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Working Paper

2023/10/TOM

From Black to Grey: Improving Access to Antimalarial Drugs in the Presence of Counterfeits

Jiatao Ding
INSEAD, jiatao.ding@insead.edu

Michael Freeman
INSEAD, michael.freeman@insead.edu

Sasa Zorc
University of Virginia, ZorcS@darden.virginia.edu

In malaria-endemic countries, the limited availability of affordable antimalarial medication has contributed to the widespread distribution of inferior counterfeit drugs. We study such markets to determine how philanthropic donors can best allocate limited funds to subsidize the purchase or sales of antimalarial drugs via private-sector distribution channels. We also consider the potential effectiveness of other interventions to improve outcomes in the presence of counterfeit drugs. To examine the supply chain of antimalarials, we develop a game-theoretic model in which the retailer has a strategic choice to source legitimate drugs from a certified supplier, potentially counterfeit drugs from an uncertified supplier, or both. In contrast with the extant literature, we show that in the presence of counterfeits, employing a purchase subsidy alone may no longer be optimal. In particular, when the donor's budget is small, the donor may prefer to offer a sales subsidy that covers both legitimate and counterfeit drugs; moreover, if the drug's retail price is exogenous and demand uncertainty is high, the donor may need to refrain from offering any subsidy at all. We also evaluate five strategies that have been employed to combat counterfeit drugs (improving consumer awareness, increasing the cost of sourcing counterfeits, adopting traceability technology, cracking down on the supply, and imposing price controls) and identify the conditions under which these approaches can either improve or worsen outcomes. Finally, we perform an extensive numerical analysis, calibrating the models to malaria data from Mozambique. Our paper provides guidance as to how to improve outcomes in the presence of counterfeit drugs. Specifically, our results indicate the need for regulators and donors to understand specific market characteristics (e.g., retailers' pricing power and demand uncertainty) in order to design effective subsidy schemes and select appropriate technologies and policies to improve access to life-saving medicines.

Key Words: Counterfeit Pharmaceutical Products; Subsidies; Health Supply Chains; Drug Quality; Traceability
History: Last updated on March 21, 2023.

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Jiatao Ding, Michael Freeman
INSEAD, 1 Ayer Rajah Avenue, Singapore 138676
jiatao.ding@insead.edu, michael.freeman@insead.edu

Saša Zorc
Darden School of Business, University of Virginia, 100 Darden Blvd, Charlottesville, VA 22903, United States
ZorcS@darden.virginia.edu

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1. Introduction

Malaria is a life-threatening disease caused by parasites transmitted to people through the bites of infected female Anopheles mosquitoes. Despite the fact that malaria is preventable and curable, more than 400,000 people die of it every year, with Africa accounting for approximately 19 of every 20 cases and deaths globally (World Health Organization (WHO) 2020a). The first-line therapy for malaria infection in malaria-endemic countries is a class of drugs known as artemisinin combination therapies (ACTs). ACTs are more than 95% effective in preventing death from malaria and are well-tolerated (Shibeshi et al. 2021). The private sector distributes the majority of ACTs, due to the lack of geographic reach of public health clinics. However, unfortunately, high production and distribution costs mean a lack of affordable access to ACTs in the private sector supply chain

(Arrow et al. 2004). This lack of affordability has motivated philanthropic donors, such as the World Bank, the Clinton Health Access Initiative, and the Bill & Melinda Gates Foundation, to subsidize distribution channels to improve the availability of ACTs. Yet, due in part to funding shortfalls,[1] the price of ACTs in many regions remains high, leaving malaria one of the biggest preventable and treatable killers of children on the planet (Roser 2022).

The high demand for antimalarials, low accessibility of legitimate drugs, and funding shortfalls have fostered the prevalence of inferior counterfeit medicines. These counterfeit drugs have infiltrated legal supply chains across Africa, most notably in the private sector, and have a significant market presence. In Kenya, counterfeits account for approximately 20% of the private sector market for ACTs, while in the Democratic Republic of the Congo, the figure is as high as 42% (Brower 2017). Although the root cause of this phenomenon is multifaceted, the high cost of legitimate medicines is a major contributing factor. Wholesalers, facing high prices for such products, may resort to buying from unverified sources at discounted rates to remain competitive, unwittingly purchasing counterfeit drugs in the process. Consequently, downstream retailers seeking to sustain their businesses may be forced to source from these wholesalers, further perpetuating the problem (WHO 2017). The gravity of the situation is compounded by the deceptive nature of counterfeit drugs, making it challenging to distinguish between genuine and fake products on store shelves. Studies have shown that even trained pharmacists can struggle to differentiate between the two (Newton et al. 2011, Nyqvist et al. 2020). As a result, unsuspecting consumers who buy drugs from these retailers risk consuming counterfeit medicines, which can endanger their health and safety.

Importantly, the presence of counterfeit drugs adds another layer of complexity for donors as they endeavor to improve access to effective healthcare solutions. In particular, the efficacy of the donor's subsidy program may be contingent upon factors such as the prevalence, cost, and quality of counterfeit drugs, among other considerations. Furthermore, depending on how the subsidy scheme is managed, the donor may risk subsidizing these counterfeit medicines. The consequences of doing so and the donor's challenges are exacerbated by the fact that counterfeit drugs come in diverse forms that vary significantly in terms of their characteristics (OCDE 2020). Substandard drugs, for example, are manufactured by registered companies but do not meet approved quality standards due to poor manufacturing, packaging, or transportation issues. Conversely, falsified medicines are intentionally produced and packaged to deceive consumers, and may contain dangerously high or low levels of active ingredients, contaminants, or no active ingredient at all. The health implications of counterfeit drugs vary depending on their type. While high-quality counterfeits may offer some therapeutic benefits, overall, substandard and falsified antimalarial drugs are estimated to cause over 100,000 deaths annually in sub-Saharan Africa alone (Chutel 2017).

[1] In 2019, funding reached only $3 billion against a global target of $5.6 billion (WHO 2020b).

Motivated by these market characteristics, this paper first investigates how donors should allocate limited funds to subsidize pharmaceutical products in markets where counterfeits are present. In particular, it builds on literature comparing the effectiveness of two types of subsidies that have been considered for ACTs (Arrow et al. 2004). The first, a *purchase subsidy*, is designed to reduce a retailer's drug acquisition cost, while the second, a *sales subsidy*, increases a retailer's revenue for drugs sold to consumers. While the extant literature (Taylor and Xiao 2014, Levi et al. 2017, Olsder et al. 2020) has consistently shown purchase subsidies to be more effective than sales subsidies, to date, this stream of research has not accounted for the presence of counterfeits.

Our paper first strengthens existing results by showing that when the donor has a sufficiently large budget, using only purchase subsidies remains optimal, even in the presence of counterfeits. Importantly, such a subsidy scheme not only improves public health but can also eliminate counterfeits from the market, demonstrating an additional benefit of purchase subsidies beyond that shown in previous literature. However, unlike prior work, we find that if the donor has only a limited budget, the optimal subsidy scheme can differ depending on the donor's objective (i.e., minimizing counterfeit's market share, maximizing quantity sold, or maximizing health benefits), the cost and quality of counterfeits, consumers' awareness of counterfeits, and the uncertainty of demand. In particular, in the presence of counterfeits and funding shortfalls – which coexist in many malaria-endemic regions – the prevailing recommendation of using a purchase subsidy only is more likely to be suboptimal, and a sales subsidy is preferred. Perhaps more surprisingly, we show that with a limited budget and when demand uncertainty is high, the donor may need to refrain from offering any subsidy at all, even when counterfeits are of high quality.

Given these findings, this paper next investigates the impact of alternative interventions designed to improve outcomes in the presence of counterfeit drugs. We study five strategies that have been employed and that target different stakeholders: improving *consumer* awareness of counterfeits, increasing the cost of sourcing counterfeits for *retailers*, the adoption of traceability technology by *donors*, cracking down on uncertified distribution channels by *law enforcement*, and price controls imposed by *governments*. While we find that these efforts have positive impacts under most circumstances, some nuances exist. For example, we show that care must be taken when improving consumer awareness or increasing the cost, as the following trade-off arises: while the budget size required by the donor to switch the retailer to sourcing legitimate drugs is reduced, consumers are harmed when the retailer continues not to source such drugs. Even when the retailer purchases a combination of legitimate drugs and counterfeit drugs, we find that improving consumer awareness or increasing the cost of sourcing counterfeits could lead to worse outcomes. Moreover, we show that adopting traceability technology is surprisingly unhelpful: it has no benefit unless both price controls exist and the donor budget is small, but even then, it can potentially backfire for some model parameters. We also characterize the conditions under which the elimination of uncertified

distribution channels or the imposition of price controls helps. Lastly, we perform numerical analysis in which the models are calibrated to malaria data in Mozambique, to identify the sensitivity of our results to parameter values and understand the implications of price controls.

Overall, our paper sheds light on how subsidy schemes should be designed and implemented, as well as what measures should be taken to improve access to medicines in the presence of counterfeits.

2. Literature Review

Product subsidies. Our paper contributes to the stream of literature on product subsidies in operations management. Research on the influenza vaccine supply chain (Chick et al. 2008, Arifoğlu et al. 2012) examines subsidies for *short-shelf-life* products. In particular, Arifoğlu et al. (2012) show that combining supply-side with demand-side interventions may be beneficial. In contrast, the key finding of Taylor and Xiao (2014) is that for *long-shelf-life* products, such as antimalarial treatments, a purchase subsidy on the supply side is sufficient, and any intervention on the demand side is unnecessary when there is only one product type in the distribution channel. The finding that only supply-side interventions are required for long-shelf-life products has been further confirmed in recent works by Levi et al. (2017), in the presence of retailer competition, and Olsder et al. (2020), for rare disease treatments. In particular, Levi et al. (2017) focus on the effectiveness of uniform subsidies. They show that using only supply-side intervention not only makes sense to maximize market consumption but, even if the suppliers are heterogeneous, this simple and practical policy will likely provide the most potential benefits. Our paper contributes to this stream of literature by taking into account the additional problem of designing effective subsidy schemes in the presence of counterfeits, i.e., a lower cost but inferior quality substitute for the primary product.

The closest existing study to our work is Taylor and Xiao (2014), and we build on their model of the supply chain for malaria medicine. While the core difference in our model is the addition of counterfeit drugs and the retailer's strategic choice of source for the drugs, our work is also differentiated from theirs by incorporating multiple additional considerations. First, we consider three different possible objective functions for donors, as maximizing the quantity of drugs sold is no longer a clear objective in the presence of counterfeits. Second, we consider several strategies other than subsidies that the donor may use to combat counterfeits. Lastly, we include a numerical analysis which calibrates our model to malaria data from Mozambique.

We show that in the presence of counterfeits, the optimal subsidy scheme differs from the recommendation of Taylor and Xiao (2014) and may require either supply-side or demand-side intervention. More importantly, when the retail price is exogenous and demand uncertainty is high, the donor may need to refrain from any subsidy, as both types of subsidy can simultaneously increase the market share of counterfeits, reduce market consumption, and worsen public health.

Counterfeits. Traditionally, supply chain management research has focused on the legal exchange of goods and services. As companies offshore and outsource their operations due to globalization, counterfeits have infiltrated the legal supply chains and proliferated within established

markets. This has motivated a growing stream of work on counterfeits, and our paper contributes to this body of literature. The analytical work on counterfeits dates back to Grossman and Shapiro (1988a,b). More recently, operations management researchers have also looked into the operational implications of counterfeits. Sun et al. (2010) study a firm's outsourcing decision in which the firm benefits from lower labor costs but faces a higher risk of imitation by a foreign firm. Zhang et al. (2012) analyze a vertical differentiation model where the firm that produces legitimate products faces nondeceptive counterfeits. Hu et al. (2013) examine the impact of a grey market, which is an unauthorized channel of distribution for a supplier's authentic products. Cho et al. (2015) study counterfeiters' endogenous pricing decisions and brand-name strategies to combat counterfeiters through pricing and quality decisions. Yi et al. (2022) analyze the global supply chain's optimal strategy for anticounterfeit action. Our paper instead focuses on the downstream problem, i.e., the retailer's pricing, sourcing, and inventory decisions in the presence of a counterfeit supplier that operates alongside a legitimate supplier. Moreover, our model includes a donor whose objective is to improve consumers' access to the product through a subsidy program, and we analyze the donor's optimal decision in the presence of a counterfeit supplier.

In another related paper, Yao and Zhu (2019) consider a company that sells an authentic product in a market in which a counterfeiter may offer an imitation product. To combat counterfeiting, the company has the option to develop anticounterfeit technology to help customers distinguish genuine from fake products. Our paper also considers anticounterfeit strategies, one being the option for a donor to adopt anticounterfeit technology to ensure she does not subsidize counterfeit drugs. We show that in this case, the adoption of anticounterfeit technology could harm consumers.

Healthcare operations in developing countries. Developing countries faces numerous healthcare challenges, including a high burden of infectious diseases, inadequate healthcare infrastructure, and funding shortfalls. These issues have been significant barriers to providing quality healthcare services to those in need. However, healthcare operations researchers have increasingly been focusing on devising potential solutions to these challenges. For example, Jónasson et al. (2017) propose a two-part modeling framework to enhance the operational efficiency of early infant diagnosis networks in Mozambique. Suen et al. (2022) develop incentive programs for optimal medication adherence and conducted a numerical study in the context of the tuberculosis epidemic in India. Boutilier et al. (2022) partner with a mobile health start-up in Kenya to improve tuberculosis treatment adherence support. Gernert et al. (2022) use a game-theoretic approach to examine different business model for ambulance systems in developing economies. Gibson et al. (2023) design and implemented an optimized sample transportation system in Malawi. Our paper contributes to this literature by focusing on interventions to improve access to antimalarials in the presence of counterfeit drugs, applying this to malaria data from Mozambique.

Empirical literature on antimalarial drug subsidies. Another stream of work specifically seeks to empirically evaluate the impact of antimalarial drug subsidies in real-world applications.

Cohen et al. (2015) conduct a randomized controlled trial in Kenya to show that subsidies for ACTs significantly increase consumers' drug access. Dupas (2014) show that short-run subsidy programs could be beneficial in the long run as well. Meanwhile, studies have also examined the effect of the 2010-2011 large-scale pilot test of the Affordable Medicines Facility-malaria (AMFm), which was a private sector co-payment mechanism that provided subsidies for quality-assured ACTs. For example, ACTwatch Group et al. (2015) find that the AMFm was associated with positive and sustained improvements in quality-assured ACT availability, price and market share in Nigeria, Tanzania and Uganda, with mixed results in Kenya and Madagascar. Similarly, mixed results are documented on the impact of AMFm by Yadav et al. (2012). In these existing works, the presence and impact of counterfeits are not explicitly taken into account. Our paper performs analytical and numerical analysis of the impact of counterfeits in this setting, providing possible explanations for the mixed results observed by ACTwatch Group et al. (2015) and Yadav et al. (2012).

3. Model Setup

Our model of the supply chain for antimalarial drugs builds on the work of Taylor and Xiao (2014). We consider a retailer (he) who sells antimalarials to consumers over an infinite time horizon, with time periods indexed by $t \in \mathbb{N}$. There is also a donor (she) who offers subsidies to the retailer.

The retailer must choose whether to source drugs from a supplier who has been certified by the donor, an uncertified supplier, or both. A certified supplier offers quality-assured legitimate drugs that have been audited and approved by the international donor, which we refer to as "certified drugs". In contrast, an uncertified supplier sells drugs that have not been certified by the donor, which may include a combination of non-quality-assured legitimate drugs, substandard drugs, and falsified drugs, and we refer to these as "uncertified drugs". (Refer to Figure 1 for a diagram of the different drug types.) In many African countries, retailers opt for uncertified suppliers due to the lower wholesale prices, despite the risks to public health (WHO 2017). We model counterfeit drugs, including substandard and falsified drugs, as being indistinguishable from legitimate drugs, as the appearance and packaging of counterfeit drugs in Africa are often as good as those of the original (Newton et al. 2011, Nyqvist et al. 2020). To illustrate, in EC.1 we provide a figure of two samples of ACT drugs in Uganda with identical packaging, one being legitimate and the other counterfeit.

The sequence of events for each period t is summarized in Figure 2 and is described in detail in the rest of this section.

3.1. The Donor

Objective Functions of the Donor. While the extant literature typically models the objective of the donor as maximization of the expected drug quantity sold (Taylor and Xiao 2014), this is not necessarily the appropriate goal if counterfeit drugs are present (Institute of Medicine Committee on the Economics of Antimalarial Drugs 2004). In particular, a second possible objective that has

Figure 1 Different types of antimalarial drugs.

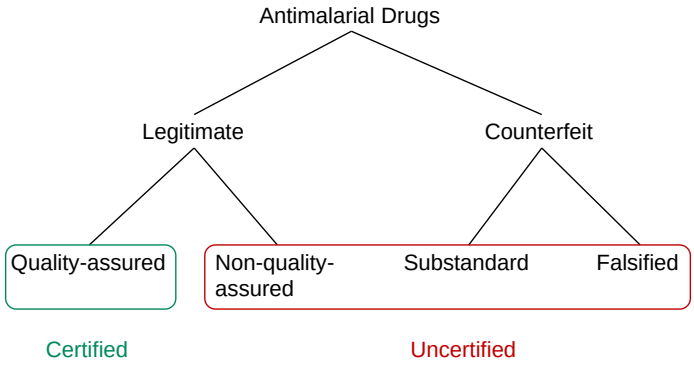
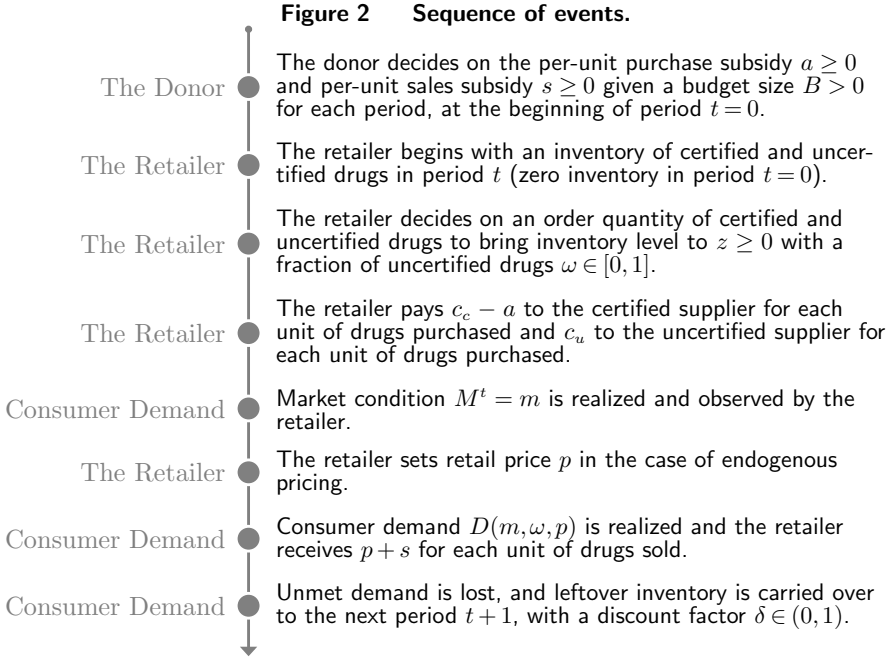


Figure 2 Sequence of events.



been used in practice is to instead minimize the market share of uncertified or counterfeit drugs. For example, in 2009, the Global Fund launched a $225 million program called the Affordable Medicines Facility–malaria (AMFm), with a primary goal being to reduce the prevalence of counterfeits on the market (PBS NewsHour 2009). This objective inherently recognizes that increasing the volume of drugs sold may not be an effective strategy when those drugs are primarily of low quality.

However, when uncertified drugs are of high quality (i.e., primarily non-quality-assured legitimate drugs), reducing their share on the market might have an adverse effect on population health, as it lowers access to unapproved but low-cost and high-quality medicines. Thus, a third possible objective, which takes into account both the quantity and quality of the drugs sold, is to maximize the overall health benefits for consumers. While this last objective is perhaps the most natural in the presence of counterfeits, to the authors' knowledge it has not been adopted in practice.

Consequently, we consider three potential objective functions for the donor: *i*) maximizing the expected quantity sold to consumers, *ii*) minimizing the market share of uncertified drugs, and *iii*) maximizing the expected health benefits created for consumers. We formalize these objectives later in (5), once all the required notation has been introduced.

Subsidy Schemes of the Donor. To achieve her given objective, at the beginning of the first period, the donor decides on the subsidy scheme consisting of a per-unit purchase subsidy $a \geq 0$ and a per-unit sales subsidy $s \geq 0$, subject to a given budget size $B > 0$ for each period. The financing mechanism of a purchase subsidy is such that the donor negotiates with the certified supplier of quality-assured legitimate drugs to achieve a reduction of a in the wholesale price for sales to the retailer (Tougher et al. 2012). Thus, the retailer can only enjoy the purchase subsidy for the certified drugs he orders, not the uncertified drugs. A sales subsidy, on the other hand, is implemented using voucher schemes at the retail level (Arrow et al. 2004). Specifically, a consumer presents a voucher to the retailer and receives a discount when purchasing the product. For each redeemed voucher, the retailer receives a subsidy of s from the donor. Hence, in contrast with a purchase subsidy, the retailer enjoys a sales subsidy for both the certified and uncertified drugs sold. (With the advancements in traceability technologies, it may become possible for the donor to offer sales subsidies exclusively on certified drugs. We consider this alternative in Section 6.3.)

3.2. The Retailer

The retailer seeks to maximize his expected profit over the infinite horizon under discount factor $\delta \in (0,1)$. In each period t, he faces uncertainty as to the market condition M^t, where $M^0, M^1, ...$ are independent and identically distributed random variables with cumulative distribution F, density f, and support $[0,\infty)$. He then makes sourcing, ordering, and pricing decisions, as we discuss next.

Sourcing and Ordering Decisions of the Retailer. Before the market uncertainty resolves, the retailer places and receives a wholesale order of drugs that takes his inventory from a starting level $x \in \mathbb{R}_{\geq 0}$ ($x = 0$ in the first period) to a new level $z \in \mathbb{R}_{\geq 0}$.[2] Importantly, the order placed by the retailer can be any combination of certified and uncertified drugs, and we denote the fraction of uncertified drugs in the retailer's inventory after receiving an order by $\omega \in [0,1]$. The inventory level for certified drugs is therefore equal to $(1-\omega)z$ and the inventory level for uncertified drugs is ωz. The fraction ω of uncertified drugs in inventory is a decision made by the retailer. Without loss of generality, we normalize the quality of certified drugs to 1 and denote the expected quality of uncertified drugs by $\theta \in [0,1)$. θ captures the retailer's estimate of the expected quality of drugs sourced from the uncertified supplier, which we assume to be unbiased. Recall that the uncertified supplier could supply any combination of non-quality-assured legitimate, substandard, and falsified

[2] To simplify the exposition, we omit the dependence of the retailer's sourcing, ordering, and pricing decisions on t; as we show later in Lemma 1, the optimal decisions of the retailer are stationary.

drugs. We focus on the case where the uncertified supplier offers at least a certain fraction of substandard or falsified drugs such that the expected quality of uncertified drugs is less than 1. This means that the expected quality of drugs on the market is $q = \omega\theta + 1 - \omega$.

For each unit of certified drugs ordered, the retailer incurs a per-unit cost $c_c > 0$ less the per-unit purchase subsidy a. For each unit of uncertified drugs ordered, the retailer incurs a per-unit cost c_u. The cost c_u can be considered to include not only the monetary cost but other costs as well, e.g., a potential penalty to the retailer if caught with counterfeit drugs. When solving the retailer's problem, we focus on the case with $c_c - a > c_u$, as otherwise he will only source certified drugs.

Pricing Decision of the Retailer. After receiving the ordered drugs, the market uncertainty for the period resolves, with the retailer observing the realization of M^t, denoted by m. Next, the retail price p is determined, such that for each unit sold, the retailer receives the price p from the consumer and the sales subsidy s from the donor. Depending on the retail context (e.g., competition, regulatory environment, etc.), the retailer may either set the price (i.e., the price is endogenous) or be a price-taker (i.e., the price is exogenous). In particular:

- Patouillard et al. (2015) show that retailers may or may not have pricing power. For example, in Cambodia, the retail prices of ACTs sold at drugstores have a very narrow range, suggesting retailers have little pricing power. By contrast, the retail prices of ACTs sold at village shops exhibit significant price variation, suggesting that when the antimalarial market is highly concentrated with weak competition, retailers may have sufficient pricing power.
- Additionally, a study on the prices of antimalarial drugs in six malaria-endemic countries noted the existence of price control in Benin, where retail prices are firmly regulated (Palafox et al. 2016). Furthermore, while price controls do not exist in the other five countries, respondents mentioned other types of pricing constraints, including compulsory pricing imposed by suppliers and the addition of recommended retail prices on product packaging.

Hence, to capture differences in retailers' pricing power, we study both settings of endogenous and exogenous retail prices in Sections 4 and 5, respectively. We show that retailers' pricing power has important implications for their sourcing decisions, donor subsidy decisions, and the impact of uncertified drugs on consumers. Moreover, we provide further insights into the conditions under which price controls could be beneficial in Section 7.

3.3. Consumer Demand

After prices are determined, the consumer demand is realized. The consumer demand, $D(m, \omega, p) = y(\omega, p)m$, in any period depends on the realization of the market condition and the expected quality and retail price of drugs on the market. Consumer demand is a function of the expected quality of drugs sold by the retailer, as uncertified drugs are deceptive and consumers cannot distinguish them from the certified ones at the time of purchasing (Newton et al. 2011, Nyqvist et al. 2020). Without loss of generality, we assume $y(\omega, p) \in [0, 1]$. In this case, the market condition m can be

interpreted as the number of customers in need of antimalarial drugs (e.g., the number of malaria cases), and $y(\omega,p)$ can be interpreted as the fraction of customers who purchase the drugs under expected quality $q=\omega\theta+1-\omega$ and retail price p.

We assume that the shape of the demand function is bivariate linear, i.e., $y(\omega,p)=1-k(1-q)-\phi p=1-k+k(\omega\theta+1-\omega)-\phi p$, where $k\in(0,1], \phi\in(0,\infty)$, and $p\in[0,\bar{p}]$; here, $\bar{p}=\frac{1-k+k(\omega\theta+1-\omega)}{\phi}$ is the price at which demand would be equal to zero. We assume that $\min\{c_c,c_u\}<\bar{p}$, i.e., that either c_c or c_u is not prohibitively high so that it is profitable for the retailer to source at least some drugs, whether certified or uncertified, in the absence of subsidy.

This demand function has several features that facilitate the analysis. First, this functional form satisfies $y(\omega,p)\in[0,1]$. Second, it is consistent with intuition that a higher quality of drugs leads to higher demand while a higher price leads to lower demand. Third, it allows us to capture the price sensitivity of consumer demand using ϕ and the consumers' counterfeit awareness with k.

The inclusion of k in the demand function is especially important in the context of this study. In particular, recent findings have shown that consumers have the ability to infer the expected quality of antimalarial drugs purchased over time and that quality improvements are often interpreted by consumers as meaning that there are fewer counterfeits on the market (Nyqvist et al. 2020). Different values of k therefore reflect different levels of consumers' counterfeit awareness, which includes awareness of the market share of counterfeits and the health cost associated with taking counterfeit drugs. On the one hand, when $k\to 0$, consumers have no awareness of counterfeit drugs and therefore consumer demand does not respond to the quality of antimalarial drugs. On the other hand, when $k\to 1$, consumers are highly sensitive to quality such that no consumer purchases antimalarial drugs with expected quality of drugs $q=0$.

Finally, at the end of each period, the realized sales are $\min(y(\omega,p)m,z)$. Unmet demand is lost if $y(\omega,p)m>z$, while leftover inventory is carried to the next period when $y(\omega,p)m\le z$.

4. Formulations and Results Under Endogenous Retail Price Scenario

We now formulate and solve the retailer's ordering (z), sourcing (ω), and pricing (p) problems under the endogenous retail price scenario and for a given subsidy scheme (a,s). Subsequently, we consider the donor's problem under different objective functions and for a given budget of size B.

4.1. Retailer's Problem

The retailer makes ordering, sourcing, and pricing decisions to maximize his expected discounted profit under a discount factor $\delta\in(0,1)$ over an infinite time horizon. Let $x\ge 0$ denote the retailer's inventory level at the beginning of a period before ordering and $h\in[0,1]$ denote the fraction of uncertified drugs in x. The retailer chooses order quantity n_c with purchase cost $(c_c-a)n_c$ incurred for certified drugs and order quantity n_u with purchase cost $c_u n_u$ incurred for uncertified drugs, where $n_c=(1-\omega)z-(1-h)x\ge 0$ and $n_u=\omega z-hx\ge 0$. After observing the realized market

condition m, the retailer chooses retail price p, which results in sales of $\min(y(\omega,p)m,z)$ and revenue $(s+p)\min(y(\omega,p)m,z)$. The leftover inventory $z-\min(y(\omega,p)m,z)$ is carried into the next period.

Let $\Upsilon\in[0,1]$ denote the random variable for the fraction of uncertified drugs in the leftover inventory and υ denote its realization. We assume that certified and uncertified drugs are sold randomly so that the expected fraction of drugs sold that are uncertified is equal to the fraction of uncertified drugs at the beginning of the period, i.e., $\mathbb{E}[\Upsilon|z-\min(y(\omega,p)m,z)>0]=\omega$. The retailer's expected discounted profit is then

$$\begin{aligned}V(x,h)=\max_{z,\omega:\omega z\geq hx,(1-\omega)z\geq(1-h)x}\{&-(c_c-a)[(1-\omega)z-(1-h)x]-c_u(\omega z-hx)\\&+\mathbb{E}_{M,\Upsilon}[\max_{p\geq0}\{(s+p)\min(y(\omega,p)M,z)+\delta V(z-\min(y(\omega,p)M,z),\Upsilon)\}]\}.\end{aligned}\tag{1}$$

The following lemma establishes the equivalence of (1) to a single-period problem.

Lemma 1. (Equivalence to a Single-Period Problem). *Problem* (1) *has the same optimal policy as the following single-period problem:*

$$\begin{aligned}\max_{z\geq0,\omega\in[0,1]}R(z,\omega)=\max_{z\geq0,\omega\in[0,1]}\{&-(c_c-a)(1-\omega)z-c_u\omega z+\mathbb{E}_M[\max_{p\geq0}\{(s+p)\min(y(\omega,p)M,z)\\&+\delta[((c_c-a)(1-\omega)+c_u\omega)(z-\min(y(\omega,p)M,z))]\}]\}.\end{aligned}\tag{2}$$

To understand the intuition behind this result, we construct the following relaxation of (1).

$$\begin{aligned}V_r(x,h)=\max_{z\geq0,\omega\in[0,1]}\{&-(c_c-a)[(1-\omega)z-(1-h)x]-c_u(\omega z-hx)\\&+\mathbb{E}_{M,\Upsilon}[\max_{p\geq0}\{(s+p)\min(y(\omega,p)M,z)+\delta V_r(z-\min(y(\omega,p)M,z),\Upsilon)\}]\}.\end{aligned}\tag{3}$$

Here, the retailer has the option to sell back any inventory he has at purchasing cost at the beginning of each period. It is straightforward to show that the retailer's optimal decision in (3) is stationary, as the underlying parameters of the problem are stationary. As a result, since the retailer starts with zero inventory in the first period and the optimal policy is stationary in (3), the inventory at the beginning of each period is always below the optimal level. Hence, he never uses the option to sell back inventory at the beginning of each period; therefore, the optimal policy for (3) is the same as the optimal policy for (1).

Meanwhile, (2) is a single-period problem where each unsold unit has salvage value $\delta(c_c-a)$ for certified drugs or salvage value δc_u for uncertified drugs. Since $V_r(x,h)$ and $\max_{z\geq0,\omega\in[0,1]}R(z,\omega)$ only differ by a constant, i.e., $\delta V_r(0,0)+(c_c-a)(1-h)x+c_uhx$, the optimal policy for (3) is the same as the optimal policy for (2). Hence, Lemma 1 holds.

Pricing Decision of the Retailer. Lemma 1 allows us to solve the relatively easier (2) rather than (1). We proceed to do so, starting by characterizing the retailer's pricing decision.

Lemma 2. (Optimal Price). *Under subsidy scheme (a,s), realized market condition m, inventory level z, and a fraction of uncertified drugs ω, the retailer's optimal price is*

$$p^*(a,s,m,z,\omega)=\begin{cases}\tilde{p}(a,s,\omega) & \text{if}\quad \tilde{p}(a,s,\omega)\geq0\quad\text{and}\quad m\leq z/y(\tilde{p}(a,s,\omega)),\\ \hat{p}(m,z,\omega)^+ & \text{otherwise.}\end{cases}$$

where $\tilde{p}(a,s,\omega)=\frac{1-k\omega(1-\theta)-\phi[s-\delta((c_c-a)(1-\omega)+c_u\omega)]}{2\phi}$, $\hat{p}(m,z,\omega)=\frac{1-k\omega(1-\theta)-z/m}{\phi}$.

In Lemma 2, $\tilde{p}(a,s,\omega)$ is the unconstrained optimal price set by the retailer, while $\hat{p}(a,s,\omega)$ is the highest price at which the retailer could clear his inventory. Note at this point that we could have $\tilde{p}(a,s,\omega)<0$ when s is sufficiently large. When this occurs, we can also show that $\hat{p}(m,z,\omega)<0$ and thus $p^*(a,s,m,z,\omega)=0$, i.e., the retailer is better off if he provides the drugs for free. This corner case is covered by our analysis in Section 5. For the rest of this section, we focus on the other case where $\tilde{p}(a,s,\omega)\geq 0$. In this case, we can easily show that $\hat{p}(a,s,\omega)\geq 0$.

If the market condition is weak, i.e., $m\leq \frac{z}{y(\tilde{p}(a,s,\omega))}$, the retailer prices at $\tilde{p}(a,s,\omega)$ to withhold stock and carries the leftover inventory to the next period. In this case, we have $\frac{\partial \tilde{p}(a,s,\omega)}{\partial \omega}=\frac{-k(1-\theta)-\phi\delta(c_c-a-c_u)}{2\phi}$, i.e., a higher fraction of uncertified drugs leads to a lower retail price. This is because a higher fraction of uncertified drugs leads to lower purchasing costs and lower demand, both of which incentivize the retailer to price more aggressively. On the other hand, we have $\frac{\partial y(\tilde{p}(a,s,\omega))}{\partial \omega}=\frac{\phi\delta(c_c-a-c_u)-k(1-\theta)}{2}$, i.e., a higher fraction of uncertified drugs may lead to higher or lower demand. While a higher fraction of uncertified drugs leads to a lower retail price, which leads to higher demand, it also leads to a lower expected quality of drugs sold, which leads to lower demand. To ascertain which effect dominates, we define the parameter r_a as follows.

Definition 1. (Cost-Quality Difference Ratio). *For a given purchase subsidy a, certified drugs with cost c_c, uncertified drugs with cost c_u and quality θ, and consumer demand with counterfeit awareness k, we define the cost-quality difference ratio as $r_a=\frac{c_c-a-c_u}{k(1-\theta)}$.*

When r_a is higher, uncertified drugs are more attractive to the retailer, as higher r_a corresponds to a higher quality of uncertified drugs, a lower counterfeit awareness of consumers, or a higher acquisition cost difference between certified and uncertified drugs. Hence, when the market condition is weak, a higher fraction of uncertified drugs leads to higher demand if and only if the cost-quality difference ratio is sufficiently high, i.e., $r_a>\frac{1}{\phi\delta}$.

Turning to the case in which the market condition is strong, i.e., $m>\frac{z}{y(\tilde{p}(a,s,\omega))}$, the retailer prices at $\hat{p}(a,s,\omega)$ to sell the entire stock. We have $\frac{\partial \hat{p}(m,z,\omega)}{\partial \omega}=-\frac{k(1-\theta)}{\phi}$, as a higher fraction of uncertified drugs leads to lower demand, which results in a lower retail price. Meanwhile, changing the fraction of uncertified drugs does not affect the quantity sold as the retailer prices to sell his entire stock.

Sourcing and Inventory Decisions of the Retailer. We now characterize the retailer's optimal sourcing and inventory decisions by the following lemma.

Lemma 3. (Optimal Sourcing and Inventory Level). *The retailer's one period expected profit $R(z,\omega)$ has a pair of unique maximizers, denoted by $\omega^*(a,s)$ and $z^*(a,s)$. The retailer performs single sourcing, i.e., $\omega^*(a,s)\in\{0,1\}$. Meanwhile, $z^*(a,s)$ is given by*

$$\int_{z^*(a,s)/y(\tilde{p}(a,s,\omega^*(a,s))}^{\infty}[s+\frac{1-k+kq-2z^*(a,s)/m}{\phi}-\delta c]f(m)dm-(1-\delta)c=0,$$

where $q=1, c=c_c-a$ *if* $\omega^*(a,s)=0$*, and* $q=\theta, c=c_u$ *if* $\omega^*(a,s)=1$*. Moreover, the retailer sources certified (uncertified) drugs only if the cost-quality difference ratio is low (high): there exists an* $\bar{r}_a>0$ *s.t. we have* $\omega^*(a,s)=0$ *if* $r_a\le\bar{r}_a$*, and* $\omega^*(a,s)=1$ *if* $r_a>\bar{r}_a$*.*[3]

Lemma 3 shows first that the retailer always performs single sourcing. When $k\to 0$, this is trivial as consumer demand does not respond to quality changes. As a result, the retailer simply sources the drugs from the supplier with a lower acquisition cost. On the other hand, when k is large, the retailer faces a trade-off: uncertified drugs have a lower acquisition cost, but they also have a lower quality, which results in lower demand. We show that the retailer still performs single sourcing despite this trade-off. This is because, in the case of endogenous retail prices, the retailer has an additional pricing lever that he can employ to adjust consumer demand and his profit margin. Hence, the retailer's optimal sourcing decision is still either certified drugs only or uncertified drugs only, whichever results in a higher expected profit. In this case, the choice of supplier depends on the cost-quality difference ratio r_a. Additionally, Lemma 3 shows that with either certified drugs only or uncertified drugs only, the retailer then chooses an optimal inventory level that equates the expected marginal contribution from stocking and selling a unit with the expected marginal cost of stocking and not selling a unit.

4.2. Donor's Problem

Recall that the donor may have different objectives in the presence of uncertified drugs, as discussed in Section 3.1. In regions where patients have poor access to drugs and where the quality differential between certified and uncertified drugs is negligible, the most straightforward objective may be to focus on increasing the quantity of drugs sold. However, in regions with a significant presence of falsified drugs, the donor might instead focus on reducing the market share of uncertified drugs due to their low quality and lack of public health benefits. Meanwhile, in regions with substandard but still partially efficacious uncertified drugs, the donor may need to take into account both the quantity and quality of drugs sold. Hence, we consider a donor who solves the following problem with three different potential objective functions, subject to the constraint that over the long run, the average per-period subsidy does not exceed the finite budget B:[4]

$$\begin{aligned}&\max_{a\ge 0,s\ge 0} D_i(a,s),\\ &\text{s.t. } (a(1-\omega^*(a,s))+s)\mathbb{E}_M[\min(y(p^*(M,z^*(a,s),\omega^*(a,s)))M,z^*(a,s))]\le B,\end{aligned}\tag{4}$$

where $i\in\{1,2,3\}$ and

$$\begin{aligned}D_1(a,s)&=1-\omega^*(a,s),\\ D_2(a,s)&=\mathbb{E}_M[\min(y(p^*(M,z^*(a,s),\omega^*(a,s)))M,z^*(a,s))],\\ D_3(a,s)&=(\omega^*(a,s)\theta+1-\omega^*(a,s))\mathbb{E}_M[\min(y(p^*(M,z^*(a,s),\omega^*(a,s)))M,z^*(a,s))].\end{aligned}\tag{5}$$

[3] When the retailer is indifferent between sourcing certified drugs only and uncertified drugs only, we assume he sources certified drugs only.

[4] Our results continue to hold when the budget constraint is based on the donor's expected discounted subsidy payment, given that the donor's discount factor is sufficiently large (Taylor and Xiao 2014).

Here D_1 is the market share of certified drugs, D_2 is the expected quantity of drugs sold, and D_3 is the expected quality-adjusted quantity of drugs sold. The quality-adjusted quantity serves as our main measure of the health benefits created for consumers.

We now characterize the donor's optimal subsidy decision by the following proposition.

PROPOSITION 1. (Optimal Subsidy). *For each possible objective function, as given by (5), there exists a pair* (a^*, s^*) *that solves (4), such that* $a^* \cdot s^* = 0$. *Moreover, there exist* $0 \le \tilde{r}_0^l \le \tilde{r}_0^m \le \tilde{r}_0^h$ *s.t.*

(i) *For any* $r_0 \in [0, \tilde{r}_0^l]$, *we have* $\omega^*(0,0) = 0$. *In this case, for any* $B \in [0, \infty)$, *we have* $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$.

(ii) *For any* $r_0 \in (\tilde{r}_0^l, \tilde{r}_0^m]$, *we have* $\omega^*(0,0) = 0$; *for any* $r_0 \in (\tilde{r}_0^m, \tilde{r}_0^h)$, *we have* $\omega^*(0,0) = 1$. *Meanwhile, for any* $r_0 \in (\tilde{r}_0^l, \tilde{r}_0^h)$, *there exist* $0 \le \tilde{B}_1^m \le \tilde{B}_2^m$, *where* $\tilde{B}_1^m = 0$ *if* $r_0 \in (\tilde{r}_0^l, \tilde{r}_0^m]$, *s.t.*

  (a) *For any* $B \in [0, \tilde{B}_1^m)$, *we have* $a^* = 0, s^* > 0$ *and* $\omega^*(a^*, s^*) = 1$, *for* $D_i(a,s), i \in \{1,2,3\}$.

  (b) *For any* $B \in [\tilde{B}_1^m, \tilde{B}_2^m)$, *we have* $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *for* $D_1(a,s)$; $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *or* $a^* = 0, s^* > 0$ *and* $\omega^*(a^*, s^*) = 1$, *for* $D_2(a,s)$ *or* $D_3(a,s)$.

  (c) *For any* $B \in [\tilde{B}_2^m, \infty)$, *we have* $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$.

(iii) *For any* $r_0 \in [\tilde{r}_0^h, \infty)$, *we have* $\omega^*(0,0) = 1$. *In this case, there exist* $0 \le \tilde{B}_1^h \le \tilde{B}_3^h \le \tilde{B}_2^h$ *s.t.*

  (a) *For any* $B \in [0, \tilde{B}_i^h)$, *we have* $a^* = 0, s^* > 0$, *and* $\omega^*(a^*, s^*) = 1$, *for* $D_i(a,s), i \in \{1,2,3\}$.

  (b) *For any* $B \in [\tilde{B}_i^h, \infty)$, *we have* $a^* > 0, s^* = 0$, *and* $\omega^*(a^*, s^*) = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$.[5]

In contrast to the well-established results in the absence of uncertified drugs, the donor's optimal subsidy may not consist solely of a purchase subsidy. Specifically, we show that the optimal subsidy may consist of a purchase subsidy only or a sales subsidy only, but not a mix of both. This follows from the earlier finding in Lemma 3 that the retailer performs single sourcing under any subsidy scheme (a, s). In particular, when the retailer sources certified drugs only, we have $s^* = 0$. This is because, in the case of single sourcing of certified drugs, a marginal increase in the purchase subsidy is more effective in increasing sales than a marginal increase in the sales subsidy. Specifically, the sales subsidy is more effective in reducing the retail price when the market condition is weak, while the purchase subsidy is more effective in increasing the inventory level. Averaging across market condition realizations, the latter effect dominates the former, which is limited to weak market conditions only (Taylor and Xiao 2014). Moreover, doing so does not change the retailer's sourcing decision, as we can show that the purchase subsidy is also more effective in increasing the retailer's profit than the sales subsidy when he performs single sourcing of certified drugs. Hence, if $s > 0$, the donor can always increase the purchase subsidy and decrease the sales subsidy to further increase the expected quantity sold. Meanwhile, when the retailer sources uncertified drugs only, we have $a^* = 0$ because a purchase subsidy cannot increase sales of uncertified drugs.

[5] When the donor is indifferent between subsidizing the retailer to source certified drugs only and uncertified drugs only, we assume she subsidizes the retailer to source certified drugs only.

When uncertified drugs on the market are of low quality and consumers' counterfeit awareness is high, the retailer will only source certified drugs, even in the absence of subsidies (Proposition 1 (i)). Moreover, for any feasible subsidy scheme, the retailer will continue to source only certified drugs. In this case, the donor's optimal subsidy consists of a purchase subsidy only, regardless of the budget size and objective function.[6]

When both the quality of uncertified drugs and consumers' counterfeit awareness are moderate, the optimal subsidy scheme is more complicated (Proposition 1 (ii)). It depends on the retailer's sourcing decision in the absence of subsidies, the objective of the donor, and the donor's budget size. The donor should use a sales subsidy only when she cannot incentivize the retailer to source certified drugs with a purchase subsidy (i.e., $B < \tilde{B}_1^m$). With a higher budget (i.e., $B \geq \tilde{B}_1^m$), to minimize the market share of uncertified drugs, the donor should use a purchase subsidy only to switch the retailer from sourcing uncertified drugs to certified drugs. On the other hand, when the donor aims to maximize the expected quantity or health benefits and the budget is not large enough (i.e., $\tilde{B}_1^m \leq B < \tilde{B}_2^m$), she might need to use either a purchase subsidy only or a sales subsidy only. The subsidy choice depends on whether it is optimal to induce the retailer to source only certified drugs with a purchase subsidy or uncertified drugs with a sales subsidy. Nevertheless, the complications disappear if the donor has a sufficiently large budget (i.e., $B \geq \tilde{B}_2^m$), in which case a purchase subsidy can close the cost gap between certified drugs and uncertified drugs and is more effective in increasing the quantity and health benefits than a sales subsidy.

When uncertified drugs on the market are of high quality or consumers' counterfeit awareness is low, the retailer only purchases uncertified drugs in the absence of subsidies (Proposition 1 (iii)). In this case, the optimal subsidy and the retailer's optimal sourcing decision in the presence of subsidies again depend on the donor's budget size and objective function. Specifically, when the donor's objective function is $D_i, i \in \{1,2,3\}$, $\tilde{B}_i^h$ is the threshold budget size, below (above) which the optimal subsidy consists of a sales (purchase) subsidy only and the retailer sources uncertified (certified) drugs only, where $\tilde{B}_1^h \leq \tilde{B}_3^h \leq \tilde{B}_2^h$. In particular:

– *Limited Budget:* In the case of a limited budget (i.e. $B < \tilde{B}_1^h$), the donor's optimal subsidy consists of a sales subsidy only, for any objective function. Specifically, a small purchase or sales subsidy is not sufficient to incentivize the retailer to switch to sourcing certified drugs. Hence, consumers are made better off when the donor offers a sales subsidy to the retailer, which acts to increase the expected quantity of uncertified drugs sold.

– *Moderate Budget:* In the case of a reasonable but not large budget ($\tilde{B}_1^h \leq B < \tilde{B}_2^h$), which subsidy is optimal depends on the objective function. When $B \geq \tilde{B}_1^h$, a purchase subsidy alone is

[6] If the objective is to minimize the market share of uncertified drugs, this can be achieved using any subsidy scheme, including one that mixes both purchase and sales subsidies. This is because only certified drugs are sourced regardless of the subsidy scheme. However, we assume in this case that a secondary objective might be to increase the quantity sold or health benefits created for consumers, in which case a purchase subsidy only is preferred.

sufficient to incentivize the retailer to switch from sourcing uncertified drugs to certified drugs. Thus, a purchase subsidy alone is optimal if the objective is to minimize the market share of uncertified drugs. However, it is not guaranteed that such an approach will increase the expected quantity or health benefits. Specifically, with high-quality uncertified drugs (i.e., $\theta \to 1$) on the market and a budget only slightly larger than $\tilde{B}_1^h$, a purchase subsidy alone can close the acquisition cost gap between certified and uncertified drugs. But this also means revoking the sales subsidy, resulting in a reduction in the expected quantity sold and the associated health benefits. Hence, if the donor's objective function is to maximize the expected quantity or health benefits for consumers, using a sales subsidy only remains optimal, and therefore we have $\tilde{B}_1^h \leq \tilde{B}_2^h$ and $\tilde{B}_1^h \leq \tilde{B}_3^h$. Furthermore, it follows intuitively that $\tilde{B}_3^h \leq \tilde{B}_2^h$, since certified drugs are assumed to be of higher quality than uncertified ones. Hence, when maximizing the health benefits is the objective, the donor may be willing to accept a lower quantity of drugs sold in order to enjoy the quality gains.

– *Large Budget:* When the donor's budget size is sufficiently large ($B \geq \tilde{B}_2^h$), a purchase subsidy alone is optimal regardless of the donor's objective. This is because, first, the retailer will only purchase certified drugs after the introduction of the subsidy. Second, the expected quantity of certified drugs sold to consumers increases more rapidly in the purchase subsidy than in the sales subsidy. Hence, when the donor has a sufficiently large budget, all three objective functions are aligned, and the optimal subsidy scheme consists of a purchase subsidy only.

4.3. Practical Implications

From a practical perspective, the results in this section have several important implications that apply when retailers have pricing power. First, we recommend that donors consider using sales subsidies rather than purchase subsidies when budgets are limited and in markets in which uncertified drugs of high quality are readily available. Given the funding shortfall at both the international and domestic levels (WHO 2020b), sales subsidies might actually be the best default option for donors when retailers have access to high-quality uncertified drugs. Importantly, this recommendation differs from the prevailing recommendation to use only purchase subsidies.

Second, we note the potential for consumers to be worse off when donors focus on the objective of eliminating uncertified drugs. Specifically, when donors spend just enough to incentivize the retailer to switch from uncertified drugs to certified drugs, the expected quantity and health benefits created for consumers may both decrease. Moreover, the potential reduction in consumer health is larger the higher the quality of those uncertified drugs. Therefore, it is important for donors to be aware of the nature of the drugs from uncertified suppliers and the implications for their specific objectives before making budget allocation decisions.

Third, we caution that when consumers' counterfeit awareness is low, the retailer is more willing to source uncertified drugs with lower expected quality. In this case, our result highlights the importance of addressing the problem of funding shortfalls, as with limited funds donors may find

themselves subsidizing these low-quality drugs under the optimal subsidy scheme. In fact, when budget limitations cannot be addressed and black market counterfeit drugs are prevalent, donors may prefer to concentrate their spending in fewer locations (rather than spreading their budget thinly) and offer purchase subsidies in order to incentivize retailers to purchase certified drugs.

Finally, our results help to explain existing empirical evidence showing that lower subsidy levels lead to a higher share of uncertified drugs on the market. For example, a study on the effect of the AMFm program (discussed earlier in Section 3.1) found that in Kenya, the market share of quality-assured legitimate ACTs significantly declined after a reduction in the subsidy level from 95% to 70% (ACTwatch Group et al. 2015). Hence, if the donor is concerned about the market share of uncertified drugs, it is important to address the problem of funding shortfalls and to be strategic about how limited funds are distributed.

5. Formulations and Results Under Exogenous Retail Price Scenario

We now analyze the retailer's optimal sourcing and inventory decisions, $\omega^*(a,s)$ and $z^*(a,s)$, and the donor's optimal subsidy decision, (a^*,s^*), under the exogenous retail price scenario.

As we show later in this section, in contrast to the endogenous retail price scenario discussed in Section 4, the retailer may perform dual sourcing, i.e., purchase both certified and uncertified drugs, with $\omega^*(a,s)\in(0,1)$ when the retail price is exogenous. In this case, the purchase subsidy covers only the certified drugs purchased by the retailer with a fraction $1-\omega^*(a,s)$, while the sales subsidy covers both the certified and uncertified drugs. We will need to take this difference in coverage into account when comparing the effect of subsidies on the retailer's decisions. More specifically, let B_a denote the budget allocated to the purchase subsidy and B_s denote the budget size allocated to the sales subsidy. When analyzing the effect of B_a and B_s on $\omega^*(a,s)$, we would like to compare $\frac{\partial\omega^*(a,s)}{\partial B_a}$ and $\frac{\partial\omega^*(a,s)}{\partial B_s}$. If $\omega^*(a,s)\in(0,1)$, we have $\frac{\partial\omega^*(a,s)}{\partial a}=\frac{\partial\omega^*(a,s)}{\partial B_a}\cdot\frac{\partial B_a}{\partial a}=\frac{\partial\omega^*(a,s)}{\partial B_a}\cdot\frac{\partial B_s}{\partial s}\cdot(1-\omega^*(a,s))$ and $\frac{\partial\omega^*(a,s)}{\partial s}=\frac{\partial\omega^*(a,s)}{\partial B_s}\cdot\frac{\partial B_s}{\partial s}$, and thus it is equivalent to compare $\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))$ and $\frac{\partial\omega^*(a,s)}{\partial s}$ instead. The same equivalence holds for the effect of B_a and B_s on $z^*(a,s)$, for which we can compare $\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s))$ and $\frac{\partial z^*(a,s)}{\partial s}$.

5.1. The Impact of Subsidies on the Retailer's Joint Sourcing and Inventory Decisions

The retailer's expected profit is given by

$$\begin{aligned}V(x,h)=\max_{z,\omega:\omega z\geq hx,(1-\omega)z\geq(1-h)x}\{&-(c_c-a)[(1-\omega)z-(1-h)x]-c_u(\omega z-hx)\\&+\mathbb{E}_{M,\Upsilon}[(s+p)\min(y(\omega,p)M,z)+\delta V(z-\min(y(\omega,p)M,z),\Upsilon)]\},\end{aligned}\tag{6}$$

where the retail price p is exogenous. Following a similar argument as Lemma 1, we can show that the retailer's problem is equivalent to the following single-period problem:

$$\begin{aligned}\max_{z\geq0,\omega\in[0,1]}R(z,\omega)=\max_{z\geq0,\omega\in[0,1]}\{&-(c_c-a)(1-\omega)z-c_u\omega z+\mathbb{E}_M[(s+p)\min(y(\omega,p)M,z)\\&+\delta[((c_c-a)(1-\omega)+c_u\omega)(z-\min(y(\omega,p)M,z))]]\}.\end{aligned}\tag{7}$$

As in Section 4, we assume that the drug acquisition costs are such that it is optimal for the retailer to stock at least some drugs, i.e., $\min\{c_c, c_u\} < p$. Now we turn to the retailer's joint optimal decisions on $\omega^*(a,s)$ and $z^*(a,s)$ for a given subsidy scheme (a,s). We focus on the case with $c_c - a > c_u$ for the retailer's decisions, as otherwise the results are straightforward. When $\omega^*(a,s) \in (0,1)$ and $z^*(a,s) \in (0,\infty)$, we will rely on the first-order approach. Thus, we assume a regularity condition that the determinant of the Hessian of $R(z,\omega)$ is positive, which ensures the joint concavity of $R(z,\omega)$ and thus also the sufficiency of first-order conditions for interior optima. For conditions on model primitives that guarantee this, refer to the proof of Lemma 4 in EC.3.

For a given subsidy scheme (a,s), $\omega^*(a,s) \in (0,1)$ and $z^*(a,s) \in (0,\infty)$ are then solutions to

$$\begin{aligned} &\int_{\frac{z^*(a,s)}{y(\omega^*(a,s))}}^{\infty} [s+p-\delta((c_c-a)(1-\omega^*(a,s))+c_u\omega^*(a,s))]f(m)dm \\ &-(1-\delta)((c_c-a)(1-\omega^*(a,s))+c_u\omega^*(a,s))=0, \end{aligned} \tag{8}$$

$$\begin{aligned} &\int_0^{\frac{z^*(a,s)}{y(\omega^*(a,s))}} \{\delta(c_c-a-c_u)y(\omega^*(a,s))-[s+p-\delta((c_c-a)(1-\omega^*(a,s))+c_u\omega^*(a,s))]k(1-\theta)\} \\ &mf(m)dm+\int_{\frac{z^*(a,s)}{y(\omega^*(a,s))}}^{\infty} \delta(c_c-a-c_u)z^*(a,s)f(m)dm+(1-\delta)(c_c-a-c_u)z^*(a,s)=0. \end{aligned} \tag{9}$$

In contrast to the case under endogenous retail pricing, the retailer may perform dual sourcing when the retail price is exogenous. This occurs because the retailer uses the fraction of uncertified drugs purchased as a means to modify his acquisition costs and adjust his profit margin. In particular, in the case of dual sourcing, $\omega^*(a,s) \in (0,1)$ is achieved when the marginal acquisition cost reduction due to higher ω equals the marginal profit loss caused by lower demand resulting from higher ω. Meanwhile, $z^*(a,s)$ is achieved when the marginal contribution from purchasing and selling an additional unit equals the marginal cost of purchasing and not selling that unit.

Effect on Sourcing. We now characterize the effect of a higher subsidy on the retailer's optimal dual-sourcing decision, with the fraction of uncertified drugs and inventory level jointly decided.

LEMMA 4. (Effect of Subsidies on Sourcing). *For a given subsidy scheme (a,s), suppose $\omega^*(a,s) \in (0,1)$. There exist $\bar{f}_{a\omega}, \bar{f}_{s\omega}, \bar{f}_{c\omega} \in [0,\infty)$ and $\bar{\delta}_{c\omega} \in [0,1]$ s.t. the directional effect of subsidies on the retailer's sourcing decision is as given in the following table.*

Effect of Subsidies on Sourcing	Conditions	Direct Effect	Indirect Effect	Total Effect
$\frac{\partial\omega^*(a,s)}{\partial j}, j \in \{a,s\}$	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{j\omega},\infty), j \in \{a,s\}$	$-$	$-/+$	$-$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0,\bar{f}_{j\omega}), j \in \{a,s\}$	$-$	$+$	$+$
$\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s)) - \frac{\partial\omega^*(a,s)}{\partial s}$	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{c\omega},\infty)$ and $\delta \in (\bar{\delta}_{c\omega},1)$	$-$	$-/+$	$-$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0,\bar{f}_{c\omega})$ and $\delta \in (\bar{\delta}_{c\omega},1)$	$-$	$+$	$+$
	$\delta \in (0,\bar{\delta}_{c\omega})$	$+$	$-/+$	$+$

First, Lemma 4 shows that when the retailer performs dual sourcing, a higher purchase or sales subsidy could lead to a higher fraction of uncertified drugs. This is because, on the one hand, higher subsidies affect the fraction of uncertified drugs through two direct channels:

- *Direct acquisition cost-saving effect*: With a higher purchase subsidy, the acquisition cost of certified drugs is reduced; hence, the retailer purchases a higher fraction of certified drugs.
- *Direct demand-enhancing effect*: With a higher purchase or sales subsidy, the retailer enjoys a higher profit margin for drugs sold. As a result, every lost unit of demand is more costly to the retailer. Hence, the retailer is made better off by purchasing a lower fraction of uncertified drugs $\omega^*(a,s)$ in order to increase consumer demand and reduce the marginal profit loss.

We note that when the discount factor δ is large, a purchase subsidy reduces the fraction of uncertified drugs via the direct channels more rapidly than a sales subsidy. This is because (i) a purchase subsidy utilizes both direct channels while a sales subsidy only uses one, and (ii) the value loss due to δ arising from a purchase subsidy is small in this case, as δ is large. On the other hand, there are two additional indirect channels through which purchase and sales subsidies affect the fraction of uncertified drugs via their impact on the inventory level:

- *Indirect demand-enhancing effect*: A higher purchase or sales subsidy leads directly to a higher inventory level (see Lemma 5 and the associated discussions for more details). With a higher inventory level, the retailer now wishes to induce additional consumer demand in order to sell out the additional units. This is achieved by reducing the fraction of uncertified drugs purchased, which boosts demand by increasing the expected quality of drugs available. This will lower the threshold market condition $\frac{z^*(a,s)}{y(\omega^*(a,s))}$ and therefore increase the stock-out probability $\int_{\frac{z^*(a,s)}{y(\omega^*(a,s))}}^{\infty} f(m)dm$, such that the retailer is more likely to sell the additional units.
- *Indirect acquisition cost-saving effect*: With a higher inventory level, the retailer also has the incentive to reduce acquisition costs, which is achieved by increasing the fraction of uncertified drugs purchased, as uncertified drugs have lower acquisition costs than certified drugs.

We can see that only the indirect acquisition cost-saving effect leads to an increase in the market share of uncertified drugs, with all of the other direct and indirect effects reducing the uncertified drugs' market share. Furthermore, Lemma 4 shows that the dominant directional effect is determined by the density distribution at the threshold market condition $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$. In particular, when $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is small, the retailer benefits little from the indirect demand-enhancing effect resulting from a lower $\omega(a,s)$. This is because when the threshold market condition falls in the low-density part of the distribution, the stock-out probability only increases marginally with a lower $\omega(a,s)$. Hence, the indirect effect associated with reducing the acquisition cost dominates, and the net indirect effect of a higher subsidy is to increase the market share of uncertified drugs. Moreover, when $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is small, a higher subsidy leads to a large increase in the inventory level. This is because a higher subsidy leads to a lower marginal cost of purchasing and not selling an additional

unit. In order to restore the FOC, the retailer then needs to stock more units to reduce the marginal contribution from purchasing and selling an additional unit. In this case, with an additional unit, the stock-out probability $\int_{\frac{z^*(a,s)}{y(\omega^*(a,s))}}^{\infty} f(m)dm$ only decreases marginally given small $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$, and therefore the marginal contribution from purchasing and selling an additional unit only decreases marginally. Hence, the retailer needs to stock significantly more units in order to restore the FOC. As a result, the net indirect effect dominates the net direct effect, and both a higher purchase and sales subsidy can result in a higher market share of uncertified drugs.

On the other hand, when the density distribution at the threshold market condition $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is large, the retailer benefits greatly from a lower $\omega(a,s)$ through the indirect demand-enhancing effect: the stock-out probability increases significantly with a lower $\omega(a,s)$, and the retailer can sell significantly more additional units. Hence, the indirect demand-enhancing effect dominates, and the net indirect effect of a higher subsidy is to reduce the fraction of uncertified drugs. The net effect of both direct and indirect channels is therefore to reduce the fraction of uncertified drugs.

Comparing the effects of a purchase versus sales subsidy on the market share of uncertified drugs, we see that the more effective solution is determined by both $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ and δ. Specifically:

- When both $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ and δ are large, the two direct effects dominate the two indirect effects. Meanwhile, a purchase subsidy is more effective in reducing the market share of uncertified drugs than a sales subsidy via the direct effect. Hence, the market share of uncertified drugs decreases more rapidly (or increases less rapidly) in a purchase subsidy than in a sales subsidy.
- When $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is small and δ remains large, there are two competing forces. On the one hand, a large δ means that a purchase subsidy is better than a sales subsidy in reducing the uncertified drugs' market share through the first three channels. On the other hand, the fourth indirect acquisition cost-saving effect channel dominates due to the small $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$. Furthermore, we know that a purchase subsidy increases the inventory level more rapidly than a sales subsidy via direct effects (see Lemma 5 and the associated discussions for more details). Since a higher inventory level results in a higher fraction of uncertified drugs via the fourth channel, a purchase subsidy can increase the market share of uncertified drugs more rapidly than a sales subsidy. Thus, we find that the market share of uncertified drugs decreases more rapidly (or increases less rapidly) in a sales subsidy than in a purchase subsidy in this case.
- Meanwhile, when δ is small, a sales subsidy reduces the market share of uncertified drugs more rapidly than a purchase subsidy through the first three channels. A sales subsidy also increases the market share of uncertified drugs less rapidly than a purchase subsidy through the fourth channel. As a result, the net effect is that a sales subsidy decreases the market share of uncertified drugs more rapidly (or increases it less rapidly) than a purchase subsidy.

Effect on Inventory Level. Next, we characterize the effect of a higher subsidy on the retailer's optimal inventory level, with the fraction of uncertified drugs and inventory level jointly decided.

LEMMA 5. (Effect of Subsidies on Inventory Level). *For a given subsidy scheme* (a, s)*, suppose* $\omega^*(a,s) \in (0,1)$*. There exist* $\bar{f}_{az}, \bar{f}_{sz}, \bar{f}_{cz} \in [0,\infty)$ *and* $\bar{\delta}_{cz} \in [0,1]$ *s.t. the directional effect of subsidies on the retailer's sourcing decision is given by the following table.*

Effect of Subsidies on Inventory Level	Conditions	Direct Effect	Indirect Effect	Total Effect
$\frac{\partial z^*(a,s)}{\partial j}, j \in \{a,s\}$	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{jz}, \infty), j \in \{a,s\}$	$+$	$-/+$	$+$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0, \bar{f}_{jz}), j \in \{a,s\}$	$+$	$-$	$-$
$\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s)) - \frac{\partial z^*(a,s)}{\partial s}$	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{cz}, \infty)$ and $\delta \in (\bar{\delta}_{cz}, 1)$	$+$	$-/+$	$+$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0, \bar{f}_{cz})$ and $\delta \in (0, \bar{\delta}_{cz})$	$+$	$-/+$	$+$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0, \bar{f}_{cz})$ and $\delta \in (\bar{\delta}_{cz}, 1)$	$+$	$-$	$-$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{cz}, \infty)$ and $\delta \in (0, \bar{\delta}_{cz})$	$+$	$-$	$-$

Again, we see that when the retailer decides on $\omega^*(a,s)$ and $z^*(a,s)$ jointly, it is possible for a higher subsidy to result in a lower inventory level. This is because subsidies first affect the inventory level through two direct channels:

- *Direct overage reducing effect*: With a higher purchase subsidy, the retailer enjoys a lower marginal cost of stocking and not selling a unit, which leads to a higher inventory level.
- *Direct underage increasing effect*: With a higher purchase or sales subsidy, the retailer enjoys a higher profit margin from stocking and selling a unit. Hence, the retailer stocks more units.

Again, we note that a purchase subsidy benefits from both direct effects but a sales subsidy utilizes only one. Hence, a purchase subsidy increases the inventory level more rapidly than a sales subsidy via the direct channels. Meanwhile, we have two additional indirect channels through which subsidies affect the inventory level by their influence on the fraction of uncertified drugs:

- *Indirect underage reducing effect*: A higher subsidy leads directly to a lower fraction of uncertified drugs. The retailer is thus incentivized to sell more units to reduce underage costs, as consumer demand is now higher. Hence, the retailer stocks more with a higher inventory level.
- *Indirect overage reducing effect*: With a lower fraction of uncertified drugs, the retailer also has a higher stocking cost and therefore a higher marginal cost of stocking excess inventory. Consequently, the retailer stocks less with a lower inventory level.

While the first three effects increase the inventory level, the last indirect overage-reducing effect decreases the inventory level. Similar to the discussion following Lemma 4, whether the net effect of a subsidy on the inventory level is positive or negative depends on the value of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$. Furthermore, whether a purchase or sales subsidy is more effective in increasing the inventory level depends on $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ and δ. We discuss the results of Lemma 5 in full detail in EC.4.

5.2. The Impact of Subsidies on the Donor's Objectives

Combining Lemmas 4 and 5, we can now characterize the impact of a higher subsidy on the expected quantity and health benefits created for consumers when the retailer performs dual sourcing.

PROPOSITION 2. (Effect of Subsidies on the Quantity and Health Benefit). *For a given subsidy scheme* (a,s)*, suppose* $\omega^*(a,s) \in (0,1)$*. Denote* $N_F := \int_{z^*(a,s)/y(\omega^*(a,s))}^{\infty} f(m)dm$ *and* $N_E := \int_0^{z^*(a,s)/y(\omega^*(a,s))} mf(m)dm$*.*

(i) $\frac{\partial D_2(a,s)}{\partial j} > 0$*, if and only if* $N_F \frac{\partial z^*(a,s)}{\partial j} > k(1-\theta)\frac{\partial \omega^*(a,s)}{\partial j} N_E, j \in \{a,s\}$.

(ii) $\frac{\partial D_2(a,s)}{\partial a}/(1-\omega^*(a,s)) > \frac{\partial D_2(a,s)}{\partial s}$*, if and only if* $N_F[\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s)) - \frac{\partial z^*(a,s)}{\partial s}] > k(1-\theta)[\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s)) - \frac{\partial \omega^*(a,s)}{\partial s}]N_E$.

(iii) $\frac{\partial D_3(a,s)}{\partial j} > 0$*, if and only if* $(\omega^*(a,s)\theta + 1 - \omega^*(a,s))\frac{\partial D_2(a,s)}{\partial j} > k(1-\theta)\frac{\partial \omega^*(a,s)}{\partial j} D_2(a,s), j \in \{a,s\}$.

(iv) $\frac{\partial D_3(a,s)}{\partial a}/(1-\omega^*(a,s)) > \frac{\partial D_3(a,s)}{\partial s}$*, if and only if* $(\omega^*(a,s)\theta + 1 - \omega^*(a,s))[\frac{\partial D_2(a,s)}{\partial a}/(1-\omega^*(a,s)) - \frac{\partial D_2(a,s)}{\partial s}] > k(1-\theta)[\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s)) - \frac{\partial \omega^*(a,s)}{\partial s}]D_2(a,s)$.

Proposition 2 (i) characterizes the conditions under which a higher subsidy leads to a higher expected quantity sold to consumers under dual sourcing. In particular, this occurs when the impact of a purchase or sales subsidy on the inventory level is sufficiently higher than the impact of the subsidy on the market share of uncertified drugs. For the inventory level, we can see that the effect of $\frac{\partial z^*(a,s)}{\partial j}, j \in \{a,s\}$ on $D_2(a,s)$ is moderated by the stock-out probability N_F, as the inventory level only affects the expected quantity sold to consumers when the market condition is strong. By contrast, for the uncertified drugs' market share, the effect of $\frac{\partial \omega^*(a,s)}{\partial j}, j \in \{a,s\}$ on $D_2(a,s)$ is moderated by $k(1-\theta)N_E$, as the market share of uncertified drugs only affects the expected quantity sold to consumers when the market condition is weak, and the magnitude of the effect depends on the quality of the uncertified drugs and consumers' counterfeit awareness. Proposition 2 (ii) then follows from (i) and specifies when a purchase subsidy increases the expected quantity sold to consumers more rapidly (or decreases it less rapidly) than a sales subsidy.

Next, Proposition 2 (iii) characterizes the conditions under which a higher subsidy leads to a higher expected health benefits for consumers. This occurs when the impact of a purchase or sales subsidy on the expected quantity is sufficiently higher than the impact of the subsidy on the market share of uncertified drugs. For the expected quantity, we can see that the effect of $\frac{\partial D_2(a,s)}{\partial j}, j \in \{a,s\}$ on $D_3(a,s)$ is moderated by $(\omega^*(a,s)\theta + 1 - \omega^*(a,s))$, which captures the quality dimension of $D_3(a,s)$. Meanwhile, for the market share of uncertified drugs, we see that the effect of $\frac{\partial \omega^*(a,s)}{\partial j}, j \in \{a,s\}$ on $D_3(a,s)$ is moderated by $k(1-\theta)D_2(a,s)$, which captures the quantity dimension of $D_3(a,s)$ in addition to the quality dimension. Proposition 2 (iv) follows from (iii) and characterizes the conditions under which a purchase subsidy increases the expected health benefits for consumers more rapidly (or decreases it less rapidly) than a sales subsidy.

Lastly, Proposition 2 shows that whenever a higher purchase or sales subsidy leads to a higher inventory level and lower market share of uncertified drugs, it will also increase the expected

quantity and health benefits for consumers. In other words, the three objective functions are aligned. This could happen when $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is sufficiently large, such as when the market condition uncertainty is small. On the other hand, if increasing the size of the purchase or sales subsidy leads to a lower inventory level and higher market share of uncertified drugs, a larger subsidy will worsen all of the three objective functions of the donor. This could happen when $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is sufficiently small, such as when the market condition uncertainty is large.

5.3. Donor's Problem

We now characterize the donor's optimal subsidy scheme given an exogenous retail price.

PROPOSITION 3. (Optimal Subsidy). *Let* (a^*, s^*) *denote the donor's optimal subsidy scheme. There exist* $0 \le \hat{r}_0^l \le \hat{r}_0^{ml} < \hat{r}_0^{mh} \le \hat{r}_0^h$ *s.t.*

(i) *If* $r_0 \in [0, \hat{r}_0^l]$, *we have* $\omega^*(0,0) = 0$. *In this case, for any* $B \in [0, \infty)$, *we have* $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$.

(ii) *If* $r_0 \in (\hat{r}_0^{ml}, \hat{r}_0^{mh})$, *we have* $\omega^*(0,0) \in (0,1)$. *In this case, there exist* $0 \le \hat{B}_1^m \le \hat{B}_2^m$ *s.t.*

  (a) *For any* $B \in [0, \hat{B}_1^m)$, *we could have* $a^* = 0, s^* > 0$, *or* $a^* > 0, s^* = 0$, *or* $a^* = s^* = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$. *Detailed conditions for each scenario follow from Lemma 4 and Proposition 2.*

  (b) *For any* $B \in [\hat{B}_2^m, \infty)$, *we have* $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$.

(iii) *If* $r_0 \in [\hat{r}_0^h, \infty)$, *we have* $\omega^*(0,0) = 1$. *In this case, there exist* $0 \le \hat{B}_1^h \le \hat{B}_2^h$ *s.t.*

  (a) *For any* $B \in [0, \hat{B}_1^h)$, *we have* $a^* = 0, s^* > 0$ *and* $\omega^*(a^*, s^*) = 1$, *for* $D_i(a,s), i \in \{1,2,3\}$.

  (b) *For any* $B \in [\hat{B}_2^h, \infty)$, *we have* $a^* > 0, s^* = 0$ *and* $\omega^*(a^*, s^*) = 0$, *for* $D_i(a,s), i \in \{1,2,3\}$.

Proposition 3 (i) shows that, similar to Proposition 1 (i), with black uncertified drugs of low quality (e.g., falsified drugs) on the market or when consumers' counterfeit awareness is high, the retailer only purchases certified drugs in the absence of a subsidy. In this case, the donor's optimal subsidy always consists of a purchase subsidy only.

Meanwhile, Proposition 3 (ii) occurs when there are substandard uncertified drugs on the market or when consumers' counterfeit awareness is moderate. In this case, the retailer performs dual sourcing and purchases a mixture of certified and uncertified drugs in the absence of a subsidy. Proposition 3 (ii) shows that when the conditions for dual sourcing arise and when the donor has only a limited budget, she may need to refrain from providing any subsidy, as a higher purchase or sales subsidy could worsen all three objectives. Notably, this is even the case when the uncertified drugs are not subsidized, i.e., when the donor provides only a purchase subsidy. Meanwhile, there are also conditions under which using a sales subsidy could improve any of the objective functions more rapidly than a purchase subsidy and vice versa. While it is challenging to specify all of the conditions for all of the objectives in practice, Proposition 3 (ii) shows that this challenge in selecting the optimal subsidy strategy is resolved when the budget is sufficiently large, in which case it is optimal to use a purchase subsidy only, for any of the objectives.

Lastly, we see in Proposition 3 (iii) that, similar to Proposition 1 (iii), with grey uncertified drugs of high quality (e.g., non-quality-assured legitimate drugs) or a low consumers' counterfeit awareness, the retailer only purchases uncertified drugs in the absence of a subsidy. In this case, for any of the three objective functions, the donor's optimal subsidy consists of only a sales subsidy when the donor's budget size is relatively small and only a purchase subsidy when her budget size is sufficiently large. We note that Proposition 3 does not cover the entire parameter space for r_0 and B. We therefore perform an extensive numerical analysis with malaria data in Mozambique in EC.8 to provide a more complete understanding of the optimal subsidy scheme in this case.

5.4. Practical Implications

Our analysis highlights the potential complications associated with implementing an optimal subsidy scheme in the presence of uncertified drugs, and it examines how these challenges are exacerbated with exogenous retail prices. Deciding on the optimal subsidy scheme in this setting requires an additional understanding of specific market characteristics, unlike in the case where uncertified drugs are not present or where retail prices are endogenous. In particular, when identifying the optimal subsidy with exogenous prices, we must further take into account demand characteristics such as demand uncertainty and product characteristics such as the value loss from carrying unsold products to the next period. Determining these market characteristics is not always easy, making it challenging to identify the optimal subsidy scheme when retail prices are exogenous.

The results in this section also highlight the potential unintended consequences when retailers have limited pricing power: specifically, a higher subsidy could result in a higher market share of uncertified drugs, lower availability of drugs, or even lower health benefits. Perhaps more surprisingly, all three objective functions can be worsened simultaneously when consumer demand is highly uncertain. This is in contrast to the setting with endogenous retail prices and implies that it may be necessary for donors to refrain from offering any subsidy program at all when price controls are in effect. Since deciding whether or not to intervene under these conditions critically depends on the uncertainty of market demand, donors should be acutely aware of local market characteristics.[7]

The phenomenon highlighted above may help to explain observations from practice, specifically those relating to the effectiveness of the AMFm program. In their evaluation of the AMFm program, ACTwatch Group et al. (2015) found that results in Madagascar were highly unfavorable, with the availability of quality-assured ACTs in the private sector falling significantly in the post-AMFm implementation period. Interestingly, retailers in Madagascar also typically have limited pricing power (AMFm Independent Evaluation Team 2012). Thus, the failure of the AMFm program in Madagascar might be explained by our finding that a subsidy program can worsen the prevailing situation in the presence of uncertified drugs and in an exogenous retail price scenario. Hence, we recommend that donors consider the extent to which retailers have pricing power when designing and implementing subsidy programs in markets in which uncertified drugs are available.

[7] Note that we show later in our numerical analysis that the distribution of demand can be highly variable, suggesting that the conditions under which subsidies may be disadvantageous could arise in the real world.

6. Alternative (Non-Subisidy) Strategies

Up to this point, we have been focusing on the use of subsidies as a means to improve outcomes in markets characterized by the presence of counterfeit drugs. However, several other interventions have been discussed or adopted in practice to combat the issues associated with counterfeit drugs. In this section, we examine the impact of four interventions before turning to a fifth in the numerical analysis in Section 7. Specifically, in this section we focus on: increasing *consumer* awareness of counterfeits, increasing the cost of sourcing counterfeits for *retailers*, the adoption of traceability technology by *donors*, and a crackdown on uncertified distribution by *law enforcement*.

6.1. Increasing Consumers' Counterfeit Awareness

One strategy to tackle the problem of uncertified drugs is to improve consumers' awareness of the issues surrounding counterfeits, such as the prevalence of counterfeits on the market or the health cost associated with consuming counterfeits (Nyqvist et al. 2020). In our model, improving consumers' counterfeit awareness is captured by a higher k.

In the case of single sourcing, the impact of increasing consumers' counterfeit awareness points to a trade-off. On the one hand, higher k reduces r_a and therefore lowers the attractiveness of uncertified drugs to the retailer. As a result, the budget size required by the donor to switch the retailer from sourcing uncertified drugs to sourcing certified drugs is smaller. On the other hand, when there are only uncertified drugs on the market, greater consumer awareness results in a reduction in the expected quantity of drugs sold as well as a deterioration in the health benefits for consumers due to the fall in consumer demand that follows from a higher k. This is the case even when we have grey uncertified drugs on the market with high θ.

Meanwhile, when the retailer performs dual sourcing, we can characterize the impact of higher consumer awareness on the retailer's decisions with the following proposition.

PROPOSITION 4. (Effect of Counterfeit Awareness on Retailer's Decisions Under Dual Sourcing). *For a given subsidy scheme* (a,s)*, suppose* $\omega^*(a,s) \in (0,1)$*. There exist* $\bar{f}_{kz} \in [0,\infty)$ *and* $\bar{\omega}_{kz} \in [0,1]$ *s.t. the directional effect of consumers' counterfeit awareness on the retailer's sourcing and inventory level decisions are given by the following table.*

Effect of Counterfeit Awareness	Conditions	Direct Effect	Indirect Effect	Total Effect
$\frac{\partial \omega^*(a,s)}{\partial k}$	unconditional	$-$	$-/+$	$-$
$\frac{\partial z^*(a,s)}{\partial k}$	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{kz},\infty)$ and $\omega^*(a,s) \in (0,\bar{\omega}_{kz})$	$-$	$+$	$+$
	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0,\bar{f}_{kz})$ and $\omega^*(a,s) \in (0,\bar{\omega}_{kz})$	$-$	$-/+$	$-$
	$\omega^*(a,s) \in (\bar{\delta}_{kz},1)$	$-$	$-/+$	$-$

Proposition 4 shows that under dual sourcing, while higher counterfeit awareness always reduces the market share of uncertified drugs, it leads to a higher inventory level only if the market

condition uncertainty is low and the fraction of uncertified drugs is low. (See EC.6 for an explanation of the mechanisms underlying these effects.) Importantly, this means that increasing consumers' counterfeit awareness does not always lead to a higher quantity sold or increased health benefits. Instead, similar to Proposition 2, Proposition 4 implies that higher counterfeit awareness results in a higher quantity sold to consumers if and only if the effect on the inventory level is sufficiently greater than the effect on the fraction of uncertified drugs. Furthermore, higher counterfeit awareness leads to higher health benefits for consumers if and only if the effect of counterfeit awareness on the quantity sold is sufficiently greater than the effect on the fraction of uncertified drugs.

Overall, these results indicate that care must be taken when increasing consumers' counterfeit awareness to avoid unintended adverse consequences for consumers. Interestingly, this need for caution could hold even in markets characterized by the presence of low-quality uncertified drugs.

6.2. Increasing the Acquisition Cost of Uncertified Drugs

Another widely used approach to combat the counterfeit drug problem is to increase the cost of sourcing counterfeit drugs. This could be achieved by, e.g., imposing heavier penalties or increasing the surveillance of retailers so that those who source counterfeits are more likely to be caught. In this way, sourcing uncertified drugs becomes less profitable for the retailer, i.e., c_u increases. While this may seem a natural strategy to adopt, its implications are more nuanced.

Specifically, increasing cost of sourcing uncertified drugs presents a similar trade-off as improving consumers' counterfeit awareness. In particular, when the retailer performs single sourcing, a higher c_u reduces r_a and, therefore, the attractiveness of uncertified drugs to the retailer. As a result, the budget size required by the donor to switch the retailer from sourcing uncertified drugs to sourcing certified drugs is smaller. On the other hand, when there are only uncertified drugs on the market, it is clear that higher c_u reduces the expected quantity sold as well as the health benefits created for consumers. Meanwhile, when the retailer performs dual sourcing, we can characterize the impact of a higher c_u on the retailer's decisions by the following proposition.

Proposition 5. (Effect of Higher Acquisition Cost of Uncertified Drugs on Retailer's Decisions Under Dual Sourcing). *For a given subsidy scheme (a,s), suppose $\omega^*(a,s) \in (0,1)$. There exist $\bar{f}_{c_u z} \in [0,\infty)$ and $\bar{\theta}_{c_u \omega}, \bar{\theta}_{c_u z} \in [0,1]$ s.t. the directional effect of higher uncertified drugs acquisition cost on the retailer's sourcing and inventory level decisions is given by the following table.*

Effect of Acquisition Cost of Uncertified Drugs	Conditions	Direct Effect	Indirect Effect	Total Effect
$\frac{\partial \omega^*(a,s)}{\partial c_u}$	$\theta \in (\bar{\theta}_{c_u \omega}, 1)$	$-/+$	$-/+$	$-$
	$\theta \in (0, \bar{\theta}_{c_u \omega})$	$-/+$	$-/+$	$+$
$\frac{\partial z^*(a,s)}{\partial c_u}$	$f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{c_u z}, \infty)$ and $\theta \in (\bar{\theta}_{c_u z}, 1)$	$-$	$+$	$+$
	$\theta \in (0, \bar{\theta}_{c_u z})$ or $(f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0, \bar{f}_{c_u z})$ and $\theta \in (\bar{\theta}_{c_u z}, 1))$	$-$	$-/+$	$-$

Proposition 5 shows that under dual sourcing, the market share of uncertified drugs can increase after increasing the cost of sourcing uncertified drugs; surprisingly, this tends to occur when the quality of uncertified drugs is low. Furthermore, a higher uncertified drugs acquisition cost will result in the retailer stocking at a higher inventory level only if the market condition uncertainty is low and the quality of uncertified drugs is high. (See EC.7 for an explanation of the mechanisms underlying these effects.) Together these insights suggest that increasing the acquisition cost associated with uncertified drugs is more likely to be beneficial in markets characterized by low demand uncertainty and relatively high-quality uncertified drugs. To determine the impact of higher cost of sourcing uncertified drugs on the expected quantity sold and the health benefits created for consumers, we can again construct a proposition similar to Proposition 2.

6.3. Eliminating Subsidies on Uncertified Drugs: Traceability Technology

Traceability technologies, including mobile and RFID-based solutions, have also been proposed as a solution to the problem of counterfeit drugs (Mackey and Nayyar 2017). In particular, blockchain technology is maturing as a revolutionary technology framework and may better ensure a "digitized" drug supply chain that is more transparent and protected from counterfeit drug infiltration. Taking advantage of these technological advances, regulators and pharmaceutical firms have been developing and deploying traceability technology for the detection of counterfeit drugs.

In the context of subsidizing antimalarial drugs, one challenge with the use of sales subsidies is that they are unable to discriminate between certified and uncertified drugs, unlike purchase subsidies, which are only applied to certified drugs. Thus, one solution is to adopt traceability technology that allows the donor to detect uncertified drugs, thus ensuring that the sales subsidies only apply to certified drugs. However, we show that the adoption of traceability technology by the donor may make things worse:

PROPOSITION 6. (Effect of Traceability Technology Adoption). *There exist* $\tilde{B}_{tt}, \hat{B}_{tt} \geq 0$ *s.t.:*

(i) *Suppose the retail price is endogenous. The adoption of traceability technology has no impact if* $B \geq \tilde{B}_{tt}$*; the adoption of traceability technology either has no impact or reduces the expected quantity of drugs sold and the health benefits created for consumers if* $B < \tilde{B}_{tt}$.

(ii) *Suppose the retail price is exogenous. The adoption of traceability technology has no impact if* $B \geq \hat{B}_{tt}$*; the adoption of traceability technology could reduce, increase or have no impact on any of the three objectives of the donor if* $B < \hat{B}_{tt}$.

Proposition 6 shows that when the donor has a sufficiently large budget, the adoption of traceability has no impact. This follows from the fact that when the donor has a large enough budget, the optimal subsidy scheme consists of a purchase subsidy only and the retailer does not source uncertified drugs. In this case, the donor is already not subsidizing uncertified drugs in the absence of traceability technology. Hence, the adoption of traceability technology has no impact.

By contrast, when the donor has a limited budget, the impact of traceability technology adoption depends on the retailer's pricing power. When the retail price is endogenous, the adoption of traceability technology has no benefit for consumers; in fact, it could hurt consumers. In particular, when the retailer sources uncertified drugs only in the absence of traceability technology, this suggests that the optimal subsidy scheme consists of a sales subsidy only and requires subsidizing these uncertified drugs. However, in this case, the adoption of traceability technology could force the donor to subsidize only certified drugs with a purchase subsidy. This is a feasible yet suboptimal scheme in the absence of traceability technology. Hence, the adoption of traceability technology reduces the quantity of drugs sold and the health benefits created for consumers. When the retail price is exogenous, the retailer could perform dual sourcing. In this case, the impact of traceability technology adoption is more nuanced and depends on both product and market characteristics.[8]

These results point to a need for caution in the adoption of traceability technology. Even when the technology is available at zero cost, it can harm consumers. Since in reality the technology is likely to be expensive to develop and implement and it could be challenging for donors to identify conditions under which the adoption of such technology helps, our results suggest that instead, donors may want to invest their limited resources in expanding the budget size available for the subsidy program. This is because, as per Proposition 1 and Proposition 3, with a sufficiently large budget the donor can offer a purchase subsidy that is capable of simultaneously eliminating uncertified drugs and maximizing health benefits created for consumers, a win-win situation. Finally, we note that the adoption of traceability technology at the consumer level, such that consumers can identify uncertified drugs at the time of purchase, might be an alternative implementation of this technology. While not specifically related to the activities of donors and thus outside the scope of this paper, future research might look to examine the implications of adopting such technologies for tackling the problem of counterfeits.

6.4. Eliminating Uncertified Drugs: Does Their Presence Harm Consumers?

Given the mixed results in Sections 6.1 to 6.3, we now study the conditions under which further steps should be taken to crack down directly on the supply of uncertified drugs. For example, law enforcement might strengthen customs inspections and border patrols to prevent uncertified drugs from entering local markets, or they might invest in policing to identify and shut down uncontrolled or street laboratories responsible for counterfeits. Taken to the extreme, these actions may make it essentially impossible for retailers to source uncertified drugs. We consider when this hard-line approach may pay dividends, i.e., when the presence of uncertified drugs harms consumers.

PROPOSITION 7. (Effect of Uncertified Drugs' Presence on Consumer Health). *Suppose* $c_c - a > c_u$. *There exist* $\bar{\theta} < \underline{\theta} \in [0,1)$ *and* $\bar{k} < \underline{k} \in (0,1]$ *s.t. for any* $\theta \leq \bar{\theta}$ *and* $k \leq \bar{k}$, *the presence of uncertified drugs hurts consumer health; for any* $\theta \leq \bar{\theta}$ *and* $k \geq \underline{k}$, *the presence of uncertified drugs has no impact; for any* $\theta \geq \underline{\theta}$, *the presence of uncertified drugs benefits consumer health.*

[8] Detailed conditions can be obtained in a manner comparable to Lemma 4 and Proposition 2.

Proposition 7 shows that whether the presence of uncertified drugs on the market harms consumer health depends on their quality and consumers' counterfeit awareness. On the one hand, when uncertified drugs are of low quality and consumers' counterfeit awareness is low, the retailer could source uncertified drugs to save on acquisition costs with little loss of consumer demand. In this case, consumers are harmed by the presence of uncertified drugs due to their low quality. However, if consumers' counterfeit awareness is high, the retailer is better off sourcing only certified drugs, as sourcing uncertified drugs would reduce consumer demand significantly. In this case, the presence of uncertified drugs has no impact. On the other hand, higher quality uncertified drugs confer a benefit to consumers independent of consumers' counterfeit awareness. When the quality of uncertified drugs is close to that of certified drugs, the reduced acquisition cost of uncertified drugs leads the retailer to offer a lower retail price (when prices are endogenous) and stock more inventory. In this case, consumers benefit from the presence of uncertified drugs.

Finally, we note that Proposition 7 does not cover the entire parameter space for θ and k. We therefore present extensive numerical results with malaria data from Mozambique in EC.8 to complement the analysis. Generally, we find that under endogenous retail prices, the same insights hold regardless of the values of θ and k. The results under exogenous retail prices are more complicated, and our numerical analysis shows that determining the magnitudes of the benefits of eliminating uncertified drugs requires a more comprehensive understanding of the market characteristics.

7. Numerical Analysis

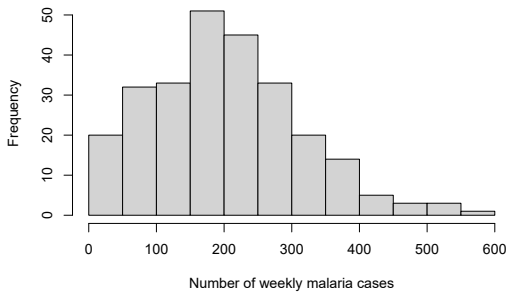
In this section, we numerically explore the retailer's sourcing and inventory decisions and the donor's optimal subsidy decisions when the models are calibrated to malaria data from Mozambique. The goal of this analysis is twofold. First, as some results do not hold for all parameter values – including Proposition 3 in Section 5.3 and Proposition 7 in Section 6.4 – we aim to explore their sensitivity to a range of realistic parameter values. The results from this sensitivity analysis are documented in EC.8. Second, while in Sections 4 and 5 we analyzed the cases of endogenous and exogenous prices respectively, finding the optimal strategies for each, identifying the circumstances under which it may be beneficial for *governments* to implement price controls was not a tractable question. Therefore, we aim to shed light on this question numerically in this section.

7.1. Data Description

An important feature of the market for ACTs is the uncertainty of demand (Bitran and Martorell 2009). The number of malaria cases at a narrow geographic level served by a retail outlet can vary significantly and is challenging to predict (Alemu et al. 2012). However, as we show in Sections 5 and 6, under exogenous retail prices, demand uncertainty plays a key role in the retailer's sourcing and ordering decisions; it must therefore also play a role in any evaluation of the impact of the donor's subsidy decisions in the presence of uncertified drugs. To capture this demand uncertainty

in a realistic setting, we use data on the number of malaria cases in six villages in Mozambique from Ferrão et al. (2021), which provides data on weekly malaria cases from 2015 to 2019. The numerical analysis in this section uses data from Sussunden (Figure 3 shows the histogram of weekly malaria cases), with similar results achieved when using data from the other five villages.

Figure 3 Histogram of weekly malaria cases in Sussunden, Mozambique from 2015 to 2019.



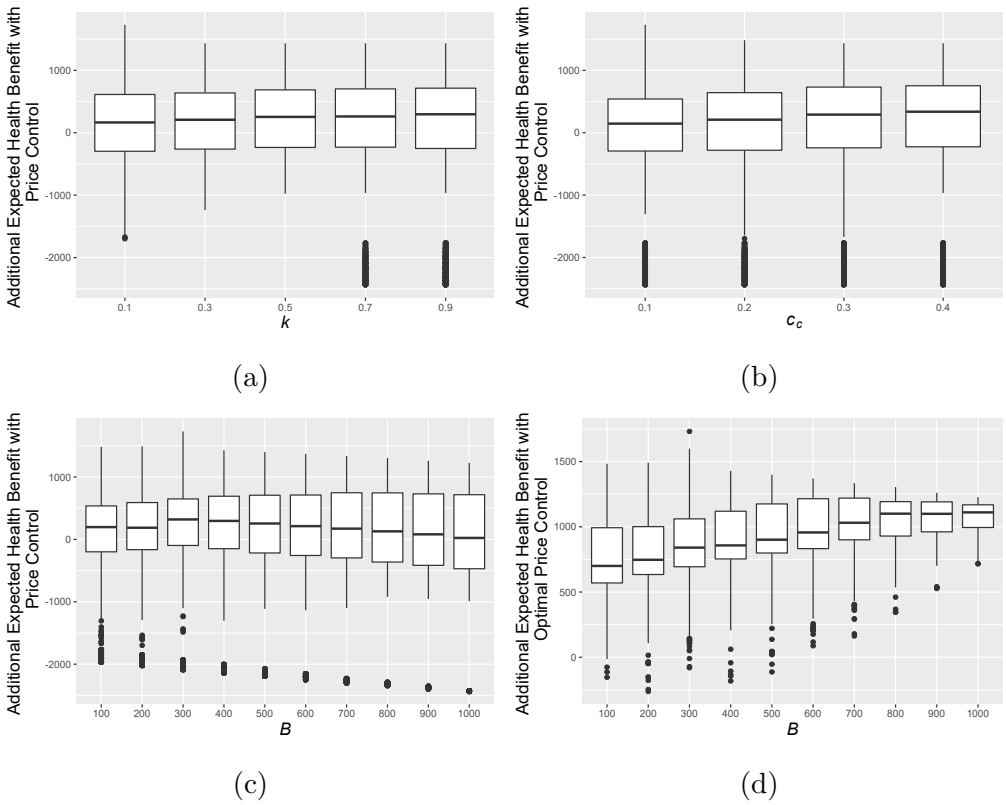
The retail price of ACTs is reported based on adult equivalent treatment doses (AETDs), a standardized unit that allows for comparison of products (Palafox et al. 2016). Based on drug information provided by the Mayo Foundation for Medical Education and Research (2022), we assume that each malaria case in our numerical analysis requires a three-day treatment course of six AETDs. We also assume that the retailer restocks every month (Watsierah and Ouma 2014). Then, after aggregating the weekly malaria case data to infer monthly demand for ACTs, we can estimate the pdf of the market condition, M. To do this, we use the R package PDFEstimator's `estimatePDF`, which is a non-parametric density estimator based on the maximum-entropy method.

In Mozambique, the retail price of ACTs per AETD ranges from 0.86 USD to 1.72 USD (Alonso et al. 2017). Meanwhile, the lowest price of ACTs internationally documented by Alonso et al. (2017) is 0.42 USD; this is similar to the ACT wholesale price in Cambodia, which is 0.41 USD (Patouillard et al. 2015). As an approximation, we henceforth assume the wholesale price for certified drugs is equal to $c_c = 0.41$ USD in this numerical analysis. For the other model parameters, we use hypercube sampling and the following range of values: discount factor $\delta \in [0.1, 0.9]$, price sensitivity of consumer demand $\phi = 0.5$,[9] wholesale price of uncertified drugs $c_u \in [\$0.1, \$0.4]$, normalized quality of uncertified drugs $\theta \in [0.1, 0.9]$, and consumers' counterfeit awareness $k \in [0.1, 0.9]$. For the analysis with an exogenous retail price, we assume $p \in [\$0.5, \$1.4]$. We sample 5,000 parameter sets under endogenous retail price and 50,000 sets under exogenous retail price.[10]

[9] As price sensitivity is not the focus of this analysis, we fix a specific value that leads to a plausible range of demand given the possible exogenous retail prices.

[10] Note that the case of exogenous retail price has an additional parameter (the exogenous price), thus more samples are required to obtain the same granularity over the parameter space.

Figure 4 Additional expected health benefits with price control for different (a) counterfeit awareness k; (b) acquisition cost of uncertified drugs c_u; (c) budget size B; (d) budget size B where prices are set optimally by the regulator to maximize expected health benefits.



7.2. Imposing Price Control: The Potential Benefit of Fixed Retail Price

In some malaria-endemic countries, the retail prices of antimalarial drugs are regulated to improve drug accessibility (Palafox et al. 2016). In this subsection, we study the conditions under which regulators might prefer to impose price controls, i.e., when the expected health benefits created for consumers under exogenous retail prices are higher than that under endogenous retail prices.

Figure 4 (a), (b) and (c) plot the additional health benefits against different parameters. Figures 4 (a) and (b) show that as consumers' awareness k or the acquisition cost of uncertified drugs c_u increases, the additional health benefits associated with price controls increase. This is because, with higher k or c_u, uncertified drugs are less attractive to the retailer, and so there are associated additional health benefits that come from the retailer sourcing more certified drugs. Figure 4 (c) shows that the additional health benefits associated with price controls are highest when the budget size is moderate. This is because with a small budget size, price control gives the retailer the incentive to source more uncertified drugs to increase their profit margin. Meanwhile, with a large budget size, the retailer would like to lower prices to stimulate demand and benefit from the large subsidy. However, price controls limit the retailer's flexibility to reduce the retail price.

In practice, the regulator may have a good understanding of the market characteristics, such that they might be able to decide on the regulated price to maximize the expected health benefits created for consumers. Hence, in Figure 4 (d), we set the regulator's chosen retail price equal to the retail price that provides the highest expected health benefits to consumers given the associated values of the model parameters. We find that, on the one hand, the trend for k and c_u remains the same as in Figures 4 (a) and 4 (b). On the other hand, we first see in Figure 4 (d) that when retail prices are set optimally by the central planner, consumers almost always benefit more than they do when retailers have pricing power. Second, we notice that there is no longer a trade-off associated with a larger budget size B, and a larger B leads to greater expected health benefits when retail prices are set optimally. This is because a larger budget size B incentivizes the retailer to stock more units of higher-quality drugs, while the regulator can now also lower the retail price to boost consumer demand as B increases. More importantly, we notice that as B increases, the variation between the additional expected health benefits arising under different parameter values decreases. In other words, the benefits associated with a price set optimally by the regulator become more robust across different parameter value combinations as the budget size increases. Hence, when the donor has a sufficiently large budget, price controls can be particularly effective if retail prices are chosen effectively by the central planner.

8. Conclusions

Malaria remains one of the major preventable life-threatening diseases despite the availability of effective treatments and continuous efforts to improve their affordability. The low accessibility of legitimate drugs has led to the prevalence of counterfeits in malaria-endemic countries, further complicating the situation. The objective of this paper has been to examine the implications of the presence of counterfeit drugs on the subsidy strategies of philanthropic donors as well as to explore other interventions designed to improve consumers' access to high-quality antimalarial drugs.

First, our results indicate that in the presence of funding shortfalls it may be optimal for donors to subsidize uncertified drugs via a sales subsidy. This is in contrast to the prevailing recommendation for donors to use only purchase subsidies when uncertified drugs are not present. In fact, when budgets are limited, we find that there are conditions under which offering any subsidy at all can result in worse outcomes. More generally, we show that the optimal type of subsidy depends on various market characteristics (e.g., retailers' pricing power, demand uncertainty, consumers' counterfeit awareness), indicating that any subsidy scheme needs be customized to its specific context. However, the additional complexity associated with selecting the optimal subsidy scheme in the presence of uncertified drugs is largely resolved when the donor has a sufficiently large budget, in which case it is always optimal to use purchase subsidies that incentivize retailers to source from certified suppliers. This finding highlights the importance of addressing the problem of funding shortfalls in markets in which uncertified drugs are prevalent.

Second, our analysis draws specific attention to the importance of accounting for the pricing power of retailers and the uncertainty of demand when choosing strategies to improve outcomes in markets characterized by the presence of uncertified drugs. While pricing regulation exists in many malaria-endemic countries, with the objective of lowering the retail price and improving drug access, we show that the implications associated with imposing pricing regulations are nuanced due to the presence of uncertified drugs. Specifically, we show that under pricing regulation, strategies that aim to improve consumers' access to high-quality drugs – including increasing the subsidy size, improving consumers' counterfeit awareness, and increasing the cost of sourcing uncertified drugs – could lead to worse outcomes. Furthermore, these adverse outcomes are more likely to occur when demand uncertainty is high.

Lastly, our analysis cautions against overreliance on the promises of emerging technology. We show that adopting traceability technology to eliminate the subsidy on uncertified drugs may hurt consumers. Hence, regulators need to carefully evaluate the consequences of technology adoption and whether these effects are aligned with the intended objectives.

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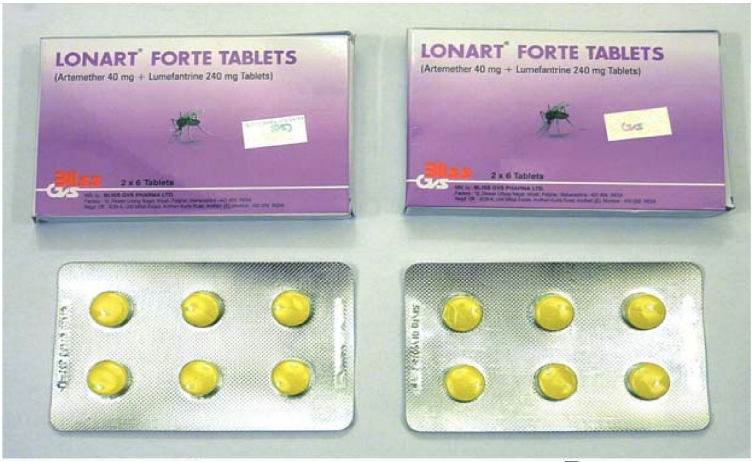
Electronic Companion

EC.1. Examples of Drugs Samples for Both Legitimate and Counterfeit ACTs.

We model counterfeit drugs as deceptive counterfeits as they are very often indistinguishable from the legitimate products with identical packaging. Below is a sample figure of the ACTs in Uganda from Nyqvist et al. (2020), where sample A is a counterfeit drug and sample B is a legitimate one.

Figure EC.1 Examples of ACT drug samples.



EC.2. Proofs of Section 4.

Proof of Lemma 1. Consider a relaxation of (1), where the retailer has the option to sell back leftover inventory at purchasing cost at the beginning of each period, as characterized by (3). It is clear that retailer's optimal decision in (3) is stationary and $V_r(x,h) = V_r(0,0) + (c_c - a)(1-h)x + c_u hx$, for any $x \geq 0, h \in [0,1]$. Let z_r^* and ω_r^* denote the retailer's optimal policy in (3). We then have

$$\begin{aligned} V_r(x,h) &= \delta V_r(0,0) + (c_c - a)(1-h)x + c_u hx + \max_{z\geq 0, \omega \in [0,1]} \{-(c_c - a)(1-\omega)z - c_u \omega z \\ &\quad + \mathbb{E}_{M,\Upsilon}[\max_{p\geq 0}\{(s+p)\min(y(\omega,p)M, z) + \delta[((c_c - a)(1-\Upsilon) + c_u\Upsilon)(z - \min(y(\omega,p)M, z))]\}] \} \\ &= \delta V_r(0,0) + (c_c - a)(1-h)x + c_u hx + \max_{z\geq 0, \omega \in [0,1]} R(z,\omega), \end{aligned}$$

where $\max_{z\geq 0,\omega\in[0,1]} R(z,\omega)$ is characterized by (2).

Problem (2) is equivalent to a single-period problem where each unsold unit has salvage value $\delta(c_c - a)$ for certified drugs, or salvage value δc_u for uncertified drugs. Let z^* and ω^* denote the retailer's optimal policy in (2). Since the retailer starts with zero inventory in the first period and the optimal policy is stationary in (3), he never uses the option to sell back inventory at the

beginning of each period, and therefore z_r^* and ω_r^* are optimal in (1) as well. Meanwhile, since $V_r(x,f)$ and $\max_{z\geq 0,\omega\in[0,1]} R(z,\omega)$ only differ by a constant, i.e., $\delta V_r(0,0)+(c_c-a)(1-h)x+c_uhx$, we have $z_r^*=z^*$ and $\omega_r^*=\omega^*$. Hence, z^* and ω^* are the optimal policy for (1) as well. □

Proof of Lemma 2. From (2), the optimal price $p^*(a,s,m,z,\omega)$ is given by

$$p^*(a,s,m,z,\omega)=\arg\max_{p\geq 0}\{[s+p-\delta((c_c-a)(1-\omega)+c_u\omega)]\min(y(\omega,p)m,z)\}. \tag{EC.1}$$

Let $\hat{p}(m,z,\omega)$ be the p that solves $y(\omega,p)m=z$, which is then given by $\hat{p}(m,z,\omega)=\frac{1-k\omega(1-\theta)-z/m}{\phi}$, and note that the objective of (EC.1) is increasing on $p\in[0,\hat{p}(m,z,\omega)]$. Thus, as $y(\omega,p)$ is decreasing in p, we can write (EC.1) as

$$p^*(a,s,m,z,\omega)=\arg\max_{p\geq(\hat{p}(m,z,\omega))^+}\{[s+p-\delta((c_c-a)(1-\omega)+c_u\omega)]y(\omega,p)m\}. \tag{EC.2}$$

The intuition of this constraint is that $\hat{p}(m,z,\omega)$ is the highest price at which the retailer can clear inventory; setting the price lower than that would still clear inventory, but also decrease revenue.

Differentiating the objective of (EC.2) with respect to p yields

$$m(1-k+k(\omega\theta+1-\omega)-2\phi p-\phi[s-\delta((c_c-a)(1-\omega)+c_u\omega)]), \tag{EC.3}$$

which is decreasing in p (weakly if $m=0$, but strictly otherwise), so the objective of (EC.2) is concave. Equating (EC.3) to zero and solving for p yields $\tilde{p}(a,s,\omega):=\frac{1-k\omega(1-\theta)-\phi[s-\delta((c_c-a)(1-\omega)+c_u\omega)]}{2\phi}$. If this price satisfies $\tilde{p}(a,s,\omega)\geq(\hat{p}(m,z,\omega))^+$, then it is also the solution to the constrained problem (EC.2), otherwise (EC.2) has a corner solution $(\hat{p}(m,z,\omega))^+$. Note that the condition $\tilde{p}(a,s,\omega)\geq(\hat{p}(m,z,\omega))^+$ can also be expressed as $\tilde{p}(a,s,\omega)\geq 0$ & $m\leq z/y(\omega,\tilde{p}(a,s,\omega))$, as $\tilde{p}(a,s,\omega)\geq\hat{p}(m,z,\omega)\Leftrightarrow my(\omega,\tilde{p}(a,s,\omega)\geq my(\omega,\hat{p}(m,z,\omega))\Leftrightarrow my(\omega,\tilde{p}(a,s,\omega))\geq z$. Hence, the statement of the lemma follows. □

Proof of Lemma 3. For a given subsidy scheme (a,s), the retailer's expected profit is

$$\begin{aligned}R(z,\omega;a,s)=&\int_0^{\frac{z}{y(\tilde{p}(a,s,\omega))}}[s+\tilde{p}(a,s,\omega)-\delta((c_c-a)(1-\omega)+c_u\omega)]y(\tilde{p}(a,s,\omega))mf(m)dm\\&+\int_{\frac{z}{y(\tilde{p}(a,s,\omega))}}^{\infty}[s+\hat{p}(m,z,\omega)-\delta((c_c-a)(1-\omega)+c_u\omega)]zf(m)dm\\&-(1-\delta)((c_c-a)(1-\omega)+c_u\omega)z.\end{aligned}$$

The second-order derivative of $R(z,\omega;a,s)$ w.r.t ω is

$$\frac{\partial^2R(z,\omega;a,s)}{\partial\omega^2}=\int_0^{\frac{z}{y(\tilde{p}(a,s,\omega))}}\frac{1}{2\phi}[\phi\delta(c_c-a-c_u)-k(1-\theta)]^2mf(m)dm\geq 0,$$

for any $z\geq 0$. Hence, the retailer performs single sourcing, i.e., $\omega^*(a,s)\in\{0,1\}$. Meanwhile, the second-order derivative of $R(z,\omega;a,s)$ w.r.t z is

$$\frac{\partial^2R(z,\omega;a,s)}{\partial z^2}=\int_{\frac{z}{y(\tilde{p}(a,s,\omega))}}^{\infty}-\frac{2}{\phi m}f(m)dm<0.$$

Hence, for any $\omega \in [0,1]$, the optimal $z^*(a,s,\omega)$ is given by the FOC of $R(z,\omega;a,s)$ w.r.t z:

$$\frac{\partial R(z,\omega;a,s)}{\partial z} = \int_{\frac{z}{y(\tilde{p}(a,s,\omega))}}^{\infty} [s + \frac{1-k+k(\omega\theta+1-\omega)-2z/m}{\phi} - \delta((c_c-a)(1-\omega)+c_u\omega)]f(m)dm$$
$$-(1-\delta)((c_c-a)(1-\omega)+c_u\omega) = 0.$$

When $\omega^*(a,s) = 0$, we then have $z^*(a,s)$ given by

$$\int_{z^*(a,s)/y(\tilde{p}(a,s,0))}^{\infty} [s + \frac{1-2z^*(a,s)/m}{\phi} - \delta(c_c-a)]f(m)dm = (1-\delta)(c_c-a).$$

When $\omega^*(a,s) = 1$, we then have $z^*(a,s)$ given by

$$\int_{z^*(a,s)/y(\tilde{p}(a,s,1))}^{\infty} [s + \frac{1-k+k\theta-2z^*(a,s)/m}{\phi} - \delta c_u]f(m)dm = (1-\delta)c_u.$$

Moreover, we have the first-order derivative of $R(z,\omega;a,s)$ w.r.t ω being

$$\begin{aligned}\frac{\partial R(z,\omega;a,s)}{\partial \omega} = &\int_0^{\frac{z}{y(\tilde{p}(a,s,\omega))}} \frac{-k(1-\theta)+\phi\delta(c_c-a-c_u)}{2\phi} \\ &\cdot\{1-k+k(\omega\theta+1-\omega)+\phi[s-\delta((c_c-a)(1-\omega)+c_u\omega)]\}mf(m)dm \\ &+\int_{\frac{z}{y(\tilde{p}(a,s,\omega))}}^{\infty} \frac{-k(1-\theta)+\phi\delta(c_c-a-c_u)}{\phi} zf(m)dm + (1-\delta)(c_c-a-c_u)z.\end{aligned} \tag{EC.4}$$

Recall we define $r_a := \frac{c_c-a-c_u}{k(1-\theta)}$. Clearly, we have $\frac{\partial R(z,\omega;a,s)}{\partial \omega} < 0$ when $r_a \to 0^+$, and therefore $\omega^*(a,s) = 0$. Meanwhile, we have $\frac{\partial R(z,\omega;a,s)}{\partial \omega} > 0$ when $r_a \to \infty$, and therefore $\omega^*(a,s) = 1$.

Suppose for a given $r_a > 0$, we have $\omega^*(a,s) = 0$. We then have

$$\begin{aligned}\frac{\partial R(z^*(a,s),0;a,s)}{\partial a}\bigg|_{z=z^*(a,s),\omega=0,p=p^*(a,s,m,z,\omega)} = &\int_0^{\frac{z^*(a,s)}{y(\tilde{p}(a,s,\omega))}} \delta(1-\omega)y(\tilde{p}(a,s,\omega))mf(m)dm \\ &+\int_{\frac{z^*(a,s)}{y(\tilde{p}(a,s,\omega))}}^{\infty} \delta(1-\omega)zf(m)dm + (1-\delta)(1-\omega)z > 0.\end{aligned}$$

Hence, in this case, higher a leads to a higher expected profit under the retailer's optimal decisions with $\omega = 0$ than the expected profit under the retailer's optimal decisions with $\omega = 1$. Similarly, we have $\frac{\partial R(z^*(a,s),0;a,s)}{\partial c_c}\Big|_{z=z^*(a,s),\omega=0,p=p^*(a,s,m,z,\omega)} < 0$.

On the other hand, suppose for a given $r_a > 0$, we have $\omega^*(a,s) = 1$ instead. We then have $\frac{\partial R(z^*(a,s),1;a,s)}{\partial \theta}\Big|_{z=z^*(a,s),\omega=1,p=p^*(a,s,m,z,\omega)} > 0$, $\frac{\partial R(z^*(a,s),1;a,s)}{\partial k}\Big|_{z=z^*(a,s),\omega=1,p=p^*(a,s,m,z,\omega)} < 0$, and $\frac{\partial R(z^*(a,s),1;a,s)}{\partial c_u}\Big|_{z=z^*(a,s),\omega=1,p=p^*(a,s,m,z,\omega)} < 0$. Hence, in this case, higher θ, lower k or lower c_u leads to a higher expected profit under the retailer's optimal decisions with $\omega = 1$ than the expected profit under the retailer's optimal decisions with $\omega = 0$.

Hence, there exists $\bar{r}_a > 0$ s.t. we have $\omega^*(a,s) = 0$ if $r_a \le \bar{r}_a$, and $\omega^*(a,s) = 1$ if $r_a > \bar{r}_a$. □

Proof of Proposition 1. We first prove $a^* \cdot s^* = 0$. Suppose there exists optimal (a^*, s^*), such that $a^* > 0$, $s^* > 0$, and $\omega^*(a^*, s^*) = 0$, that is, the retailer purchases certified drugs only. Meanwhile, for a given subsidy scheme (a, s), suppose we have $\omega^*(a, s) = 0$. In this case, we have

$$\begin{aligned}
\frac{\partial R(z^*(a,s),0)}{\partial s} &= \int_0^{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}} \{y(\tilde{p}(a,s,0)) + \frac{\partial \tilde{p}(a,s,0)}{\partial s}[y(\tilde{p}(a,s,0)) - \phi(s+\tilde{p}(a,s,0) - \delta(c_c - a))]\} \\
&\quad \cdot mf(m)dm + \int_{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}}^{\infty} z^*(a,s) f(m)dm \\
&\quad + \{\int_{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}}^{\infty} [s + \frac{1 - 2z^*(a,s)/m}{\phi} - \delta(c_c - a)] f(m)dm - (1-\delta)(c_c - a)\} \frac{\partial z^*(a,s)}{\partial s} \\
&= \int_0^{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}} y(\tilde{p}(a,s,0)) m f(m)dm + \int_{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}}^{\infty} z^*(a,s) f(m)dm \\
&= D_2(a,s),
\end{aligned}$$

since we have $y(\tilde{p}(a,s,0)) - \phi(s + \tilde{p}(a,s,0) - \delta(c_c - a)) = 0$ given the FOC of $\tilde{p}(a,s,0)$, and $\int_{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}}^{\infty} [s + \frac{1-2z^*(a,s)/m}{\phi} - \delta(c_c - a)] f(m)dm - (1-\delta)(c_c - a) = 0$ given the FOC of $z^*(a,s)$.

Similarly, we have

$$\begin{aligned}
\frac{\partial R(z^*(a,s),0)}{\partial a} &= \int_0^{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}} \delta y(\tilde{p}(a,s,0)) m f(m)dm + \int_{\frac{z^*(a,s)}{y(\tilde{p}(a,s,0))}}^{\infty} \delta z^*(a,s) f(m)dm + (1-\delta) z^*(a,s) \\
&= \delta D_2(a,s) + (1-\delta) z^*(a,s).
\end{aligned}$$

Since the quantity sold is upper-bounded by the inventory level, we have $D_2(a,s) \le \delta D_2(a,s) + (1-\delta) z^*(a,s)$. Hence, $R(z^*(a+b, s-b), 0)$ is increasing in b, or in other words, if the donor shifts a part of budget from s to a, the retailer's profit will increase, as long he still sources certified drugs only ($\omega = 0$). Meanwhile, $R(z^*(a+b, s-b), 1)$ is weakly decreasing in b, since a retailer who sources only uncertified drugs ($\omega = 1$) benefits from s, but not from a. Hence, if $\omega^*(a^*, s^*) = 0$, then $\omega(a^* + b, s^* - b) = 0$ for any $b \ge 0$, i.e., the retailer still purchases certified drugs if a part of the budget is shifted from s to a. It then follows directly from Proposition 1 of Taylor and Xiao (2014) that the donor can increase the quantity of drugs sold, as well as the health benefits to the population, by shifting the entire budget into a, which is contradictory to the optimality of (a^*, s^*). Hence $a^* > 0$ and $s^* > 0$ is sub-optimal if the donor aims to maximize quantity or health benefits. Note that if the donor's objective is to maximize the market share of certified drugs, she will be indifferent between $a^* > 0, s^* > 0$ and $a^* > 0, s^* = 0$ as both will lead to the donor sourcing certified drugs only.

Suppose now that (a^*, s^*) is optimal and $\omega^*(a^*, s^*) = 1$, that is, the retailer purchases uncertified drugs only. In this case, $(0, s^*)$ is also optimal because $\omega^*(a^*, s^*) = 1$ implies $\omega^*(0, s^*) = 1$, so both the objective and the constraint of (4) are equal under (a^*, s^*) and $(0, s^*)$.

To facilitate the exposition, we start with parts (i) and (iii) of Proposition 1, followed by part (ii), for the rest of this proof.

(i) When $r_0 \to 0^+$, we have $\omega^*(0,0) = 0$ by Lemma 3. In this case, the retailer purchases certified drugs only in the absence of subsidy. Moreover, when $r_0 \to 0^+$, the retailer strictly prefers sourcing certified drugs only for any feasible subsidy scheme (a,s) and we have $\omega^*(a,s) = 0$ by (EC.4). Meanwhile, when $r_0 \to 0^+$, it is straightforward to show that the retailer's expected profit $R(z,\omega;a,s)$ is continuous in r_0. Hence, there exist an $\tilde{r}_0^l \geq 0$ s.t. if $r_0 \in [0,\tilde{r}_0^l]$, for any $B \in [0,\infty)$, the optimal subsidy scheme is $a^* > 0, s^* = 0$ (Taylor and Xiao 2014), for any $D_i(a,s), i \in \{1,2,3\}$, and we have $\omega^*(a^*,s^*) = 0$.

(iii) When $r_0 \to \infty$, we have $\omega^*(0,0) = 1$ by Lemma 3. In this case, the retailer purchases uncertified drugs only in the absence of subsidy. Moreover, when $r_0 \to \infty$, the retailer strictly prefers sourcing uncertified drugs only, for any feasible subsidy scheme (a,s), and we have $\omega^*(a,s) = 1$ by (EC.4). Meanwhile, when $r_0 \to \infty$, it is straightforward to show that the retailer's expected profit $R(z,\omega;a,s)$ is continuous in r_0. Hence, there exist an $\tilde{r}_0^h \geq 0$ s.t. for any $r_0 \in [\tilde{r}_0^h,\infty)$, we have $\omega^*(0,0) = 1$.

This implies that there exists $\tilde{B}_1^h \geq 0$ s.t. for any budget size $B < \tilde{B}_1^h$, we have $\omega^*(a,s) = 1$ for any $a,s > 0$ s.t. $a \cdot s = 0$ and $(a(1-\omega^*(a,s)) + s)\mathbb{E}_m[\min(y(p^*(m,z^*(a,s),\omega^*(a,s)))m, z^*(a,s))] \leq B$. In this case, a purchase subsidy only cannot subsidize drugs as the retailer purchases uncertified drugs only. Hence, the optimal subsidy scheme consists of $a^* = 0, s^* > 0$, for any $D_i(a,s), i \in \{1,2,3\}$, and we have $\omega^*(a^*,s^*) = 1$.

As the budget size further increases above $\tilde{B}_1^h$, the donor can now use a purchase subsidy only such that the retailer purchases certified drugs only under subsidy, with $(a(1-\omega^*(a,0)) + 0)\mathbb{E}_m[\min(y(p^*(m,z^*(a,0),\omega^*(a,0)))m, z^*(a,0))] = B$. Hence, the optimal subsidy scheme is $a^* > 0, s^* = 0$ if $D_1(a,s)$. However, in this case, $s = 0$ might be suboptimal if $D_2(a,s)$ or $D_3(a,s)$: in the case with $\theta \to 1$, a purchase subsidy is relatively small such that it only closes the cost gap between certified drugs and uncertified drugs, i.e., $c_c - a$ and c_u, with little improvement on the expected quantity or health benefits. Hence, the optimal subsidy scheme might remain as $a^* = 0, s^* > 0$ if $D_2(a,s)$ or $D_3(a,s)$.

As the budget size further increases above $\tilde{B}_3^h$ where $\tilde{B}_1^h \leq \tilde{B}_3^h$, the optimal subsidy scheme consists of a purchase subsidy only and the retailer purchases certified drugs only after subsidy if $D_3(a,s)$. This is because a purchase subsidy increases expected quantity more rapidly than a sales subsidy (Taylor and Xiao 2014) and certified drugs have higher quality than uncertified drugs. Hence, when the donor's budget size is sufficiently high, i.e., $B \geq \tilde{B}_3^h$, the optimal subsidy scheme consists of a purchase subsidy only if the donor's objective function is to maximize expected health benefits.

As the budget size further increases above $\tilde{B}_2^h$ where $\tilde{B}_3^h \leq \tilde{B}_2^h$, the optimal subsidy scheme consists of a purchase subsidy only and the retailer purchases certified drugs only after subsidy as well if $D_2(a,s)$: maximizing expected quantity may require a higher budget size to switch to

certified drugs only and a purchase subsidy only than maximizing expected health benefit, as the effect of subsidizing uncertified drugs is not discounted by the quality differential when the donor's objective function is to maximize expected quantity sold to consumers.

(ii) Let $\tilde{r}_0^m$ denote the r_0 s.t. the retailer is indifferent between sourcing certified drugs and uncertified drugs.

For any $r_0 \in (\tilde{r}_0^m, \tilde{r}_0^h)$, we have $\omega^*(0,0) = 1$ by Lemma 3. In this case, the retailer purchases uncertified drugs only in the absence of subsidy. Then, there exists $\tilde{B}_1^m \geq 0$ s.t. for any budget size $B < \tilde{B}_1^m$, we have $\omega^*(a,s) = 1$ for any $a, s > 0$ s.t. $a \cdot s = 0$ and $(a(1-\omega^*(a,s)) + s)\mathbb{E}_m[\min(y(p^*(m, z^*(a,s), \omega^*(a,s)))m, z^*(a,s))] \leq B$. In this case, a purchase subsidy only cannot subsidize drugs as the retailer purchases uncertified drugs only. Hence, for any $B \in [0, \tilde{B}_1^m)$, the optimal subsidy scheme consists of $a^* = 0, s^* > 0$, for any $D_i(a,s), i \in \{1,2,3\}$, and we have $\omega^*(a^*, s^*) = 1$.

Moreover, with $B \in [\tilde{B}_1^m, \infty)$, the donor can use a purchase subsidy only to incentivize the retailer to source certified drug only, and we have $a^* > 0$ and $s^* = 0$ and $\omega^*(a^*, s^*) = 0$ if $D_1(a,s)$. If the donor aims to maximize expected quantity or health benefits, the optimal subsidy scheme further depends on the budget size. In particular, when the donor has a sufficiently large budget, i.e., $B \in [\tilde{B}_2^m, \infty)$ where $\tilde{B}_2^m \geq \tilde{B}_1^m$, we have $a^* > 0, s^* = 0$ if the donor aims to maximize the expected quantity or health benefits. This is because a purchase subsidy of size $c_c - c_u$ closes the cost gap between certified and uncertified drugs, and using the remaining purchase subsidy increases expected quantity or health benefits more than using a sales subsidy only, regardless we have the retailer sourcing certified drugs or uncertified drugs under a sales subsidy only. On the other hand, when the donor has a smaller budget, i.e., $B \in [\tilde{B}_1^m, \tilde{B}_2^m)$, the retailer might source uncertified drugs under a sales subsidy only, which could lead to a higher expected quantity or health benefits than using a purchase subsidy only. Hence, there exist $0 \leq \tilde{B}_1^m \leq \tilde{B}_2^m$ s.t. for any $B \in [\tilde{B}_1^m, \tilde{B}_2^m)$, we have $a^* > 0, s^* = 0$ and $\omega^*(a^*, s^*) = 0$, or $a^* = 0, s^* > 0$ and $\omega^*(a^*, s^*) = 1$, for $D_2(a,s)$ or $D_3(a,s)$; for any $B \in [\tilde{B}_2^m, \infty)$, we have $a^* > 0, s^* = 0$ and $\omega^*(a^*, s^*) = 0$, for $D_2(a,s)$ or $D_3(a,s)$.

On the other hand, for any $r_0 \in (\tilde{r}_0^l, \tilde{r}_0^m]$, we have $\omega^*(0,0) = 0$. In this case, for any feasible subsidy scheme $(a,0)$, we have $\omega^*(a,0) = 0$ by Lemma 3 and therefore we have $a^* > 0, s^* = 0$ for any $B \in [0, \infty)$ if $D_1(a,s)$. Then, the structural results for $r_0 \in (\tilde{r}_0^l, \tilde{r}_0^m]$ remain the same as the results for $r_0 \in (\tilde{r}_0^m, \tilde{r}_0^h)$, except we have $\tilde{B}_1^m = 0$ if $r_0 \in (\tilde{r}_0^l, \tilde{r}_0^m]$. □

EC.3. Proofs of Section 5.

Proof of Lemma 4. For a given subsidy scheme (a,s) and retail price p, suppose $\omega^*(a,s) \in (0,1)$. The retailer's expected profit is

$$R(z,\omega;a,s) = \int_0^{\frac{z}{y(\omega)}} [s + p - \delta((c_c - a)(1-\omega) + c_u\omega)]y(\omega)mf(m)dm + \int_{\frac{z}{y(\omega)}}^{\infty} [s + p - \delta((c_c - a)(1-\omega) + c_u\omega)]zf(m)dm - (1-\delta)((c_c - a)(1-\omega) + c_u\omega)z.$$

Denote the first-order derivatives w.r.t to z and ω as

$$F_1(z,\omega;a,s)=\frac{\partial R(z,\omega;a,s)}{\partial z}=\int_{\frac{z(a,s)}{y(\omega(a,s))}}^{\infty}[s+p-\delta((c_c-a)(1-\omega(a,s))+c_u\omega(a,s))]f(m)dm$$
$$-(1-\delta)((c_c-a)(1-\omega(a,s))+c_u\omega(a,s)), \tag{EC.5}$$

and

$$F_2(z,\omega;a,s)=\frac{\partial R(z,\omega;a,s)}{\partial \omega}=$$
$$\int_0^{\frac{z(a,s)}{y(\omega(a,s))}}\{\delta(c_c-a-c_u)y(\omega(a,s))-[s+p-\delta((c_c-a)(1-\omega(a,s))+c_u\omega(a,s))]k(1-\theta)\}mf(m)dm$$
$$+\int_{\frac{z(a,s)}{y(\omega(a,s))}}^{\infty}\delta(c_c-a-c_u)z(a,s)f(m)dm+(1-\delta)(c_c-a-c_u)z(a,s). \tag{EC.6}$$

Define matrix J as

$$J=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial z} & \frac{\partial F_1(z,\omega;a,s)}{\partial \omega}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial z} & \frac{\partial F_2(z,\omega;a,s)}{\partial \omega}\end{bmatrix},$$

where

$$\frac{\partial F_1(z,\omega;a,s)}{\partial z}=-\frac{1}{y(\omega(a,s))}N_Mf(\frac{z(a,s)}{y(\omega(a,s))}),$$
$$\frac{\partial F_1(z,\omega;a,s)}{\partial \omega}=(\delta N_F+1-\delta)(c_c-a-c_u)-\frac{z(a,s)}{y(\omega(a,s))^2}k(1-\theta)N_Mf(\frac{z(a,s)}{y(\omega(a,s))}),$$
$$\frac{\partial F_2(z,\omega;a,s)}{\partial z}=(\delta N_F+1-\delta)(c_c-a-c_u)-\frac{z(a,s)}{y(\omega(a,s))^2}k(1-\theta)N_Mf(\frac{z(a,s)}{y(\omega(a,s))}),$$
$$\frac{\partial F_2(z,\omega;a,s)}{\partial \omega}=-2k(1-\theta)\delta(c_c-a-c_u)N_E-\frac{z(a,s)^2}{y(\omega(a,s))^3}k^2(1-\theta)^2N_Mf(\frac{z(a,s)}{y(\omega(a,s))}),$$

with $N_E:=\int_0^{\frac{z(a,s)}{y(\omega(a,s))}}mf(m)dm$, $N_F:=\int_{\frac{z(a,s)}{y(\omega(a,s))}}^{\infty}f(m)dm$, $N_M:=s+p-\delta((c_c-a)(1-\omega(a,s))+c_u\omega(a,s))$. We assume that $\text{Det}(J)\Big|_{z\in(0,\infty),\omega\in(0,1)}>0$. Moreover, since $\frac{\partial F_1(z,\omega;a,s)}{\partial z}<0$, we have that $R(z,\omega;a,s)$ is jointly concave in z,ω, which is a sufficient to guarantee that FOCs characterize the interior optimal solutions if $z^*(a,s)\in(0,\infty)$ and $\omega^*(a,s)\in(0,1)$.

Then we have

$$\begin{aligned}\text{Det}(J)\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}&=\frac{\partial F_1(z,\omega;a,s)}{\partial z}\cdot\frac{\partial F_2(z,\omega;a,s)}{\partial \omega}-\frac{\partial F_1(z,\omega;a,s)}{\partial \omega}\cdot\frac{\partial F_2(z,\omega;a,s)}{\partial z}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}\\&=\frac{1}{y(\omega^*(a,s))}N_Mf(\frac{z^*(a,s)}{y(\omega^*(a,s))})2k(1-\theta)\delta(c_c-a-c_u)N_E\\&+\frac{z^*(a,s)}{y(\omega^*(a,s))^2}N_Mf(\frac{z^*(a,s)}{y(\omega^*(a,s))})2k(1-\theta)(\delta N_F+1-\delta)(c_c-a-c_u)\\&-[(\delta N_F+1-\delta)(c_c-a-c_u)]^2>0.\end{aligned}$$

Define F_a as

$$F_a=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial a}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial a}\end{bmatrix},$$

where

$$\frac{\partial F_1(z,\omega;a,s)}{\partial a} = (\delta N_F + 1 - \delta)(1-\omega(a,s)),$$

$$\frac{\partial F_2(z,\omega;a,s)}{\partial a} = -\delta[y(\omega(a,s)) + (1-\omega(a,s))k(1-\theta)]N_E - (\delta N_F + 1 - \delta)z(a,s).$$

Define $J_{a\omega}$ as

$$J_{a\omega} = \begin{bmatrix} \frac{\partial F_1(z,\omega;a,s)}{\partial z} & \frac{\partial F_1(z,\omega;a,s)}{\partial a} \\ \frac{\partial F_2(z,\omega;a,s)}{\partial z} & \frac{\partial F_2(z,\omega;a,s)}{\partial a} \end{bmatrix}.$$

We then have

$$\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s)) = -\frac{\text{Det}(J_{a\omega})/(1-\omega(a,s))}{\text{Det}(J)}\bigg|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$-\frac{\text{Det}(J_{a\omega})}{1-\omega(a,s)}\bigg|_{z=z^*(a,s),\omega=\omega^*(a,s)} = -\frac{1}{y(\omega^*(a,s))}N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})[\delta\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}N_E + \delta k(1-\theta)N_E$$
$$+ (\delta N_F + 1 - \delta)\frac{z^*(a,s)}{1-\omega^*(a,s)} + (\delta N_F + 1 - \delta)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)] + (\delta N_F + 1 - \delta)^2(c_c - a - c_u).$$

Define F_s as

$$F_s = \begin{bmatrix} \frac{\partial F_1(z,\omega;a,s)}{\partial s} \\ \frac{\partial F_2(z,\omega;a,s)}{\partial s} \end{bmatrix},$$

where

$$\frac{\partial F_1(z,\omega;a,s)}{\partial s} = N_F,$$

$$\frac{\partial F_2(z,\omega;a,s)}{\partial s} = -k(1-\theta)N_E.$$

Define $J_{s\omega}$ as

$$J_{s\omega} = \begin{bmatrix} \frac{\partial F_1(z,\omega;a,s)}{\partial z} & \frac{\partial F_1(z,\omega;a,s)}{\partial s} \\ \frac{\partial F_2(z,\omega;a,s)}{\partial z} & \frac{\partial F_2(z,\omega;a,s)}{\partial s} \end{bmatrix}.$$

We then have

$$\frac{\partial \omega^*(a,s)}{\partial s} = -\frac{\text{Det}(J_{s\omega})}{\text{Det}(J)}\bigg|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$-\text{Det}(J_{s\omega})\bigg|_{z=z^*(a,s),\omega=\omega^*(a,s)} = -\frac{1}{y(\omega^*(a,s))}N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})[k(1-\theta)N_E + \frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)N_F]$$
$$+ N_F(\delta N_F + 1 - \delta)(c_c - a - c_u).$$

Note that both $-\text{Det}(J_{a\omega})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}$ and $-\text{Det}(J_{s\omega})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}$ are linear in $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ with a negative coefficient for $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$. Hence, there exists $\bar{f}_{j\omega} \geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) > \bar{f}_{j\omega}$, we have $\frac{\partial \omega^*(a,s)}{\partial j} < 0$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) < \bar{f}_{j\omega}$, we have $\frac{\partial \omega^*(a,s)}{\partial j} > 0; j \in \{a,s\}$.

Moreover, we have

$$\begin{aligned}
&\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s)) < \frac{\partial\omega^*(a,s)}{\partial s}\\
\iff & -\frac{\mathrm{Det}(J_{a\omega})}{1-\omega(a,s)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)} < -\mathrm{Det}(J_{s\omega})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}\\
\iff & -\frac{N_M}{y(\omega^*(a,s))}f(\frac{z^*(a,s)}{y(\omega^*(a,s))})[\delta y(\omega^*(a,s))\frac{N_E}{1-\omega^*(a,s)}-(1-\delta)k(1-\theta)N_E\\
&+(\delta N_F+1-\delta)\frac{z^*(a,s)}{1-\omega^*(a,s)}+\frac{z^*(a,s)}{1-\omega^*(a,s)}k(1-\theta)(1-\delta)(1-N_F)]\\
&+(1-\delta)(1-N_F)(\delta N_F+1-\delta)(c_c-a-c_u)<0,
\end{aligned} \tag{EC.7}$$

where

$$\begin{aligned}
&\delta y(\omega^*(a,s))\frac{N_E}{1-\omega^*(a,s)}-(1-\delta)k(1-\theta)N_E+(\delta N_F+1-\delta)\frac{z^*(a,s)}{1-\omega^*(a,s)}\\
&+\frac{z^*(a,s)}{1-\omega^*(a,s)}k(1-\theta)(1-\delta)(1-N_F)>0\\
\iff & [\frac{N_E y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta)N_E+\frac{N_F z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)]\delta\\
&>k(1-\theta)N_E-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F).
\end{aligned}$$

If $\frac{N_E y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta)N_E+\frac{N_F z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)>0$, we have $\bar{\delta}_{c\omega}=\max\{0,\frac{k(1-\theta)N_E-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)}{\frac{N_E y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta)N_E+\frac{N_F z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)}\}$. In this case, if $\delta>\bar{\delta}_{c\omega}$, we have the coefficient of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ in the LHS of the last inequality of (EC.7) being negative. Hence, $\exists\ \bar{f}_{c\omega}\geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})>\bar{f}_{c\omega}$, we have the LHS of the last inequality of (EC.7) being negative and therefore $\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))<\frac{\partial\omega^*(a,s)}{\partial s}$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})<\bar{f}_{c\omega}$, we have the LHS of the last inequality of (EC.7) being positive and therefore $\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))>\frac{\partial\omega^*(a,s)}{\partial s}$. If $\delta<\bar{\delta}_{c\omega}$, we have the coefficient of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ in the LHS of the last inequality of (EC.7) being positive. Hence, we have the LHS of the last inequality of (EC.7) being positive and therefore $\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))>\frac{\partial\omega^*(a,s)}{\partial s}$.

If $\frac{N_E y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta)N_E+\frac{N_F z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)=0$, we have $k(1-\theta)N_E-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)<0$ and therefore $\bar{\delta}_{c\omega}=0$.

If $\frac{N_E y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta)N_E+\frac{N_F z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)<0$, we have $k(1-\theta)N_E-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)<\frac{N_E y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta)N_E+\frac{N_F z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{1-\omega^*(a,s)}-\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)(1-N_F)<0$ and therefore $\bar{\delta}_{c\omega}=0$.

In both cases, if $\delta>\bar{\delta}_{c\omega}$, we have the coefficient of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ in the LHS of the last inequality of (EC.7) being negative. Hence, there exists $\bar{f}_{c\omega}\geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})>\bar{f}_{c\omega}$, we have the LHS of the last inequality of (EC.7) being negative and therefore $\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))<\frac{\partial\omega^*(a,s)}{\partial s}$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})<\bar{f}_{c\omega}$, we have the LHS of the last inequality of (EC.7) being positive and therefore $\frac{\partial\omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))>\frac{\partial\omega^*(a,s)}{\partial s}$. □

Proof of Lemma 5. For a given subsidy scheme (a,s), suppose $\omega^*(a,s)\in(0,1)$. Define J_{az} as

$$J_{az}=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial a} & \frac{\partial F_1(z,\omega;a,s)}{\partial \omega}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial a} & \frac{\partial F_2(z,\omega;a,s)}{\partial \omega}\end{bmatrix},$$

where $F_1(z,\omega;a,s)$ and $F_2(z,\omega;a,s)$ are given by (EC.5) and (EC.6). We then have

$$\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s))=-\frac{\mathrm{Det}(J_{az})/(1-\omega(a,s))}{\mathrm{Det}(J)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$\begin{aligned}
&-\frac{\mathrm{Det}(J_{az})}{(1-\omega^*(a,s))}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}\\
&=(\delta N_F+1-\delta)[2k(1-\theta)\delta(c_c-a-c_u)N_E+\frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k^2(1-\theta)^2N_Mf(\frac{z^*(a,s)}{y(\omega^*(a,s))})]\\
&-[(\delta N_F+1-\delta)(c_c-a-c_u)-\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_Mf(\frac{z^*(a,s)}{y(\omega^*(a,s))})]\\
&\cdot[\delta N_E(\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}+k(1-\theta))+(\delta N_F+1-\delta)\frac{z^*(a,s)}{1-\omega^*(a,s)}].
\end{aligned}$$

Define J_{sz} as

$$J_{sz}=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial s} & \frac{\partial F_1(z,\omega;a,s)}{\partial \omega}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial s} & \frac{\partial F_2(z,\omega;a,s)}{\partial \omega}\end{bmatrix}.$$

We then have

$$\frac{\partial z^*(a,s)}{\partial s}=-\frac{\mathrm{Det}(J_{sz})}{\mathrm{Det}(J)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$\begin{aligned}
&-\mathrm{Det}(J_{sz})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}\\
&=N_F[2k(1-\theta)\delta(c_c-a-c_u)N_E+\frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k^2(1-\theta)^2N_Mf(\frac{z^*(a,s)}{y(\omega^*(a,s))})]\\
&-[(\delta N_F+1-\delta)(c_c-a-c_u)-\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_Mf(\frac{z^*(a,s)}{y(\omega^*(a,s))})]N_Ek(1-\theta).
\end{aligned}$$

Note that both $-\mathrm{Det}(J_{az})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}$ and $-\mathrm{Det}(J_{sz})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}$ are linear in $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ with a positive coefficient for $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$. Hence, there exists $\bar{f}_{jz}\geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})>\bar{f}_{jz}$, we have $\frac{\partial\omega^*(a,s)}{\partial j}>0$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})<\bar{f}_{jz}$, we have $\frac{\partial\omega^*(a,s)}{\partial j}<0; j\in\{a,s\}$.

Moreover, we have

$$\begin{aligned}
&\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s)) > \frac{\partial z^*(a,s)}{\partial s}\\
\iff & -\frac{\mathrm{Det}(J_{az})}{1-\omega(a,s)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)} > -\mathrm{Det}(J_{sz})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}\\
\iff & f(\frac{z^*(a,s)}{y(\omega^*(a,s))})\{(1-\delta)(1-N_F)\frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k^2(1-\theta)^2N_M+\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_M \qquad \text{(EC.8)}\\
&[\delta N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}+(\delta N_F+1-\delta)\frac{z^*(a,s)}{1-\omega^*(a,s)}-(1-\delta)N_Ek(1-\theta)]\}\\
&+(1-\delta)(1-N_F)2\delta(c_c-a-c_u)k(1-\theta)N_E-(\delta N_F+1-\delta)(c_c-a-c_u)\\
&[\delta N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}+(\delta N_F+1-\delta)\frac{z^*(a,s)}{1-\omega^*(a,s)}-(1-\delta)N_Ek(1-\theta)]>0,
\end{aligned}$$

where

$$\begin{aligned}
&(1-\delta)(1-N_F)\frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k^2(1-\theta)^2N_M+\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_M\\
&[\delta N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}+(\delta N_F+1-\delta)\frac{z^*(a,s)}{1-\omega^*(a,s)}-(1-\delta)N_Ek(1-\theta)]>0\\
\iff & [-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)+N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}-(1-N_F)\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)]\delta>\\
&-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)-\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta).
\end{aligned}$$

If $-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)+N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}-(1-N_F)\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)>0$, we have $\bar{\delta}_{cz}=\max\{0,\frac{-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)-\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)}{-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)+N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}-(1-N_F)\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)}\}$. In this case, if $\delta>\bar{\delta}_{cz}$, we have the coefficient of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ in the LHS of the last inequality of (EC.8) being positive. Hence, $\exists\ \bar{f}_{cz}\geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})>\bar{f}_{cz}$, we have the LHS of the last inequality of (EC.8) being positive and therefore $\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s))>\frac{\partial z^*(a,s)}{\partial s}$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})<\bar{f}_{cz}$, we have the LHS of the last inequality of (EC.8) being negative and therefore $\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))<\frac{\partial \omega^*(a,s)}{\partial s}$. If $\delta<\bar{\delta}_{cz}$, we have the coefficient of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ in the LHS of the last inequality of (EC.8) being negative. Hence, $\exists\ \bar{f}_{cz}\geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})>\bar{f}_{cz}$, we have the LHS of the last inequality of (EC.8) being negative and therefore $\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s))<\frac{\partial z^*(a,s)}{\partial s}$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})<\bar{f}_{cz}$, we have the LHS of the last inequality of (EC.8) being positive and therefore $\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))>\frac{\partial \omega^*(a,s)}{\partial s}$.

If $-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)+N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}-(1-N_F)\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)=0$, we have $-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)-\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)<0$ and therefore $\bar{\delta}_{cz}=0$.

If $-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)+N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}-(1-N_F)\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)<0$, we have $-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)-\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)<-(1-N_F)\frac{z^*(a,s)}{y(\omega^*(a,s))}k(1-\theta)+N_E\frac{y(\omega^*(a,s))}{1-\omega^*(a,s)}-(1-N_F)\frac{z^*(a,s)}{1-\omega^*(a,s)}+N_Ek(1-\theta)<0$ and therefore $\bar{\delta}_{cz}=0$.

In both cases, if $\delta>\bar{\delta}_{cz}$, we have the coefficient of $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ in the LHS of the last inequality of (EC.8) being positive. Hence, there exists $\bar{f}_{cz}\geq 0$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})>\bar{f}_{cz}$, we have the LHS

of the last inequality of (EC.8) being positive and therefore $\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s)) > \frac{\partial z^*(a,s)}{\partial s}$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) < \bar{f}_{cz}$, we have the LHS of the last inequality of (EC.8) being negative and therefore $\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s)) < \frac{\partial \omega^*(a,s)}{\partial s}$. □

Proof of Proposition 2. Suppose $\omega^*(a,s) \in (0,1)$. We have

$$\frac{\partial D_2(a,s)}{\partial a}/(1-\omega^*(a,s)) = -k(1-\theta)[\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))]N_E + N_F[\frac{\partial z^*(a,s)}{\partial a}/(1-\omega^*(a,s))], \tag{EC.9}$$

and

$$\frac{\partial D_2(a,s)}{\partial s} = -k(1-\theta)\frac{\partial \omega^*(a,s)}{\partial s}N_E + N_F\frac{\partial z^*(a,s)}{\partial s}. \tag{EC.10}$$

Proposition 2 (i) and (ii) follow from (EC.9) and (EC.10). Moreover, we have

$$\begin{aligned}\frac{\partial D_3(a,s)}{\partial a}/(1-\omega^*(a,s)) = &-(1-\theta)[\frac{\partial \omega^*(a,s)}{\partial a}/(1-\omega^*(a,s))]D_2(a,s) \\ &+(\omega^*(a,s)\theta+1-\omega^*(a,s))[\frac{\partial D_2(a,s)}{\partial a}/(1-\omega^*(a,s))],\end{aligned} \tag{EC.11}$$

and

$$\frac{\partial D_3(a,s)}{\partial s} = -(1-\theta)\frac{\partial \omega^*(a,s)}{\partial s}D_2(a,s) + (\omega^*(a,s)\theta+1-\omega^*(a,s))\frac{\partial D_2(a,s)}{\partial s}. \tag{EC.12}$$

Proposition 2 (iii) and (iv) follow from (EC.11) and (EC.12). □

Proof of Proposition 3. (i) When $r_0 \to 0^+$, we have (EC.6) being negative for any $z \geq 0$. Hence, we have $\omega^*(0,0) = 0$. Moreover, when $r_0 \to 0^+$, the retailer strictly prefers sourcing certified drugs only for any feasible subsidy scheme (a,s) and we have $\omega^*(a,s) = 0$ by (EC.6). Meanwhile, when $r_0 \to 0^+$, it is straightforward to show that the retailer's expected profit $R(z,\omega;a,s)$ is continuous in r_0. Hence, the results follow.

(ii) When $r_0 \to 0^+$, we have (EC.6) being negative for any $z \geq 0$ and therefore $\omega^*(0,0) = 0$. When $r_0 \to \infty$, we have (EC.6) being positive for any $z \geq 0$ and therefore $\omega^*(0,0) = 1$. Hence, there exist $0 \leq \hat{r}_0^{ml} < \hat{r}_0^{mh}$ s.t. if $r_0 \in (\hat{r}_0^{ml}, \hat{r}_0^{mh})$, we have $\omega^*(0,0) \in (0,1)$.

(ii) (a) The results follow from Lemma 4 and Proposition 2.

(ii) (b) When either a or s is sufficiently large, we have (EC.6) being negative for any $z \geq 0$ and therefore $\omega^*(a,s) = 0$. Meanwhile, when the retailer sources certified drugs only, a purchase subsidy increases quantity more rapidly than a sales subsidy under exogenous retail price (Taylor and Xiao 2014). Hence, the results follow.

(iii) When $r_0 \to \infty$, we have (EC.6) being positive for any $z \geq 0$. Hence, we have $\omega^*(0,0) = 1$. Moreover, when $r_0 \to \infty$, the retailer strictly prefers sourcing uncertified drugs only for any feasible subsidy scheme (a,s) and we have $\omega^*(a,s) = 1$ by (EC.6). Meanwhile, when $r_0 \to \infty$, it is straightforward to show that the retailer's expected profit $R(z,\omega;a,s)$ is continuous in r_0.

(iii) (a) The results follow same arguments as Proposition 1 (iii) (a).

(iii) (b) The results follow same arguments as Proposition 1 (iii) (b). □

EC.4. Detailed Discussion of Lemma 5.

We discuss here in detail the total effect of subsidies on inventory level under dual sourcing. In particular, when the density distribution at the threshold market condition $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is small, the retailer benefits little from a higher $z(a,s)$ through the indirect underage reducing effect: the stock-out probability does not increase much with a higher $z(a,s)$. Hence, the indirect effect of reducing stocking cost dominates and the net indirect effect of higher subsidy is to reduce inventory level. Moreover, in this case, a higher subsidy leads to a large decrease in the market share of uncertified drugs: the retailer is incentivized to stock a higher fraction of certified drugs, and he needs to stock a lot higher to reduce the stock-out probability given little demand at the threshold market condition. As a result, the net indirect effect dominates the net direct effect, and a higher purchase or sales subsidy leads to a lower inventory level.

On the other hand, when the density distribution at the threshold market condition $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is large, retailer benefits a lot from a higher $z(a,s)$: the stock-out probability increases significantly with a higher $z(a,s)$, and the retailer could sell many more units. Hence, the indirect effect of selling more dominates and the net indirect effect of a higher subsidy is to increase inventory level. The net effect of both direct and indirect channels is therefore to increase the inventory level.

Moreover, when $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ and δ are large, the direct effect dominates and the direct effect of purchase subsidy is stronger in increasing inventory level than the direct effect of sales subsidy. Hence, in this case, the inventory level increases more rapidly (or decreases less rapidly) in a purchase subsidy than a sales subsidy. Meanwhile, when $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ and δ are small, the indirect effect dominates. With small δ, the indirect effect of purchase subsidy is weaker in reducing the market share of uncertified drugs than the indirect effect of sales subsidy. Due to this, inventory level decreases less rapidly (or increases more rapidly) in a purchase subsidy than a sales subsidy. Hence, the disadvantage of a purchase subsidy in reducing the market share of uncertified drugs could in fact lead to an advantage in increasing inventory level.

Instead, if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is small and δ is large, the direct effects of purchase subsidy and sales subsidy are similar, while the indirect overage reducing effect dominates the net indirect effect. With large δ, purchase subsidy reduces the market share of uncertified drugs more rapidly than sales subsidy. Due to this, inventory level decreases more rapidly in purchase subsidy than sales subsidy. Hence, the advantage of a purchase subsidy in reducing the market share of uncertified drugs could in fact lead to a disadvantage in reducing inventory level. Meanwhile, if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is large and δ is small, the indirect underage reducing effect dominates the net indirect effect. With small δ, a purchase subsidy reduces the market share of uncertified drugs less rapidly than a sales subsidy. Due to this, inventory level increases less rapidly in a purchase subsidy than a sales subsidy.

EC.5. Proofs of Section 6.

Proof of Proposition 4. For a given subsidy scheme (a,s), suppose $\omega^*(a,s)\in(0,1)$ with exogenous retail price. Define F_k as

$$F_k=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial k}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial k}\end{bmatrix},$$

where

$$\frac{\partial F_1(z,\omega;a,s)}{\partial k}=-\frac{z(a,s)\omega(a,s)(1-\theta)}{y(\omega(a,s))^2}N_M f(\frac{z(a,s)}{y(\omega(a,s))}),$$

and

$$\frac{\partial F_2(z,\omega;a,s)}{\partial k}=-\delta[y(\omega(a,s))+(1-\omega(a,s))k(1-\theta)]N_E-(\delta N_F+1-\delta)z(a,s).$$

Define $J_{k\omega}$ as

$$J_{k\omega}=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial z} & \frac{\partial F_1(z,\omega;a,s)}{\partial k}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial z} & \frac{\partial F_2(z,\omega;a,s)}{\partial k}\end{bmatrix}.$$

We then have

$$\frac{\partial\omega^*(a,s)}{\partial k}=-\frac{\mathrm{Det}(J_{k\omega})}{\mathrm{Det}(J)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$\begin{aligned}
&-\mathrm{Det}(J_{k\omega})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)}=-\frac{z^*(a,s)\omega^*(a,s)(1-\theta)}{y(\omega^*(a,s))^2}N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})[(\delta N_F+1-\delta)(c_c-a-c_u)\\
&-\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})]-\frac{N_M}{y(\omega^*(a,s))}f(\frac{z^*(a,s)}{y(\omega^*(a,s))})\\
&\cdot\{N_E[\delta(c_c-a-c_u)\omega^*(a,s)(1-\theta)+N_M(1-\theta)]+\frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k\omega^*(a,s)(1-\theta)^2N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})\}\\
&=-\frac{z^*(a,s)\omega^*(a,s)(1-\theta)}{y(\omega^*(a,s))^2}N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})(\delta N_F+1-\delta)(c_c-a-c_u)\\
&-\frac{N_M}{y(\omega^*(a,s))}f(\frac{z^*(a,s)}{y(\omega^*(a,s))})N_E[\delta(c_c-a-c_u)\omega^*(a,s)(1-\theta)+N_M(1-\theta)]<0.
\end{aligned}$$

Hence, we have $\frac{\partial\omega^*(a,s)}{\partial k}<0$.

Define J_{kz} as

$$J_{kz}=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial k} & \frac{\partial F_1(z,\omega;a,s)}{\partial\omega}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial k} & \frac{\partial F_2(z,\omega;a,s)}{\partial\omega}\end{bmatrix}.$$

We then have

$$\frac{\partial z^*(a,s)}{\partial k}=-\frac{\mathrm{Det}(J_{kz})}{\mathrm{Det}(J)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$-\text{Det}(J_{kz})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)} = [(\delta N_F+1-\delta)(c_c-a-c_u) - \frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})]$$
$$\cdot\{N_E[-\delta(c_c-a-c_u)\omega^*(a,s)(1-\theta)-N_M(1-\theta)] - \frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k\omega^*(a,s)(1-\theta)^2N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})\}$$
$$-[2k(1-\theta)\delta(c_c-a-c_u)N_E + \frac{z^*(a,s)^2}{y(\omega^*(a,s))^3}k\omega^*(a,s)(1-\theta)^2N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})]$$
$$\cdot\frac{z^*(a,s)}{y(\omega^*(a,s))^2}\omega^*(a,s)(1-\theta)N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$$
$$=\{N_EN_M-(c_c-a-c_u)[\delta N_E+(\delta N_F+1-\delta)\frac{z^*(a,s)}{y(\omega^*(a,s))}]\omega^*(a,s)\}\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)^2N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$$
$$-(\delta N_F+1-\delta)(c_c-a-c_u)N_E[\delta(c_c-a-c_u)\omega^*(a,s)(1-\theta)+N_M(1-\theta)].$$

Hence, there exist $\bar{f}_{kz}\in[0,\infty)$ and $\bar{\omega}_{kz}\in[0,1]$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})\in(\bar{f}_{kz},\infty)$ and $\omega^*(a,s)\in(0,\bar{\omega}_{kz})$, we have $\frac{\partial z^*(a,s)}{\partial k}>0$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})\in(0,\bar{f}_{kz})$ and $\omega^*(a,s)\in(0,\bar{\omega}_{kz})$, or $\omega^*(a,s)\in(\bar{\omega}_{kz},1)$, we have $\frac{\partial z^*(a,s)}{\partial k}<0$. □

Proof of Proposition 5. For a given subsidy scheme (a,s), suppose $\omega^*(a,s)\in(0,1)$ with exogenous retail price. Define F_{c_u} as

$$F_{c_u}=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial c_u}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial c_u}\end{bmatrix},$$

where

$$\frac{\partial F_1(z,\omega;a,s)}{\partial c_u}=-\omega(a,s)(\delta N_F+1-\delta),$$

and

$$\frac{\partial F_2(z,\omega;a,s)}{\partial c_u}=N_E[-\delta y(\omega(a,s))+\delta\omega(a,s)k(1-\theta)]-(\delta N_F+1-\delta)z(a,s).$$

Define $J_{c_u\omega}$ as

$$J_{c_u\omega}=\begin{bmatrix}\frac{\partial F_1(z,\omega;a,s)}{\partial z} & \frac{\partial F_1(z,\omega;a,s)}{\partial c_u}\\ \frac{\partial F_2(z,\omega;a,s)}{\partial z} & \frac{\partial F_2(z,\omega;a,s)}{\partial c_u}\end{bmatrix}.$$

We then have

$$\frac{\partial\omega^*(a,s)}{\partial c_u}=-\frac{\text{Det}(J_{c_u\omega})}{\text{Det}(J)}\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$-\text{Det}(J_{c_u\omega})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)} = -\omega^*(a,s)(\delta N_F+1-\delta)[(\delta N_F+1-\delta)(c_c-a-c_u)$$
$$-\frac{z^*(a,s)}{y(\omega^*(a,s))^2}k(1-\theta)N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})] + \frac{N_M}{y(\omega^*(a,s))}f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$$
$$\cdot\{N_E[-\delta y(\omega^*(a,s))+\delta\omega^*(a,s)k(1-\theta)]-(\delta N_F+1-\delta)z^*(a,s)\}$$

Hence, there exists $\bar{\theta}_{c_u\omega}\in[0,1]$ s.t. if $\theta\in(\bar{\theta}_{c_u\omega},1)$, we have $\frac{\partial\omega^*(a,s)}{\partial c_u}<0$; if $\theta\in(0,\bar{\theta}_{c_u\omega})$, we have $\frac{\partial\omega^*(a,s)}{\partial c_u}>0$.

Define $J_{c_u z}$ as

$$J_{c_u z} = \begin{bmatrix} \frac{\partial F_1(z,\omega;a,s)}{\partial c_u} & \frac{\partial F_1(z,\omega;a,s)}{\partial \omega} \\ \frac{\partial F_2(z,\omega;a,s)}{\partial c_u} & \frac{\partial F_2(z,\omega;a,s)}{\partial \omega} \end{bmatrix}.$$

We then have

$$\frac{\partial z^*(a,s)}{\partial c_u} = -\frac{\text{Det}(J_{c_u z})}{\text{Det}(J)}\bigg|_{z=z^*(a,s),\omega=\omega^*(a,s)},$$

where

$$\begin{aligned}
&-\text{Det}(J_{c_u z})\Big|_{z=z^*(a,s),\omega=\omega^*(a,s)} = [(\delta N_F + 1 - \delta)(c_c - a - c_u) - \frac{z^*(a,s)}{y(\omega^*(a,s))^2} k(1-\theta) N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})] \\
&\cdot \{N_E[-\delta y(\omega^*(a,s)) + \delta\omega^*(a,s)k(1-\theta)] - (\delta N_F + 1 - \delta) z^*(a,s)\} \\
&- [2k(1-\theta)\delta(c_c - a - c_u)N_E + \frac{z^*(a,s)^2}{y(\omega^*(a,s))^3} k\omega^*(a,s)(1-\theta)^2 N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})]\omega^*(a,s)(\delta N_F + 1 - \delta) \\
&= \frac{z^*(a,s)}{y(\omega^*(a,s))^2} k(1-\theta) N_M f(\frac{z^*(a,s)}{y(\omega^*(a,s))})[N_E \delta y(\omega^*(a,s)) - N_E \delta\omega^*(a,s)k(1-\theta) \\
&+ (\delta N_F + 1 - \delta) z^*(a,s) - f(\frac{z^*(a,s)}{y(\omega^*(a,s))})k(1-\theta)\omega^*(a,s)(\delta N_F + 1 - \delta)] - (\delta N_F + 1 - \delta)](c_c - a - c_u) \\
&\cdot [N_E \delta y(\omega^*(a,s)) + (\delta N_F + 1 - \delta) z^*(a,s)] - k(1-\theta)\delta(c_c - a - c_u) N_E \omega^*(a,s)(\delta N_F + 1 - \delta).
\end{aligned}$$

Hence, there exist $\bar{f}_{c_u z} \in [0,\infty)$ and $\bar{\theta}_{c_u z} \in [0,1]$ s.t. if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (\bar{f}_{c_u z}, \infty)$ and $\theta \in (\bar{\theta}_{c_u z}, 1)$, we have $\frac{\partial z^*(a,s)}{\partial c_u} > 0$; if $f(\frac{z^*(a,s)}{y(\omega^*(a,s))}) \in (0, \bar{f}_{c_u z})$ and $\theta \in (\bar{\theta}_{c_u z}, 1)$, or $\theta \in (0, \bar{\theta}_{c_u z})$, we have $\frac{\partial z^*(a,s)}{\partial c_u} < 0$. □

Proof of Proposition 6. (i) Suppose the retail price is endogenous. When the donor has a sufficient budget, we have $a^* > 0$, $s^* = 0$ and $\omega^*(a^*, s^*) = 0$ in the absence of traceability technology, given the results in Proposition 1. Hence, there is no benefit for adopting traceability technology by the donor: the donor does not subsidize uncertified drugs even in the absence of traceability technology.

When the donor has a limited budget, we could have either $a^* > 0$, $s^* = 0$ and $\omega^*(a^*, s^*) = 0$, or $a^* = 0$, $s^* > 0$ and $\omega^*(a^*, s^*) = 1$, in the absence of traceability technology, given the results in Proposition 1. For the former case, again, there is no benefit for adopting traceability technology by the donor. For the latter case, it is optimal for the donor to subsidize uncertified drugs in the absence of traceability technology. In this case, the adoption of traceability technology forces the donor to subsidize certified drugs with a purchase subsidy only, which is a feasible yet suboptimal subsidy scheme in the absence of traceability technology if the donor aims to maximize the expected quantity or the health benefits. Hence, when the donor has a limited budget, the adoption of traceability technology either has no impact or reduces the expected quantity of drugs sold and the health benefits created for consumers.

(ii) Suppose the retail price is exogenous. When the donor has a sufficient budget, we have $a^* > 0$, $s^* = 0$ and $\omega^*(a^*, s^*) = 0$ in the absence of traceability technology, given the results in Proposition 3. Hence, there is no benefit for adopting traceability technology by the donor.

When the donor has a limited budget, the retailer could perform dual sourcing. In this case, the impact of traceability technology adoption is more nuanced and depends on both product

and market characteristics, similar to Lemma 4 and Proposition 2. While we omit the detailed conditions under which the adoption of traceability technology reduces, increases, or has no impact on the donor's three potential objectives, they can be obtained in a manner comparable to Lemma 4 and Proposition 2. □

Proof of Proposition 7. Suppose $c_c - a > c_u$. When $\theta \to 0$ and $k \to 0$, the retailer sources uncertified drugs only with $\omega^*(a,s) = 1$ as $\theta \to 0$ does not hurt consumer demand. In this case, the health benefits for consumers are reduced by the presence of uncertified drugs due to $\theta \to 0$.

When $\theta \to 0$ and $k \to 1$, the retailer sources certified drugs only with $\omega^*(a,s) = 1$ as $\theta \to 0$ and $k \to 1$ leads to significantly lower consumer demand when the retailer sources uncertified drugs. In this case, the presence of uncertified drugs has no impact.

When $\theta \to 1$, the retailer sources uncertified drugs only with $\omega^*(a,s) = 1$ as $\theta \to 1$ does not hurt consumer demand. In this case, the consumers benefit from higher health benefits associated with the presence of uncertified drugs due to $c_c - a > c_u$ and $\theta \to 1$, which leads to higher expected quantity sold. □

EC.6. Detailed Discussion of Proposition 4.

Proposition 4 shows that under dual sourcing, higher counterfeit awareness always reduces the fraction of uncertified drugs on the market. First, the direct effect of higher counterfeit awareness on the fraction of uncertified drugs is that the retailer wants to purchase a lower fraction of uncertified drugs to *i*) increase demand when the market condition is weak and *ii*) increase demand by lowering the threshold market condition to achieve a higher stock-out probability. Meanwhile, the direct effect of higher consumer awareness on the inventory level is that the retailer wants to stock fewer units due to lower demand. In this case, with a lower inventory level, we also have the two indirect channels of *iii*) reverse *indirect demand-enhancing effect* that leads to a higher fraction of uncertified drugs and *iv*) reverse *indirect acquisition cost-saving effect* that leads to a lower fraction of uncertified drugs. In particular, the direct effect of a lower fraction of uncertified drugs to increase demand with a higher stock-out probability cancels out the reverse indirect demand-enhancing effect. As a result, the net effect of higher k is such that it reduces $\omega^*(a,s)$ under dual sourcing.

On the other hand, Proposition 4 also shows that higher counterfeit awareness may increase or reduce inventory level under dual sourcing, depending on the density distribution at the threshold market condition and the fraction of uncertified drugs sourced by the retailer. When $\omega^*(a,s)$ is small, consumer demand is not sensitive to higher k. In this case, with small $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$, given the direct effect that higher consumer awareness reduces the fraction of uncertified drugs, the indirect overage reducing effect dominates the indirect underage reducing effect. Hence the net indirect effect due to the lower fraction of uncertified drugs is a reduction in inventory level. Since the direct effect of higher consumer awareness is also a reduction in inventory level, the net total effect is

that higher k leads to lower $z^*(a,s)$. Instead, with large $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ while $\omega^*(a,s)$ remains small, the indirect underage reducing effect dominates the indirect overage reducing effect. Hence, the net indirect effect is an increase in inventory level. Since $\omega^*(a,s)$ is small and consumer demand is not sensitive to higher k, the direct effect of higher counterfeit awareness on reducing the inventory level is small. Hence, the net total effect is that higher k leads to higher $z^*(a,s)$.

When $\omega^*(a,s)$ is large, consumer demand is highly sensitive to higher k. In this case, the direct effect that reduces inventory level is larger than the indirect underage reducing effect that increases inventory level due to large $\omega^*(a,s)$. Hence, when $\omega^*(a,s)$ is large, higher k always leads to lower $z^*(a,s)$ for any $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$.

EC.7. Detailed Discussion of Proposition 5.

Proposition 5 shows that interestingly, under dual sourcing, a higher acquisition cost of sourcing uncertified drugs reduces the fraction of uncertified drugs if their quality is high. First, the direct effect of higher acquisition cost of uncertified drugs on the fraction of uncertified drugs is that *i*) the retailer wants to purchase a lower fraction of uncertified drugs due to their higher acquisition cost and *ii*) the retailer wants to purchase a higher fraction of uncertified drugs to increase profit margin. Meanwhile, the direct effect on the inventory level is a reduction in inventory level due to higher acquisition cost. With lower inventory level, we also have the two indirect channels of *iii*) reverse *indirect demand-enhancing effect* that leads to a higher fraction of uncertified drugs and *iv*) reverse *indirect acquisition cost-saving effect* that leads to a lower fraction of uncertified drugs. When θ is large, both the direct effect, which is an increased profit margin, and the reverse indirect demand-enhancing effect are small, and therefore the net effect of higher c_u is a decrease in $\omega^*(a,s)$.

On the other hand, a higher acquisition cost of sourcing uncertified drugs may increase or reduce inventory level under dual sourcing, depending on both the density distribution at the threshold market condition and the quality of uncertified drugs. When θ is large, a higher acquisition cost of sourcing uncertified drugs leads to a lower fraction of uncertified drugs through the direct effect, which leads to higher inventory level via indirect underage reducing effect and lower inventory level via indirect overage reducing effect. When $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is small, the indirect overage reducing effect dominates and the net indirect effect is that a higher acquisition cost of sourcing uncertified drugs reduces the inventory level. Together with the direct effect on the inventory level, the net total effect is that higher c_u leads to a lower $z^*(a,s)$. When $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$ is large, the indirect underage reducing effect dominates the rest of the effects and therefore the net total effect is that higher c_u leads to a higher $z^*(a,s)$.

When θ is small instead, higher c_u leads to a higher fraction of uncertified drugs through the direct effect, which leads to a lower inventory level via reverse indirect underage reducing effect and a higher inventory level via reverse indirect overage reducing effect. In this case, the direct effect of

higher c_u on reducing the inventory level is larger than the reverse indirect overage reducing effect on increasing the inventory level due to small θ. Hence, when θ is small, higher c_u always leads to lower $z^*(a,s)$ for any $f(\frac{z^*(a,s)}{y(\omega^*(a,s))})$.

EC.8. Numerical Analysis: Sensitivity Analysis

In this section, we provide the sensitivity analysis for Proposition 3 and Proposition 7, with the setup of the numerical analysis as presented in Section 7.1.

EC.8.1. Optimal Subsidy Scheme Under Exogenous Retail Price Scenario

First, we analyze the characteristics of the optimal subsidy scheme when the retailer performs dual sourcing, as Proposition 3 does not cover the entire parameter space. We show that, out of the 50,000 instances with an exogenous retail price, the retailer performs dual sourcing in 7,317 instances, while the optimal subsidy scheme requires both purchase subsidy and sales subsidy in 3,841 instances. These instances are more likely to occur with a small to medium budget size (75% with $B \leq 600$) and low cost-quality difference ratio (77% with $r_a \leq 1$). Hence, our numerical analysis suggests that under dual sourcing, the optimal subsidy scheme is likely to consist of both a purchase subsidy and a sales subsidy, especially with a small budget size and low cost-quality difference ratio.

EC.8.2. Additional health benefits of Eliminating Uncertified Drugs

Next, we would like to use the numerical results to better understand the additional health benefits when uncertified drugs are eliminated, as Proposition 7 does not cover the entire parameter space for θ and k, and it does not provide the magnitude of the additional health benefits. Figures EC.2 and EC.3 plot the additional expected health benefits associated with eliminating uncertified drugs under endogenous retail price and exogenous retail price scenarios, respectively. Note that we only plot the instances in which the retailer sources at least some uncertified drugs, as otherwise clearly there is no need to eliminate uncertified drugs. We notice from Figure EC.2 that under the endogenous retail price scenario, when θ is low, the retailer only sources uncertified drugs if k is low as well. As θ increases, the retailer starts to source uncertified drugs with higher k. Meanwhile, the additional health benefits associated with eliminating uncertified drugs are larger when θ is smaller or k is larger, and elimination of uncertified drugs is not preferable when we have grey uncertified drugs of high quality. These observations hold for different values of θ and k and are consistent with Proposition 7. Hence, under an endogenous price scenario, elimination of uncertified drugs is required when we have low-quality uncertified drugs on the market and consumers' counterfeit awareness is low.

The results under an exogenous retail price scenario are more complicated. One major difference we can observe by comparing Figure EC.3 with Figure EC.2 is that under exogenous retail pricing, the retailer may source uncertified drugs under various values of θ and k, even with high-quality

Figure EC.2 Additional expected health benefits of eliminating uncertified drugs under an endogenous retail price scenario when the retailer sources at least some uncertified drugs.

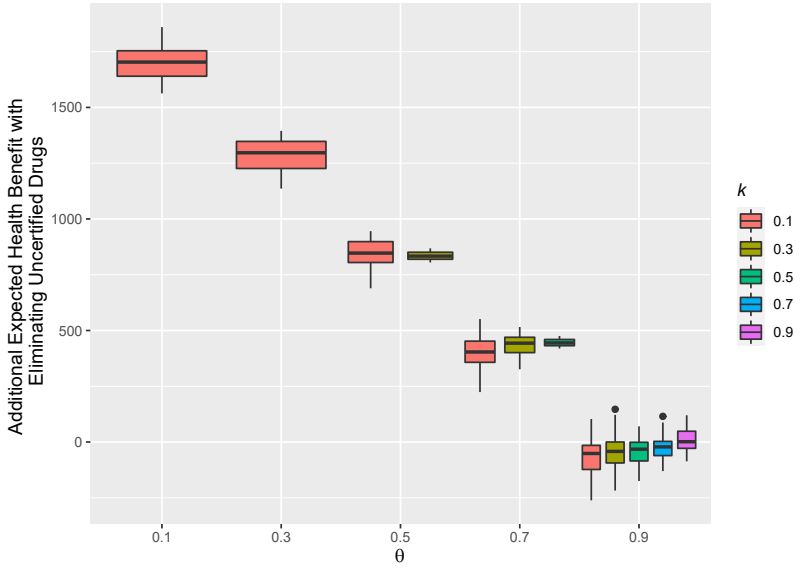
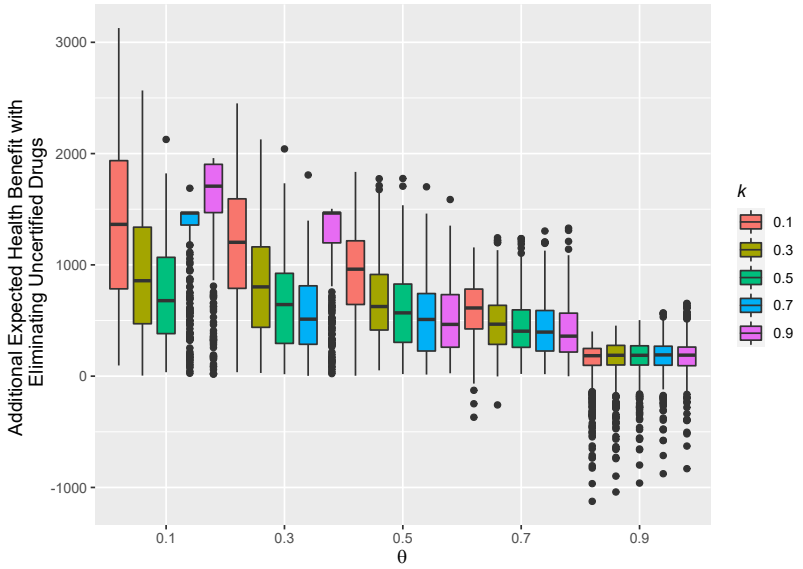


Figure EC.3 Additional expected health benefits of eliminating uncertified drugs under an exogenous retail price scenario when the retailer sources at least some uncertified drugs.



uncertified drugs on the market. Moreover, in this case, while the additional health benefits associated with eliminating uncertified drugs are still higher with lower θ, the role of k is more nuanced, and higher additional health benefits may be achieved with either lower k or higher k. Hence, under exogenous retail pricing, eliminating uncertified drugs could be preferable under various values of θ and k, while ascertaining the magnitude of the benefit associated with eliminating uncertified drugs requires a more detailed understanding of the market characteristics.