\overline{r}_1 = \arg\max_{r_1} r_1 \left(1 - \frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right)$,

$$\begin{aligned}\Pi_1(r_1; F_2(\cdot)) &= r_1 \left(1 - \frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) - \frac{k}{u_1} \\ &< \overline{r}_1 \left(1 - \frac{\overline{r}_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) - \frac{k}{u_1} \\ &= \frac{(\alpha u_1 + \beta(2 - 2\alpha u_2) - 2)^2}{16\alpha\beta u_2} - \frac{k}{u_1};\end{aligned}$$

—If $r_1 > \overline{r}_1 = 1 - \frac{\alpha u_1}{2}$,

$$\Pi_1(r_1; F_2(\cdot)) = 0 - \frac{k}{u_1} < \frac{(\alpha u_1 + \beta(2 - 2\alpha u_2) - 2)^2}{16\alpha\beta u_2} - \frac{k}{u_1};$$

—If $r_1 < \underline{r}_1 = \frac{(\alpha u_1 + \beta(2 - 2\alpha u_2) - 2)^2}{16\alpha\beta u_2}$,

$$\Pi_1(r_1; F_2(\cdot)) = r_1 - \frac{k}{u_1} < \frac{(\alpha u_1 + \beta(2 - 2\alpha u_2) - 2)^2}{16\alpha\beta u_2} - \frac{k}{u_1}.$$

Hence, (EC.35)-(EC.36) constitute a mixed-strategy Nash equilibrium, and the equilibrium profits are equal to

$$\pi_1(u_1; u_2) = \frac{(\alpha u_1 + \beta(2 - 2\alpha u_2) - 2)^2}{16\alpha\beta u_2} - \frac{k}{u_1}, \; \pi_2(u_2; u_1) = \frac{(2\beta + \alpha u_1 + 2\alpha\beta u_2 - 2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}. \tag{EC.39}$$

- If $u_2 > \frac{2\beta - \alpha u_1 + 2}{2\alpha\beta}$, consider the following distributions

$$F_1(r_1) = \begin{cases} 1 - \frac{(\alpha u_1 + 2\alpha\beta u_2 - 2)^2}{8\alpha^3\beta u_2^3 h(r_1)} - (1-\beta)\frac{1}{2\alpha u_2 h(r_1)} & \underline{r}_1 \le r_1 < \overline{r}_1 \\ 1 & r_1 = \overline{r}_1 \end{cases} \tag{EC.40}$$

$$F_2(r_2) = \left(1 - \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2 h^{-1}(r_2)}\right) \Big/ \left(\frac{h^{-1}(r_2)}{\alpha\beta u_2} + \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right), \ \underline{r}_2 \le r_2 \le \overline{r}_2. \tag{EC.41}$$

with supports

$$\underline{r}_1 = \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2}, \ \overline{r}_1 = 1 - \frac{\alpha u_1}{2}, \ \underline{r}_2 = \frac{(\alpha u_1 + 2\alpha\beta u_2 - 2)^2}{8\alpha^3\beta u_2^3} + (1-\beta)\frac{1}{2\alpha u_2}, \ \overline{r}_2 = \frac{1}{2\alpha u_2}.$$

Given $F_1(\cdot)$, by (EC.31), Firm 2's expected profit is:

—If $r_2 \in [\underline{r}_2, \overline{r}_2]$,

$$\begin{aligned} \Pi_2(r_2; F_1(\cdot)) &= r_2 \cdot \mathbb{P}(r_2 \le h(r_1)) + 0 - \frac{k}{u_2} \\ &= r_2 \cdot \mathbb{P}\left(r_1 \ge h^{-1}(r_2)\right) - \frac{k}{u_2} \\ &= r_2 \cdot \left(1 - \lim_{r \to h^{-1}(r_2)} F_1(r)\right) - \frac{k}{u_2} \\ &= \frac{(\alpha u_1 + 2\alpha\beta u_2 - 2)^2}{8\alpha^3\beta u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}. \end{aligned}$$

—If $r_2 > \overline{r}_2$,

$$\Pi_2(r_2; F_1(\cdot)) = 0 - \frac{k}{u_2} < \frac{(\alpha u_1 + 2\alpha\beta u_2 - 2)^2}{8\alpha^3\beta u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}.$$

—If $r_2 < \underline{r}_2$,

$$\Pi_2(r_2; F_1(\cdot)) = r_2 - \frac{k}{u_2} < \frac{(\alpha u_1 + 2\alpha\beta u_2 - 2)^2}{8\alpha^3\beta u_2^3} + (1-\beta)\frac{1}{2\alpha u_2} - \frac{k}{u_2}.$$

Given $F_2(\cdot)$, by (EC.30), Firm 1's expected profit is:

—If $r_1 \in [\underline{r}_1, \overline{r}_1]$

$$\begin{aligned} \Pi_1(r_1; F_2(\cdot)) &= r_1 \cdot \mathbb{P}(r_2 > h(r_1)) + r_1\left(1 - \frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) \cdot \mathbb{P}(r_2 \le h(r_1)) - \frac{k}{u_1} \\ &= r_1 \cdot (1 - F_2(h(r_1))) + r_1\left(1 - \frac{r_1}{\alpha\beta u_2} - \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) \cdot F_2(h(r_1)) - \frac{k}{u_1} \\ &= r_1 - r_1\left(\frac{r_1}{\alpha\beta u_2} + \frac{\alpha u_1 - 2(1-\beta)}{2\alpha\beta u_2}\right) \cdot F_2(h(r_1)) - \frac{k}{u_1} \\ &= \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2} - \frac{k}{u_1}. \end{aligned}$$

—If $r_1 > \overline{r}_1 = 1 - \frac{\alpha u_1}{2}$,

$$\Pi_1(r_1; F_2(\cdot)) = 0 - \frac{k}{u_1} < \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2} - \frac{k}{u_1}.$$

—If $r_1 < \underline{r}_1 = \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2}$,

$$\Pi_1(r_1; F_2(\cdot)) = r_1 - \frac{k}{u_1} < \frac{(2-\alpha u_1)(\alpha u_2 - 1)}{2\alpha u_2} - \frac{k}{u_1}.$$

Hence, (EC.40)-(EC.41) constitute a mixed-strategy Nash equilibrium, and the corresponding profits are equal to:

$$\pi_1(u_1;u_2)=\frac{(2-\alpha u_1)(\alpha u_2-1)}{2\alpha u_2}-\frac{k}{u_1},\quad \pi_2(u_2;u_1)=\frac{(\alpha u_1+2\alpha\beta u_2-2)^2}{8\alpha^3\beta u_2^3}+(1-\beta)\frac{1}{2\alpha u_2}-\frac{k}{u_2}.$$

Summarizing all cases yields (EC.24)-(EC.25). $\square$

Lemma EC.11. *When the publishers share a fraction β of the same content, for any given publication cycles $\mathbf{u}$, suppose that $\alpha u_1<\alpha u_2\leq 1$. The pricing game (9) does not have a pure-strategy Nash equilibrium, but it has a mixed-strategy Nash equilibrium yielding the following profits:*

$$\pi_1^\beta(u_1;u_2)=\begin{cases}(1-\beta)(1-\frac{\alpha u_1}{2})-\frac{k}{u_1} & u_2\leq\Phi(u_1)\\ \frac{(\alpha u_1+\beta(2-2\alpha u_2)-2)^2}{16\alpha\beta u_2}-\frac{k}{u_1} & u_2>\Phi(u_1)\end{cases},$$

$$\pi_2^\beta(u_2;u_1)=\begin{cases}(1-\beta)\left(1-\frac{\alpha u_2}{2}\right)-\frac{k}{u_2} & u_2\leq\Phi(u_1)\\ \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3u_2^3}+(1-\beta)\left(1-\frac{\alpha u_2}{2}\right)-\frac{k}{u_2} & u_2>\Phi(u_1)\end{cases},$$

where $\Phi(u_1)$ is defined by (EC.26).

Proof. Throughout the proof, we omit the arguments $\mathbf{u}$ in the functions since the cycles are fixed in the pricing game. Since $\alpha u_1<\alpha u_2\leq 1$, the customers' optimal consumption portfolios are given by Lemma EC.9. Using (EC.20), we obtain that $\frac{\partial U^\beta(0,1)}{\partial d}\geq 0\Leftrightarrow r_1\leq 1-\frac{\alpha u_1}{2}$ and $U^\beta(1,1)<U^\beta\left(1-\frac{u_1}{u_2},1\right)$ if and only if $r_1>(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$. Thus, (7) consists in approximating $\delta^\beta(r_1)$ in (EC.19) with $\tilde{\delta}^\beta(r_1)$, as given by (EC.27). Let $\Lambda^\beta(\tilde{\delta}^\beta(r_1),r_1)$ be given by (EC.28) and $(\Lambda^\beta)^{-1}(r_2)$ be given by (EC.29) with $v(u_2,1)=1-\frac{\alpha u_2}{2}$ by (2) since $\alpha u_2\leq 1$.

Because $\Lambda^\beta(\tilde{\delta}^\beta(r_1),r_1)$ is increasing, when $(1-\beta)(1-\frac{\alpha u_1}{2})<r_1\leq 1+\beta(\alpha u_2-1)-\frac{\alpha u_1}{2}$, $r_2\leq\Lambda^\beta(\tilde{\delta}^\beta(r_1),r_1)\Rightarrow r_2\leq 1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2$. Accordingly, by Lemma EC.9, Firm 1's profit can be expressed as:

- If $r_2\leq 1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2$,

$$\Pi_1(r_1;r_2)=\begin{cases}r_1-\frac{k}{u_1} & r_1\leq\max\left\{(1-\beta)\left(1-\frac{\alpha u_1}{2}\right),(\Lambda^\beta)^{-1}(r_2)-\epsilon\right\}\\ r_1\tilde{\delta}^\beta(r_1)-\frac{k}{u_1} & \max\left\{(1-\beta)\left(1-\frac{\alpha u_1}{2}\right)+\epsilon,(\Lambda^\beta)^{-1}(r_2)\right\}\leq r_1<1-\frac{\alpha u_1}{2}+\beta(\alpha u_2-1)\\ 0-\frac{k}{u_1} & r_1\geq 1-\frac{\alpha u_1}{2}+\beta(\alpha u_2-1)\end{cases}.$$

- If $1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2<r_2\leq 1-\frac{\alpha u_2}{2}$,

$$\Pi_1(r_1;r_2)=\begin{cases}r_1-\frac{k}{u_1} & r_1<r_2+\frac{\alpha}{2}(u_2-u_1)\\ 0-\frac{k}{u_1} & r_1\geq r_2+\frac{\alpha}{2}(u_2-u_1)\end{cases}.$$

- If $r_2>1-\frac{\alpha u_2}{2}$,

$$\Pi_1(r_1;r_2)=\begin{cases}r_1-\frac{k}{u_1} & r_1\leq 1-\frac{\alpha u_1}{2}\\ 0-\frac{k}{u_1} & r_1>1-\frac{\alpha u_1}{2}\end{cases}.$$

Following a similar analysis to the proof of Lemma EC.8's, Firm 1's best response is then:

- If $u_2\leq\Phi(u_1)$,

$$r_1^\beta(r_2)=\begin{cases}(1-\beta)(1-\frac{\alpha u_1}{2}) & r_2\leq(1-\beta)\left(1-\frac{\alpha u_2}{2}\right)+\frac{\alpha\beta u_1^2}{8u_2}\\ (\Lambda^\beta)^{-1}(r_2)-\epsilon & (1-\beta)\left(1-\frac{\alpha u_2}{2}\right)+\frac{\alpha\beta u_1^2}{8u_2}<r_2\leq 1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2\\ r_2+\frac{\alpha}{2}(u_2-u_1)-\epsilon & 1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2<r_2\leq 1-\frac{\alpha u_2}{2}\\ 1-\frac{\alpha u_1}{2} & r_2>1-\frac{\alpha u_2}{2}\end{cases}.$$

- If $u_2 > \Phi(u_1)$,

$$r_1^\beta(r_2) = \begin{cases} \frac{1}{4}(2-\alpha u_1 + 2\beta(\alpha u_2 - 1)) & r_2 \leq \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) \\ (\Lambda^\beta)^{-1}(r_2) - \epsilon & \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3 u_2^3} + (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) < r_2 \leq 1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2 \\ r_2 + \frac{\alpha}{2}(u_2 - u_1) - \epsilon & 1-\beta+\left(\beta-\frac{1}{2}\right)\alpha u_2 < r_2 \leq 1-\frac{\alpha u_2}{2} \\ 1-\frac{\alpha u_1}{2} & r_2 > 1-\frac{\alpha u_2}{2} \end{cases},$$

where $\epsilon > 0$ is an arbitrarily small amount.

Firm 2's profit can be expressed as:

- If $r_1 \leq (1-\beta)(1-\frac{\alpha u_1}{2})$,

$$\Pi_2(r_2; r_1) = \begin{cases} r_2 & r_2 \leq (1-\beta)(1-\frac{\alpha u_2}{2}) \\ 0 & r_2 > (1-\beta)(1-\frac{\alpha u_2}{2}) \end{cases}.$$

- If $(1-\beta)(1-\frac{\alpha u_1}{2}) < r_1 \leq 1-\frac{\alpha u_1}{2} + \beta(\alpha u_2 - 1)$,

$$\Pi_2(r_2; r_1) = \begin{cases} r_2 & r_2 \leq \Lambda^\beta(\tilde{\delta}(r_1), r_1) \\ 0 & r_2 > \Lambda^\beta(\tilde{\delta}(r_1), r_1) \end{cases}.$$

- If $1-\frac{\alpha u_1}{2} + \beta(\alpha u_2 - 1) < r_1 \leq 1-\frac{\alpha u_1}{2}$,

$$\Pi_2(r_2; r_1) = \begin{cases} r_2 & r_2 \leq r_1 - \frac{\alpha}{2}(u_2 - u_1) \\ 0 & r_2 > r_1 - \frac{\alpha}{2}(u_2 - u_1) \end{cases}.$$

- If $r_1 > 1-\frac{\alpha u_1}{2}$,

$$\Pi_2(r_2; r_1) = \begin{cases} r_2 & r_2 \leq 1-\frac{\alpha u_2}{2} \\ 0 & r_2 > 1-\frac{\alpha u_2}{2} \end{cases}.$$

Therefore, Firm 2's best response is:

$$r_2^\beta(r_1) = \begin{cases} (1-\beta)\left(1-\frac{\alpha u_2}{2}\right) & r_1 \leq (1-\beta)\left(1-\frac{\alpha u_1}{2}\right) \\ (\Lambda^\beta)^{-1}(r_2) & (1-\beta)\left(1-\frac{\alpha u_1}{2}\right) < r_1 \leq 1-\frac{\alpha u_1}{2} + \beta(\alpha u_2 - 1) \\ r_1 - \frac{\alpha}{2}(u_2 - u_1) & 1-\frac{\alpha u_1}{2} + \beta(\alpha u_2 - 1) < r_1 \leq 1-\frac{\alpha u_1}{2} \\ 1-\frac{\alpha u_2}{2} & r_1 > 1-\frac{\alpha u_1}{2} \end{cases}.$$

The analysis hereafter is similar to that of Lemma EC.10. The pure-strategy Nash equilibrium when $u_2 \leq \Phi(u_1)$ is similar to the pure-strategy equilibrium in Lemma EC.10 with $r_1^\beta = (1-\beta)\left(1-\frac{\alpha u_1}{2}\right)$ and $r_2^\beta = (1-\beta)\left(1-\frac{\alpha u_2}{2}\right)$, yielding equilibrium profits $\pi_i^\beta(u_i; u_{-i}) = (1-\beta)\left(1-\frac{\alpha u_i}{2}\right)$ for $i \in \{1,2\}$, similar to (EC.34). Otherwise, when $u_2 > \Phi(u_1)$, there only exists a mixed-strategy Nash equilibrium characterized by distributions (EC.35)-(EC.36) and supports (EC.37)-(EC.38), yielding equilibrium profits (EC.39). In (EC.35), (EC.38), and (EC.39), since $\alpha u_2 \leq 1$, $v(u_2, 1) = 1-\frac{\alpha u_2}{2}$ by (2). □

LEMMA EC.12. *Suppose that $u_1^\beta \leq u_2^\beta$. If $\pi_1^\beta(u_1^\beta; u_2^\beta) \geq 0$ and $u_2^\beta \geq \frac{1}{\alpha}$, then, $\alpha u_1^\beta \leq 1$.*

Proof. We prove by showing that setting $u_1 > \frac{1}{\alpha}$ is strictly dominated.

Customer Choice. Suppose $u_1 \geq \frac{1}{\alpha}$. Thus, $u_2 \geq \frac{1}{\alpha}$ and $q(u_1) = q(u_2) = q := \frac{1}{2\alpha}$. Let $n := \frac{u_2}{u_1}$ and $p_i := r_i u_i$ for $i \in \{1,2\}$. Accordingly, the customer's utility (13) when buying $d \in \{0,1,\ldots,n\}$ units Product 1 and $w \in \{0,1\}$ units Product 2 every u_1 period is

$$\hat{U}^\beta(d,w) = \begin{cases} d(q-p_1) + w(q-p_2) & d \leq n-1 \\ d(q-p_1) + w((1-\beta)q - p_2) & d = n \end{cases}.$$

Customers always buy Product 1 when Product 2 is not released if and only if $p_1 \le q$. At the release times of Product 2, customers have three options: (i) Buy Product 1 only; (ii) Buy Product 2 only; (iii) Buy both products. Accordingly, customers prefer

(i) Buying Product 1 if $q \ge p_1$ and

- $(i) \succ (ii)$
$$q - p_1 > q - p_2 \Leftrightarrow p_1 < p_2; \text{ and}$$
- $(i) \succ (iii)$
$$q - p_1 > (2-\beta)q - p_1 - p_2 \Leftrightarrow p_2 > (1-\beta)q.$$

(ii) Buying Product 2 if $q \ge p_2$ and

- $(ii) \succeq (i)$
$$q - p_2 > q - p_1 \Leftrightarrow p_2 \le p_1; \text{ and}$$
- $(ii) \succ (iii)$
$$q - p_2 > (2-\beta)q - p_1 - p_2 \Leftrightarrow p_1 > (1-\beta)q.$$

(iii) Buying both if $q \ge \max\{p_1, p_2\}$ and

- $(iii) \succeq (i)$
$$(2-\beta)q - p_1 - p_2 \ge q - p_1 \Leftrightarrow p_2 \le (1-\beta)q; \text{ and}$$
- $(iii) \succeq (ii)$
$$(2-\beta)q - p_1 - p_2 \ge q - p_2 \Leftrightarrow p_1 \le (1-\beta)q.$$

In summary, given prices p_1 and p_2 and cycles u_1 and u_2, the customers choose to consume (d^β, w^β) units of each product in a cycle of duration u_2:

$$(d^\beta, w^\beta) = \begin{cases} (n,1) & p_1 \le (1-\beta)q, p_2 \le (1-\beta)q \\ (n-1,1) & q \ge p_1 > (1-\beta)q, p_2 \le p_1 \\ (n,0) & q \ge p_2 > (1-\beta)q, p_1 < p_2 \text{ or } p_2 > q \ge p_1 \\ (0,1) & p_1 > q \ge p_2 \\ (0,0) & p_1 > q, p_2 > q \end{cases}. \tag{EC.42}$$

Pricing Game. For any $i \in \{1,2\}$, setting $p_i > q$ is strictly dominated because it generates zero revenue. Hence, we restrict our attention to $p_1 \le q$ for $i \in \{1,2\}$. From (EC.42), Firm 1's average profit can be written as:

- If $p_2 \le (1-\beta)q$,
$$\Pi_1(p_1;p_2) = -\frac{k}{u_1} + \frac{1}{u_2}\begin{cases} np_1 & p_1 \le (1-\beta)q \\ (n-1)p_1 & (1-\beta)q < p_1 \le q \\ 0 & p_1 > q \end{cases}; \tag{EC.43}$$
- If $(1-\beta)q < p_2 \le q$,
$$\Pi_1(p_1;p_2) = -\frac{k}{u_1} + \frac{1}{u_2}\begin{cases} np_1 & p_1 < p_2 \\ (n-1)p_1 & p_2 \le p_1 \le q \\ 0 & p_1 > q \end{cases}. \tag{EC.44}$$

Therefore, for $\epsilon > 0$ arbitrarily small, Firm 1's best response is:

- If $\beta \le \frac{1}{n}$,

$$p_1^\beta(p_2) = \begin{cases} (1-\beta)q & p_2 \le (1-\beta)q \\ p_2 - \epsilon & (1-\beta)q < p_2 \le q \end{cases};$$

- If $\beta > \frac{1}{n}$,

$$p_1^\beta(p_2) = \begin{cases} q & p_2 \le (1-\frac{1}{n})q \\ p_2 - \epsilon & (1-\frac{1}{n})q < p_2 \le q \end{cases}.$$

Similarly, from (EC.42), Firm 2's average profit can be written as:

- If $p_1 \le (1-\beta)q$,

$$\Pi_2(p_2;p_1) = -\frac{k}{u_2} + \frac{1}{u_2}\begin{cases} p_2 & p_2 \le (1-\beta)q \\ 0 & (1-\beta)q < p_2 \le q \end{cases}; \tag{EC.45}$$

- If $(1-\beta)q < p_1 \le q$,

$$\Pi_2(p_2;p_1) = -\frac{k}{u_2} + \frac{1}{u_2}\begin{cases} p_2 & p_2 \le p_1 \\ 0 & p_2 > p_1 \end{cases}. \tag{EC.46}$$

Accordingly, Firm 2's best response function is:

$$p_2^\beta(p_1) = \begin{cases} (1-\beta)q & p_1 \le (1-\beta)q \\ p_1 - \epsilon & (1-\beta)q < p_1 \le q \end{cases}.$$

Next, we next characterize the different Nash equilibria based on the value of β.

- When $\beta \le \frac{1}{n}$, the best responses cross at $(p_1^\beta, p_2^\beta) = ((1-\beta)q, (1-\beta)q)$; the corresponding equilibrium profits are

$$\pi_1^\beta(u_1;u_2) = \frac{1}{u_2}n(1-\beta)q - \frac{k}{u_1} = \frac{1-\beta}{2\alpha u_1} - \frac{k}{u_1};$$
$$\pi_2^\beta(u_2;u_1) = \frac{1}{u_2}(1-\beta)q - \frac{k}{u_2} = \frac{1-\beta}{2\alpha u_2} - \frac{k}{u_2}.$$

- When $\beta > \frac{1}{n}$, the best responses do not cross. We construct a mixed-strategy Nash equilibrium with distributions F_i and supports $p_i \in \left[\underline{p}_i, \overline{p}_i\right]$. Consider the following distributions:

$$F_1(p_1) = \begin{cases} 1 - \frac{(n-1)}{np_1}q & \frac{(n-1)}{n}q \le p_1 < q \\ 1 & p_1 = q \end{cases}, \quad F_2(p_2) = n - \frac{(n-1)q}{p_2},\ \frac{(n-1)}{n}q \le p_2 \le q \tag{EC.47}$$

with the following supports

$$\underline{p}_i = \frac{n-1}{n}q, \quad \overline{p}_i = q, \quad \forall i \in \{1,2\}.$$

Given $F_2(\cdot)$, by (EC.43)-(EC.44), Firm 1's expected profit is:

—If $\frac{n-1}{n}q \le p_1 \le q$,

$$\begin{aligned}\Pi_1(p_1;F_2) &= \frac{1}{u_2}\left(np_1(1-F_2(p_1)) + (n-1)p_1F_2(p_1)\right) - \frac{k}{u_1} \\ &= \frac{1}{u_2}\left(np_1 - p_1F_2(p_1)\right) - \frac{k}{u_1} \\ &= \frac{(n-1)q}{u_2} - \frac{k}{u_1};\end{aligned}$$

—If $p_1 > q$,

$$\Pi_1(p_1;F_2) = 0 - \frac{k}{u_1} < \frac{(n-1)q}{u_2} - \frac{k}{u_1};$$

—If $p_1 < \frac{n-1}{n}q$,

$$\Pi_1(p_1; F_2) = \frac{np_1}{u_2} - \frac{k}{u_1} < \frac{(n-1)q}{u_2} - \frac{k}{u_1}.$$

Given $p_1 \sim F_1(\cdot)$, by (EC.45)-(EC.46), Firm 2's expected profit is:

—If $\frac{n-1}{n}q \le p_2 \le q$,

$$\begin{aligned}\Pi_2(p_2; F_1) &= \frac{1}{u_2} p_2 \cdot \mathbb{P}(p_1 \ge p_2) - \frac{k}{u_2} \\ &= \frac{1}{u_2} p_2 \cdot \left(1 - \lim_{p \to p_2^-} F_1(p)\right) - \frac{k}{u_2} \\ &= \frac{1}{u_2} p_2 \cdot \left(1 - \left(1 - \frac{(n-1)q}{np_2}\right)\right) - \frac{k}{u_2} \\ &= \frac{n-1}{n}\frac{q}{u_2} - \frac{k}{u_2};\end{aligned}$$

—If $p_2 > q$,

$$\Pi_2(p_2; F_1) = 0 - \frac{k}{u_2} < \frac{n-1}{n}\frac{q}{u_2} - \frac{k}{u_2};$$

—If $p_2 < \frac{n-1}{n}q$,

$$\Pi_2(p_2; F_1) = \frac{p_2}{u_2} - \frac{k}{u_2} < \frac{n-1}{n}\frac{q}{u_2} - \frac{k}{u_2}.$$

Hence, when $\beta > \frac{1}{n}$ and $u_1 < u_2$, (EC.47) is a mixed-strategy Nash equilibrium, and the equilibrium profits are

$$\begin{aligned}\pi_1^\beta(u_1; u_2) &= \frac{(n-1)q}{u_2} - \frac{k}{u_1} = \frac{1}{2\alpha}\left(\frac{1}{u_1} - \frac{1}{u_2}\right) - \frac{k}{u_1}, \\ \pi_2^\beta(u_2; u_1) &= \frac{(n-1)}{n}\frac{q}{u_2} - \frac{k}{u_2} = \frac{1}{2\alpha}\frac{u_2 - u_1}{u_2^2} - \frac{k}{u_2}.\end{aligned}$$

Combining the two cases ($\beta \le \frac{1}{n}$ and $\beta > \frac{1}{n}$), when $u_2 \ge u_1 \ge \frac{1}{\alpha}$, the equilibrium profits are:

$$\pi_1^\beta(u_1; u_2) = \begin{cases} \frac{1}{2\alpha}\left(\frac{1}{u_1} - \frac{1}{u_2}\right) - \frac{k}{u_1} & u_1 < \beta u_2 \\ \frac{1-\beta}{2\alpha u_1} - \frac{k}{u_1} & \beta u_2 \le u_1 \le u_2 \end{cases}; \quad \pi_2^\beta(u_2; u_1) = \begin{cases} \frac{1-\beta}{2\alpha u_2} - \frac{k}{u_2} & u_1 \le u_2 \le \frac{u_1}{\beta} \\ \frac{1}{2\alpha}\frac{u_2 - u_1}{u_2^2} - \frac{k}{u_2} & u_2 > \frac{u_1}{\beta} \end{cases}. \quad \text{(EC.48)}$$

Publishing Game. When $u_1 < \beta u_2$, $\pi_1^\beta(u_1; u_2)$ is decreasing (since $\alpha k < 0.5$ by assumption). When $\beta u_2 \le u_1 \le u_2$, $\pi_1^\beta(u_1; u_2)$ is either nonincreasing and nonnegative if $1 - \beta \ge 2\alpha k$ or increasing and negative if $1 - \beta < 2\alpha k$. Hence, when $1 - \beta \ge 2\alpha k$, $\pi_1^\beta(u_1; u_2)$ is nonincreasing, and it is optimal for Firm 1 to set $u_1^\beta \le \frac{1}{\alpha}$. When $1 - \beta < 2\alpha k$, it is optimal for Firm 1 to set $u_1^\beta \le \frac{1}{\alpha}$ if $\max_{u_1 \le \frac{1}{\alpha}} \pi_1^\beta(u_1; u_2) \ge \pi_1^\beta(u_2; u_2) = \frac{1-\beta}{2\alpha u_2} - \frac{k}{u_2}$. Since the latter is negative when $1 - \beta < 2\alpha k$, it is sufficient to require that $\max_{u_1 \le \frac{1}{\alpha}} \pi_1^\beta(u_1; u_2^\beta) \ge 0$ for Firm 1 to set $u_1^\beta \le \frac{1}{\alpha}$. Since $\pi_1^\beta(u_1^\beta; u_2^\beta) \ge 0$ if $u_2^\beta \ge \frac{1}{\alpha}$ by assumption, we must have that $\max_{u_1 \le \frac{1}{\alpha}} \pi_1^\beta(u_1; u_2^\beta) \ge 0$. □

Lemma EC.13. *Suppose $0 < \beta \le 1$ and $\frac{1-\beta}{2} \le \alpha k < 0.5$. The following set of equations*

$$u_1 = \sqrt{\frac{2ku_2}{\alpha u_2 - 1}}, \quad u_2 = \frac{(2 - \alpha u_1)\left(2\beta + \sqrt{\beta(4\beta + 6\alpha k - 3)}\right)}{2\alpha\beta(1 - 2\alpha k)} \quad \text{(EC.49)}$$

has a unique solution $\left(u_1^{\beta,2}, u_2^{\beta,2}\right)$ such that $0 < u_1 < \frac{1}{\alpha}$. Moreover, $\sqrt{\frac{2k}{\alpha}} < u_1^{\beta,2} < \min\left\{2\sqrt{\frac{k}{\alpha}}, \frac{1}{\alpha}\right\}$ and $u_2^{\beta,2} > \frac{2}{\alpha}$.

Proof. Combining the two equations in (EC.49), the solution, if it exists, is a root to the following cubic equation:

$$g^{\beta}(u_1) = 3\alpha^2 u_1^3 - u_1^2\left(-4\alpha\beta + 6\alpha + 2\alpha\sqrt{\beta(4\beta+6\alpha k-3)}\right) - 6\alpha k u_1 + 12k. \tag{EC.50}$$

Since $\frac{\partial g^{\beta}\left(\frac{1}{\alpha}\right)}{\partial \alpha k} = 6 - \frac{6\beta}{\sqrt{\beta(4\beta+6\alpha k-3)}} > 0$ since $\frac{1-\beta}{2} \le \alpha k < 0.5$ by assumption, we obtain that

$$g^{\beta}\left(\frac{1}{\alpha}\right) = \frac{4\beta - 2\sqrt{\beta(4\beta+6\alpha k-3)} + 6\alpha k - 3}{\alpha} < g^{\beta}\left(\frac{1}{\alpha}\right)\Big|_{\alpha k=0.5} = 0.$$

Additionally,

$$g^{\beta}(0) = 12k > 0,\ \lim_{u_1\to\infty} g^{\beta}(u_1) > 0.$$

Hence, because $g^{\beta}(u_1)$ is cubic, there is only one root to $g^{\beta}(u_1)=0$ in $u_1 \in \left(0, \frac{1}{\alpha}\right)$. Let $\left(u_1^{\beta,2}, u_2^{\beta,2}\right)$ denote the corresponding solution. We obtain that $u_1^{\beta,2} > \sqrt{\frac{2k}{\alpha}}$ because

$$g^{\beta}\left(\sqrt{\frac{2k}{\alpha}}\right) = k(8\beta - 4\sqrt{\beta(4\beta+6\alpha k-3)}) > 0 \text{ when } \alpha k < 0.5.$$

Additionally, $u_1^{\beta,2} < 2\sqrt{\frac{k}{\alpha}}$ because

$$\begin{aligned} g^{\beta}\left(2\sqrt{\frac{k}{\alpha}}\right) &= 4k\left(4\beta - 2\sqrt{\beta(4\beta+6\alpha k-3)} + 3\sqrt{\alpha k} - 3\right) \\ &\le 4k\left(2\beta + 3\sqrt{\frac{1-\beta}{2}} - 3\right) && \text{when } \alpha k \ge \frac{1-\beta}{2} \\ &< 0, \end{aligned}$$

given that $2\beta + 3\sqrt{\frac{1-\beta}{2}} - 3$ is concave, maximized at $\beta = \frac{23}{32}$, and negative at its peak.

Consequently,

$$\begin{aligned} u_2^{\beta,2} &= \frac{(2-\alpha u_1^{\beta,2})\left(2\beta + \sqrt{\beta(4\beta+6\alpha k-3)}\right)}{2\alpha\beta(1-2\alpha k)} \\ &> \frac{(2-2\sqrt{\alpha k})\left(2\beta + \sqrt{\beta(4\beta+6\alpha k-3)}\right)}{2\alpha\beta(1-2\alpha k)} && \text{since } u_1^{\beta,2} < 2\sqrt{\frac{k}{\alpha}} \\ &\ge \frac{(2-2\sqrt{\alpha k})\cdot 3\beta}{2\alpha\beta(1-2\alpha k)} && \text{when } \alpha k \ge \frac{1-\beta}{2} \\ &\ge \frac{3(2+\sqrt{2})}{4\alpha} && \text{since } 0 < \alpha k < 0.5 \\ &> \frac{2}{\alpha} \quad \square \end{aligned}$$

LEMMA EC.14. *If* $0 < \beta < \frac{1}{2}$ *and* $0 < \alpha k < 0.5$, *then* $\sqrt{\frac{2\alpha k}{1-\beta}} \le \frac{2(1-\beta)\left(1+2\left(1+\sqrt{2}\right)\beta\right)}{1+4\beta-4\beta^2}$.

Proof. For $0 < \beta < \frac{1}{2}$ and $\alpha k > 0$,

$$\begin{aligned} \sqrt{\tfrac{2\alpha k}{1-\beta}} &\le \frac{2(1-\beta)\left(1+2\left(\sqrt{2}+1\right)\beta\right)}{1+4\beta-4\beta^2} \\ \Leftrightarrow \quad \alpha k &\le \frac{2(1-\beta)^3\left(1+2\left(\sqrt{2}+1\right)\beta\right)^2}{(1+4\beta-4\beta^2)^2} := \rho(\beta). \end{aligned}$$

Since

$$\frac{\partial\rho(\beta)}{\partial\beta}=\frac{2(1-\beta)^2\left(2\left(\sqrt{2}+1\right)\beta+1\right)\eta(\beta)}{1+(4(1-\beta)\beta)^3},$$

in which

$$\eta(\beta):=8\left(\sqrt{2}+1\right)\beta^3-4\left(2\sqrt{2}+3\right)\beta^2+2\left(1-5\sqrt{2}\right)\beta+4\sqrt{2}-7,$$

and

$$\eta(0)<0,\quad \frac{\partial\eta(\beta)}{\partial\beta}=24\left(\sqrt{2}+1\right)\beta^2-8\left(2\sqrt{2}+3\right)\beta+2\left(1-5\sqrt{2}\right)<0 \text{ for } 0<\beta<0.5,$$

we get $\frac{\partial\rho(\beta)}{\partial\beta}<0$ for $0<\beta<1/2$ and therefore $\rho(\beta)>\rho(1/2)=\frac{(2+\sqrt{2})^2}{16}\approx 0.73$. Since $\alpha k<0.5$ by assumption, $\alpha k<\rho(\beta)$ is guranteed. $\square$

Lemma EC.15. *When* $0<\beta<\frac{1}{2}$,

$$\frac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}\right]\le\frac{1-\beta}{2}.$$

Proof.

$$\begin{aligned}
&\tfrac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}\right] &&\le\frac{1-\beta}{2}\\
\Leftrightarrow\quad &\left(-4\beta^3+11\beta^2-8\beta+2\right)-2(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)} &&\le\beta^2(1-\beta)\\
\Leftrightarrow\quad &\underbrace{2(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}}_{>0} &&\ge\underbrace{-3\beta^3+10\beta^2-8\beta+2}_{>0}\\
\Leftrightarrow\quad &3\beta^6-8\beta^5+4\beta^4 &&\ge 0\\
\Leftrightarrow\quad &3\beta^2-8\beta+4 &&\ge 0,
\end{aligned}$$

which is always true when $0<\beta<\frac{1}{2}$. $\square$

Lemma EC.16. *When* $0<\beta<1$ *and* $0<\alpha k<0.5$, $(1-\beta)-\sqrt{2(1-\beta)\alpha k}>\frac{\beta(1-2\alpha k)}{2(1-\beta)}$ *if and only if* $0<\beta<\frac{1}{2}$ *and*

$$\alpha k<\frac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)^{\frac{3}{2}}\sqrt{-3\beta^3+8\beta^2-5\beta+1}\right]. \tag{EC.51}$$

Proof. Given $0<\beta<1$ and $0<\alpha k<0.5$,

$$\begin{aligned}
&(1-\beta)-\sqrt{2(1-\beta)\alpha k} &&>\frac{\beta(1-2\alpha k)}{2(1-\beta)}\\
\Leftrightarrow\ &2(1-\beta)^2-2(1-\beta)\sqrt{2(1-\beta)\alpha k} &&>\beta(1-2\alpha k)\\
\Leftrightarrow\ &2(1-\beta)^2-\beta(1-2\alpha k) &&>2(1-\beta)\sqrt{2(1-\beta)\alpha k}.
\end{aligned}\tag{EC.52}$$

The validity of (EC.52) depends on whether its left-hand side is greater than 0. To be specific,

- If $\beta<\frac{1}{2}$, $2(1-\beta)^2-\beta(1-2\alpha k)>2(1-\beta)^2-\beta>0$. Therefore, the left-hand side of (EC.52) is positive, which yields:

$$\begin{aligned}
\text{(EC.52)}\Leftrightarrow\ &\left(2(1-\beta)^2-\beta(1-2\alpha k)\right)^2-2^2(1-\beta)^2 2(1-\beta)\alpha k &&>0\\
\Leftrightarrow\ &4\beta^2(\alpha k)^2+\left(-4\beta^2-8(1-\beta)^3+8\beta(1-\beta)^2\right)\alpha k+\beta^2+4(1-\beta)^4-4\beta(1-\beta)^2 &&>0.
\end{aligned}$$

  Since the left-hand side is convex quadratic in αk and is equal to $-4(1-\beta)^3\beta<0$ when $\alpha k=0.5$, (EC.52) is equivalent to (EC.51).

- If $\beta \geq \frac{1}{2}$, the left-hand side of (EC.52) is greater than 0 if and only if $\alpha k > \frac{-2\beta^2+5\beta-2}{2\beta}$. Hence, we discuss in two cases:

  — When $\alpha k \leq \frac{-2\beta^2+5\beta-2}{2\beta}$, (EC.52) does not hold.

  — When $\alpha k > \frac{-2\beta^2+5\beta-2}{2\beta}$, (EC.52) is equivalent to (EC.51). For (EC.51) to hold, we need:

$$
\begin{aligned}
&\tfrac{-2\beta^2+5\beta-2}{2\beta} && < \frac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)^{\frac{3}{2}}\sqrt{-3\beta^3+8\beta^2-5\beta+1}\right]\\
\Leftrightarrow\quad & \beta(-2\beta^2+5\beta-2) && < \left(-4\beta^3+11\beta^2-8\beta+2\right)-2(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}\\
\Leftrightarrow\quad & (1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)} && < (1-\beta)^3\\
\Leftrightarrow\quad & -3\beta^3+8\beta^2-5\beta+1 && < (1-\beta)^3\\
\Leftrightarrow\quad & \beta(-2\beta^2+5\beta-2) && < 0.
\end{aligned}
$$

  However, $-2\beta^2+5\beta-2 \geq 0$ for all $\frac{1}{2} \leq \beta < 1$. Therefore, (EC.52) does not hold.

Combining the two cases ($\beta \leq \frac{1}{2}$ and $\beta > \frac{1}{2}$) completes the proof. $\square$

Lemma EC.17. *When* $0 < \beta < \frac{1}{2}$, $\frac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}\right]$ *is decreasing in* β.

Proof. Taking the first-order derivative, we have

$$
\begin{aligned}
&\frac{d\frac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}\right]}{d\beta}\\
=&\frac{(1-\beta)(-6\beta^4+5\beta^3+16\beta^2-16\beta+4+4\left(\beta^2+\beta-1\right)\sqrt{(1-\beta)(-3\beta^3+8\beta^2-5\beta+1)})}{2\beta^3\sqrt{(1-\beta)(-3\beta^3+8\beta^2-5\beta+1)}}.
\end{aligned}
$$

We prove that the first-order derivative is negative for all $0 < \beta < \frac{1}{2}$ by contradiction. Suppose not, we should have

$$
\begin{aligned}
&\underbrace{-6\beta^4+5\beta^3+16\beta^2-16\beta+4}_{>0} && \geq \underbrace{-4\left(\beta^2+\beta-1\right)}_{>0}\sqrt{(1-\beta)(-3\beta^3+8\beta^2-5\beta+1)}\\
\Leftrightarrow\quad & (-6\beta^4+5\beta^3+16\beta^2-16\beta+4)^2 && \geq 4^2\left(\beta^2+\beta-1\right)^2(1-\beta)(-3\beta^3+8\beta^2-5\beta+1)\\
\Leftrightarrow\quad & (2-\beta)\beta^3(1-2\beta)(6\beta^3+5\beta^2-6\beta+4) \leq 0, &&
\end{aligned}
$$

which is a contradiction since $6\beta^3+5\beta^2-6\beta+4 > 0$ for all $0 < \beta < \frac{1}{2}$. $\square$

Proof of Theorem 3. Without loss of generality, denote Firm 1 such that $u_1^\beta \leq u_2^\beta$. Suppose *a priori* that $\pi_1^\beta(u_1^\beta; u_2^\beta) \geq 0$, an assumption we will check *a posteriori*. By Lemma EC.12, if $\alpha u_2^\beta \geq 1$, $\alpha u_1^\beta \leq 1$. Combining Lemmas EC.10 and EC.11, the profit expressions when $u_1 < u_2$ are

$$
\pi_1^\beta(u_1;u_2)=\begin{cases}(1-\beta)(1-\frac{\alpha u_1}{2})-\frac{k}{u_1} & u_2 \leq \Phi(u_1)\\ \frac{(\alpha u_1+\beta(2-2\alpha u_2)-2)^2}{16\alpha\beta u_2}-\frac{k}{u_1} & \Phi(u_1) < u_2 \leq \max\{\Phi(u_1),\frac{2\beta-\alpha u_1+2}{2\alpha\beta}\}\\ \frac{(2-\alpha u_1)(\alpha u_2-1)}{2\alpha u_2}-\frac{k}{u_1} & u_2 > \max\{\Phi(u_1),\frac{2\beta-\alpha u_1+2}{2\alpha\beta}\}\end{cases}, \tag{EC.53}
$$

$$
\pi_2^\beta(u_2;u_1)=\begin{cases}(1-\beta)v(u_2,1)-\frac{k}{u_2} & u_2 \leq \Phi(u_1)\\ \frac{(2\beta+\alpha u_1+2\alpha\beta u_2-2)^4}{512\alpha^3\beta^3 u_2^3}+(1-\beta)v(u_2,1)-\frac{k}{u_2} & \Phi(u_1) < u_2 \leq \max\{\Phi(u_1),\frac{2\beta-\alpha u_1+2}{2\alpha\beta}\}\\ \frac{(\alpha u_1+2\alpha\beta u_2-2)^2}{8\alpha^3\beta u_2^3}+(1-\beta)v(u_2,1)-\frac{k}{u_2} & u_2 > \max\{\Phi(u_1),\frac{2\beta-\alpha u_1+2}{2\alpha\beta}\}\end{cases}, \tag{EC.54}
$$

in which, by (2), $v(u_2,1)=1-\frac{\alpha u_2}{2}$ if $u_2\le\frac{1}{\alpha}$ and $v(u_2,1)=\frac{1}{2\alpha u_2}$ otherwise, and $\Phi(u_1)$ is defined by (EC.26).

When $u_1=u_2$, the profit expressions are obtained either directly from (EC.48) if $\alpha u_1\ge 1$ or using a similar argument if $\alpha u_1<1$ to yield:

$$\pi_i^\beta(u_i;u_{-i})=(1-\beta)v(u_i,1)-\frac{k}{u_i}. \tag{EC.55}$$

Define

$$t_1^{-1}(\beta):=\frac{1-\beta}{2} \tag{EC.56}$$

$$t_2^{-1}(\beta):=\frac{1}{\beta^2}\left[0.5\left(-4\beta^3+11\beta^2-8\beta+2\right)-(1-\beta)\sqrt{(1-\beta)\left(-3\beta^3+8\beta^2-5\beta+1\right)}\right]. \tag{EC.57}$$

By Lemma EC.17, $t_2^{-1}(\beta)$ is decreasing in β when $\beta\in(0,1/2)$. Hence, when $\beta<1/2$, $\alpha k<t_2^{-1}(\beta)$ if and only if $\beta<t_2(\alpha k)$.

Next, we next characterize the equilibrium in two regions of the parameter values, and check that $\pi_1^\beta(u_1^\beta;u_2^\beta)\ge 0$.

Region 1:

$$0<\beta<\frac{1}{2} \text{ and } \alpha k<t_2^{-1}(\beta). \tag{EC.58}$$

By Lemma EC.15, if (EC.58) holds, $\alpha k<t_1^{-1}(\beta)$. Consider the point

$$u_1^{\beta,1}=u_2^{\beta,1}=u^{\beta,1}:=\sqrt{\frac{2k}{(1-\beta)\alpha}}. \tag{EC.59}$$

Since $\alpha k<t_1^{-1}(\beta)$, $u^{\beta,1}<\frac{1}{\alpha}$. Accordingly, by (EC.55), the corresponding profits are

$$\pi_1^{\beta,1}=\pi_2^{\beta,1}=(1-\beta)-\sqrt{2(1-\beta)\alpha k}, \tag{EC.60}$$

which are positive since $\alpha k<t_1^{-1}(\beta)$.

Because

$$u^{\beta,1}\le\Phi(u^{\beta,1})\Leftrightarrow\frac{\alpha u^{\beta,1}-2(1-\beta)}{2\alpha\beta}\le\sqrt{\frac{(1-\beta)(2-\alpha u^{\beta,1})u^{\beta,1}}{\alpha\beta}}\Rightarrow\alpha u^{\beta,1}\le\frac{2+2\beta-4\beta^2+4\sqrt{2}(1-\beta)\beta}{1+4(1-\beta)\beta}$$

and because the latter condition holds when $\alpha k<0.5$ and (EC.58) holds by Lemma EC.14, we obtain that $u_2^{\beta,1}\le\Phi(u_1^{\beta,1})$.

Next, we next show that (EC.59) constitutes an equilibrium by showing that no firm has an incentive to deviate. First, $u^{\beta,1}$ maximizes $(1-\beta)\left(1-\frac{\alpha u_i}{2}\right)-\frac{k}{u_i}$ for $i\in\{1,2\}$, so it attains a local maximum for both firms (since $u^{\beta,1}<\frac{1}{\alpha}$). In fact, by (EC.53)-(EC.54), this is a dominant strategy for Firm i as long as $u_i\le u_{-i}^{\beta,1}$ and $u_{-i}^{\beta,1}\le\Phi(u_i)$ or as long as $u_i>u_{-i}$ and $u_i\le\Phi(u_{-i}^{\beta,1})$, for any $i=1,2$. In the following, we assume, without loss of generality (since firms choose the same cycle in equilibrium), that $u_1\le u_2$.

Second, Firm 1 has no incentive to set u_1 such that $u_2^{\beta,1}>\Phi(u_1)$ while keeping $u_1\le u_2^{\beta,1}$ because, since $\beta<1/2$ by (EC.58),

$$\begin{aligned}
u_2^{\beta,1}>\Phi(u_1)\Rightarrow&\qquad u_2^{\beta,1}>\frac{2-2\beta-\alpha u_1}{2\alpha\beta}+u_1\\
\Leftrightarrow&\quad(1-2\beta)\alpha u_1>2(1-\beta)-2\alpha\beta u_2^{\beta,1}\\
\Rightarrow&(1-2\beta)\alpha u_2^{\beta,1}>2(1-\beta)-2\alpha\beta u_2^{\beta,1}\\
\Leftrightarrow&\qquad u_2^{\beta,1}>\frac{2(1-\beta)}{\alpha},
\end{aligned}$$

which never holds since $u_2^{\beta,1} < \frac{1}{\alpha}$

Third, Firm 2 has no incentive to set u_2 such that $u_2 > \Phi(u_1^{\beta,1})$. To see this, let $u_2^l(u_1) := \frac{2-2\beta-\alpha u_1}{2\alpha\beta} + u_1$ and note that $\Phi(u_1) \geq u_2^l(u_1)$. Because $u_2^l(u_1)$ is linear and $\beta < 0.5$, $u_2^l(u_1^{\beta,1}) > u_2^l(0) = \frac{1-\beta}{\alpha\beta}$. For any u_2, Firm 2's profit is bounded from above by the average net information minus the publishing cost, i.e., by $\frac{1}{2\alpha u_2} - \frac{k}{u_2}$, which is decreasing in u_2 since $\alpha k < 0.5$ by assumption. Hence, for any $u_2 > \Phi(u_1)$, since $\Phi(u_1^{\beta,1}) \geq u_2^l(u_1^{\beta,1}) > u_2^l(0)$, $\pi_2(u_2; u_1^{\beta,1}) < \frac{1}{2\alpha u_2^l(0)} - \frac{k}{u_2^l(0)}$. On the other hand, by Lemma EC.16,

$$(1-\beta) - \sqrt{2(1-\beta)\alpha k} > \frac{1}{2\alpha u_2^l(0)} - \frac{k}{u_2^l(0)}$$

if and only if (EC.58) holds. Since $\pi_2^{\beta,1} = (1-\beta) - \sqrt{2(1-\beta)\alpha k}$ by (EC.60), we conclude that Firm 2 does not have a profitable deviation such that $u_2 > \Phi(u_1^{\beta,1})$.

Hence, no deviation is profitable, therefore, $(u_1^{\beta,1}, u_2^{\beta,1})$ is a Nash equilibrium. One can show its uniqueness by applying a similar logic to the argument above; details are omitted.

Region 2:

$$0 < \beta \leq 1 \text{ and } \frac{1-\beta}{2} \leq \alpha k < 0.5. \tag{EC.61}$$

Consider the pair $\left(u_1^{\beta,2}, u_2^{\beta,2}\right)$ solving (EC.49). By Lemma EC.13, when (EC.61) holds, the solution $\left(u_1^{\beta,2}, u_2^{\beta,2}\right)$ is unique on $0 < u_1 < \frac{1}{\alpha}$, $\sqrt{\frac{2k}{\alpha}} < u_1^{\beta,2} < \min\left\{2\sqrt{\frac{k}{\alpha}}, \frac{1}{\alpha}\right\}$, and $u_2^{\beta,2} > \frac{2}{\alpha}$. Moreover, $\left(u_1^{\beta,2}, u_2^{\beta,2}\right)$ is feasible in (10) since $2u_1^{\beta,2} < u_2^{\beta,2}$. Furthermore,

$$\begin{aligned}
\Phi(u_1^{\beta,2}) &= \frac{1}{2\alpha\beta}\left(2\left(1-\beta+\sqrt{\beta(1-\beta)\alpha u_1^{\beta,2}(2-\alpha u_1^{\beta,2})}\right) - \alpha u_1^{\beta,2}(1-2\beta)\right) \\
&\leq \frac{1}{2\alpha\beta}\left(2\left(1-\beta+\sqrt{(1-\beta)\beta}\right) - \alpha u_1^{\beta,2}(1-2\beta)\right) && \text{since } \alpha u_1^{\beta,2}(2-\alpha u_1^{\beta,2}) < 1 \\
&< \frac{1}{2\alpha\beta} \underbrace{\left(2\left(1-\beta+\sqrt{(1-\beta)\beta}\right)+1\right)}_{\text{concave, maximized at } \beta=0.5\text{, and the maximum is } 3} && \text{since } \alpha u_1^{\beta,2} < 1 \text{ and } \beta \leq 1 \\
&< \frac{6-3\alpha u_1^{\beta,2}}{2\alpha\beta} && \text{since } u_1^{\beta,2} < 1/\alpha \\
&\leq \frac{(2-\alpha u_1^{\beta,2})\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)}{2\alpha\beta(1-2\alpha k)} && \text{since } \frac{d\left(\frac{2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}}{1-2\alpha k}\right)}{d\alpha k} \geq 0 \\
& && \text{and } \alpha k \geq \frac{1-\beta}{2} \text{ under (EC.61)} \\
&= u_2^{\beta,2}(u_1^{\beta,2}) && \text{by (EC.49).}
\end{aligned}$$

Therefore, $u_2^{\beta,2} > \Phi(u_1^{\beta,2})$. On the other hand,

$$\begin{aligned}
u_2^{\beta,2} &= \frac{(2-\alpha u_1^{\beta,2})\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)}{2\alpha\beta(1-2\alpha k)} && \text{by (EC.49)} \\
&\geq \frac{6-3\alpha u_1^{\beta,2}}{2\alpha\beta} && \text{under (EC.61)} \\
&> \frac{2+2\beta-\alpha u_1^{\beta,2}}{2\alpha\beta} && \text{since } \beta \leq 1 \text{ and } \alpha u_1^{\beta,2} < 1 \text{ by Lemma EC.13.}
\end{aligned}$$

Thus, by plugging $u_2^{\beta,2}(u_1^{\beta,2})$ from (EC.49) into the profit functions (EC.53)-(EC.54) when $u_2 > \Phi(u_1)$ and $u_2 > \frac{2\beta-\alpha u_1+2}{2\alpha\beta}$, we obtain the equilibrium profits as a function of $u_1^{\beta,2}$:

$$\pi_1^{\beta,2}(u_1^{\beta,2}) = 1 - \frac{\alpha u_1^{\beta,2}}{2} - \frac{1}{3}\left(2\beta - \sqrt{\beta(4\beta+6\alpha k-3)}\right) - \frac{k}{u_1^{\beta,2}}; \tag{EC.62}$$

$$\pi_2^{\beta,2}(u_1^{\beta,2}) = \frac{2\beta^2(1-2\alpha k)^2\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}+2\alpha k-1\right)}{(2-\alpha u_1^{\beta,2})\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)^3}. \tag{EC.63}$$

One the one hand, note that $\pi_1^{\beta,2}\left(\frac{1}{\alpha}\right) = \frac{1}{2} - \frac{1}{3}\left(2\beta - \sqrt{\beta(4\beta+6\alpha k-3)}\right) - \alpha k$ is positive since it is strictly decreasing in αk and equal to 0 when $\alpha k = 0.5$. Because $\pi_1^{\beta,2}(u_1)$ is strictly decreasing for $u_1 > \sqrt{2k/\alpha}$ and $\sqrt{2k/\alpha} < u_1^{\beta,2} < 1/\alpha$ by Lemma EC.13, we have $\pi_1^{\beta,2}(u_1^{\beta,2}) > \pi_1^{\beta,2}\left(\frac{1}{\alpha}\right) > 0$. On the other hand, $\pi_2^{\beta,2}(u_1^{\beta,2}) > 0$ since $2\beta + \sqrt{\beta(4\beta+6\alpha k-3)} + 2\alpha k - 1$ is increasing in αk and is thus greater than $2\beta > 0$ (evaluated at $\alpha k = (1-\beta)/2$, the lower bound on αk under (EC.61)).

Next, we next show that $\left(u_1^{\beta,2}, u_2^{\beta,2}\right)$ constitutes an equilibrium by showing that no firm has an incentive to deviate. We consider the following scenarios:

(i) $u_2 > \Phi(u_1)$ and $u_2 \geq 2u_1$;

(ii) $u_1 > \Phi(u_2)$ and $u_1 \geq 2u_2$;

(iii) $u_2 \leq \Phi(u_1)$ and $u_2 \geq 2u_1$;

(iv) $u_1 \leq \Phi(u_2)$ and $u_1 \geq 2u_2$;

(v) $u_1 = u_2$.

Case (i): $u_2 > \Phi(u_1)$ and $u_2 \geq 2u_1$. We consider in turn deviations by Firm 2 and Firm 1 within the region $u_2 > \Phi(u_1)$ and $u_2 \geq 2u_1$. First, consider Firm 2's profit expressions (EC.54) when $u_2 > \Phi(u_1^{\beta,2})$ and $u_2 \geq 2u_1^{\beta,2}$.

- When $u_2 \leq \frac{2\beta-\alpha u_1^{\beta,2}+2}{2\alpha\beta}$,

$$\frac{\partial \pi_2(u_2;u_1^{\beta,2})}{\partial u_2} = \begin{cases} \frac{(-3\alpha u_1^{\beta,2}+2\beta(\alpha u_2-3)+6)(\alpha u_1^{\beta,2}+2(\beta+\alpha\beta u_2-1))^3}{512\alpha^3\beta^3u_2^4} - \frac{\alpha(1-\beta)}{2} + \frac{k}{u_2^2} & u_2 \leq \frac{1}{\alpha} \\ \frac{(-3\alpha u_1^{\beta,2}+2\beta(\alpha u_2-3)+6)(\alpha u_1^{\beta,2}+2(\beta+\alpha\beta u_2-1))^3}{512\alpha^3\beta^3u_2^4} + \frac{1}{u_2^2}\left(k - \frac{1-\beta}{2\alpha}\right) & u_2 > \frac{1}{\alpha} \end{cases}.$$

  Note that

  — $\alpha u_1^{\beta,2} + 2(\beta+\alpha\beta u_2 - 1) > \alpha u_1^{\beta,2} + 2(\beta-1) + 2(1-\beta) - \alpha u_1^{\beta,2} + 2\alpha\beta u_1^{\beta,2} \geq 0$ since $u_2 > \Phi(u_1^{\beta,2}) \geq \frac{2-2\beta-\alpha u_1^{\beta,2}}{2\alpha\beta} + u_1^{\beta,2}$;

  — $-3\alpha u_1^{\beta,2} + 2\beta(\alpha u_2 - 3) + 6 > -3\alpha u_1^{\beta,2} + 2\beta(2\alpha u_1^{\beta,2} - 3) + 6 = 3(1-\beta)(2-\alpha u_1^{\beta,2}) + \beta\alpha u_1^{\beta,2} > 0$ since $u_2 \geq 2u_1^{\beta,2}$;

  — $-\frac{\alpha(1-\beta)}{2} + \frac{k}{u_2^2} \geq -\frac{\alpha(1-\beta)}{2} + \alpha^2 k \geq 0$ under (EC.61);

  — $k - \frac{1-\beta}{2\alpha} \geq 0$ under (EC.61).

  As a result, $\pi_2(u_2; u_1^{\beta,2})$ monotonically increases in u_2 when $u_2 \leq \frac{2\beta-\alpha u_1^{\beta,2}+2}{2\alpha\beta}$. Therefore, $u_2 > \frac{2\beta-\alpha u_1^{\beta,2}+2}{2\alpha\beta}$ in equilibrium.

- When $u_2 > \frac{2\beta-\alpha u_1^{\beta,2}+2}{2\alpha\beta}$,

$$\frac{\partial \pi_2(u_2;u_1^{\beta,2})}{\partial u_2} = \frac{-4\alpha\beta u_2(\alpha u_2(1-2\alpha k) + 2\alpha u_1^{\beta,2} - 4) - 3(\alpha u_1^{\beta,2}-2)^2}{8\alpha^3\beta u_2^4}.$$

The numerator is concave quadratic in u_2 and $\partial\pi_2(u_2;u_1^{\beta,2})/\partial u_2$ is positive at $u_2=(2\beta-\alpha u_1^{\beta,2}+2)/(2\alpha\beta)$. Thus, the first-order optimality conditions are necessary and sufficient (within the region $u_2>\Phi(u_1^{\beta,2})$ and $u_2>u_1^{\beta,2}$), yielding $u_2^{\beta,2}(u_1^{\beta,2})$ as given by (EC.49).

As a result, Firm 2 has no incentive to deviate from $u_2^{\beta,2}$ within the region $u_2>\Phi(u_1^{\beta,2})$ and $u_2\geq 2u_1^{\beta,2}$.

Second, consider Firm 1's profit expressions (EC.53) when $u_2^{\beta,2}>\Phi(u_1)$ and $u_2^{\beta,2}\geq 2u_1$.

- If $0<\beta\leq\frac{\sqrt{2}}{2}$, since

$$\begin{aligned}
u_2^{\beta,2}(u_1^{\beta,2}) &= \frac{(2-\alpha u_1^{\beta,2})\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)}{2\alpha\beta(1-2\alpha k)} \\
&> \frac{2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}}{2\alpha\beta(1-2\alpha k)} && \text{since } u_1^{\beta,2}<1/\alpha \\
&\geq \frac{2+\frac{1}{\beta}}{2\beta\alpha} && \text{under (EC.61)} \\
&\geq \frac{1+\beta}{\beta\alpha} && \text{when } \beta\leq\frac{\sqrt{2}}{2},
\end{aligned}$$

the expression $\frac{2(1+\beta)-2\beta\alpha u_2^{\beta,2}}{\alpha}$ is negative. Hence, $u_1>\frac{2(1+\beta)-2\beta\alpha u_2^{\beta,2}}{\alpha}$. Thus, differentiating Firm 1's profit in (EC.53) when $u_1>\frac{2(1+\beta)-2\beta\alpha u_2^{\beta,2}}{\alpha}$, we have

$$\frac{\partial\pi_1(u_1;u_2^{\beta,2})}{\partial u_1}=\frac{1}{2}\left(-\alpha+\frac{2k}{u_1^2}+\frac{1}{u_2^{\beta,2}}\right).$$

Since $\frac{\partial\pi_1(u_1;u_2^{\beta,2})}{\partial u_1}$ monotonically decreases in u_1, and

$$\lim_{u_1\to 0}\frac{\partial\pi_1(u_1;u_2^{\beta,2})}{\partial u_1}>0,\quad \frac{\partial\pi_1(u_1;u_2^{\beta,2})}{\partial u_1}\Big|_{u_1=u_2^{\beta,2}}=\frac{2\alpha k-(\alpha u_2^{\beta,2})^2+\alpha u_2^{\beta,2}}{2\alpha(u_2^{\beta,2})^2}<\frac{1-(\alpha u_2^{\beta,2})^2+\alpha u_2^{\beta,2}}{2\alpha(u_2^{\beta,2})^2}<0$$

when $\alpha k<0.5$ and $\alpha u_2^{\beta,2}\geq(1+\beta)/\beta$, Firm 1's best response (within $u_2^{\beta,2}>\Phi(u_1)$ and $u_2^{\beta,2}\geq 2u_1$) can be obtained by solving the first-order optimality condition, yielding $u_1^{\beta,2}(u_2^{\beta,2})$ as given by (EC.49).

- If $\frac{\sqrt{2}}{2}<\beta\leq 1$, differentiate Firm 1's profit in (EC.53) when $u_2>\Phi(u_1)$ with respect to u_1:

$$\frac{\partial\pi_1(u_1;u_2^{\beta,2})}{\partial u_1}=\begin{cases}\frac{k}{u_1^2}+\frac{\alpha u_1+\beta(2-2\alpha u_2^{\beta,2})-2}{8\beta u_2^{\beta,2}} & u_1\leq\frac{2\beta-2\alpha\beta u_2^{\beta,2}+2}{\alpha\beta}\\ \frac{1}{2}\left(-\alpha+\frac{2k}{u_1^2}+\frac{1}{u_2^{\beta,2}}\right) & u_1>\frac{2\beta-2\alpha\beta u_2^{\beta,2}+2}{\alpha\beta}\end{cases}.$$

Firm 1's profit is pseudo-concave. Thus, Firm 1 has a single best response, which can be obtained from solving the first-order optimality condition. When $\beta=1$, since $2\beta-2\alpha\beta u_2^{\beta,2}+2=4-2\alpha u_2^{\beta,2}<0$, there does not exist any $u_1>0$ such that $u_1\leq(2\beta-2\alpha\beta u_2^{\beta,2}+2)/(\alpha\beta)$. Thus, we focus on $0<\beta<1$ for this case.

—If $u_1\leq\frac{2\beta-2\alpha\beta u_2^{\beta,2}+2}{\alpha\beta}$ (which implies that $u_2^{\beta,2}<\frac{1+\beta}{\beta}$), $\pi_1(u_1;u_2^{\beta,2})$ is strictly concave since

$$\begin{aligned}
\frac{\partial^2\pi_1(u_1;u_2^{\beta,2})}{\partial u_1^2} &= \frac{\alpha}{8\beta u_2^{\beta,2}}-\frac{2k}{u_1^3} \\
&< \frac{\alpha^2}{16\beta}-\frac{2k}{u_1^3} && \text{since } \alpha u_2^{\beta,2}>2 \\
&< \frac{\alpha^2}{16\beta}-\frac{\alpha^3 k}{4(1-\beta)^3} && \text{since } u_1\leq\frac{2(1+\beta)-2\beta\alpha u_2^{\beta,2}}{\alpha}<\frac{2(1-\beta)}{\alpha} \\
&\leq \frac{\alpha^2}{16}\left(\frac{1}{\beta}-\frac{2}{(1-\beta)^2}\right) && \text{since } \alpha k\leq\frac{1-\beta}{2} \text{ under (EC.61)} \\
&< 0 && \text{since } \sqrt{2}/2<\beta<1.
\end{aligned}$$

Evaluating $\frac{\partial \pi_1(u_1;u_2^{\beta,2})}{\partial u_1}$ at the threshold, we have

$$\begin{aligned}
\frac{\partial \pi_1(u_1;u_2^{\beta,2})}{\partial u_1}\Big|_{u_1=\frac{2(1+\beta)-2\beta\alpha u_2^{\beta,2}}{\alpha}} &= \frac{1}{2}\left(\alpha\left(\frac{2\alpha k}{(2\beta-2\alpha\beta u_2^{\beta,2}+2)^2}-1\right)+\frac{1}{u_2^{\beta,2}}\right)\\
&> \frac{1}{2}\left(\alpha\left(\frac{2\alpha k}{(2\beta-4\beta+2)^2}-1\right)+\frac{\beta\alpha}{1+\beta}\right) && \text{as } 2<\alpha u_2^{\beta,2}<\frac{1+\beta}{\beta}\\
&\geq \frac{1}{2}\left(\alpha\left(\frac{1-\beta}{(2\beta-4\beta+2)^2}-1\right)+\frac{\beta\alpha}{1+\beta}\right) && \text{under (EC.61)}\\
&= \frac{\alpha\beta}{4(1-\beta)(1+\beta)}\\
&> 0.
\end{aligned}$$

Therefore, $\frac{\partial \pi_1(u_1;u_2^{\beta,2})}{\partial u_1}>0$, and Firm 1's profit monotonically increases for all $u_1 \leq \frac{2\beta-2\alpha\beta u_2^{\beta,2}+2}{\alpha\beta}$. Hence, in equilibrium, $u_1 > (2\beta-2\alpha\beta u_2^{\beta,2}+2)/(\alpha\beta)$.

—If $u_1 > \frac{2\beta-2\alpha\beta u_2^{\beta,2}+2}{\alpha\beta}$, the profit-maximizing publication cycle $u_1^{\beta,2}(u_2^{\beta,2})$ is given by (EC.49).

As a result, Firm 1 has no incentive to deviate from $u_1^{\beta,2}$ within the region $u_2^{\beta,2}>\Phi(u_1)$ and $u_2^{\beta,2}\geq 2u_1$.

Case (ii): $u_1>\Phi(u_2)$ and $u_1 \geq 2u_2$. We consider, in turn, deviations by Firm 1 and Firm 2 within the region $u_1>\Phi(u_2)$ and $u_1\geq 2u_2$. First, we check that Firm 1 has no incentive to set u_1 such that $u_1>\Phi(u_2^{\beta,2})$ and $u_1>u_2^{\beta,2}$. Suppose that $u_1\geq 2u_2^{\beta,2}$. Since $u_2^{\beta,2}>\frac{1}{\alpha}$, Firm 1's profit is bounded by the average information accumulated, given by (2), minus the publication cost, thus,

$$\begin{aligned}
\pi_1(u_1;u_2^{\beta,2}) &\leq \frac{1}{2\alpha u_1}-\frac{k}{u_1}\\
&\leq \frac{1}{4\alpha u_2^{\beta,2}}-\frac{k}{2u_2^{\beta,2}} && \text{since } \alpha k<0.5 \text{ and } u_1\geq 2u_2^{\beta,2}\\
&= \frac{\beta(1-2\alpha k)^2}{2(2-\alpha u_1^{\beta,2})\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)} && \text{using (EC.49)}\\
&< \frac{\beta(1-2\alpha k)^2}{2\left(2\beta+\sqrt{\beta(4\beta+6\alpha k-3)}\right)} && \text{since } u_1^{\beta,2}<1/\alpha\\
&= \underbrace{\frac{\beta-2\alpha k\beta}{\sqrt{\beta(4\beta+6\alpha k-3)}}}_{\leq\beta\leq 1 \text{ under (EC.61)}}\cdot\underbrace{\frac{1}{6}\left(-4\beta+2\sqrt{\beta(4\beta+6\alpha k-3)}-3-6\alpha k+6\right)}_{>0 \text{ since this term is increasing in } \alpha k \text{ and by (EC.61)}}\\
&\leq \frac{1}{6}\left(-4\beta+2\sqrt{\beta(4\beta+6\alpha k-3)}-3\alpha u_1^{\beta,2}-\frac{6k}{u_1^{\beta,2}}+6\right) && \text{since } \frac{d\left(-3\alpha u_1-\frac{6k}{u_1}\right)}{du_1}<0 \text{ for } u_1>\sqrt{\frac{2k}{\alpha}}\\
& && \text{and } u_1^{\beta,2}>\sqrt{2k/\alpha} \text{ by Lemma EC.13}\\
&= \pi_1^{\beta,2}(u_1^{\beta,2};u_2^{\beta,2}).
\end{aligned}$$

Therefore, Firm 1 has no incentive to set u_1 such that $u_1>\Phi(u_2^{\beta,2})$ and $u_1>u_2^{\beta,2}$. Next, we show that Firm 2 has no incentive to set u_2 such that $u_1^{\beta,2}>\Phi(u_2)$ and $u_1^{\beta,2}>u_2$. When $u_1^{\beta,2}>u_2$, Firm 2's profits are similar to Firm 1's profit in (EC.53). Using that $u_1^{\beta,2}>\Phi(u_2)$ and that $u_2<u_1^{\beta,2}\leq\frac{2+2\beta-\alpha u_1^{\beta,2}}{2\alpha\beta}$ (since $u_1^{\beta,2}<1/\alpha$) yields:

$$\pi_2\left(u_2;u_1^{\beta,2}\right)=\frac{(\alpha u_2+\beta(2-2\alpha u_1^{\beta,2})-2)^2}{16\alpha\beta u_1^{\beta,2}}-\frac{k}{u_2}. \tag{EC.64}$$

If $\beta<0.5$, since $u_1^{\beta,2}>\Phi(u_2)>\frac{2-2\beta-\alpha u_2}{2\alpha\beta}+u_2$ and $u_1^{\beta,2}<1/\alpha$, we have

$$\frac{1}{\alpha}>\frac{2-2\beta-\alpha u_2}{2\alpha\beta}+u_2 \Rightarrow \alpha u_2>1,$$

which is a contradiction since $u_2 < u_1^{\beta,2} \leq 1/\alpha$ by assumption. Therefore, we restrict our subsequent analysis to $0.5 \leq \beta \leq 1$. Thus, using (EC.64), consider the following cubic equation:

$$\begin{aligned}h(u_2) &:= \pi_2(u_2; u_1^{\beta,2}) \cdot 16\alpha\beta u_1^{\beta,2} u_2 \\ &= \alpha^2 u_2^3 - 4\alpha(1-\beta(1-\alpha u_1^{\beta,2}))u_2^2 + 4\left(1-\beta(1-\alpha u_1^{\beta,2})\right)^2 u_2 - 16\alpha\beta k u_1^{\beta,2}.\end{aligned}$$

On the one hand, $h(0) < 0$. On the other hand, $h\left(\frac{2(1-\beta(1-\alpha u_1^{\beta,2}))}{\alpha}\right) < 0$ and $\frac{2(1-\beta(1-\alpha u_1^{\beta,2}))}{\alpha} \geq 2u_1^{\beta,2}$. Hence, for Firm 2 to make a positive profit somewhere in $u_2 \in (0, u_1^{\beta,2})$, the discriminant of $h(u_2)$ must be positive. The discriminant is equal to $256\alpha^4\beta k u_1^{\beta,2}(1-2\alpha k)\Delta(\beta)$, in which

$$\Delta(\beta) := -\beta\left(3\alpha u_1^{\beta,2}(9\alpha k - 2) - 2\beta^2(\alpha u_1^{\beta,2} - 1)^3 - 6\beta(\alpha u_1^{\beta,2} - 1)^2 + 6\right) + 2.$$

It is easy to check that $\Delta(\cdot)$ is monotonically decreasing in β. Note that

$$\begin{aligned}\Delta\left(\frac{1}{2}\right) &= \frac{1}{4}\left(1 + \alpha u_1^{\beta,2}(3-54\alpha k) + 3(\alpha u_1^{\beta,2})^2 + (\alpha u_1^{\beta,2})^3\right) \\ &\leq \frac{1}{4}\left(1 + 6\alpha k + 3\sqrt{2}\sqrt{\alpha k} - 52\sqrt{2}(\alpha k)^{\frac{3}{2}}\right) && \text{since } \frac{d\left(1 + x(3-54\alpha k) + 3x^2 + x^3\right)}{dx} < 0 \\ & && \text{and } \alpha u_1^{\beta,2} > \sqrt{2\alpha k} \\ &< 0 && \text{when } \frac{1-\beta}{2} \leq \alpha k < 0.5 \text{ and } \beta = \frac{1}{2}.\end{aligned}$$

Thus, $\Delta(\beta) < 0, \forall 0.5 \leq \beta \leq 1$. As a result, $h(u_2) < 0, \forall u_2 < u_1^{\beta,2}, 0.5 \leq \beta \leq 1$, which implies that $\pi_2(u_2; u_1^{\beta,2}) < 0$, $\forall u_2 < u_1^{\beta,2}$, $\forall 0.5 \leq \beta < 1$, i.e., Firm 2 cannot make a positive profit. Hence, Firm 2 has no incentive to set u_2 such that $u_1^{\beta,2} > \Phi(u_2)$ and $u_1^{\beta,2} > u_2$.

Case (iii): $u_2 \leq \Phi(u_1)$ and $u_2 \geq 2u_1$. If $u_2^{\beta,2} \leq \Phi(u_1)$ and $u_2^{\beta,2} \geq 2u_1$, $\pi_1(u_1; u_2^{\beta,2}) = (1-\beta)(1-\alpha u_1/2) - k/u_1$, which is smaller than $\max_u (1-\beta)(1-\alpha u/2) - k/u = (1-\beta) - \sqrt{2(1-\beta)\alpha k} \leq 0$ by (EC.61). Hence, Firm 1 has no incentive to set u_1 such that $u_2^{\beta,2} \leq \Phi(u_1)$ and $u_2^{\beta,2} \geq 2u_1$. Now, consider Firm 2. If $u_2 \leq \Phi(u_1^{\beta,2})$ and $u_2 \geq 2u_1^{\beta,2}$ and $u_2 \leq 1/\alpha$, then $\pi_2(u_2; u_1^{\beta,2}) = (1-\beta)(1-\alpha u_2/2) - k/u_2$, which is also nonpositive using a similar argument. If $u_2 \leq \Phi(u_1^{\beta,2})$ and $u_2 \geq 2u_1^{\beta,2}$ and $u_2 > 1/\alpha$, then $\pi_2(u_2; u_1^{\beta,2}) = (1-\beta)\frac{1}{2\alpha u_2} - k/u_2$, which is strictly increasing in u_2 under (EC.61), showing that there is no best response in that region. Combining these two cases, Firm 2 has no incentive to set u_2 such that $u_2 \leq \Phi(u_1^{\beta,2})$ and $u_2 \geq 2u_1^{\beta,2}$.

Case (iv): $u_1 \leq \Phi(u_2)$ and $u_1 \geq 2u_2$. Consider first Firm 1. Similar to Firm 2 in Case (iii), its profit, when $u_1 \leq \Phi(u_2^{\beta,2})$ and $u_1 \geq 2u_2^{\beta,2}$, is $\pi_1(u_1; u_2^{\beta,2}) = (1-\beta)\frac{1}{2\alpha u_1} - \frac{k}{u_1}$, which is strictly increasing in u_1 under (EC.61), showing that there is no best response in that region. Hence, Firm 1 has no incentive to set u_1 such that $u_1 \leq \Phi(u_2^{\beta,2})$ and $u_1 \geq 2u_2^{\beta,2}$. Now consider Firm 2. Similar to Firm 1 in Case (iii), its profit is $\pi_2(u_2; u_1^{\beta,2}) = (1-\beta)(1-\alpha u_2/2) - k/u_2$, which is nonpositive under (EC.61). Hence, Firm 2 has no incentive to set u_2 such that $u_1^{\beta,2} \leq \Phi(u_2)$ and $u_1 \geq 2u_2^{\beta,2}$.

Case (v): $u_1 = u_2$. By Lemma EC.12, setting $\alpha u_1 \leq 1$ is a best response to $u_2^{\beta,2}$ since $\alpha u_2^{\beta,2} \geq 1$. Hence, Firm 1 has no incentive to set $u_1 = u_2^{\beta,2}$. Now, consider Firm 2. If it sets $u_2 = u_1^{\beta,2}$, then $u_2 < 1/\alpha$. By (EC.55), Firm 2's profit is $\pi_2(u_1^{\beta,2}; u_1^{\beta,2}) = (1-\beta)(1-\alpha u_1^{\beta,2}/2) - k/u_1^{\beta,2}$, which is smaller than or equal to $\max_u (1-\beta)(1-\alpha u/2) - k/u = (1-\beta) - \sqrt{2(1-\beta)\alpha k} \leq 0$ by (EC.61). Hence, Firm 2 has no incentive to set $u_2 = u_1^{\beta,2}$.

Hence, no deviation is profitable. Therefore, assuming that Firm 1 publishes more frequently, $(u_1^{\beta,2}, u_2^{\beta,2})$ is a Nash equilibrium. Since the roles of the firms can be inverted, there exists a pair of Nash equilibria. One can show uniqueness of this pair of equilibria by applying a similar logic to the argument above; details are omitted. □

Proof of Proposition 5. For notational simplicity, we use superscript β in place of $(\beta, 2)$.

- Applying the implicit function theorem to the pair of best responses in (EC.49), we obtain:

$$\begin{bmatrix} 1 & \frac{\sqrt{k}}{\sqrt{2u_2^\beta}(\alpha u_2^\beta-1)^{3/2}} \\ \frac{3}{4\beta-2\sqrt{\beta(4\beta+6\alpha k-3)}} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial u_1^\beta}{\partial\beta} \\ \frac{\partial u_2^\beta}{\partial\beta} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3(2-\alpha u_1^\beta)}{4\alpha\beta\sqrt{\beta(4\beta+6\alpha k-3)}} \end{bmatrix},$$

  yielding

$$\begin{bmatrix} \frac{\partial u_1^\beta}{\partial\beta} \\ \frac{\partial u_2^\beta}{\partial\beta} \end{bmatrix} = \frac{1}{D} \begin{bmatrix} 1 & -\frac{\sqrt{k}}{\sqrt{2u_2^\beta}(\alpha u_2^\beta-1)^{3/2}} \\ -\frac{3}{4\beta-2\sqrt{\beta(4\beta+6\alpha k-3)}} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{3(2-\alpha u_1^\beta)}{4\alpha\beta\sqrt{\beta(4\beta+6\alpha k-3)}} \end{bmatrix}$$
$$= \frac{1}{D}\frac{3(2-\alpha u_1^\beta)}{4\alpha\beta\sqrt{\beta(4\beta+6\alpha k-3)}} \begin{bmatrix} -\frac{\sqrt{k}}{\sqrt{2u_2^\beta}(\alpha u_2^\beta-1)^{3/2}} \\ 1 \end{bmatrix},$$

  where

$$D := 1 - \frac{3}{4\beta-2\sqrt{\beta(4\beta+6\alpha k-3)}} \cdot \frac{\sqrt{k}}{\sqrt{2u_2^\beta}(\alpha u_2^\beta-1)^{3/2}} \tag{EC.65}$$
$$= 1 - \frac{1}{\alpha u_2^\beta - 1}\sqrt{\frac{3\alpha k}{2(2-\alpha u_1^\beta)\left(-4\beta+2\left(\sqrt{\beta(4\beta+6\alpha k-3)}+3\right)-3\alpha u_1^\beta\right)}} \quad \text{using (EC.49)}$$
$$> 0$$

  since $\alpha u_1^\beta \le 1$ by Lemma EC.12, $\alpha u_2^\beta > 2$ by Lemma EC.6, $\alpha k < 0.5$ and

$$-4\beta + 2\left(\sqrt{\beta(4\beta+6\alpha k-3)}+3\right) - 3\alpha u_1^\beta \ge -4\beta + 2(\beta+3) - 3\alpha u_1^\beta \quad \text{under (EC.61)}$$
$$> 1 \quad \text{by Lemma EC.12 and since } \beta < 1.$$

  Thus, $\frac{\partial u_1^\beta}{\partial\beta} < 0$ and $\frac{\partial u_2^\beta}{\partial\beta} > 0$.

- Now, let us consider the profits in (EC.62). By the envelope theorem,

$$\frac{\mathrm{d}\pi_1(u_1^\beta; u_2^\beta)}{\mathrm{d}\beta} = \frac{\partial\pi_1(u_1^\beta; u_2^\beta)}{\partial u_2^\beta}\frac{\partial u_2^\beta}{\partial\beta} + \frac{\partial\pi_1(u_1^\beta; u_2^\beta)}{\partial\beta}.$$

  As, using the profit expressions from (EC.62),

$$\frac{\partial\pi_1(u_1^\beta; u_2^\beta)}{\partial u_2^\beta} = \frac{2-\alpha u_1^\beta}{2\alpha {u_2^\beta}^2} > 0, \quad \frac{\partial u_2^\beta}{\partial\beta} > 0, \quad \frac{\partial\pi_1(u_1^\beta; u_2^\beta)}{\partial\beta} = 0,$$

  we have

$$\frac{\mathrm{d}\pi_1(u_1^\beta; u_2^\beta)}{\mathrm{d}\beta} > 0.$$

  Similarly,

$$\frac{\mathrm{d}\pi_2(u_2^\beta; u_1^\beta)}{\mathrm{d}\beta} = \frac{\partial\pi_2(u_2^\beta; u_1^\beta)}{\partial u_1^\beta}\frac{\partial u_1^\beta}{\partial\beta} + \frac{\partial\pi_2(u_2^\beta; u_1^\beta)}{\partial\beta}.$$

As, using again the profit expressions from (EC.63),

$$\frac{\partial \pi_2(u_2^\beta; u_1^\beta)}{\partial u_1^\beta} = \frac{\alpha u_1^\beta + 2\alpha\beta u_2^\beta - 2}{4\alpha^2 \beta {u_2^\beta}^3} > 0, \quad \frac{\partial u_1^\beta}{\partial \beta} < 0,$$

and

$$\frac{\partial \pi_2(u_1^\beta, u_2^\beta)}{\partial \beta} = -\frac{(2-\alpha u_1^\beta)^2}{8\alpha^3\beta^2 {u_2^\beta}^3} < 0,$$

we have

$$\frac{\mathrm{d}\pi_2(u_1^\beta, u_2^\beta)}{\mathrm{d}\beta} < 0.$$

- It is straightforward that $u_1^{\beta,1}$ and $u_2^{\beta,1}$ increase in β and that the corresponding profits $\pi_1^{\beta,1}$ and $\pi_2^{\beta,1}$ in (EC.60) decrease in β. $\square$

EC.4. Publishers with Different Fixed Costs

THEOREM EC.1. *Suppose that $k_1 \le k_2$. There exists an equilibrium in the publishing game (10) in which Firm 1 publishes faster than Firm 2. The equilibrium publication cycles solve:*

$$u_1^k = \sqrt{\frac{2k_1 u_2^k}{\alpha u_2^k - 1}}, \quad u_2^k = \frac{\left(\sqrt{6\alpha k_2 + 1} + 2\right)(2-\alpha u_1^k)}{2\alpha(1-2\alpha k_2)}. \tag{EC.66}$$

Proof. Since the pricing game does not involve the fixed publication cost, the pricing equilibria are identical to those presented in Lemma EC.2 (when $\alpha u_1 \le 1 < \alpha u_2$) and Lemma EC.5 (when $\alpha u_1 < \alpha u_2 \le 1$), with with k replaced by k_i in $\pi_i(u_i; u_{-i})$ for $i \in \{1, 2\}$.

In the publishing game, the best response functions (EC.66) can be derived as in Theorem 2; following the rationale of Lemma EC.6, the best responses (EC.66) cross only once with $u_1^k \in \left[0, \frac{1}{\alpha}\right]$, and like Theorem 2, the leapfrogging incentives are only weaker because Firm 2 has a larger fixed cost. $\square$

Proof of Proposition 6. • Applying the implicit function theorem to the pair of best responses (EC.66), we get

$$\begin{bmatrix} 1 & \frac{\sqrt{k_1}}{\sqrt{2u_2^k}(\alpha u_2^k - 1)^{3/2}} \\ \frac{3}{4 - 2\sqrt{6\alpha k_2 + 1}} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial u_1^k}{\partial k_1} \\ \frac{\partial u_2^k}{\partial k_1} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{u_2^k}{2k_1(\alpha u_2^k - 1)}} \\ 0 \end{bmatrix},$$

which yields

$$\begin{bmatrix} \frac{\partial u_1^k}{\partial k_1} \\ \frac{\partial u_2^k}{\partial k_1} \end{bmatrix} = \frac{1}{D} \sqrt{\frac{u_2^k}{2k_1(\alpha u_2^k - 1)}} \begin{bmatrix} 1 \\ -\frac{\sqrt{k_1}}{\sqrt{2u_2^k}(\alpha u_2^k - 1)^{3/2}} \end{bmatrix},$$

where $D = 1 - \frac{3}{4 - 2\sqrt{6\alpha k_2 + 1}} \cdot \frac{\sqrt{k_1}}{\sqrt{2u_2^k}(\alpha u_2^k - 1)^{3/2}} > 0$ following the same rationale as in (EC.65).

Thus, $\frac{\partial u_1^k}{\partial k_1} > 0$ and $\frac{\partial u_2^k}{\partial k_1} < 0$. Similarly, one can show that $\frac{\partial u_1^k}{\partial k_2} < 0$ and $\frac{\partial u_2^k}{\partial k_2} > 0$.

- Similar to Lemma EC.2, the equilibrium profits are:

$$\pi_1^k(u_1^k; u_2^k) = \frac{(2-\alpha u_1^k)(\alpha u_2^k - 1)}{2\alpha u_2^k} - \frac{k_1}{u_1^k}; \quad \pi_2^k(u_2^k; u_1^k) = \frac{(\alpha u_1^k + 2\alpha u_2^k - 2)^2}{8\alpha^3 {u_2^k}^3} - \frac{k_2}{u_2^k}.$$

Taking derivatives, we have

$$\frac{\partial \pi_1^k}{\partial u_2^k} = \frac{2 - \alpha u_1^k}{2\alpha {u_2^k}^2} > 0, \quad \frac{\partial \pi_2^k}{\partial u_1^k} = \frac{\alpha u_1^k + 2\alpha u_2^k - 2}{4\alpha^2 {u_2^k}^3} > 0,$$

and, for $i=1,2$,

$$\frac{\partial \pi_i^k}{\partial k_i} = -\frac{1}{u_i^k} < 0.$$

Applying the envelope theorem,

$$\frac{\mathrm{d}\pi_i^k}{\mathrm{d}k_i} = \underbrace{\frac{\partial \pi_i^k}{\partial u_{-i}^k}}_{+} \cdot \underbrace{\frac{\partial u_{-i}^k}{\partial k_i}}_{-} + \underbrace{\frac{\partial \pi_i^k}{\partial k_i}}_{-} < 0$$

$$\frac{\mathrm{d}\pi_{-i}^k}{\mathrm{d}k_i} = \underbrace{\frac{\partial \pi_{-i}^k}{\partial u_i^k}}_{+} \cdot \underbrace{\frac{\partial u_i^k}{\partial k_i}}_{+} + \underbrace{\frac{\partial \pi_{-i}^k}{\partial k_i}}_{0} > 0. \quad \square$$

EC.5. Publishers with Different Time Sensitivity

Theorem EC.2. *Suppose that $\alpha_1 \le \alpha_2$. There exists an equilibrium in the publishing game (10) in which Firm 1 publishes faster than Firm 2. The equilibrium cycles solve:*

$$u_1^\alpha = \sqrt{\frac{2\alpha_2 k u_2^\alpha}{\alpha_1(\alpha_2 u_2^\alpha - 1)}}, \quad u_2^\alpha = \frac{\left(\sqrt{6\alpha_2 k+1}+2\right)(\alpha_1 u_1^\alpha - 2)}{2\alpha_2(2\alpha_2 k - 1)}. \tag{EC.67}$$

Proof. The proof is similar to that of Theorem 2. Similar to Lemma EC.3 and Corollary EC.6, one can show that in equilibrium $\alpha_1 u_1^\alpha \le 1$ and $\alpha_2 u_2^\alpha > 1$. Hence, we obtain, similar to Lemma EC.2, the following equilibrium profits:

$$\pi_1(u_1;u_2) = \begin{cases} \frac{(\alpha_1 u_1 - 2\alpha_2 u_2)^2}{16\alpha_2 u_2} - \frac{k}{u_1} & u_1 \le \frac{4-2\alpha_2 u_2}{\alpha_1} \\ \frac{(2-\alpha_1 u_1)(\alpha_2 u_2 - 1)}{2\alpha_2 u_2} - \frac{k}{u_1} & u_1 > \frac{4-2\alpha_2 u_2}{\alpha_1} \end{cases}; \quad \pi_2(u_1;u_2) = \begin{cases} \frac{(\alpha_1 u_1 + 2\alpha_2 u_2)^4}{512\alpha_2 u_2^3} - \frac{k}{u_2} & u_2 \le \frac{4-\alpha_1 u_1}{2\alpha_2} \\ \frac{(\alpha_1 u_1 + 2\alpha_2 u_2 - 2)^2}{8\alpha_2^3 u_2^3} - \frac{k}{u_2} & u_2 > \frac{4-\alpha_1 u_1}{2\alpha_2} \end{cases}. \tag{EC.68}$$

By analyzing the firms' profits, we obtain their best responses (EC.67), which are analogous to (11). In particular, we have $\sqrt{\frac{2k}{\alpha_1}} < u_1^\alpha < 2\sqrt{\frac{k}{\alpha_1}}$, similar to Lemma EC.6.

The proof of existence and uniqueness of the Nash equilibrium proceeds in three steps: establish that the best responses cross exactly once, show that Firm 2 has no incentive to set $u_2 \le u_1^\alpha$, and show that Firm 1 has no incentive to set $u_1 \ge u_2^\alpha$. All three steps are similar to the proof of Theorem 2. $\square$

Proof of Proposition 7. *Change in α_1.* We first characterize $u_1^\alpha(\alpha_1)$, then $u_2^\alpha(\alpha_1)$, and finally $\pi_1^\alpha(\alpha_1)$ and $\pi_2^\alpha(\alpha_1)$.

- Solving (EC.67) for u_1^α, we obtain that u_1^α must solve $g^\alpha(u_1)=0$, in which

$$g^\alpha(u_1) := -2\alpha_1^2 u_1^2\left(\sqrt{6k\alpha_2 k+1}+1\right) - 6\alpha_1^2 k u_1 + 12\alpha_1 k + 3\alpha_1^3 u_1^3. \tag{EC.69}$$

Since $\frac{\partial g^\alpha(u_1)}{\partial u_1^\alpha}$, equal to $\alpha_1\left(u_1\left(9\alpha_1 u_1 - 4\left(\sqrt{6\alpha_2 k+1}+1\right)\right) - 6k\right)$, is convex in u_1, its maximum is attained at either end of the boundaries. Because $0 < u_1^\alpha < 2\sqrt{\frac{k}{\alpha_1}}$ (see the proof of Theorem EC.2) and $\alpha_2 \ge \alpha_1$,

$$\frac{\partial g(u_1^\alpha)}{\partial u_1^\alpha} \le \max\left\{ \frac{\partial g(u_1^\alpha)}{\partial u_1^\alpha}\Big|_{u_1^\alpha=0}, \frac{\partial g(u_1^\alpha)}{\partial u_1^\alpha}\Big|_{u_1^\alpha=2\sqrt{\frac{k}{\alpha_1}}} \right\}$$

$$= \max\left\{\underbrace{-6\alpha_1 k}_{<0}, 12\alpha_1 k - 4\sqrt{2}\sqrt{\alpha_1 k}\left(\sqrt{6\alpha_2 k+1}+1\right)\right\}$$

$$\leq \max\left\{\underbrace{-6\alpha_1 k}_{<0}, \sqrt{\alpha_1 k}\underbrace{\left(12\sqrt{\alpha_1 k} - 4\sqrt{2}\left(\sqrt{6\cdot\alpha_1 k+1}+1\right)\right)}_{\text{increasing, maximum=-8.4 when } \alpha_1 k=0.5}\right\}$$

$$<0.$$

Thus,

$$\frac{\partial g^\alpha(u_1^\alpha)}{\partial u_1^\alpha} < 0. \tag{EC.70}$$

Since $\frac{1}{u_1}\frac{\partial g^\alpha(u_1)}{\partial \alpha_1}$, equal to $2\left(u_1\left(3\alpha_1 u_1 - \sqrt{6\alpha_2 k+1}-1\right)-3k\right)$, is convex and $0 \leq u_1^\alpha \leq \min\left\{2\sqrt{\frac{k}{\alpha_1}}, \frac{1}{\alpha_1}\right\}$ (see the proof of Theorem EC.2) and $\alpha_2 \geq \alpha_1$, we obtain

$$\frac{1}{u_1^\alpha}\frac{\partial g^\alpha(u_1^\alpha)}{\partial \alpha_1} \leq \max\left\{\frac{1}{u_1^\alpha}\frac{\partial g(u_1^\alpha)}{\partial \alpha_1}\Big|_{u_1^\alpha=0}, \frac{1}{u_1^\alpha}\frac{\partial g(u_1^\alpha)}{\partial \alpha_1}\Big|_{u_1^\alpha=2\sqrt{\frac{k}{\alpha_1}}}\right\}$$

$$= \max\left\{-6k, 6k - 2\sqrt{\frac{2k}{\alpha_1}}\left(\sqrt{6\alpha_2 k+1}+1\right)\right\}$$

$$\leq \max\left\{-6k, \sqrt{\frac{k}{\alpha_1}}\underbrace{\left[6\sqrt{\alpha_1 k} - 2\sqrt{2}\left(\sqrt{6\cdot\alpha_1 k+1}+1\right)\right]}_{\text{concave, maximized at } \alpha_1 k=0.5 \text{ and the maximum is } -4.24}\right\}$$

$$< 0,$$

which implies that

$$\frac{\partial g^\alpha(u_1^\alpha)}{\partial \alpha_1} < 0.$$

Thus, by the implicit function theorem,

$$\frac{\partial u_1^\alpha}{\partial \alpha_1} = -\frac{\frac{\partial g^\alpha(u_1^\alpha)}{\partial \alpha_1}}{\frac{\partial g^\alpha(u_1^\alpha)}{\partial u_1^\alpha}} < 0.$$

- Substituting αu_1 by $x = \alpha u_1$ in (EC.69), we have

$$g^\alpha(x) = 12\alpha_1 k - 2x^2\left(\sqrt{6\alpha_2 k+1}+1\right) - 6\alpha_1 kx + 3x^3.$$

Let $x^\alpha = \alpha u_1^\alpha$. Since

$$\frac{\partial g^\alpha(x^\alpha)}{\partial \alpha_1} = -6k(x^\alpha - 2) > 0$$

and

$$\frac{\partial g^\alpha(x^\alpha)}{\partial x^\alpha} = -6\alpha_1 k - 4x^\alpha\left(\sqrt{6\alpha_2 k+1}+1\right) + 9(x^\alpha)^2$$

$$\leq \max\Big\{-6\alpha_1 k - 4x^\alpha\left(\sqrt{6\alpha_2 k+1}+1\right) + 9(x^\alpha)^2\Big|_{x^\alpha=0},$$

$$-6\alpha_1 k - 4x^\alpha\left(\sqrt{6\alpha_2 k+1}+1\right) + 9(x^\alpha)^2\Big|_{x^\alpha=2\sqrt{\alpha_1 k}}\Big\}$$

$$\leq \max\left\{-6\alpha_1 k, 30\alpha_1 k - 8\sqrt{\alpha_1 k}\left(\sqrt{6\alpha_2 k+1}+1\right)\right\}$$

$$\leq \max\left\{-6\alpha_1 k, \sqrt{\alpha_1 k}\underbrace{\left[30\sqrt{\alpha_1 k} - 8\left(\sqrt{6\cdot\alpha_1 k+1}+1\right)\right]}_{\text{increasing and negative when } \alpha_1 k=0.5}\right\}$$

$$< 0,$$

we obtain, by the implicit function theorem, that

$$\frac{\partial x^\alpha}{\partial \alpha_1} = -\frac{\frac{\partial g^\alpha(x^\alpha)}{\partial \alpha_1}}{\frac{\partial g^\alpha(x^\alpha)}{\partial x^\alpha}} > 0.$$

Because

$$u_2^\alpha = \frac{\left(\sqrt{6\alpha_2 k+1}+2\right)(2-\alpha_1 u_1^\alpha)}{2\alpha_2(1-2\alpha_2 k)} = \frac{\left(\sqrt{6\alpha_2 k+1}+2\right)(2-x^\alpha)}{2\alpha_2(1-2\alpha_2 k)},$$

we have

$$\frac{\partial u_2^\alpha}{\partial \alpha_1} = -\frac{\sqrt{6\alpha_2 k+1}+2}{2\alpha_2(1-2\alpha_2 k)} \cdot \frac{\partial x^\alpha}{\partial \alpha_1} < 0.$$

- Consider Firm 1's profit in (EC.68). By the envelope theorem,

$$\begin{aligned}\frac{\mathrm{d}\pi_1^\alpha}{\mathrm{d}\alpha_1} &= \frac{\partial \pi_1^\alpha}{\partial u_2^\alpha}\frac{\partial u_2^\alpha}{\partial \alpha_1} + \frac{\partial \pi_1^\alpha}{\partial \alpha_1} \\ &= \underbrace{\frac{2-\alpha_1 u_1^\alpha}{2\alpha_2 {u_2^\alpha}^2}}_{+\text{ since } \alpha_1 u_1^\alpha \le 1} \cdot \underbrace{\frac{\partial u_2^\alpha}{\partial \alpha_1}}_{-} + \underbrace{\frac{u_1^\alpha}{2}\left(\frac{1}{\alpha_2 u_2^\alpha}-1\right)}_{-\text{ since } \alpha_2 u_2^\alpha > 1} < 0.\end{aligned}$$

Plugging the inverse of Firm 2's best response in (EC.67) into Firm 2's profit in (EC.68), we obtain

$$g(u_2^\alpha) := \pi_2^\alpha({u_2^\alpha}^{-1}(u_2^\alpha), u_2^\alpha) = \frac{-6\alpha_2 k + \sqrt{6\alpha_2 k+1}+1}{9\alpha_2 u_2^\alpha}.$$

Thus, by taking the total derivatives, we have

$$\begin{aligned}\frac{\mathrm{d}g(u_2^\alpha)}{\mathrm{d}\alpha_1} &= \frac{\partial g(u_2^\alpha)}{\partial u_2^\alpha}\frac{\partial u_2^\alpha}{\partial \alpha_1} + \frac{\partial g(u_2^\alpha)}{\partial \alpha_1} \\ &= \underbrace{-\frac{-6\alpha_2 k + \sqrt{6\alpha_2 k+1}+1}{9\alpha_2 (u_2^\alpha)^2}}_{-} \cdot \underbrace{\frac{\partial u_1^\alpha}{\partial \alpha_1}}_{-} + 0 > 0.\end{aligned}$$

Change in α_2. We first characterize $u_1^\alpha(\alpha_2)$, then $u_2^\alpha(\alpha_2)$, and finally $\pi_1^\alpha(\alpha_2)$ and $\pi_2^\alpha(\alpha_2)$.

- Consider again (EC.69). As

$$\frac{\partial g^\alpha(u_1^\alpha)}{\partial \alpha_2} = -\frac{6\alpha_1 k u_1^2}{\sqrt{6\alpha_2 k+1}} < 0$$

and $\frac{\partial g^\alpha(u_1^\alpha)}{\partial u_1^\alpha} < 0$ by (EC.70), we obtain, by the implicit function theorem,

$$\frac{\partial u_1^\alpha}{\partial \alpha_2} = -\frac{\frac{\partial g^\alpha(u_1^\alpha)}{\partial \alpha_2}}{\frac{\partial g^\alpha(u_1^\alpha)}{\partial u_1^\alpha}} < 0.$$

- Applying the implicit function theorem to (EC.67), we get

$$\begin{bmatrix} 1 & \frac{\alpha_1\sqrt{\alpha_2 k}}{\sqrt{2u_2}(\alpha_1(\alpha_2 u_2-1))^{3/2}} \\ \frac{2\alpha_1+\alpha_1\sqrt{6\alpha_2 k+1}}{2\alpha_2-4\alpha_2^2 k} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial u_1^\alpha}{\partial \alpha_2} \\ \frac{\partial u_2^\alpha}{\partial \alpha_2} \end{bmatrix} = \begin{bmatrix} -\frac{\alpha_1\sqrt{k u_2}}{\sqrt{2\alpha_2}(\alpha_1(\alpha_2 u_2-1))^{3/2}} \\ \frac{\left(2\sqrt{6\alpha_2 k+1}-\alpha_2 k\left(18\alpha_2 k+8\sqrt{6\alpha_2 k+1}+1\right)+1\right)(2-\alpha_1 u_1)}{2\alpha_2^2(2\alpha_2 k-1)^2\sqrt{6\alpha_2 k+1}} \end{bmatrix},$$

which yields that

$$\begin{bmatrix} \frac{\partial u_1^\alpha}{\partial \alpha_2} \\ \frac{\partial u_2^\alpha}{\partial \alpha_2} \end{bmatrix} = \frac{1}{D}\begin{bmatrix} 1 & -\frac{\alpha_1\sqrt{\alpha_2 k}}{\sqrt{2u_2}(\alpha_1(\alpha_2 u_2-1))^{3/2}} \\ -\frac{2\alpha_1+\alpha_1\sqrt{6\alpha_2 k+1}}{2\alpha_2-4\alpha_2^2 k} & 1 \end{bmatrix} \begin{bmatrix} -\frac{\alpha_1\sqrt{k u_2}}{\sqrt{2\alpha_2}(\alpha_1(\alpha_2 u_2-1))^{3/2}} \\ \frac{\left(2\sqrt{6\alpha_2 k+1}-\alpha_2 k\left(18\alpha_2 k+8\sqrt{6\alpha_2 k+1}+1\right)+1\right)(2-\alpha_1 u_1)}{2\alpha_2^2(2\alpha_2 k-1)^2\sqrt{6\alpha_2 k+1}}, \end{bmatrix}$$

where $D := 1 - \frac{\alpha_1\sqrt{\alpha_2 k}}{\sqrt{2u_2}(\alpha_1(\alpha_2 u_2-1))^{3/2}} \cdot \frac{2\alpha_1+\alpha_1\sqrt{6\alpha_2 k+1}}{2\alpha_2-4\alpha_2^2 k} > 0$ following the same rationale as in (EC.65). When $\alpha_2 k < 0.23$, $2\sqrt{6\alpha_2 k+1} - \alpha_2 k\left(18\alpha_2 k + 8\sqrt{6\alpha_2 k+1}+1\right)+1 > 0$ and therefore $\frac{\partial u_2^\alpha}{\partial \alpha_2} < 0$.

- Plugging Firm 2's best response in (EC.67) into Firm 1's profit in (EC.68), we obtain the equilibrium profit

$$f(u_1^\alpha) := \pi_1^\alpha(u_1^\alpha, u_2^\alpha(u_1^\alpha)) = \frac{1}{6}\left(2\sqrt{6\alpha_2 k+1} - \frac{6k}{u_1^\alpha} - 3\alpha_1 u_1^\alpha + 2\right).$$

Taking the total derivatives, we get

$$\begin{aligned}\frac{\mathrm{d}f(u_1^\alpha)}{\mathrm{d}\alpha_2} &= \frac{\partial f(u_1^\alpha)}{\partial u_1^\alpha}\frac{\partial u_1^\alpha}{\partial \alpha_2} + \frac{\partial f(u_1^\alpha)}{\partial \alpha_2}\\ &= \underbrace{\left(\frac{k}{{u_1^\alpha}^2} - \frac{\alpha_1}{2}\right)}_{-,\ \text{since } u_1^\alpha > \sqrt{\frac{2k}{\alpha_1}}} \cdot \underbrace{\frac{\partial u_1^\alpha}{\partial \alpha_2}}_{-} + \underbrace{\frac{k}{\sqrt{6\alpha_2 k+1}}}_{+} > 0.\end{aligned}$$

Consider Firm 2's profit in (EC.68). By the envelope theorem,

$$\begin{aligned}\frac{\mathrm{d}\pi_2^\alpha}{\mathrm{d}\alpha_2} &= \frac{\partial \pi_2^\alpha}{\partial u_1^\alpha}\frac{\partial u_1^\alpha}{\partial \alpha_2} + \frac{\partial \pi_2^\alpha}{\partial \alpha_2}\\ &= \underbrace{\frac{\alpha_1(\alpha_1 u_1^\alpha + 2\alpha_2 u_2^\alpha - 2)}{4\alpha_2^3 {u_2^\alpha}^3}}_{+,\ \text{since } \alpha_2 u_2^\alpha > 1} \cdot \underbrace{\frac{\partial u_1^\alpha}{\partial \alpha_2}}_{-} - \frac{(\alpha_1 u_1^\alpha + 2\alpha_2 u_2^\alpha - 2)(3\alpha_1 u_1^\alpha + 2\alpha_2 u_2^\alpha - 6)}{8\alpha_2^4 {u_2^\alpha}^3} < 0,\end{aligned}$$

since

$$\begin{aligned}3\alpha_1 u_1^\alpha + 2\alpha_2 u_2^\alpha - 6 &= \frac{\left(6\alpha_2 k + \sqrt{6\alpha_2 k+1} - 1\right)(2 - \alpha_1 u_1^\alpha)}{1 - 2\alpha_2 k} && \text{using (EC.67)}\\ &> 0 && \text{since } \alpha_1 u_1^\alpha \le 1. \quad \square\end{aligned}$$