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Fairness Concerns in Heterogeneous Teams: Utility, Reward, and Income

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Problem Definition: In homogeneous teams, stimulating inequality aversion among agents is a silver bullet: it leads to higher output while being fair, on all possible dimensions of fairness: utility, reward, and income. Is this result still true in heterogeneous teams when agents might respond differently to incentives?
Methodology/Results: We consider a principal two-agent model in which agents of different abilities are inequality-averse between each other, on one of the following three types of fairness consideration: utility, reward, and income. We analytically show that whether inequality aversion benefits the principal depends on the type of fairness consideration and the degree of agent heterogeneity. Specifically, we find that stimulating utility fairness does not benefit the principal and gives rise to inequalities when the agents are highly heterogeneous; but that it does benefit the principal without creating inequalities when agents are only moderately heterogeneous. Stimulating reward fairness wins on both fronts: It always benefits the principal and results in no inequalities. Stimulating income fairness hurts potentially on both fronts: It always lowers the principal's payoff and inequalities may arise.
Managerial Implications: Our research indicates that stimulating inequality aversion (e.g., through performance and pay transparency) can backfire under the wrong type of fairness consideration and/or if the team is too heterogeneous. In general, stimulating inequality aversion when workers compare their rewards is a safer bet than when they compare their utilities; and it should be avoided when they compare their incomes**.**

Keywords: Fairness; Principal-agent; Teams; Moral Hazard; Inequality Aversion; Heterogeneity; Pay Transparency

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1. Introduction

Many organizations have experimented with greater process transparency (Buell 2019, Guda et al. 2023), in particular on their workers' performance and wages (Blanes i Vidal and Nossol 2011, Song et al. 2018) because it often improves performance by eliciting social comparisons (Roels and Su 2014, Long and Nasiry 2020, Cullen and Pakzad-Hurson 2023), although not always (Turkoglu and Tucker 2022).

Central to the criteria of interpersonal comparison are three core principles of distributive justice: utility, reward, and income. These principles are inspired by Moulin (2004), who formalized four fairness principles for distributive justice: compensation, reward, exogenous rights, and fitness. The utility fairness principle, which relates to compensation, aims to ensure equality *ex-post* by equalizing individual utilities. The reward fairness principle, which relates to equity, entitles individuals to the "fruit of their own labor" (Moulin 2004,

p. 22). The income fairness principle, which relates to the exogenous rights, centers around "equality *ex-ante*, in the sense that we have an equal claim to resources regardless of the way they affect our welfare or that of others" (Moulin 2004, p. 22). The fitness principle aims at maximizing the total utility; since it is not relational, it cannot be used as a basis of comparison among agents, and we ignore it from hereon.

In homogeneous teams, a principal's contracting strategy is straightforward: an equal-sharing rule is both fair on all dimensions (agents' utilities are equal, their rewards are proportional to their efforts, and their incomes are equal) and beneficial to the principal (Bartling and von Siemens 2010a, Gill and Stone 2015). In equilibrium, agents have no "mental burden" associated with inequality aversion, despite being sensitive to it. Moreover, a principal always benefits from stimulating greater inequality aversion because it leads to higher output.

How about heterogeneous teams? Is stimulating inequality aversion still benefiting the principal and resulting in fair outcomes when team members have different abilities and thus respond differently to incentives? To what extent does it depend on the type of fairness consideration at work?

To illustrate this conundrum, consider in Table 1 a scenario where two agents contribute differently to a common project (top row of left panel), leading to a total output of 5 units, with different costs for their respective contributions (bottom row of left panel). What could be a fair split?

- According to utility fairness, a split of $(2,3)$ would be fair to equate Agent 1's utility (revenue-cost), equal to $2-1$, to Agent 2's, equal to $3-2$.
- According to reward fairness, a split of $(3,2)$ would be fair to equate Agent 1's reward, i.e., income-to-contribution ratio, equal to $3/3$, to Agent 2's, equal to $2/2$.
- According to income fairness, a split of $(2.5,2.5)$ would be fair to equate the agents' incomes.

Table 1 An example of fair split.

Data	Agent 1	Agent 2
Contribution	3	2
Cost	1	2

Fairness Consideration	Agent 1	Agent 2
Utility	2	3
Reward	3	2
Income	2.5	2.5

Even though principals might have some discretion to stimulate more or less inequality aversion, e.g., through greater pay transparency (Long and Nasiry 2020) or relative performance feedback (Song et al. 2018), what type of fairness consideration is the most salient is often outside their control and typically a function of the team characteristics and the nature of tasks. In particular, agents anticipating future interactions tend to compare themselves more on income and on reward (Deutsch 1975). Also, utility fairness considerations are more prevalent among friends than non-friends (Deutsch 1975). In this research, we consider an environment where a particular type of fairness applies, based on the characteristics of the team and the project, and study whether stimulating more concerns for this type of fairness may benefit the principal.

We consider a principal overseeing a team of two agents with heterogeneous abilities (Long and Nasiry 2020) who work on a common task. (They might also work independently on individual tasks, which are not subject to social comparison.) The joint output of the common task is split, as in Holmström (1982), and the agents are subject to fairness concerns between each other—specifically, feelings of envy and guilt, based on one of three fairness principles. Similar to Fehr and Schmidt (1999), agents compare some form of payoff (e.g., utility, income) to a reference point, be it the other agent's payoff or the amount considered to be deserved. In line with the literature, we assume their efforts are perfect substitutes to disentangle the issue of fairness from co-production, but our results carry over to more general settings. The timeline is as follows: In the first stage, the principal offers a contract; in the second stage, agents choose their effort levels, simultaneously and noncooperatively, to maximize their individual utility. We assume that the output is contractible, unlike the agents' efforts. We focus on affine output-sharing contracts with limited liability, as is common in many intra-organizational settings (e.g., profit-sharing agreements).

Within this setting, we examine, under each principle of fairness, the following questions: Would it be in the principal's interest to stimulate guilt or envy among agents? Does the optimal contract result in inequalities between agents?

We find that whether a principal should stimulate guilt or envy among agents and whether it gives rise to inequalities depends on the principle of fairness at work, as summarized in Table 2.

Utility. When agents compare their utilities, the result depends on the degree of agent heterogeneity. On the one hand, when the team is quite heterogeneous, inequality aversion does not change the principal's payoff, but gives rise to inequalities between agents. It is

Table 2 **Effect of stimulating inequality aversion on the principal's payoff.**

	Agent heterogeneity	
Object of comparison	Mild	Strong
Utility	↑	–
Reward	↑	↑
Income	↓	↓

Here, "↑" denotes increasing payoff, "↓" denotes decreasing payoff, and "–" denotes no change in payoff.

indeed optimal for the principal to reward the most able agent and not the other. While the former suffers from guilt, the latter suffers from envy. On the other hand, when the team is moderately homogeneous, inequality aversion might benefit the principal without creating inequalities, but this requires careful engineering of their contract. Specifically, the principal needs to balance incentives so that the most able agent exerts more effort than under inequality neutrality, but without making the least able agent drop his effort below the inequality-neutrality effort level.

Reward. When workers compare their rewards, inequality aversion always increases the payoff for the principal and results in a fair allocation of rewards, as in homogeneous teams—but this again requires some careful contract engineering.

Income. When agents compare their incomes, inequality aversion definitely hurts as it always leads to a lower payoff for the principal. It also leads to an unfair allocation if agents are quite heterogeneous and not very inequality-averse, similar to utility fairness. Indeed, it is optimal for the principal to allocate some or all reward to only the most able agent and only a fixed-fee payment or nothing to the least able agent.

The difference in outcomes across the different types of fairness stems from differences in the game structure: While the effort choice game (or a transformation thereof) is supermodular in the case of utility fairness or reward fairness, it is submodular in the case of income fairness. In supermodular games, an agent exerts more effort in response to the other agent's higher effort. This enables agents to put in efforts that exceed the risk-neutral levels in equilibrium, thus achieving a higher output. In submodular games, in contrast, the efforts respond to each other in opposite directions, rendering the previous mechanism ineffective.

In sum, our research indicates that inequality aversion needs to be carefully manipulated in the context of heterogeneous teams. Stimulating inequality aversion can backfire under the wrong type of fairness consideration and/or if the team is too heterogeneous. In general, stimulating inequality aversion when agents compare their rewards is a safer bet than when they compare their utilities; and it should be avoided when they compare their incomes.

2. Literature Review

Our study contributes to the literatures on fairness principles, on the management of team operations, and on principal-agent models with inequality-averse agents.

2.1. Fairness Principles

The principles of distributive justice stem from Adams' equity theory (Adams 1965), which argues that one's reward should be proportional to one's input. This theory has been extended by Deutsch (1975) and Cook and Hegtvedt (1983) to incorporate "need" and "equality": "need" implies the resources should be allocated to those who need it most, while "equality" means that people should receive the same amount irrespective of their input. Inspired by this stream of work, Moulin (2004) formalized the fairness principles to "compensation," "reward," and "exogenous-rights," which serve as the theoretical basis for this paper.

The three fairness principles have been analytically modeled in various forms; see Table 3 in Appendix for a summary. In economics, utility fairness has been studied by Demougin and Fluet (2003), Itoh (2004), Gill and Stone (2015), among others. Reward fairness has been studied by Gill and Stone (2015) in a general formulation; it also relates to the Shapley value, which rewards agents for their incremental contribution (Moulin 2004, Chapter 5). And income fairness has been modeled by Itoh (2004) and Hellmann and Wasserman (2017), among others. In operations management, fairness concerns have been primarily studied in the context of decentralized supply chains (Cui et al. 2007, Avcı et al. 2014, Wu and Niederhoff 2014, Qi et al. 2020) or pricing (Cohen et al. 2022). Overall, these works appear to focus on one or two types of fairness. We contribute to this literature by comparing outcomes across all three types of fairness.

2.2. Management of Team Operations

A growing body of literature in people-centric operations studies team dynamics, such as incentives to collaborate, help each other, and share knowledge (Siemsen et al. 2007, Özkan-Seely et al. 2015, Rahmani et al. 2017, Crama et al. 2019). Although fairness concerns are often not explicitly modeled in this literature, they naturally arise in teams. Of particular interest to our study is the effect of social comparison on team performance, e.g., feedback design (Roels and Su 2014, Chan et al. 2014, Song et al. 2018, Tan and Staats 2020, Long and Nasiry 2020, Niewoehner and Staats 2022, Cullen and Pakzad-Hurson 2023). Social comparison as a performance improvement mechanism has also been studied empirically, both in lab experiments (Charness et al. 2014, Tafkov 2013, Hannan et al. 2008) and in field experiments (Blanes i Vidal and Nossol 2011, Cowgill 2015, Song et al. 2018). Unlike previous studies, which largely promote greater transparency, we demonstrate that stimulating inequality aversion (through greater transparency) needs careful crafting. It could backfire with heterogeneous agents under the wrong type of fairness consideration.

2.3. Principal-Agent Problems with Inequality-Averse Agents

We embed fairness considerations into a principal-agent model involving team production (Holmström 1982), building on the seminal work by Fehr and Schmidt (1999) on envy and guilt, which has received substantial empirical support (see, e.g., Bolton and Ockenfels 2000). The literature can be classified into two streams, depending on whether individual outputs are verifiable or whether it is only the team output that is so.

2.3.1. Individual Outputs. When individual outputs are verifiable, the effect of envy and guilt depends on whether efforts are verifiable or not.

When efforts are not verifiable, envy is known to have two effects, resulting in an ambiguous effect on output: On one hand, envy motivates agents to increase effort (at least when agents are homogeneous); on the other hand, it might require compensation due to agents' disutility when comparing incomes (Itoh 2004, Neilson and Stowe 2010, Kragl 2015). In contrast, guilt has always a negative impact on output: It is known to discourage agents when individual performance measures are used and agents are homogeneous (Itoh 2004); and similar to envy, guilt requires compensation due to agents' disutility when comparing incomes (Neilson and Stowe 2010). However, these effects may vanish when at least one agent's effort is verifiable; that is, envy (resp. guilt) may no longer motivate (resp. demotivate) agents to exert high effort (Demougin et al. 2006, Rey-Biel 2008).

Further studies investigate the interplay among inequality aversion, risk aversion (Englmaier and Wambach 2010), limited liability (Bartling and von Siemens 2010b), and reference types (Bartling 2011).

2.3.2. Team Outputs. When only the team output is verifiable, inequality aversion is well known to increase team output when the team is homogeneous (Bartling and von Siemens 2010a, Gill and Stone 2015).

Considering heterogeneous agents like we do, Kölle et al. (2011) and Kölle et al. (2016) take the perspective of a social planner (or, alternatively, a principal dealing with agents that have no limited liability) offering a symmetric output-sharing contract to agents who are averse to inequalities in utility. Within their context, introducing *ex-ante* wealth differences is needed to increase team output. However, their proposed symmetric-share contractual arrangement is in general suboptimal for the principal; it is, in particular, inefficient when agents are inequality-neutral since the principal would then want to give shares only to the most able agent.

We contribute to this literature in the following ways. First, we do not restrict the principal's choice of contract (besides respecting liability constraints). Second, we operationalize in the context of heterogeneous teams the principle of reward fairness, which is widely prevalent in the workplace (Matuson 2022).

3. Model

We next present the model components, formulate the effort choice game and the principal's optimal contract design problem, and introduce the benchmark of inequality-neutral agents.

3.1. Model Components

We consider a principal and two heterogeneous agents indexed by $i = 1, 2$. Agent i exerts effort $e_i \geq 0$ at a cost $C_i(e_i) = \frac{1}{2}c_i e_i^2$. The team's output P is an additive function of the agents' efforts, i.e., efforts are perfect substitutes. Accordingly, $P(e_1, e_2) = k_1 e_1 + k_2 e_2$, with $k_1, k_2 > 0$. Let $k := (k_2/c_2)/(k_1/c_1)$ denote the relative ability of Agent 2. We assume that Agent 1 has higher ability, i.e., $k \leq 1$. Without loss of generality, we normalize the problem by setting $c_1 = c_2 = k_1 = 1$, so that $k_2 = k$. Similar to Gill and Stone (2015) and Kölle et al. (2011, 2016), we do not consider randomness in output, but our results can be generalized when the output is subject to a random shock and agents compare their *ex-ante* utilities.

The joint output is verifiable, unlike the agents' individual efforts. We assume the principal offers an affine output-sharing contract parameterized by $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, in which γ_i denotes Agent i's output share and f_i denotes his fixed-fee payment. Thus, Agent i's income is $I_i(e_i, e_{-i}, \gamma_i, f_i) = \gamma_i P(e_1, e_2) + f_i$ for $i = 1, 2$. Although such contracts are not optimal, they are common in practice (e.g., profit-sharing agreements in a medical group practice). As is common in intra-organizational settings (Long and Nasiry 2020), we assume that agents have limited liabilities, i.e., $f_i \geq 0$ and $\gamma_i \geq 0$ for $i = 1, 2$. Agent i's nominal utility is thus $v_i(e_i, e_{-i}, \Phi) = \gamma_i P(e_1, e_2) + f_i - \frac{1}{2}e_i^2$, where $-i := 3 - i$ for $i \in \{1, 2\}$. The principal receives the residual output; hence, the principal's profit is $\pi_i(e_i, e_{-i}, \Phi) = (1 - \gamma_1 - \gamma_2)P(e_1, e_2) - f_1 - f_2$.

Agents are subject to envy and guilt as in Fehr and Schmidt (1999) and we consider three objects of comparison: utility (superscript 'U'), reward ('R'), and income ('I'). That is, agents experience disutilities when they get either more or less of the object of reference than the other agent. Accordingly, Agent i's total utility under utility, reward, or income fairness is respectively

$$
\begin{aligned}
u_i^U(e_i; e_{-i}, \Phi) =& v_i(e_i, e_{-i}, \gamma_i, f_i) - \alpha[v_i(e_i, e_{-i}, \gamma_i, f_i) - v_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i})]^+ \\
& - \beta[v_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i}) - v_i(e_i, e_{-i}, \gamma_i, f_i)]^+, \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
u_i^R(e_i; e_{-i}, \Phi) =& v_i(e_i, e_{-i}, \gamma_i, f_i) \\
& - \alpha\left[I_i(e_i, e_{-i}, \gamma_i, f_i) - \frac{k_i e_i}{P(e_i, e_{-i})}(I_i(e_i, e_{-i}, \gamma_i, f_i) + I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i}))\right]^+ \qquad (2) \\
& - \beta\left[\frac{k_i e_i}{P(e_i, e_{-i})}(I_i(e_i, e_{-i}, \gamma_i, f_i) + I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i})) - I_i(e_i, e_{-i}, \gamma_i, f_i)\right]^+,
\end{aligned}
$$

$$
\begin{aligned}
u_i^I(e_i; e_{-i}, \Phi) =& v_i(e_i, e_{-i}, \gamma_i, f_i) - \alpha[I_i(e_i, e_{-i}, \gamma_i, f_i) - I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i})]^+ \qquad (3) \\
& - \beta[I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i}) - I_i(e_i, e_{-i}, \gamma_i, f_i)]^+,
\end{aligned}
$$

where α (for "ahead") measures the level of guilt, β (for "behind") measures the level of envy, $[\cdot]^+ := \max\{\cdot, 0\}$, and $i = 1, 2$. The term $(k_i e_i / P) \cdot (I_i + I_{-i})$ in (2) represents the income that Agent i believes he deserves given his relative contribution $k_i e_i / P$. We restrict the guilt parameter to $\alpha \in [0, 1/2]$ as in Rey-Biel (2008). Otherwise if $\alpha < 0$, agents would be inequality-seeking, which could arise under competition but not under cooperation (McClintock et al. 1984, p. 187-188), and would be contrary to our concern for fairness; and if $\alpha > 1/2$ agents who are averse to utility or income inequality would be willing to transfer their income to the other agent. Similarly, we restrict the envy parameter to $\beta \in [0, 1]$ as in

Rey-Biel (2008) and Demougin et al. (2006). Otherwise if $\beta < 0$, agents would be inequality-seeking; and if $\beta > 1$, an envious agent who is averse to utility inequality would gain more utility from envy than from the income itself. These parameter value ranges have also been found to be the most relevant in empirical studies (Nunnari and Pozzi 2022). Further, we assume that $\beta \geq \alpha$ (Fehr and Schmidt 1999) since people tend to suffer more from an inequality to their disadvantage than an inequality to their advantage (Loewenstein et al. 1989). For simplicity of exposition, we assume that $\alpha + \beta \leq 1$. In practice, envy and guilt are correlated (Nunnari and Pozzi 2022), so, throughout the paper, when we refer to stimulating greater inequality aversion, we mean increasing both α and β.

For each fairness principle, we solve the problem backward by first solving for the agents' equilibrium efforts and then the principal's profit-maximizing contract. Throughout our analysis, we ignore the agents' participation constraints (i.e., agents may earn negative utility in equilibrium), but these could be incorporated at the cost of a more cumbersome analysis.

3.2. Equilibrium Efforts

Given an output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, each agent maximizes his utility given the other agent's choice of effort. Under a given fairness principle $F \in \{U, R, I\}$, in equilibrium:

$$e_i^* = \max_{e_i \geq 0} u_i^F(e_i; e_{-i}^*, \gamma_i, f_i), \quad i \in \{1, 2\}, . \tag{4}$$

Assumption 1 **(Equilibrium Selection Rule)**. *If there are multiple equilibria in (4), we select the Pareto-dominating equilibrium.*

3.3. Optimal Contract

Anticipating the equilibrium effort, the principal offers a revenue sharing contract parameterized by $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$. Given fairness principle $F \in \{U, R, I\}$, the principal's problem is

$$\begin{aligned} \max_{(\gamma_1, \gamma_2, f_1, f_2)} \quad & (1 - \gamma_1 - \gamma_2)(e_1^* + k e_2^*) - f_1 - f_2, & \\ \text{s.t.} \quad & f_i \geq 0 & i = 1, 2, \\ & \gamma_1 + \gamma_2 \leq 1,\ 0 \leq \gamma_i \leq 1 & i = 1, 2, \\ & e_i^* \in \arg\max_{e_i \geq 0} u_i^F(e_i; e_{-i}^*, \gamma_i, f_i) & i = 1, 2. \end{aligned} \tag{5}$$

Although fixed-transfer payments are in principle allowed, we show that one can restrict our attention to contracts without fixed-transfer payments, i.e., $f_1 = f_2 = 0$, under both utility and reward fairness, without loss of optimality. We do so in three steps. First, given a Nash equilibrium to the effort game (4) under contract Φ, say, (e_1^f, e_2^f), we construct a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ with no fixed-transfer payments that leads to not only a higher profit for the principal at (e_1^f, e_2^f), but to higher best responses by the agents. Second, we show that any effort game (4) with no fixed-transfer payments, and in particular under Φ', is supermodular. Finally, combining these two results, we show that, under contract Φ', there exists an equilibrium (e_1^0, e_2^0) such that $e_1^0 \geq e_1^f$ and $e_2^0 \geq e_2^f$, and thus $\pi(e_1^0, e_2^0, \Phi') \geq \pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$, where at least one of the two inequalities is strict.

In contrast, under income fairness, game (4) is submodular, so the argument does not apply. As a result, the optimal fixed-transfer payments are not necessarily zero. We optimize the contract design problem (5) by sequentially finding the optimal fixed-transfer payments (f_1, f_2) and then the optimal shares (γ_1, γ_2).

3.4. Benchmark: Inequality-Neutral Agents

When agents are inequality-neutral (superscript 'N'), i.e, when $\alpha = \beta = 0$, agents' equilibrium choices of effort solving (4) are $e_1^N := \gamma_1$ and $e_2^N := k\gamma_2$. The next proposition shows that the maximum output when agents are inequality-neutral is attained by evenly sharing the output with the most able agent and giving nothing to the other agent.

PROPOSITION 1 **(Benchmark)**. *When agents are inequality-neutral, the principal allocates shares to only the most able agent and offers an output-sharing contract* $(\gamma_1, f_1) = (1/2, 0)$.

In the following three sections, we present the optimal contract under utility, reward, and income fairness. We accord a more detailed presentation to utility fairness, as the analyses of the latter two cases exhibit substantial similarities, albeit leading to different results.

4. Utility Fairness

Considering utility fairness, we first characterize the agents' equilibrium efforts and then the principal's optimal contract.

4.1. Equilibrium Efforts

Under utility fairness, agents tend to choose efforts to reduce the gap in utilities. Game (4) turns out to be transformable into a supermodular game by eliminating the strictly dominated strategies. Thus, in equilibrium, agents' efforts are strategic complements (despite the additive production function) and tend to move in the same direction.

Relative to the inequality-neutral benchmark, envy demotivates effort, while guilt motivates effort. To see this, recall from (1) that Agent i's total utility takes the form of $v_i - \alpha[v_i - v_{-i}]^+ - \beta[v_{-i} - v_i]^+$. Suppose that Agent i is envious, i.e., that $v_i < v_{-i}$. If he puts in more effort than the inequality-neutral benchmark, not only does his own nominal utility decrease, but his teammate's nominal utility increases due to the larger output, enlarging the disutility caused by envy. Therefore, envy demotivates effort. In contrast, suppose that Agent i is guilty, i.e., that $v_i > v_{-i}$. If he puts in more effort than the inequality-neutral benchmark, his own nominal utility decreases while his teammate's nominal utility increases, reducing the utility gap. Thus, guilt motivates effort.

To characterize the equilibrium efforts, we define the following notations:

$$\begin{aligned} e_1^\alpha &:= e_1^N + \tfrac{\alpha}{1-\alpha}\gamma_2, \quad e_1^\beta := \left[e_1^N - \tfrac{\beta}{1+\beta}\gamma_2\right]^+, \\ e_2^\alpha &:= e_2^N + \tfrac{\alpha}{1-\alpha}k\gamma_1, \ e_2^\beta := \left[e_2^N - \tfrac{\beta}{1+\beta}k\gamma_1\right]^+. \end{aligned} \tag{6}$$

Superscripts α and β respectively refer to the distortion induced by guilt or envy: While e_i^α maximizes $v_i - \alpha[v_i - v_{-i}]$, e_i^β maximizes $v_i - \beta[v_{-i} - v_i]$ (subject to $e_i \geq 0$). Agent i's utility function (1) turns out to be piecewise quadratic, as we explain next. On the one hand, when $v_i > v_{-i}$, the agent's utility is $v_i - \alpha[v_i - v_{-i}]$, which is quadratic and maximized at e_i^α. On the other hand, when $v_i < v_{-i}$, the agent's utility is $v_i - \beta[v_{-i} - v_i]$, which is quadratic and maximized at e_i^β. Because guilt leads to higher effort than the inequality-neutral level and envy leads to lower effort than the inequality-neutral level, $e_i^\alpha \geq e_i^N \geq e_i^\beta$ for $i = 1, 2$. The remaining case arises when $v_i = v_{-i}$. Let $\overline{e}_i(e_{-i})$ denote Agent i's effort level that equalizes agents' nominal utilities given that Agent $-i$ exerts e_{-i}, i.e., $v_i(\overline{e}_i(e_{-i}), e_{-i}) = v_{-i}(e_{-i}, \overline{e}_i(e_{-i}))$ for $i = 1, 2$. Whenever it is a best response, $\overline{e}_i(e_{-i})$ lies between e_i^β and e_i^α, i.e., $e_i^\alpha \geq \overline{e}_i(e_{-i}) \geq e_i^\beta$ for $i = 1, 2$.

As a result, any $e_i \notin [e_i^\beta, e_i^\alpha]$ is strictly dominated. Thus, we can restrict the strategy space to this interval without loss of generality. In this restricted action space, the game is supermodular. As discussed in §3.3, the optimal contract turns out to be such that $f_1, f_2 = 0$. Accordingly, we focus our presentation of the equilibrium efforts to such contracts. (See Lemma EC.1 for a characterization under general contracts with fixed-transfer payments. All proofs and supporting results appear in the electronic companion.)

Lemma 1. *Under utility fairness, there exist thresholds $\theta_{AB} > \theta_{BC} > \theta_{CD}$ such that given an output-sharing contract $\Phi = (\gamma_1, \gamma_2, 0, 0)$:*

A. *If $\gamma_2/\gamma_1 \geq \theta_{AB}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt:* $e_1^* = e_1^\beta$, $e_2^* = e_2^\alpha$;

B. *If $\theta_{BC} \leq \gamma_2/\gamma_1 \leq \theta_{AB}$, there exists a continuum of Nash equilibria such that $v_1 = v_2$:* $e_2^* \in \left[\max\left\{e_2^\beta, \bar{e}_2\left(e_1^\beta\right)\right\}, e_2^\alpha\right]$ *and* $e_1^* = \bar{e}_1(e_2^*)$;

C. *If $\theta_{CD} \leq \gamma_2/\gamma_1 \leq \theta_{BC}$, there exists a continuum of Nash equilibria such that $v_1 = v_2$:* $e_1^* \in \left[\max\left\{e_1^\beta, \bar{e}_1\left(e_2^\beta\right)\right\}, e_1^\alpha\right]$ *and* $e_2^* = \bar{e}_2(e_1^*)$;

D. *If $\gamma_2/\gamma_1 \leq \theta_{CD}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences guilt and Agent 2 experiences envy:* $e_1^* = e_1^\alpha$, $e_2^* = e_2^\beta$.

Two types of equilibria emerge. On the one hand, Cases A and D involve extremely uneven payoff distributions, which leave one agent guilty and the other envious: while the guilty agent puts in more effort to counteract his sense of guilt, the envious agent reduces his effort. In particular, when $\gamma_i > 0$ and $\gamma_{-i} = 0$, i.e., Agent i exerts e_i^N and Agent $-i$ exerts zero effort despite feelings of envy and guilt.

On the other hand, Cases B and C are associated with a more even payoff distribution. In these cases, agents' effort choices are dominated by their fairness concerns: agents' efforts move hand in hand to equalize their utilities, consistent with the supermodular nature of the game. In particular, one agent (namely, Agent 2 in Case B and Agent 1 in Case C) puts in a "base" effort and the other chooses his effort to equalize their utilities. The agent who sets the base effort effectively dictates the level of output, i.e., acts as a bottleneck for team production. Consequently, the largest (resp., smallest) output among all equilibria in Cases B and C is achieved when the base effort is chosen to be e^α (resp., e^β), consistent with the motivating (resp., demotivating) role of guilt (resp., envy).

We next state an equilibrium selection rule based on Pareto-optimality (Mas-Colell et al. 1995, p. 313) for the discussion of equilibrium output.

Lemma 2. *When a continuum of Nash equilibria exists in Lemma 1, the equilibrium with maximum effort from both agents is Pareto-optimal.*

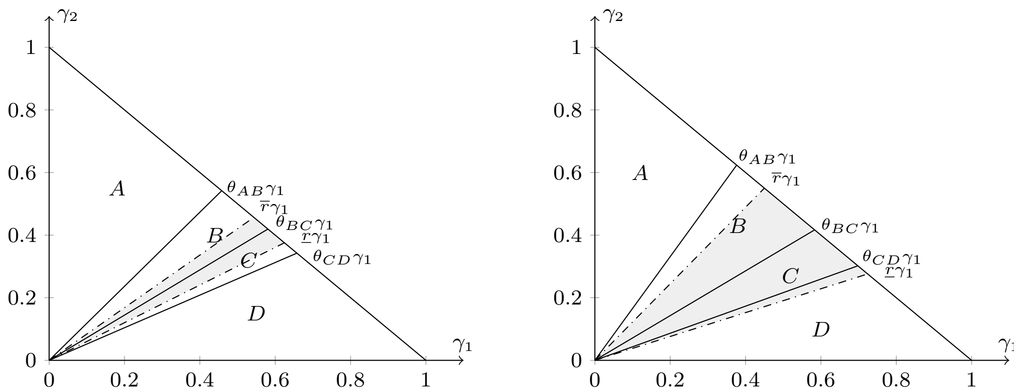
4.2. Optimal Contract

We first characterize how inequality aversion affects output because the principal's payoff turns out to be proportional to it since the optimal contract does not involve fixed-transfer payments. We then present the optimal contract and finally study some comparative statics with respect to k and α.

4.2.1. Effect of Fairness Concerns on Output. Compared with inequality-neutral agents, endogenous fairness concerns could increase or decrease total output depending on the payoff distribution.

LEMMA 3. *Under utility fairness, for a given output-sharing contract $\Phi = (\gamma_1, \gamma_2, 0, 0)$, agents generate more output than if they were inequality-neutral if and only if $\underline{r}(\alpha) \leq \gamma_2/\gamma_1 \leq \overline{r}(\alpha)$, for some thresholds $\underline{r}(\alpha)$ and $\overline{r}(\alpha)$, with $0 \leq \underline{r}(\alpha) \leq \theta_{BC} \leq \overline{r}(\alpha) \leq \theta_{AB}$. Moreover, $\underline{r}(\alpha)$ decreases and $\overline{r}(\alpha)$ increases in α.*

Figure 1 Output comparison under different output-sharing contracts, under utility fairness.



Note. In both plots, $c = 1$, $\beta = 0.5$, $k = 0.7$; in the left panel, $\alpha = 0.1$; in the right panel, $\alpha = 0.3$. Regions A-D correspond to Cases A-D in Lemma 1. The shaded area corresponds to higher output.

According to Lemma 3, total output with inequality-averse agents could be higher or lower than that with inequality-neutral agents depending on the contract. Note that this is also true when agents are homogeneous ($k = 1$): In this case, $\theta_{BC} = 1$ and stimulating inequality aversion is beneficial when $\gamma_1 = \gamma_2$ but not necessarily otherwise. When agents are heterogeneous, offering a symmetric contract may not only be suboptimal, but it may also result in smaller output since $\overline{r}(\alpha) < 1$ when k or α is small.

Because $\overline{r}(\alpha) - \underline{r}(\alpha)$ increases in α, the range of values for γ_2/γ_1 that leads to a higher output than in the inequality-neutral case increases with α, as is illustrated by comparing the left and right panels of Figure 1, respectively corresponding to $\alpha = 0.1$ and $\alpha = 0.3$. Hence, greater inequality aversion (in the form of greater guilt) expands the set of contracts that can achieve a higher output; whether the contract selected by the principal lies in this range remains to be shown. However, the range of values of γ_2/γ_1 that lead to a higher output than in the inequality-neutral case remains a strict subset of $[0, \infty)$ even if

α approaches its upper limit (1/2), so not every contract leads to a higher output, even in the limiting case.

4.2.2. Optimal Contract. While Lemma 3 characterizes the output for any given contract, the principal may want to offer a different contract when agents are inequality-averse than when they are inequality-neutral. The next theorem characterizes the optimal contract offered in equilibrium. To simplify the exposition, define $\hat{\gamma} := \frac{1-(1-\alpha)^2(1-k^2)}{2\left(1-2(1-\alpha)^2(1-k^2)+\sqrt{k^2+(1-k^2)^2(1-\alpha)^2}\right)}$ and $\kappa(\alpha) := (1-\alpha)\sqrt{\frac{1-2\alpha}{1-2\alpha+\alpha^2}}$.

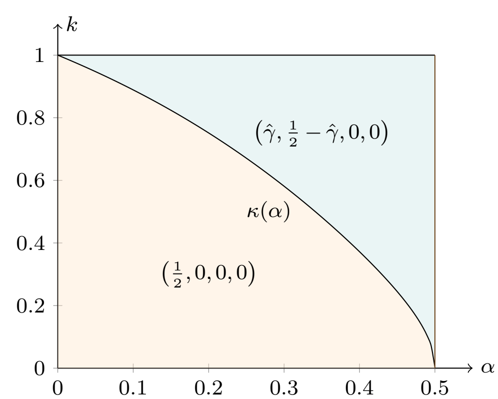
THEOREM 1. *Under utility fairness, the principal's best output-sharing contract is to offer fixed-transfer payments $f_1^* = f_2^* = 0$ and output shares*

$$(\gamma_1^*, \gamma_2^*) = \begin{cases} \left(\frac{1}{2}, 0\right) & \text{if } k \le \kappa(\alpha), \\ \left(\hat{\gamma}, \frac{1}{2} - \hat{\gamma}\right) & \text{if } k > \kappa(\alpha). \end{cases} \tag{7}$$

When the degree of guilt aversion α increases, the principal's optimal payoff π^* increases. Moreover, when $k > \kappa(\alpha)$, $v_1(e_1^*, e_2^*) = v_2(e_1^*, e_2^*)$ under the optimal contract.

The total output share given by the principal to the agent satisfies $\gamma_1^* + \gamma_2^* = 1/2$, as if they were inequality-neutral. Hence, the fact that agents are averse to utility inequality does not require additional compensation from the principal.

Figure 2 **Optimal contracts under utility fairness.**



By Theorem 1, the principal's payoff is maximized by allocating shares to either one agent (when $\gamma_1 = \frac{1}{2}$ and $\gamma_2 = 0$) or both agents depending on their level of heterogeneity and their level of inequality aversion; see Figure 2. When agents are mildly heterogeneous and highly inequality-averse, i.e., when $k > \kappa(\alpha)$, rewarding both agents is more efficient.

By attempting to reduce their inequalities in utilities, agents exert higher effort, thus mitigating team moral hazard and boosting output. Otherwise, i.e., when $k \leq \kappa(\alpha)$, the principal's payoff is maximized by rewarding only the most able agent as in the inequality-neutral case, rendering the same output as in the inequality-neutral case.

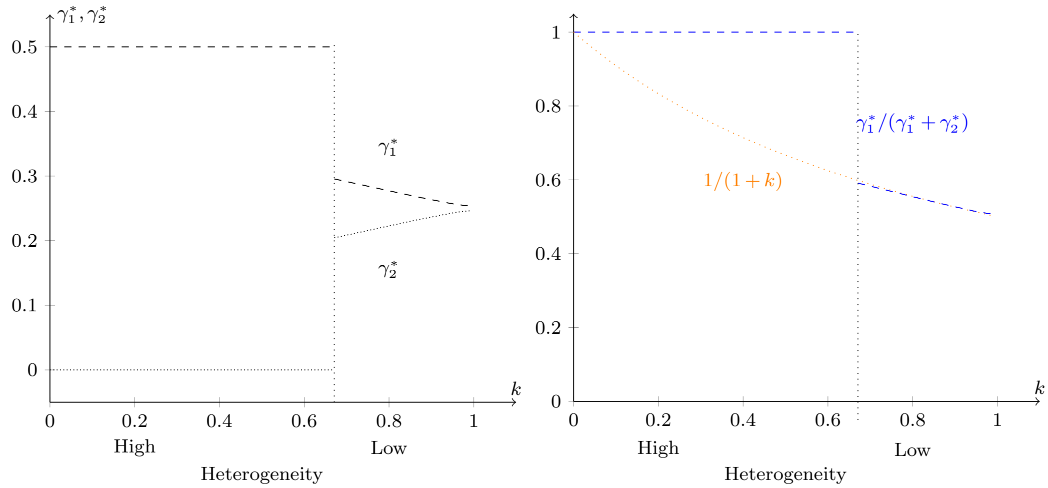
When $k > \kappa(\alpha)$, the optimal contract satisfies $\gamma_1^*/\gamma_2^* = \theta_{BC}$ (see Figure 1). That is, the equilibrium effort lies at the boundary between Cases B and C in Lemma 1. Given our equilibrium selection rule (Assumption 1), both agents exert the maximum possible effort e_i^α for $i = 1, 2$. This equilibrium, which maximizes the agents' utilities $v_i - \alpha[v_i - v_{-i}]$, is as if each were ahead of the other. At this level, no agent is attempting to equalize his utility to the other and, therefore, no agent acts as a bottleneck on the other. This explains why setting $\gamma_1^* = \gamma_2^* = \theta_{BC}$ is efficient. Since the agents' utilities are equal in equilibrium, they do not have any mental burden associated with their inequality aversion. Therefore, this optimal contract not only increases the principal's payoff (relative to the inequality-neutral case), it is also envy- and guilt-free.

When $k > \kappa(\alpha)$, the principal benefits from stimulating more guilt: Since e_i^α increases in α for $i = 1, 2$, the more guilt, the higher the output, and thus, the higher the principal's payoff. In the limit, when $\alpha = 1/2$, the equilibrium efforts are $e_1^* = \gamma_1 + \gamma_2$ and $e_2^* = k(\gamma_1 + \gamma_2)$; that is, the team moral hazard is completely eliminated at the highest possible level of guilt. Moreover, the principal's optimal profit is jointly convex in k and α (Lemma EC.6). Thus, when team members' abilities are less heterogeneous, the principal has increasing returns on stimulating fairness concerns.

In contrast, when $k \leq \kappa(\alpha)$, the most able agent (who is the only one who is rewarded) experiences guilt in equilibrium, the other agent experiences envy, and the principal's payoff remains identical to what she would get with inequality-neutral agents. In this case, stimulating inequality aversion is detrimental.

4.2.3. Comparative Statics. The optimal contract is such that greater agent heterogeneity brings greater disparity in output shares. In particular, when agents are more heterogeneous (k decreases), γ_1^* increases and γ_2^* decreases, to discontinuously jump to $\gamma_1^* = 1$ when only the most able agent is rewarded (Figure 3, left panel). In fact, when $k > \kappa(\alpha)$ so that both agents receive a share of the output, their relative shares closely track their relative abilities (Figure 3, right panel). Whether only one or both agents receive an output share, the more able agent is given a higher share than the other agent. This

Figure 3 **Optimal contract under utility fairness.**



Note. Here, $\alpha = 0.25$.

finding is consistent with Tan and Staats (2020) who empirically find that, in restaurants, more tables are assigned to high-speed waiters, and this uneven assignment results in an increase in sales.

Although one might conjecture that a higher level of inequality aversion induces a more even payoff division (in the spirit of the principle of income fairness, see §6), we find that it is actually the opposite that holds under utility fairness. Specifically, when $k > \kappa(\alpha)$, if α increases, $\gamma_1^*(\alpha)$ increases and $\gamma_2^*(\alpha)$ decreases. Similar to Cohen et al. (2022), who consider different fairness principles (albeit different) and reach an impossibility result, we find a natural tension between utility fairness and income fairness: The more averse the agents are to inequalities in utility, the higher their discrepancy in incomes (assuming no fixed fee). As discussed in §3.3, the structures of the games are also different: supermodular under utility fairness and submodular under income fairness. Hence, in heterogeneous teams, there is no silver bullet: any initiative that stimulates utility fairness will result in income inequalities.

4.2.4. Summary. Compared to inequality-neutral agents, stimulating utility fairness concerns could increase or decrease the principal's payoff if the agents' contract is not properly engineered. Even under the optimal contract, stimulating fairness concerns may be detrimental: when $k \leq \kappa(\alpha)$, only one agent receives an output share, making both agents experience a disutility due to envy or guilt, without offering the principal a higher payoff. Hence, before stimulating fairness under aversion to utility inequality, principals need to account for the agents' degree of heterogeneity and carefully design their contract.

5. Reward Fairness

Considering now reward-fairness concerned agents, we first describe their equilibrium efforts and then identify the principal's optimal contract.

5.1. Equilibrium Efforts

Under reward fairness, the effort choice game (4) is supermodular across the entire action space (Lemma EC.8) when fixed-transfer payments are zero (which is optimal by Lemma EC.9), similar to the case of utility fairness. Thus, the equilibrium structure and the optimal contract exhibit similarities to those derived under utility fairness. In particular, the optimal fixed-transfer payments are zero and the agents' efforts are strategic complements.

To characterize the equilibrium efforts, we define the following notations:

$$\begin{aligned} e_1^\alpha &:= e_1^N + \alpha\gamma_2, \quad e_1^\beta := \left[e_1^N - \beta\gamma_2\right]^+, \\ e_2^\alpha &:= e_2^N + k\alpha\gamma_1, \; e_2^\beta := \left[e_2^N - k\beta\gamma_1\right]^+. \end{aligned} \tag{8}$$

Additionally, let $\overline{e}_i(e_{-i})$ denote Agent i's effort level that guarantees agents' incomes being proportional to their contributions given that Agent $-i$ exerts e_{-i}, i.e., $\frac{I_i(\overline{e}_i(e_{-i}),e_{-i},\gamma_i,f_i)}{I_{-i}(e_{-i},\overline{e}_i(e_{-i}),\gamma_{-i},f_{-i})} = \frac{k_i e_i}{k_{-i}e_{-i}}$ for $i=1,2$.

Lemma 4. *Under reward fairness, there exist thresholds $\overline{\eta}_{AB} > \underline{\eta}_{AB} > \eta_{BC}$ such that given an output-sharing contract $\Phi = (\gamma_1,\gamma_2,0,0)$:*

A. *If $\underline{\eta}_{AB} \le \gamma_2/\gamma_1 \le \overline{\eta}_{AB}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: $e_1^* = e_1^\beta$, $e_2^* = e_2^\alpha$;*

B. *If $\eta_{BC} \le \gamma_2/\gamma_1 \le \underline{\eta}_{AB}$ or $\gamma_2/\gamma_1 \ge \overline{\eta}_{AB}$, there exists a continuum of Nash equilibria such that $\frac{e_1^*}{ke_2^*} = \frac{\gamma_1}{\gamma_2}$: $e_2^* \in \left[\max\left\{e_2^\beta, \overline{e}_2\left(e_1^\beta\right)\right\}, e_2^\alpha\right]$ and $e_1^* = \overline{e}_1\left(e_2^*\right)$;*

C. *If $\gamma_2/\gamma_1 \le \eta_{BC}$, there exists a continuum of Nash equilibria such that $\frac{e_1^*}{ke_2^*} = \frac{\gamma_1}{\gamma_2}$: $e_1^* \in \left[\max\left\{e_1^\beta, \overline{e}_1\left(e_2^\beta\right)\right\}, e_1^\alpha\right]$ and $e_2^* = \overline{e}_2\left(e_1^*\right)$;*

The structure of equilibrium efforts is parallel to that of Lemma 1. However, Case B arises on two disconnected ranges of values of γ_2/γ_1, unlike the equilibrium efforts under utility fairness, in which it only arises in one range. This is because even with a very low share γ_1, it is possible for Agent 1 to achieve reward fairness by putting in a low effort, whereas it is impossible for him to equate his utility to Agent 2's. By the same logic, Case D does not arise under reward fairness when $f_1 = f_2 = 0$.

The Pareto-optimal equilibrium is similar to that under utility fairness.

LEMMA 5. *When a continuum of Nash equilibria exists in Lemma 4, the equilibrium with maximum effort from both agents is Pareto-optimal.*

5.2. Optimal Contract

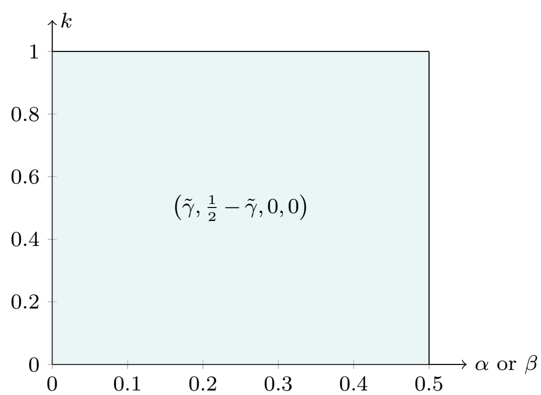
We next characterize the principal's optimal contract. Like the utility-fairness case, the contract depends on the level of guilt α; moreover, both agents always receive a revenue share, as illustrated in Figure 4. To simplify the exposition, define $\tilde{\gamma} := \frac{\alpha}{\sqrt{4\alpha^2 k^2 + (1-k^2)^2} - 1 + 2\alpha + k^2}$.

THEOREM 2. *Under reward fairness, the principal's best output-sharing contract is to offer fixed-transfer payments* $f_1^* = f_2^* = 0$ *and output shares*

$$(\gamma_1^*, \gamma_2^*) = \left(\tilde{\gamma}, \frac{1}{2} - \tilde{\gamma}\right).$$

The principal's payoff with reward fairness is higher than without inequality aversion. Moreover, $\gamma_1^*/\gamma_2^* = e_1^*/(ke_2^*)$.

Figure 4 Optimal contract under reward fairness.



Similar to utility fairness, reward fairness helps mitigate team moral hazard. This is because agents are more likely to put in effort when they see the other agent doing the same (due to the supermodular nature of the game), ensuring that their contributions align with their incomes. Moreover, it is always optimal for the principal to offer a fair contract that leads to higher profits. Similar to the utility fairness case, the principal faces increasing returns from stimulating fairness concerns when the team is less heterogeneous.

In contrast to utility fairness, under which stimulating inequality aversion could raise fairness issues without increasing the principal's payoff (namely, when $k \le \kappa(\alpha)$, see Theorem 1), stimulating reward fairness is always beneficial (under the optimal contract) under reward fairness. This is because reward fairness is easier to achieve than utility fairness: Even when the agents' shares are highly asymmetric, i.e., $\gamma_i^* \gg \gamma_{-i}^*$, the agents can achieve reward fairness by adjusting their efforts, while it would be impossible for them to equalize their utilities at any effort level. Not only is the principal always better off with inequality aversion than without, but also the agents never experience a disutility associated with inequality aversion. This is in contrast to the case of utility fairness, which required agents to be only moderately heterogeneous (i.e., $k > \kappa(\alpha)$, see Theorem 1).

In sum, we find that stimulating reward fairness pays off in all teams, irrespective of their degree of heterogeneity, without resulting in feelings of envy or guilt in equilibrium, and the principal incurs increasing marginal returns on doing so.

6. Income Fairness

Considering last income-fairness, we first describe their equilibrium efforts and then identify the principal's optimal contract.

6.1. Equilibrium Efforts

Under income fairness, the effort choice game (4) is submodular—agents' efforts act as strategic substitutes and tend to move in opposite directions. This is in stark contrast to the cases of utility and reward fairness, under which the game (or a transformed version thereof) was supermodular. Still, the equilibrium efforts take a similar four-region structure as in Lemma 1. To characterize the equilibrium efforts, we define the following notations:

$$\begin{aligned} e_1^\alpha &:= \left[e_1^N - \alpha(\gamma_1 - \gamma_2)\right]^+, \; & e_1^\beta &:= e_1^N + \beta(\gamma_1 - \gamma_2), \\ e_2^\alpha &:= e_2^N + k\alpha(\gamma_1 - \gamma_2), & e_2^\beta &:= \left[e_2^N - k\beta(\gamma_1 - \gamma_2)\right]^+. \end{aligned} \tag{9}$$

Additionally, let $\overline{e}_i(e_{-i})$ denote Agent i's effort level that guarantees agents' incomes being equal given that Agent $-i$ exerts e_{-i}, i.e., $I_i(\overline{e}_i(e_{-i}), e_{-i}, \gamma_i, f_i) = I_{-i}(e_{-i}, \overline{e}_i(e_{-i}), \gamma_{-i}, f_{-i})$ for $i = 1, 2$. Unlike the cases of utility- and reward-fairness, $\overline{e}_i(e_{-i})$ is downward-sloping. The next lemma is expressed in terms of $f_1 - f_2$, unlike Lemmas 1 and 4, which were expressed in terms of γ_1/γ_2 (subject to $f_1 = f_2 = 0$), but its structure is parallel to them.

LEMMA 6. *Under income fairness, there exist thresholds $\zeta_{AB} < \zeta_{BC} < \zeta_{CD}$ such that given an output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ such that $\gamma_1 \geq \gamma_2$:*

A. If $f_1 - f_2 \leq \zeta_{AB}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: $e_1^* = e_1^\beta$, $e_2^* = e_2^\alpha$;

B. If $\zeta_{AB} \le f_1 - f_2 \le \zeta_{BC}$, there exists a continuum of Nash equilibria such that $\gamma_1(e_1^* + ke_2^*) + f_1 = \gamma_2(e_1^* + ke_2^*) + f_2$: $e_2^* \in \left[e_2^\beta, \max\{e_2^\alpha, \overline{e}_2(e_1^\alpha)\}\right]$ and $e_1^* = \overline{e}_1(e_2^*)$;

C. If $\zeta_{BC} \le f_1 - f_2 \le \zeta_{CD}$, there exists a continuum of Nash equilibria such that $\gamma_1(e_1^* + ke_2^*) + f_1 = \gamma_2(e_1^* + ke_2^*) + f_2$: $e_2^* \in \left[\max\left\{e_2^\beta, \overline{e}_2\left(e_1^\beta\right)\right\}, e_2^\alpha\right]$ and $e_1^* = \overline{e}_1(e_2^*)$;

D. If $f_1 - f_2 \ge \zeta_{CD}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences guilt and Agent 2 experiences envy: $e_1^* = e_1^\alpha$, $e_2^* = e_2^\beta$.

When $f_1 = f_2 = 0$, only Case D arises; in this case, the equilibrium efforts under income fairness are lower than those under inequality neutrality. To see this, consider the following two effects. On the one hand, if Agent i receives a higher output share than the other agent, he feels guilty and therefore puts in less effort to shrink the pay difference $(\gamma_i - \gamma_{-i})(e_1 + ke_2)$. On the other hand, if Agent i receives a smaller share than the other agent, he is envious and also puts in less effort to shrink the pay difference $(\gamma_{-i} - \gamma_i)(e_1 + ke_2)$. Thus, when $f_1 = f_2 = 0$, the output achievable when agents are averse to income inequality is always lower than when they are inequality-neutral.

6.2. Optimal Contract

We next characterize the optimal contract, which unlike the optimal contracts under utility fairness and reward fairness, depends on both the levels of envy and guilt. See Figure 5 for a graphical depiction.

THEOREM 3. *Under income fairness, the principal's best output-sharing contract Φ is such that*

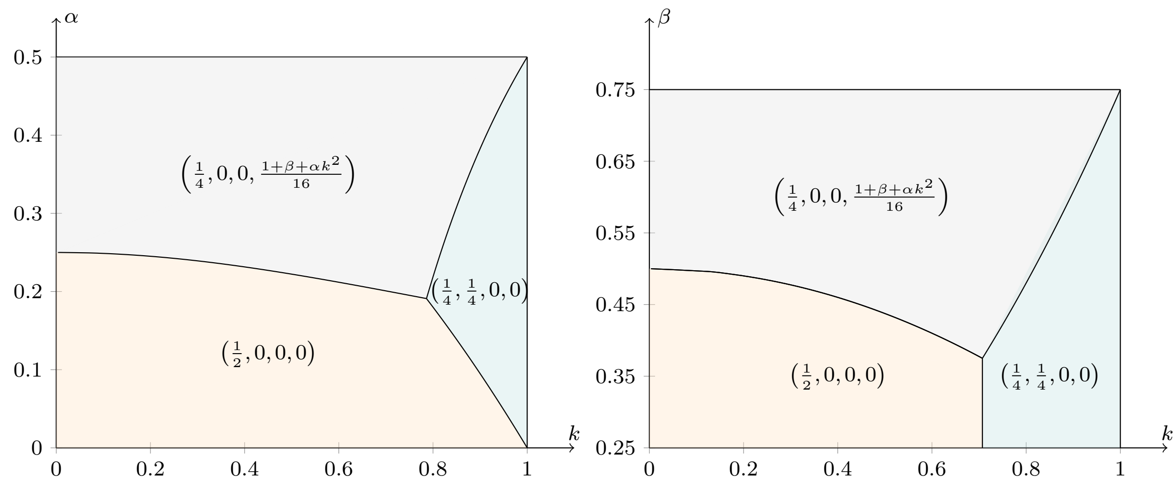
$$(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \begin{cases} \left(\frac{1}{4}, \frac{1}{4}, 0, 0\right), & \max\left\{\sqrt{1-2\alpha}, \sqrt{\frac{\beta}{1-\alpha}}\right\} < k \le 1, \\ \left(\frac{1}{4}, 0, 0, \frac{1+\beta+\alpha k^2}{16}\right), & \max\left\{\sqrt{\frac{1-2\alpha-\beta}{\alpha}}, 0\right\} \le k \le \sqrt{\frac{\beta}{1-\alpha}}, \\ \left(\frac{1}{2}, 0, 0, 0\right), & 0 \le k \le \min\left\{\sqrt{1-2\alpha}, \sqrt{\frac{1-2\alpha-\beta}{\alpha}}\right\}. \end{cases} \tag{10}$$

The principal's payoff under income fairness is less than that without inequality aversion.

Much like utility fairness, the principal optimally allocates shares to one or both agents depending on their level of heterogeneity and inequality aversion. However, there are a few notable differences, as we explain next.

First, the optimal contract takes three forms depending on the level of heterogeneity and inequality aversion. When the level of heterogeneity is low, the optimal contract is

Figure 5 **Optimal contracts under income fairness.**



(a) principal, two-agent team with $\beta = 0.5$ (b) principal, two-agent team with $\alpha = 0.25$

$\gamma_1 = \gamma_2 = \frac{1}{4}$, which is also fair under income fairness. However, when agents are quite heterogeneous and not very inequality-averse (in terms of either guilt or envy), which corresponds to the third case in Theorem 3, it is optimal to evenly share the reward with only the most able agent and give nothing to the other, as in the utility-fairness case when $k \geq \kappa(\alpha)$, resulting in disutilities in equilibrium due to both envy and guilt. And, finally, when agents are quite heterogeneous but very inequality-averse, which corresponds to the second case in Theorem 3, it is optimal to give Agent 1 a 25%-share of the reward and a fixed-transfer payment of the same amount to Agent 2. Unlike the other contracts considered so far, the principal allocates only a quarter, and not half, of the output to the agents. Although Agent 2 receives only a fixed payment, he exerts positive effort in equilibrium, being purely motivated by equalizing incomes.

Second, contrary to utility or reward fairness, both envy and guilt simultaneously influence the design of the optimal contract and the resulting payoff under income fairness. In the second case of Theorem 1, Agent 1's total income equates 25% of the reward, taking the form of shares, whereas Agent 2's income equates 25% of the output, taking the form of a fixed fee. While the outcome is income-fair in equilibrium, given that Agent 2 receives upfront a fixed fee while Agent 1 needs to work toward collecting his income, Agent 1 is motivated by envy, while Agent 2 is motivated by guilt. Thus, both envy and guilt affect effort exertion, and the resulting output is affected by the level of envy and guilt.

In sum, stimulating income fairness always leads to a lower payoff for the principal due to the corresponding decrease in effort; and when the team is relatively heterogeneous, it always results in feelings of envy and guilt in equilibrium.

7. Conclusion

Fairness concerns often arise in the management of heterogeneous teams: while over-using the more able agents seems more efficient, such uneven involvement might demotivate other agents due to inequality aversion. In homogeneous teams, giving symmetric shares to agents is both fair and optimal for the principal; and as shown by past research, stimulating greater inequality aversion increases the principal's payoff. Is it still the case in heterogeneous teams?

To answer this question, we consider a principal two-agent setup, involving substitutable efforts, when agents have heterogeneous abilities and are subject to aversion to inequality in utility, reward, or income.

Should a principal stimulate inequality aversion in the same manner as they would in homogeneous teams? We find that the decision hinges on the specific type of fairness consideration and degree of heterogeneity. Under utility and reward fairness, stimulating inequality aversion might lead to higher payoff, but it requires a careful engineering of the contract (Lemma 3, Theorem 2). In contrast, it always hurts under income fairness (Theorem 3).

Do agents experience inequalities in equilibrium under the optimal contract? Under reward fairness, never. Under utility and income fairness, sometimes—and always in the cases where only one agent receives a share of output.

Hence, stimulating inequality aversion as a way to "freely" boost performance (in the sense that agents do not incur any disutility from their inequality aversion) may work well in homogeneous teams, but it certainly does not apply universally. The impact of fairness concerns, the contract design, and whether fairness can be achieved, all depend on the team members' object of fairness comparison, which is intimately shaped by the underlying nature of the task and team characteristics. A caveat to our analysis is that it focuses on the Pareto-optimal equilibrium selection rule (Assumption 1).

Our stylized model can be extended in a few ways. First, future research could study fairness concerns in larger teams, with more than two agents. Second, we consider an additive production function to isolate the effect of fairness concerns, but in many settings, team output could be nonlinear and involve complementarities (coproduction). While these two extensions can be easily accommodated in the context of homogeneous teams (Gill and Stone 2015), their treatment in heterogeneous teams becomes quickly intractable. However, we conjecture, based on preliminary numerical evidence, that the effect of fairness concerns to mitigate team moral hazard would persist with nonlinear production functions. Third, one could apply the model to explore the impact of fairness concerns on other operational aspects, for example, on the agents' willingness to collaborate or help each other (Siemsen et al. 2007) and how to best manage them. Fourth, we considered fairness considerations, operationalized as guilt and envy (Fehr and Schmidt 1999), but agents could adopt other, non-fair behaviors, such as seeking to accentuate inequalities if they are competitive.

In contrast to most research to date, which has praised the benefits of stimulating greater inequality aversion through greater performance and pay transparency, our study identifies important boundary conditions, namely, the degree of team heterogeneity and the type of fairness consideration. We hope that our results will invite managers to reflect on the potential downsides of social comparison before committing to greater performance transparency.

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Appendix: Fairness Concern Types in the Literature

Table 3 Fairness concern types in the literature.

	Income Inequality	Utility Inequality	Reward Inequality
Individual Outputs			
Fehr and Schmidt (1999)	X		
Demougin and Fluet (2003)		X	
Itoh (2004)	X	X	
Goel and Thakor (2006)	X		
Rey-Biel (2008)	X	X	
Neilson and Stowe (2010)	X	X	
Englmaier and Wambach (2010)		X	
Bartling and von Siemens (2010b)	X		
Bartling (2011)	X		
Kragl (2015)		X	
Team Outputs			
Bartling and von Siemens (2010a)		X	
Gill and Stone (2015)		X	X
Kölle et al. (2016)		X	
Hellmann and Wasserman (2017)	X		
Supply Chain Coordination			
Cui et al. (2007)	X		
Wu et al. (2008)	X		
Avcı et al. (2014)	X		

An "X" in merged cells of "income inequality" and "utility inequality" refers to settings with no cost concerns, so that utility and income are equal.

Electronic Companion: Proofs and Supplementary Results

In the electronic companion, we prove results concerning inequality-averse agents who compare their utilities (§EC.1), their incomes relative to their contributions (rewards) (§EC.2), or their incomes (§EC.3).

EC.1. Utility Fairness

We first characterize the equilibrium efforts given an affine output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$. Lemma 1 is a special case when $f_1 = f_2 = 0$.

LEMMA EC.1. *Under utility fairness, there exist thresholds $\Theta_{AB}(\gamma_1,\gamma_2) \le \Theta_{BC}(\gamma_1,\gamma_2) \le \Theta_{CD}(\gamma_1,\gamma_2)$ such that given any contract $\Phi = (\gamma_1,\gamma_2,f_1,f_2)$, the agents' equilibrium efforts are*

(A) If $f_1 - f_2 \le \Theta_{AB}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: $e_1^* = e_1^\beta$, $e_2^* = e_2^\alpha$;

(B) If $\Theta_{AB} \le f_1 - f_2 \le \Theta_{BC}$, there exists a continuum of Nash equilibria such that $v_1 = v_2$: $e_2^* \in \left[\max\left\{e_2^\beta, \bar{e}_2\left(e_1^\beta\right)\right\}, e_2^\alpha\right]$ and $e_1^* = \bar{e}_1\left(e_2^*\right)$;

(C) If $\Theta_{BC} \le f_1 - f_2 \le \Theta_{CD}$, there exists a continuum of Nash equilibria such that $v_1 = v_2$: $e_1^* \in \left[\max\left\{e_1^\beta, \bar{e}_1\left(e_2^\beta\right)\right\}, e_1^\alpha\right]$ and $e_2^* = \bar{e}_2\left(e_1^*\right)$;

(D) If $f_1 - f_2 \ge \Theta_{CD}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences guilt and Agent 2 experiences envy: $e_1^* = e_1^\alpha$, $e_2^* = e_2^\beta$.

Proof. The proof is organized as follows. We first simplify the agents' utilities (1); then, we analyze the agents' best responses solving (4); finally, we identify the Nash equilibrium.

Utilities. First, consider Agent 1. In (1), the equation $v_2 - v_1 = (\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2) - (\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2) = 0$ (convex in e_1) has two roots (assuming $r_- \le r_+$) in e_1:

$$r_{+,-} = \gamma_1 - \gamma_2 \pm \sqrt{(\gamma_1-\gamma_2+e_2)^2 - 2(1-k)e_2(\gamma_1-\gamma_2) + 2(f_1-f_2)}.$$

Thus,

$$u_1(e_1; e_2, \gamma_1, \gamma_2, f_1, f_2) = \begin{cases} \gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2 - \beta((\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2) - (\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2)), & e_1 \le r_-, \\ \gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2 - \alpha((\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2) - (\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2)), & r_- \le e_1 \le r_+, \\ \gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2 - \beta((\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2) - (\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2)), & e_1 \ge r_+. \end{cases}$$

Since the first branch $\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2-\beta((\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2)-(\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2))$ (concave in e_1) is maximized at $\frac{\gamma_1\beta+\gamma_1-\gamma_2\beta}{1+\beta} > \gamma_1-\gamma_2 > r_-$, any effort level $e_1 < r_-$ is strictly dominated. Furthermore, the second branch $\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2-\alpha((\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2)-(\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2))$ is maximized at $\frac{\gamma_1(1-\alpha)+\gamma_2\alpha}{1-\alpha} > \gamma_1 > r_-$. Thus, the following utility function leads to the same best responses as the original utility function for Agent 1.

$$u_1(e_1; e_2, \gamma_1, \gamma_2, f_1, f_2) = \begin{cases} u_1^\alpha := \gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2 - \alpha((\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2) - (\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2)), & e_1 \le \bar{e}_1(e_2), \\ u_1^\beta := \gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2 - \beta((\gamma_2(e_1+ke_2)+f_2-\frac{1}{2}e_2^2) - (\gamma_1(e_1+ke_2)+f_1-\frac{1}{2}e_1^2)), & e_1 \ge \bar{e}_1(e_2), \end{cases} \tag{EC.1}$$

where $\overline{e}_1(e_2) := r_+$, which is the only value of e_1 such that $v_1 - v_2 = 0$ and $e_1 \geq (\gamma_1 - \gamma_2)$. Similarly, we consider the following utility for Agent 2, which leads to the same best responses as the original utility (1):

$$u_2(e_1; e_2, \gamma_1, \gamma_2, f_1, f_2)$$
$$= \begin{cases} u_2^\alpha := \gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2 - \alpha((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2)), & e_2 \leq \overline{e}_2(e_1), \\ u_2^\beta := \gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2 - \beta((\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2) - (\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2)), & e_2 \geq \overline{e}_2(e_1), \end{cases} \tag{EC.2}$$

where $\overline{e}_2(e_1)$ is the only value of e_2 such that $v_1 - v_2 = 0$ and $e_2 \geq k(\gamma_2 - \gamma_1)$. Moreover, $\overline{e}_2(e_1)$ and $\overline{e}_1(e_2)$ are inverse functions to each other.

Best responses. Given a contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, each agent chooses his effort to maximize his utility by solving (4). Define the following constants

$$\begin{aligned} t_{21} &= -k(\gamma_1 - \gamma_2) + \sqrt{\tfrac{\gamma_2^2}{(1+\beta)^2} - 2(f_1 - f_2) - (1 - k^2)(\gamma_1 - \gamma_2)^2}; \\ t'_{21} &= -k(\gamma_1 - \gamma_2) - \sqrt{\tfrac{\gamma_2^2}{(1+\beta)^2} - 2(f_1 - f_2) - (1 - k^2)(\gamma_1 - \gamma_2)^2}; \\ t_{22} &= -k(\gamma_1 - \gamma_2) + \sqrt{\tfrac{\gamma_2^2}{(1-\alpha)^2} - 2(f_1 - f_2) - (1 - k^2)(\gamma_1 - \gamma_2)^2}; \\ t'_{22} &= -k(\gamma_1 - \gamma_2) - \sqrt{\tfrac{\gamma_2^2}{(1-\alpha)^2} - 2(f_1 - f_2) - (1 - k^2)(\gamma_1 - \gamma_2)^2}; \\ \tilde{t}_{21} &= -k(\gamma_1 - \gamma_2) + \sqrt{-2(f_1 - f_2) + k^2(\gamma_1 - \gamma_2)^2}; \\ \tilde{t}'_{21} &= -k(\gamma_1 - \gamma_2) - \sqrt{-2(f_1 - f_2) + k^2(\gamma_1 - \gamma_2)^2}; \end{aligned}$$

where the first digit i in the double subscript refers to Agent i; the second digit refers to the order of distance from $k(\gamma_2 - \gamma_1)$, i.e., $t_{i1} \leq t_{i2}$ for $i = 1, 2$; the group of values denoted by an apostrophe is smaller than the group without it: $t'_{i2} \leq t'_{i1} \leq t_{i1} \leq t_{i2}$; and the $\tilde{t}_{ij}$ refers to an alternative threshold instead of t_{ij} in some boundary cases.

Since Agent 1's simplified utility function is piece-wise quadratic and $\partial u_1^\beta / \partial e_1 \,|_{e_1 = \overline{e}_1(e_2)} < \partial u_1^\alpha / \partial e_1 \,|_{e_1 = \overline{e}_1(e_2)}$, his maximum utility is attained either at the optima of these quadratic functions or at the breakpoint. We define the values of e_1 maximizing u_1^α and u_1^β by the following:

$$e_1^\alpha := \gamma_1 + \frac{\alpha}{1-\alpha}\gamma_2, \quad e_1^\beta := \gamma_1 - \frac{\beta}{1+\beta}\gamma_2. \tag{EC.3}$$

Unlike (6), e_1^α and e_1^β defined by (EC.3) are not restricted to be nonnegative. Agent 1's best responses can be summarized into three cases, depending on, first, the value of γ_1/γ_2, and then, the slopes of u_1^β and u_1^α at the threshold $e_1 = \overline{e}_1(e_2)$:

- If $\gamma_1/\gamma_2 \geq \beta/(1+\beta)$,
  - — When $t'_{21} < e_2 < t_{21}$, we have $\frac{\partial u_1^\alpha}{\partial e_1}|_{e_1 = \overline{e}_1(e_2)} > \frac{\partial u_1^\beta}{\partial e_1}|_{e_1 = \overline{e}_1(e_2)} > 0$. Thus, Agent 1's utility is maximized when $e_1 > \overline{e}_1(e_2)$ on $u^\beta(e_1)$. Taking the first-order optimality condition, the optimal effort level is e_1^β.
  - — When $t'_{22} < e_2 < t'_{21}$ or $t_{21} < e_2 < t_{22}$, we have $\frac{\partial u_1^\beta}{\partial e_1}|_{e_1 = \overline{e}_1(e_2)} < 0 < \frac{\partial u_1^\alpha}{\partial e_1}|_{e_1 = \overline{e}_1(e_2)}$. Correspondingly, Agent 1's utility is maximized at the threshold $\overline{e}_2(e_1)$.
  - — When $e_2 < t'_{22}$ or $e_2 > t_{22}$, we have $\frac{\partial u_1^\beta}{\partial e_1}|_{e_1 = \overline{e}_1(e_2)} < \frac{\partial u_1^\alpha}{\partial e_1}|_{e_1 = \overline{e}_1(e_2)} < 0$. Thus, Agent 1's utility is maximized when $e_1 < \overline{e}_1(e_2)$ on $u^\alpha(e_1)$. Taking the first-order optimality condition, the optimal effort level is e_1^α.

- If $\gamma_1/\gamma_2 \le \beta/(1+\beta)$, $e_1^\beta \le 0$ and therefore $u^\beta(e_1)$ is decreasing for all $e_1 \ge 0$. Considering efforts are non-negative, Agent 1's best-response functions have the same structure as the best-responses when $\gamma_1/\gamma_2 \ge \beta/(1+\beta)$:
  - —When $\tilde{t}'_{21} < e_2 < \tilde{t}_{21}$, we have $\bar{e}_1(e_2) < 0$. Therefore, Agent 1's utility is monotonically decreasing for any $e_1 \ge 0$, and the optimal effort level is 0.
  - —When $t'_{22} < e_2 < \tilde{t}'_{21}$ or $\tilde{t}_{21} < e_2 < t_{22}$, we have $\bar{e}_1(e_2) > 0$ and $\frac{\partial u_1^\alpha}{\partial e_1}|_{e_1=\bar{e}_1(e_2)} > 0 > \frac{\partial u_1^\beta}{\partial e_1}|_{e_1=\bar{e}_1(e_2)}$. Correspondingly, Agent 1's utility is maximized at the threshold $\bar{e}_2(e_1)$.
  - —When $e_2 < t'_{22}$ or $e_2 > t_{22}$, Agent 1's utility is maximized at e_1^α.

In summary,

if $\gamma_1/\gamma_2 \ge \beta/(1+\beta)$,
$$e_1^*(e_2) = \begin{cases} e_1^\beta, & t'_{21} \le e_2 \le t_{21}, \\ \bar{e}_1(e_2), & t'_{22} \le e_2 \le t'_{21} \text{ or } t_{21} \le e_2 \le t_{22}, \\ e_1^\alpha, & e_2 \le t'_{22} \text{ or } e_2 \ge t_{22}, \end{cases}$$
if $\gamma_1/\gamma_2 \le \beta/(1+\beta)$,
$$e_1^*(e_2) = \begin{cases} 0, & \tilde{t}'_{21} \le e_2 \le \tilde{t}_{21}, \\ \bar{e}_1(e_2), & t'_{22} \le e_2 \le \tilde{t}'_{21} \text{ or } \tilde{t}_{21} \le e_2 \le t_{22}, \\ e_1^\alpha, & e_2 \le t'_{22} \text{ or } e_2 \ge t_{22}. \end{cases}$$

Similarly, Agent 2's best response is

if $\gamma_2/\gamma_1 \ge \beta/(1+\beta)$,
$$e_2^*(e_1) = \begin{cases} e_2^\beta, & t'_{11} \le e_1 \le t_{11}, \\ \bar{e}_2(e_1), & t'_{12} \le e_1 \le t'_{11} \text{ or } t_{11} \le e_1 \le t_{12}, \\ e_2^\alpha, & e_1 \le t'_{12} \text{ or } e_1 \ge t_{12}, \end{cases}$$
if $\gamma_2/\gamma_1 \le \beta/(1+\beta)$,
$$e_2^*(e_1) = \begin{cases} 0, & \tilde{t}'_{11} \le e_1 \le \tilde{t}_{11}, \\ \bar{e}_2(e_1), & t'_{12} \le e_1 \le \tilde{t}'_{11} \text{ or } \tilde{t}_{11} \le e_1 \le t_{12}, \\ e_2^\alpha, & e_1 \le t'_{12} \text{ or } e_1 \ge t_{12}. \end{cases}$$

Equilibrium. From the best response functions, we can see that Agent i's best-response is between $\max\{e_i^\beta, 0\}$ and e_i^α. That is, any effort level $e_i < \max\{e_i^\beta, 0\}$, for $i = 1, 2$, is strictly dominated. Additionally, it is easy to see that: 1) $t'_{12} < t'_{11} < e_1^\beta$ and $t'_{22} < t'_{21} < e_2^\beta$; 2) when $\gamma_1/\gamma_2 \le \beta/(1+\beta)$, $\tilde{t}'_{11} \le 0$; 3) when $\gamma_2/\gamma_1 \le \beta/(1+\beta)$, $\tilde{t}'_{21} \le 0$. Consequently, in the equilibrium discussion, we consider only the best responses to non-dominated strategies listed below.

- If $\beta/(1+\beta) \le \gamma_1/\gamma_2 \le (1+\beta)/\beta$,
$$e_1^*(e_2) = \begin{cases} e_1^\beta, & e_2^\beta \le e_2 \le t_{21}, \\ \bar{e}_1(e_2), & t_{21} \le e_2 \le t_{22}, \\ e_1^\alpha, & e_2 \ge t_{22}, \end{cases} \quad e_2^*(e_1) = \begin{cases} e_2^\beta, & e_1^\beta \le e_1 \le t_{11}, \\ \bar{e}_2(e_1), & t_{11} \le e_1 \le t_{12}, \\ e_2^\alpha, & e_1 \ge t_{12}. \end{cases}$$
- If $\gamma_1/\gamma_2 \le \beta/(1+\beta)$,
$$e_1^*(e_2) = \begin{cases} 0, & e_2^\beta \le e_2 \le \tilde{t}_{21}, \\ \bar{e}_1(e_2), & \tilde{t}_{21} \le e_2 \le t_{22}, \\ e_1^\alpha, & e_2 \ge t_{22}, \end{cases} \quad e_2^*(e_1) = \begin{cases} e_2^\beta, & 0 \le e_1 \le t_{11}, \\ \bar{e}_2(e_1), & t_{11} \le e_1 \le t_{12}, \\ e_2^\alpha, & e_1 \ge t_{12}. \end{cases}$$
- If $\gamma_1/\gamma_2 \ge (1+\beta)/\beta$,
$$e_1^*(e_2) = \begin{cases} e_1^\beta, & 0 \le e_2 \le t_{21}, \\ \bar{e}_1(e_2), & t_{21} \le e_2 \le t_{22}, \\ e_1^\alpha, & e_2 \ge t_{22}, \end{cases} \quad e_2^*(e_1) = \begin{cases} 0, & e_1^\beta \le e_1 \le \tilde{t}_{11}, \\ \bar{e}_2(e_1), & \tilde{t}_{11} \le e_1 \le t_{12}, \\ e_2^\alpha, & e_1 \ge t_{12}. \end{cases}$$

Combining the best responses, since $\bar{e}_1(\cdot)$ and $\bar{e}_2(\cdot)$ are inverse functions to each other, there are only three ways the best responses can intersect. In order to characterize these crossing points in terms of $f_1 - f_2$, define

$$\begin{aligned} \Theta_{AB} &:= \frac{1}{2}\left[\frac{\gamma_2^2}{(1+\beta)^2} - \frac{k^2\gamma_1^2}{(1-\alpha)^2} - (1-k^2)(\gamma_1-\gamma_2)^2\right], \\ \Theta_{BC} &:= \frac{1}{2}\left[\frac{\gamma_2^2 - k^2\gamma_1^2}{(1-\alpha)^2} - (1-k^2)(\gamma_1-\gamma_2)^2\right], \\ \Theta_{CD} &:= \frac{1}{2}\left[\frac{\gamma_2^2}{(1-\alpha)^2} - \frac{k^2\gamma_1^2}{(1+\beta)^2} - (1-k^2)(\gamma_1-\gamma_2)^2\right], \\ \tilde{\Theta}_{AB} &:= \frac{k^2}{2}\left((\gamma_1-\gamma_2)^2 - \frac{\gamma_1^2}{(1-\alpha)^2}\right), \\ \tilde{\Theta}_{CD} &:= -\frac{1}{2}\left((\gamma_1-\gamma_2)^2 - \frac{\gamma_2^2}{(1-\alpha)^2}\right), \end{aligned} \tag{EC.4}$$

such that $\Theta_{AB} \le \Theta_{BC} \le \Theta_{CD}$ and $\tilde{\Theta}_{AB} \le \Theta_{BC} \le \tilde{\Theta}_{CD}$. The effort equilibrium can be summarized below.

- When $\beta/(1+\beta) \le \gamma_1/\gamma_2 \le (1+\beta)/\beta$,
  - —If $f_1 - f_2 \le \Theta_{AB}$, then $e_2^\alpha \le t_{21}$ and $e_1^\beta \ge t_{12}$. The corresponding equilibrium efforts are $(e_1^*, e_2^*) = (e_1^\beta, e_2^\alpha)$.
  - —If $\Theta_{AB} \le f_1 - f_2 \le \Theta_{BC}$, then $e_2 \ge t_{21}$ and $e_1 \ge t_{11}$. There is a continuum of equilibrium efforts: $e_2^* \in \left[\max\left\{e_2^\beta, \bar{e}_2\left(e_1^\beta\right)\right\}, e_2^\alpha\right]$ and $e_1^* = \bar{e}_1\left(e_2^*\right)$;
  - —If $\Theta_{BC} \le f_1 - f_2 \le \Theta_{CD}$. There is a continuum of equilibrium efforts: $e_1^* \in \left[\max\left\{e_1^\beta, \bar{e}_1\left(e_2^\beta\right)\right\}, e_1^\alpha\right]$ and $e_2^* = \bar{e}_2\left(e_1^*\right)$;
  - —If $f_1 - f_2 \ge \Theta_{CD}$, then the best-response functions cross only once at $(e_1^*, e_2^*) = (e_1^\alpha, e_2^\beta)$.
- When $\gamma_1/\gamma_2 \le \beta/(1+\beta)$, the analysis is similar, the results above hold after replacing e_1^β by 0 and Θ_{AB} by $\tilde{\Theta}_{AB}$.
- When $\gamma_1/\gamma_2 \le (1+\beta)/\beta$, the results in the first bullet point hold after replacing e_2^β by 0 and Θ_{CD} by $\tilde{\Theta}_{CD}$.

We re-define $e_i^\beta := \max\left\{e_i^\beta, 0\right\}$ and $e_i^\alpha := e_i^\alpha$ for $i = 1, 2$; and re-define

$$\Theta_{AB} := \begin{cases} \Theta_{AB}, & \gamma_1/\gamma_2 \ge \beta/(1+\beta), \\ \tilde{\Theta}_{AB}, & \gamma_1/\gamma_2 \le \beta/(1+\beta), \end{cases}, \ \Theta_{BC} := \Theta_{BC}, \ \Theta_{CD} := \begin{cases} \Theta_{CD}, & \gamma_1/\gamma_2 \le (1+\beta)/\beta, \\ \tilde{\Theta}_{CD}, & \gamma_1/\gamma_2 \ge (1+\beta)/\beta. \end{cases}$$

Summarizing all three cases yields the statement of Lemma EC.1. □

Proof of Lemma 1. Define the following constants, where θ_{ij} is the positive root to Θ_{ij} (defined in (EC.4)) with respect to γ_2/γ_1 for $ij \in \{AB, BC, CD\}$.

$$\begin{aligned} \theta_{AB} &:= \frac{(1+\beta)\left((1-\alpha)(1+\beta)(1-k^2)-\sqrt{1-(1-(1-\alpha)^2+(1+\beta)^2)(1-k^2)+(1+\beta)^2(1-k^2)^2}\right)}{(1-\alpha)(1-(1+\beta)^2(1-k^2))}, \\ \theta_{BC} &:= \frac{-(1-\alpha)^2\left(1-k^2\right)+\sqrt{(1-\alpha)^2+(1-\alpha)^2k^4-(1+2\alpha-4\alpha^2)k^2}}{(1-\alpha)^2k^2-(\alpha-2)\alpha}, \\ \theta_{CD} &:= \frac{(1-\beta)\left(\beta(1+\gamma)-\gamma+(1-\beta)(1+\gamma)k^2+\sqrt{(1+\gamma)^2-k^2((\beta-2)\beta+(\gamma+1)^2)+(1-\beta)^2k^4}+1\right)}{(\gamma+1)((1-\beta)^2k^2+(2-\beta)\beta)}. \end{aligned} \tag{EC.5}$$

With these constants, Lemma 1 follows directly from Lemma EC.1. □

Proof of Lemma 2. Since all points at $(e_1^*, e_2^*) = (\bar{e}_1(e_2^*), e_2^*)$ or $(e_1^*, \bar{e}_2(e_1^*))$ lead to equal nominal utilities for Agents 1 and 2, by (1), the sum of agents' utilities is in equilibrium

$$u_T := u_1 + u_2 = (\gamma_1 + \gamma_2)(e_1 + ke_2) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2.$$

Since u_T increases in e_1 for $e_1 \le \gamma_1 + \gamma_2$ and u_T increases in e_2 for $e_2 \le k(\gamma_1 + \gamma_2)$, the sum of agents' utilities—and therefore their individual utilities ($u_1 = u_2 = u_T/2$)—increases in e_1 and e_2 because, by Lemma 1, $e_1^* \le e_1^\alpha = \frac{\gamma_1(1-\alpha)+\gamma_2\alpha}{1-\alpha} \le \gamma_1 + \gamma_2$ and $e_2^* \le e_2^\alpha = \frac{k(\gamma_2(1-\alpha)+\gamma_1\alpha)}{1-\alpha} \le k(\gamma_1 + \gamma_2)$ when $\alpha \le 1/2$. □

We next present an auxiliary result. This inequality will be used in the proof of Lemma 3 to show that the equilibrium output when agents are inequality-averse is less than the output if agents were inequality-neutral.

Lemma EC.2. *When $0 < k \le 1$, $0 < \alpha \le 1/2$, $0 < \beta \le 1$, and $\alpha \le \beta$ we have, using (EC.5),*

$$\tfrac{1+\beta}{\beta}\tfrac{\alpha}{1-\alpha}k^2 \le \theta_{AB}.$$

Proof. Using (EC.5), we transform the desired inequality into the following:

$$\frac{(1+\beta)\left(-(1-\alpha)(1+\beta)(1-k^2)+\sqrt{1-\left(1-(1-\alpha)^2+(1+\beta)^2\right)\left(1-k^2\right)+(1+\beta)^2\left(1-k^2\right)^2}\right)}{(1-\alpha)\left(1-(1+\beta)^2\left(1-k^2\right)\right)} \geq \frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}k^2$$
$$\Leftrightarrow \frac{1-(2-\alpha)\alpha\left(1-k^2\right)}{(1-\alpha)(1+\beta)(1-k^2)+\sqrt{1-\left(1-(1-\alpha)^2+(1+\beta)^2\right)\left(1-k^2\right)+(1+\beta)^2\left(1-k^2\right)^2}} \geq \frac{\alpha}{\beta}k^2$$
$$\Leftrightarrow \alpha^2k^4\left((1-\alpha)^2-k^2\left(\alpha^2-2\alpha+(\beta+1)^2\right)+(\beta+1)^2k^4\right)-\left((1-\alpha)^2\beta-(\alpha-1)\alpha(\beta+1)k^4+\alpha k^2(\alpha+\beta-1)\right)^2 \leq 0.$$

The left-hand side is quadratic and decreasing in $\beta\in[\alpha,1]$. Therefore, substituting $\beta=\alpha$ into the expression, the left-hand side is less than $\alpha^2\left(1-k^2\right)\left(\alpha(2-\alpha)\left(1-k^2\right)-1\right)\left((1-\alpha)^2-\left(1-2\alpha^2\right)k^2+(1+\alpha)^2k^4\right)$, which is negative when $0<\alpha\leq 1/2$ and $0<k\leq 1$. Thus, the desired inequality holds. □

Proof of Lemma 3. When agents are inequality-neutral ('N'), given contract $\Phi_\Gamma=(\gamma_1,\gamma_2)$, their best responses are identified by the first-order optimality conditions for (4) with $\alpha=\beta=0$. By Lemma 1, the output is $P^N(e_1^N,e_2^N)=e_1^N+ke_2^N=\gamma_1+k^2\gamma_2$.

When agents are inequality-averse, we divide the parameter space $(\gamma_1,\gamma_2,f_1,f_2)$ into four regions depending on the equilibrium types characterized in Lemma EC.1 under Assumption 1:

$$\begin{aligned}
&\text{Region A}=\{(\gamma_1,\gamma_2,f_1,f_2)|\tfrac{\gamma_1}{\gamma_2}\geq\tfrac{\beta}{1+\beta},f_1-f_2\leq\Theta_{AB}(\gamma_1,\gamma_2)\text{ or }\tfrac{\gamma_1}{\gamma_2}\leq\tfrac{\beta}{1+\beta},f_1-f_2\leq\tilde{\Theta}_{AB}(\gamma_1,\gamma_2)\};\\
&\text{Region B}=\{(\gamma_1,\gamma_2,f_1,f_2)|\tfrac{\gamma_1}{\gamma_2}\geq\tfrac{\beta}{1+\beta},\Theta_{AB}(\gamma_1,\gamma_2)\leq f_1-f_2\leq\Theta_{BC}(\gamma_1,\gamma_2)\\
&\qquad\text{or }\tfrac{\gamma_1}{\gamma_2}\leq\tfrac{\beta}{1+\beta},\tilde{\Theta}_{AB}(\gamma_1,\gamma_2)\leq f_1-f_2\leq\Theta_{BC}(\gamma_1,\gamma_2)\};\\
&\text{Region C}=\{(\gamma_1,\gamma_2,f_1,f_2)|\tfrac{\gamma_1}{\gamma_2}\leq(1+\beta)/\beta,\Theta_{BC}(\gamma_1,\gamma_2)\leq f_1-f_2\leq\Theta_{CD}(\gamma_1,\gamma_2)\\
&\qquad\text{or }\tfrac{\gamma_1}{\gamma_2}\geq(1+\beta)/\beta,\Theta_{BC}(\gamma_1,\gamma_2)\leq f_1-f_2\leq\tilde{\Theta}_{CD}(\gamma_1,\gamma_2)\};\\
&\text{Region D}=\{(\gamma_1,\gamma_2,f_1,f_2)|\tfrac{\gamma_1}{\gamma_2}\leq(1+\beta)/\beta,f_1-f_2\geq\Theta_{CD}(\gamma_1,\gamma_2)\text{ or }\tfrac{\gamma_1}{\gamma_2}\geq(1+\beta)/\beta,f_1-f_2\geq\tilde{\Theta}_{CD}(\gamma_1,\gamma_2)\}.
\end{aligned}\tag{EC.6}$$

In each region, we derive the equilibrium output and compare it with the inequality-neutral output P^N.

Region A: We consider the output in two cases, depending on the parameter value of γ_1/γ_2.

Case Aa. $\gamma_1/\gamma_2\geq\beta/(1+\beta)$. In this case, by Lemma 1, the equilibrium efforts are $\left(e_1^\beta,e_2^\alpha\right)$. The total output is correspondingly

$$P^A\left(e_1^\beta,e_2^\alpha\right)=\gamma_1-\tfrac{\beta}{1+\beta}\gamma_2+k^2\gamma_2+\tfrac{\alpha}{1-\alpha}k^2\gamma_1=P^N+\left(\tfrac{\alpha}{1-\alpha}k^2\gamma_1-\tfrac{\beta}{1+\beta}\gamma_2\right).$$

Suppose $P^A\left(e_1^\beta,e_2^\alpha\right)>P^N$, then $\gamma_2<\frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}k^2\gamma_1$. However, for a contract to be in Region A, $0=f_1-f_2\leq\Theta_{AB}$, which implies $\gamma_2\geq\theta_{AB}\gamma_1$ by (EC.5). According to Lemma EC.2, $\frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}k^2\leq\theta_{AB}$, a contradiction. Thus, $P^A\left(e_1^\beta,e_2^\alpha\right)\leq P^N$.

Case Ab. $\gamma_1/\gamma_2<\beta/(1+\beta)$. By Lemma 1, the equilibrium efforts are $(0,e_2^\alpha)$. The total output is correspondingly

$$P^A\left(0,e_2^\alpha\right)=0+k^2\gamma_2+\tfrac{\alpha}{1-\alpha}k^2\gamma_1=P^N-\left(1-\tfrac{\alpha}{1-\alpha}k^2\right)\gamma_1\leq P^N$$

since $0<\alpha\leq 1/2$ and $0<k\leq 1$.

Combining Cases Aa and Ab, we have $P^A\left(e_1^\beta,e_2^\alpha\right)\leq P^N$ for all contracts with parameter values in Region A.

Region B: By Lemma 1, the equilibrium efforts are $(\bar{e}_1(e_2^\alpha),e_2^\alpha)$. The total output is correspondingly

$$\begin{aligned}P^B(\bar{e}_1(e_2^\alpha),e_2^\alpha)&=\bar{e}_1(e_2^\alpha)+ke_2^\alpha\\&=\left(1+\tfrac{\alpha}{1+\alpha}k^2\right)\gamma_1-(1-k^2)\gamma_2+\sqrt{2(f_1-f_2)+\tfrac{\gamma_1^2k^2}{(1-\alpha)^2}+\left(1-k^2\right)(\gamma_1-\gamma_2)^2}.\end{aligned}$$

Comparing $P^B(\bar{e}_1(e_2^\alpha), e_2^\alpha)$ and P^N, we get $P^B(\bar{e}_1(e_2^\alpha), e_2^\alpha) > P^N$ if and only if $\gamma_2 < \bar{r}(\alpha,k)\gamma_1$, where $\bar{r}(\alpha,k) := \left(\alpha + \sqrt{\alpha^2(1-k^2)(k^2+1)^2 - 2\alpha(1-k^4) + k^4 - k^2 + 1} + k^2 - 1\right)/((1-\alpha)k^2)$.

Region C: By Lemma 1, the equilibrium efforts are $(e_1^\alpha, \bar{e}_2(e_1^\alpha))$. The total output is correspondingly

$$\begin{aligned} P^C(e_1^\alpha, \bar{e}_2(e_1^\alpha)) &= e_1^\alpha + k\bar{e}_2(e_1^\alpha) \\ &= (1-k^2)\gamma_1 + \left(\tfrac{\alpha}{1+\alpha} + k^2\right)\gamma_2 + k\sqrt{\tfrac{\gamma_2^2}{(1-\alpha)^2} + 2(f_2 - f_1) - (1-k^2)(\gamma_1-\gamma_2)^2}. \end{aligned}$$

Comparing $P^C(e_1^\alpha, \bar{e}_2(e_1^\alpha))$ and P^N, we get $P^B(\bar{e}_1(e_2^\alpha), e_2^\alpha) \geq P^N$ if and only if

$$(1-\alpha)k\sqrt{\tfrac{\gamma_2^2}{(1-\alpha)^2} - (1-k^2)(\gamma_1-\gamma_2)^2} \geq -\gamma_2\alpha + (1-\alpha)(\gamma_1 + \gamma_2 k^2) - \gamma_1(1-\alpha)(1-k^2) - \gamma_2(1-\alpha)k^2,$$

which holds when either the right-hand side is negative, implying $\gamma_2 \geq k^2(1-\alpha)/\alpha \cdot \gamma_1$, or

$$(1-\alpha)^2k^2\left(\tfrac{\gamma_2^2}{(1-\alpha)^2} - (1-k^2)(\gamma_1-\gamma_2)^2\right) - \left(-\gamma_2\alpha + (1-\alpha)(\gamma_1+\gamma_2k^2) - \gamma_1(1-\alpha)(1-k^2) - \gamma_2(1-\alpha)k^2\right)^2 \geq 0,$$

which implies $\gamma_2 \geq \gamma_1 \cdot \frac{(1-\alpha)k}{\sqrt{(1-\alpha)^2k^6 - (1-\alpha^2)k^4 + (1+2\alpha-\alpha^2)k^2 - \alpha^2} - (1-\alpha)k^3 + k}$. In summary, $P^C(\bar{e}_1(e_2^\alpha), e_2^\alpha) \geq P^c$ if and only if $\gamma_2 \geq \min\left\{\frac{(1-\alpha)k}{\sqrt{(1-\alpha)^2k^6 - (1-\alpha^2)k^4 + (1+2\alpha-\alpha^2)k^2 - \alpha^2} - (1-\alpha)k^3 + k}, k^2(1-\alpha)/\alpha\right\}\gamma_1$.

Region D: Since $\Theta_{CD}(\gamma_1,\gamma_2) > \beta/(1+\beta)$, we need to consider two cases, depending on the values of γ_1/γ_2.

Case Da: When $\gamma_1/\gamma_2 \leq (1+\beta)/\beta$, by Lemma 1, the equilibrium efforts are (e_1^α, e_2^β).

$$P^D(e_1^\alpha, e_2^\beta) = \gamma_1 + \tfrac{\alpha}{1-\alpha}\gamma_2 + k^2\gamma_2 - \tfrac{\beta}{1+\beta}k^2\gamma_1 = P^N + \left(\tfrac{\alpha}{1-\alpha}\gamma_2 - \tfrac{\beta}{1+\beta}k^2\gamma_1\right).$$

Therefore, $P^D(e_1^\alpha, e_2^\beta) \geq P^N$ if and only if $\gamma_2 \geq \frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}k^2\gamma_1$.

Case Db: When $\gamma_1/\gamma_2 > (1+\beta)/\beta$, by Lemma 1, the equilibrium efforts are $(e_1^\alpha, 0)$. The output is correspondingly

$$P^D(e_1^\alpha, 0) = \gamma_1 + \tfrac{\alpha}{1-\alpha}\gamma_2 + 0 = P^N + \left(\tfrac{\alpha}{1-\alpha}\gamma_2 - k^2\gamma_2\right).$$

In this case, $P^D(e_1^\alpha, 0) \geq P^N$ if and only if $k \leq \sqrt{\frac{\alpha}{1-\alpha}}$.

Combining all conditions across all four Regions A-D, the output is higher under inequality aversion than under inequality neutrality if and only if $\underline{r}(\alpha,k)\gamma_1 \leq \gamma_2 \leq \bar{r}(\alpha,k)\gamma_1$, where $\bar{r}(\alpha,k)$ was defined in our analysis of Region B and

$$\underline{r}(\alpha,k) := \begin{cases} 0, & k \leq \sqrt{\tfrac{\alpha}{1-\alpha}}, \\ \tfrac{1-\alpha}{\alpha}\tfrac{\beta}{1+\beta}k^2, & \sqrt{\tfrac{\alpha}{1-\alpha}} < k \leq \omega(\alpha), \\ \tfrac{(1-\alpha)k}{\sqrt{(1-\alpha)^2k^6 - (1-\alpha^2)k^4 + (1+2\alpha-\alpha^2)k^2 - \alpha^2} - (1-\alpha)k^3 + k}, & k > \omega(\alpha), \end{cases}$$

where $\omega(\alpha) \in [0,1)$ uniquely solves

$$\frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}k^2 = \frac{(1-\alpha)k}{\sqrt{(1-\alpha)^2k^6 - (1-\alpha^2)k^4 + (1+2\alpha-\alpha^2)k^2 - \alpha^2} - (1-\alpha)k^3 + k}. \tag{EC.7}$$

When $k = \omega(\alpha)$, it is straightforward to check that $\underline{r}(\alpha,k)$ is equal to θ_{CD} defined in (EC.5).

Finally, we show that when α increases, $\bar{r}(\alpha,k)$ increases and $\underline{r}(\alpha,k)$ decreases. Differentiating $\bar{r}(\alpha,k)$ with respect to α,

$$\frac{\partial \bar{r}(\alpha,k)}{\partial\alpha} = \frac{\sqrt{\alpha^2(1-k^2)(1+k^2)^2 - 2\alpha(1-k^4) + k^4 - k^2 + 1} + \alpha - \alpha k^4 + 2k^2 - 1}{(1-\alpha)^2\sqrt{\alpha^2(1-k^2)(1+k^2)^2 - 2\alpha(1-k^4) + k^4 - k^2 + 1}} \geq 0$$

since

$$\begin{aligned}&\alpha^2(1-k^2)(1+k^2)^2 - 2\alpha(1-k^4) + k^4 - k^2 + 1 - \Big(\alpha^2(1-k^2)(1+k^2)^2 - 2\alpha(1-k^4) + k^4 - k^2 + 1\Big)^2\\ =\ & k^2(1-k^2)(3-\alpha-\alpha k^2)(1-\alpha-\alpha k^2) \geq 0.\end{aligned}$$

As for $\underline{r}(\alpha,k)$, $\underline{r}$ is continuous except for a downward jump at $\alpha = \frac{k^2}{1+k^2}$ (i.e., $k = \sqrt{\frac{\alpha}{1-\alpha}}$). The first piece is constant. The second piece is decreasing in α. The derivative of the third piece with respect to α is

$$\frac{k\left(\alpha - k\left(\sqrt{-\alpha^2-(1-\alpha^2)k^4+(1-\alpha)^2k^6+(1-(\alpha-2)\alpha)k^2} - (1-\alpha)k^3 + 2k\right)\right)}{\sqrt{-\alpha^2-(1-\alpha^2)k^4+(1-\alpha)^2k^6+(1-(\alpha-2)\alpha)k^2}\left(\sqrt{-\alpha^2-(1-\alpha^2)k^4+(1-\alpha)^2k^6+(1-(\alpha-2)\alpha)k^2} - (1-\alpha)k^3 + k\right)^2}.$$

Its denominator is positive; its numerator is positive if and only if $k < \frac{1}{\sqrt{2}}$ and $\frac{3k^2}{k^2+1} < \alpha \leq \frac{k^2}{k^2+1} + \sqrt{\frac{k^2}{(1-k^2)(1+k^2)^2}}$. We next show that the latter condition never holds. For $\underline{r}$ to take the value of the third piece, by (EC.7), we must have

$$\begin{aligned}\alpha &\leq \tfrac{\beta k}{1+\beta}\left(\sqrt{-\alpha^2-(1-\alpha^2)k^4+(1-\alpha)^2k^6+(1-\alpha^2+2\alpha)k^2} - (1-\alpha)k^3 + k\right)\\ &\leq \tfrac{k}{2}\left(\sqrt{((1-\alpha)k^3+2k)^2+(-\alpha^2+(\alpha^2+4\alpha-5)k^4-(\alpha^2-2\alpha+3)k^2)} - (1-\alpha)k^3 - 2k + 3k\right) \text{ (since } \beta \leq 1)\\ &\leq \tfrac{3k^2}{2} < \tfrac{3k^2}{1+k^2},\end{aligned}$$

in which the second inequality holds since $-\alpha^2 + (\alpha^2+4\alpha-5)k^4 - (\alpha^2-2\alpha+3)k^2 \leq 0$ when $0 \leq \beta \leq 1$ and $0 \leq k \leq 1$. Thus, the derivative of the third piece of $\underline{r}$ with respect to α is negative. Therefore, $\underline{r}(\alpha,k)$ is decreasing in α. □

Before proving Theorem 1, we present auxiliary Lemmas EC.3-EC.5. By the proof of Lemma 1, we consider only action spaces $e_i \in [e_i^\beta, e_i^\alpha]$ for $i = 1, 2$. When $f_1 = f_2 = 0$, (1) simplifies to

$$\begin{aligned}u_i(e_i; e_{-i}, \gamma_i, \gamma_{-i}, 0, 0) =& \gamma_i(e_1+ke_2) - ce_i^2 - \alpha[(\gamma_i(e_1+ke_2) - ce_i^2) - (\gamma_{-i}(e_1+ke_2) - ce_{-i}^2)]^+ \\ & - \beta[(\gamma_{-i}(e_1+ke_2) - ce_{-i}^2) - (\gamma_i(e_1+ke_2) - ce_i^2)]^+, \quad i = 1, 2,\end{aligned} \tag{EC.8}$$

where $e_i \in [e_i^\beta, e_i^\alpha]$. To prove Theorem 1, we first show that for any optimal output-sharing contract, $f_1 = f_2 = 0$; and then find the optimal contract solving (5) with $f_1 = f_2 = 0$.

We show that $f_1 = f_2 = 0$ in all optimal output-sharing contracts in three steps. First, given a Nash equilibrium (e_1^f, e_2^f) to the effort game (4) under contract Φ, we propose in Lemma EC.3 a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ with no fixed-transfer payments that leads to both higher best effort responses and higher payoff at (e_1^f, e_2^f). Second, we show in Lemma EC.4 that any effort game (4) with no fixed-transfer payments maximizing (EC.8) is supermodular. Finally, based on the supermodularity of the effort game under Φ' (Lemma EC.4) and the elevated best responses at (e_1^f, e_2^f) (Lemma EC.3, point 2), we show in Lemma EC.5 that there exists an equilibrium (e_1^0, e_2^0) such that $e_1^0 \geq e_1^f$ and $e_2^0 \geq e_2^f$, and thus $\pi(e_1^0, e_2^0, \Phi') \geq \pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$ (Lemma EC.3, point 1). As a result, we consider share-only contracts in the proof of Theorem 1.

Lemma EC.3. *Under utility fairness, given any contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ and a Nash equilibrium (e_1^f, e_2^f) to the effort game (4), there exists a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ such that*

1. *Φ' leads to greater payoff when efforts are (e_1^f, e_2^f), i.e., $\pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$.*
2. *Agent i's best response to e_{-i}^f under Φ', denoted by $B_i(e_{-i}^f; \Phi')$, is greater than e_i^f, i.e., $B_i(e_{-i}^f; \Phi') \geq e_i^f$, for $i = 1, 2$.*

When at least one of f_1 and f_2 is strictly positive and $\alpha > 0$, at least one of the three inequalities (i.e., $\pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$, and $B_i(e_{-i}^f; \Phi') \geq e_i^f$ for $i = 1, 2$) are strict.

Proof. Given contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, we construct the desired contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ in three cases, depending on the relative values of v_1 and v_2 when efforts are (e_1^f, e_2^f) under Φ.

- Case 1: $v_1 < v_2$, i.e., (e_1^f, e_2^f) is in the interior of (u_1^β, u_2^α) under Φ, where (u_1^β, u_2^α) are defined by (EC.1)-(EC.2). Depending on the values of f_1 and f_2, we consider three subcases.

  a) When $\frac{1+\beta}{\beta} f_2 \geq f_1 \geq \frac{\beta}{1+\beta} f_2$, let

$$(\gamma_1', \gamma_2') := (f_1/(e_1^f + k e_2^f), f_2/(e_1^f + k e_2^f)). \tag{EC.9}$$

  Since the function values and "fairness gaps" $(v_1 - v_2)$ at (e_1^f, e_2^f) are identical in (1) or (EC.8), i.e., under Φ or Φ', by construction, (e_1^f, e_2^f) also lies in (u_1^β, u_2^α) under Φ'. For Agent 1, since

$$\begin{aligned}
\frac{\mathrm{d}u_1^\beta(e_1; e_2, \Phi')}{\mathrm{d}e_1} &= (1+\beta)(\gamma_1' - 2ce_1) - \beta\gamma_2' && \text{plugging in (1)} \\
&\geq (1+\beta)(\gamma_1 - 2c_1 e_1) - \beta\gamma_2 && \text{by (EC.9) and } f_1 \geq \beta/(1+\beta) f_2 \\
&= \frac{\mathrm{d}u_1^\beta(e_1; e_2, \Phi)}{\mathrm{d}e_1} = 0 && \text{plugging in (EC.8)}
\end{aligned}$$

  and Agent 1's utility function (EC.8) under Φ' is quasi-concave and piece-wise quadratic by definition, it follows that $B_1(e_2^f; \Phi') \geq e_1^f$.

  As for Agent 2, similarly, we have

$$\begin{aligned}
\frac{\mathrm{d}u_2^\alpha(e_2; e_1^f, \Phi')}{\mathrm{d}e_2}\Big|_{e_2 = e_2^f} &= (1-\alpha)(k\gamma_2' - 2ce_2) + k\alpha\gamma_1' && \text{plugging in (1)} \\
&> (1-\alpha)(k\gamma_2 - 2ce_2) + k\alpha\gamma_1 && \text{since } \alpha > 0 \text{ and } f_i > 0 \text{ for at least one } i \\
&= \frac{\mathrm{d}u_2^\alpha(e_2; e_1^f, \Phi)}{\mathrm{d}e_2}\Big|_{e_2 = e_2^f} = 0 && \text{plugging in (EC.8).}
\end{aligned}$$

  Since Agent 2' utility function is quasi-concave and piece-wise quadratic, it follows that $B_2(e_1^f; \Phi') \geq e_2^f$.

  Finally, by (EC.9), $\pi(e_1^f, e_2^f, \Phi') = \pi(e_1^f, e_2^f, \Phi)$.

  b) When $f_1 < \frac{\beta}{1+\beta} f_2$, let

$$\gamma_1^c := f_1/(e_1^f + k e_2^f) + \gamma_1;\ \gamma_2^c := (1+\beta)/\beta(\gamma_1^c - \gamma_1) + \gamma_2. \tag{EC.10}$$

  We consider two subsubcases depending on whether the contract $(\gamma_1^c, \gamma_2^c, 0, 0)$ changes the relative envy or guilt of the two agents under efforts (e_1^f, e_2^f) compared to the given contract Φ.

  —If $\gamma_2^c(e_1^f + k e_2^f) - c{e_2^f}^2 - \gamma_1^c(e_1^f + k e_2^f) + c{e_1^f}^2 \geq 0$, let $\Phi' = (\gamma_1^c, \gamma_2^c, 0, 0)$. Then, (e_1^f, e_2^f) is on (u_1^β, u_2^α) under Φ' because the fairness gap $(v_1 - v_2)$ in (EC.8) is

$$\gamma_2^c(e_1^f + k e_2^f) - c{e_2^f}^2 - \gamma_1^c(e_1^f + k e_2^f) + c{e_1^f}^2 \geq 0.$$

  Following similar arguments to a), we have $B_1(e_2^f; \Phi') \geq e_1^f$, $B_2(e_1^f; \Phi') \geq e_2^f$. By (EC.10) and $f_1 < \frac{\beta}{1+\beta} f_2$, $\pi(e_1^f, e_2^f, \Phi') > \pi(e_1^f, e_2^f, \Phi)$.

—If $\gamma_2^c(e_1^f+ke_2^f)-c{e_2^f}^2-\gamma_1^c(e_1^f+ke_2^f)+c{e_1^f}^2<0$, consider the contract where $\gamma_1'>\gamma_1$ and $\gamma_2'\geq\gamma_2$ jointly solve the fairness condition $\gamma_2'(e_1^f+ke_2^f)-c{e_2^f}^2-\gamma_1'(e_1^f+ke_2^f)+c{e_1^f}^2+f_2-f_1=0$ and the budget-balancing condition $(\gamma_1'-\gamma_1+\gamma_2'-\gamma_2)(e_1^f+ke_2^f)=f_1+f_2$:

$$\gamma_1'=\frac{\gamma_2(e_1^f+ke_2^f)-c{e_2^f}^2-\gamma_1({}_1^f+ke_2^f)+c{e_1^f}^2+f_1+f_2}{2(e_1^f+ke_2^f)}+\gamma_1, \tag{EC.11}$$

$$\gamma_2'=\frac{\gamma_1(e_1^f+ke_2^f)-c{e_1^f}^2-\gamma_2(e_1^f+ke_2^f)+c{e_2^f}^2+f_1+f_2}{2(e_1^f+ke_2^f)}+\gamma_2. \tag{EC.12}$$

Since (e_1^f,e_2^f) is on the interior of (u_1^β,u_2^α) in Case 1, by (1), the fairness gap (v_1-v_2) equals

$$\gamma_2(e_1^f+ke_2^f)-c{e_2^f}^2-\gamma_1(e_1^f+ke_2^f)+c{e_1^f}^2+f_2-f_1>0,$$

and since by assumption

$$\begin{aligned} 0>&\gamma_2^c(e_1^f+ke_2^f)-c{e_2^f}^2-\gamma_1^c(e_1^f+ke_2^f)+c{e_1^f}^2 && \text{by case assumption}\\ \geq&\gamma_2(e_1^f+ke_2^f)-c{e_2^f}^2-f_1-\gamma_1(e_1^f+ke_2^f)+c{e_1^f}^2 && \text{since } f_1\geq 0,\end{aligned}$$

the existence and uniqueness of (EC.11)-(EC.12) with $\gamma_1'\geq\gamma_1$ and $\gamma_2'\geq\gamma_2$ are guaranteed.

Compared with Φ, Agent 1's best response under Φ' satisfies $B_1(e_2^f;\Phi')\geq e_1^f$ because (e_1^f,e_2^f) is on the boundary of $u_1^\alpha(e_1;e_2^f,\Phi')$ and $u_1^\beta(e_1;e_2^f,\Phi')$, and

$$\begin{aligned}\frac{\mathrm{d}u_1(e_1;e_2^f,\Phi')}{\mathrm{d}e_1}\Big|_{e_1\to e_1^f-}&=\frac{\mathrm{d}u_1^\alpha(e_1;e_2^f,\Phi')}{\mathrm{d}e_1}\Big|_{e_1=e_1^f} && \text{by (EC.8)}\\ &=(1-\alpha)(\gamma_1'-2ce_1^f)+\alpha\gamma_2' && \text{by (EC.8)}\\ &>(1-\alpha)(\gamma_1-2ce_1^f)+\alpha\gamma_2 && \text{since } \gamma_1'>\gamma_1,\ \gamma_2'\geq\gamma_2\\ &=(1+\beta)(\gamma_1-2ce_1^f)-\beta\gamma_2+(\alpha+\beta)(2ce_1^f-\gamma_1+\gamma_2) && \text{rearranging}\\ &\geq(1+\beta)(\gamma_1-2ce_1^f)-\beta\gamma_2 && \text{since } e_1^f\geq e_1^\beta\\ &=\frac{\mathrm{d}u_1^\beta(e_1;e_2^f,\Phi)}{\mathrm{d}e_1}\Big|_{e_1=e_1^f}=0 && \text{by (1).}\end{aligned}$$

As for Agent 2, as (e_1^f,e_2^f) is on the boundary of u_1^α and u_1^β under Φ', we consider the left-derivative of u_1:

$$\begin{aligned}\frac{\mathrm{d}u_2(e_2;e_1^f,\Phi')}{\mathrm{d}e_2}\Big|_{e_2\to e_2^f-}=&\frac{\mathrm{d}u_2^\alpha(e_2;e_1^f,\Phi')}{\mathrm{d}e_2}\Big|_{e_2=e_2^f} && \text{by (EC.8)}\\ =&(1-\alpha)(k\gamma_1'-2ce_2^f)+\alpha k\gamma_2' && \text{by (EC.8)}\\ =&\frac{\mathrm{d}u_2^\alpha(e_2;e_1^f,\Phi)}{\mathrm{d}e_2}\Big|_{e_2=e_2^f}+k(\alpha(\gamma_1'-\gamma_1)+(1-\alpha)(\gamma_2'-\gamma_2)) && \text{by (EC.11)-(EC.12)}\\ \geq&\frac{\mathrm{d}u_2^\alpha(e_2;e_1^f,\Phi)}{\mathrm{d}e_2}\Big|_{e_2=e_2^f}=0 && \text{since } \gamma_i'\geq\gamma_i\end{aligned}$$

Thus, $B_1(e_2^f;\Phi')>e_1^f$, $B_2(e_1^f;\Phi')\geq e_2^f$. Moreover, $\pi(e_1^f,e_2^f,\Phi')=\pi(e_1^f,e_2^f,\Phi)$ due to the budget-balancing constraint.

c) When $f_2<\frac{\beta}{1+\beta}f_1$, then let $\gamma_2^d=f_2/(e_1^f+ke_2^f)+\gamma_2$ and $\gamma_1^d=\frac{1+\beta}{\beta}(\gamma_2^d-\gamma_2)+\gamma_1$. Similar to b), when $\gamma_2^d(e_1^f+ke_2^f)-c{e_2^f}^2-\gamma_1^d(e_1^f+ke_2^f)+c{e_1^f}^2\geq 0$, let $\Phi'=(\gamma_1^d,\gamma_2^d,0,0)$; otherwise, let (γ_1',γ_2') defined by (EC.11)-(EC.12). Following the same arguments to b), we have $B_i(e_{-i}^f,\Phi')\geq e_i^f$ for $i=1,2$, and $\pi(e_1^f,e_2^f,\Phi')\geq\pi(e_1^f,e_2^f,\Phi)$, and at least one of the three inequalities are strict.

- Case 2: $v_1 > v_2$, i.e., (e_1^f, e_2^f) is on (u_1^α, u_2^β) under Φ defined by (EC.1)-(EC.2): following a similar argument to Case 1, we can construct the desired contract Φ'.
- Case 3: $v_1 = v_2$, i.e., (e_1^f, e_2^f) is on on the boundary of (u_1^β, u_2^α) and (u_1^α, u_2^β) defined by (EC.1)-(EC.2). Consider $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ defined by

$$(\gamma_i', \gamma_{-i}') := (f_i/(e_1^f + ke_2^f) + \gamma_i - \epsilon, f_{-i}/(e_1) + ke_2^f + \gamma_{-i}),$$

where $\epsilon > 0$ is arbitrarily small. (EC.9). Then for both agents u_i^α has a higher slope at (e_1^f, e_2^f) under Φ' than Φ. For example, for Agent 1,

$$\begin{aligned}
\frac{\mathrm{d}u_1^\alpha(e_1; e_2^f, \Phi')}{\mathrm{d}e_1}|_{e_1=e_1^f} &= (1-\alpha)(\gamma_1' - 2ce_1^f) + \alpha\gamma_2' && \text{by (EC.8)}\\
&> (1-\alpha)(\gamma_1 - 2ce_1^f) + \alpha\gamma_2 && \text{since } \alpha > 0 \text{ and at least one } f_i > 0\\
&= \frac{\mathrm{d}u_1^\alpha(e_1; e_2^f, \Phi)}{\mathrm{d}e_1}|_{e_1=e_1^f} && \text{by (1).}
\end{aligned}$$

Since (e_1^f, e_2^f) is a Nash equilibrium under Φ, $\frac{du_1^\alpha(e_1;e_2^f,\Phi)}{de_1}|_{e_1=e_1^f} \geq 0$. Thus, $\frac{du_1^\alpha(e_1;e_2^f,\Phi)}{de_1}|_{e_1=e_1^f} \geq 0$. Since u_i^α is on the left side of u_i^β by (EC.8), the best responses satisfy $B_i(e_{-i}) \geq e_i^f$. Moreover, by (EC.9), the principal's payoff $\pi(e_1^f, e_2^f, \Phi') = (1-\gamma_1' - \gamma_2')(e_1^f + ke_2^f) > (1-\gamma_1-\gamma_2)(e_1^f + ke_2^f) + f_1 + f_2 = \pi(e_1^f, e_2^f, \Phi)$.

Summarizing Cases 1-3 above, there always exists contract Φ' that satisfies the two criteria in the lemma. □

LEMMA EC.4. *Under utility fairness, given any contract $\Phi = (\gamma_1, \gamma_2, 0, 0)$ with no fixed-transfer payments, the corresponding effort game (4) maximizing (EC.8) is supermodular.*

Proof. In the proof, we omit the contract arguments in $u_i(e_i; e_{-i}, \gamma_i, \gamma_{-i}, 0, 0)$ and write instead $u_i(e_i, e_{-i})$. Since the action space $e_i \in [e_i^\beta, e_i^\alpha]$ is compact, and $u_i(e_i, e_{-i})$ is continuous in e_i and e_{-i} by definition, we need to show that $u_i(e_i, e_{-i})$ has increasing differences. Since the utility difference $(\gamma_i(e_1 + ke_2) - ce_i^2) - (\gamma_{-i}(e_1 + ke_2) - ce_{-i}^2)$ is increasing in e_{-i} when $e_{-i} \in [e_{-i}^\beta, e_{-i}^\alpha]$, u_i moves from u_i^β to u_i^α (defined by (EC.1)-(EC.2)) when e_{-i} increases. Thus, it is sufficient to show $\partial u_i(e_i, e_{-i})/\partial e_i$ is increasing in e_{-i} including the kink (which is still continuous). When $u_i(e_i, e_{-i}) = u_i^\beta(e_i, e_{-i})$ (when e_{-i} is small), the derivative is

$$\partial u_i(e_i, e_{-i})/\partial e_i = \partial u_i^\beta(e_i, e_{-i})/\partial e_i = (1+\beta)(\gamma_i k_i - 2ce_i) - \beta\gamma_{-i}k_i,$$

which is constant in e_{-i}. Similarly, when $u_i(e_i, e_{-i}) = u_i^\alpha(e_i, e_{-i})$ (when e_{-i} is big), the derivative is

$$\partial u_i(e_i, e_{-i})/\partial e_i = \partial u_i^\alpha(e_i, e_{-i})/\partial e_i = (1-\alpha)(\gamma_i k_i - 2ce_i) + \alpha\gamma_{-i}k_i,$$

which is also constant in e_{-i}. Hence, the derivative $\partial u_i(e_i, e_{-i})/\partial e_i$ is piecewise constant. Furthermore,

$$\begin{aligned}
\partial u_i^\alpha(e_i, e_{-i})/\partial e_i &= (1+\beta)(\gamma_i k_i - 2ce_i) - \beta\gamma_{-i}k_i - (\alpha+\beta)((\gamma_i - \gamma_{-i})k_i - 2ce_i) && \text{rearranging}\\
&\geq \partial u_i^\beta(e_i, e_{-i})/\partial e_i && \text{since } e_i \geq e_i^\beta.
\end{aligned}$$

Therefore, $\partial u_i(e_i, e_{-i})/\partial e_i$ is piecewise constant in e_{-i} with a positive jump at the transition point, when $u_i(e_i, e_{-i}) = u_i^\alpha(e_i, e_{-i}) = u_i^\beta(e_i, e_{-i})$. As a result, $\partial u_i(e_i, e_{-i})/\partial e_i$ is weakly increasing in e_{-i}, i.e., $u_i(e_i, e_{-i})$ has increasing differences when $e_i \in [e_i^\beta, e_i^\alpha]$, and the game of interest is supermodular. □

Lemma EC.5. *Under utility fairness, for any output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ with non-zero fixed-transfer payments and equilibrium efforts (e_1^f, e_2^f), there exists a Nash equilibrium (e_1^0, e_2^0) to effort game (4) under the contract in Lemma EC.3 such that $e_i^0 \geq e_i^f$ for $i = 1,2$, and thus, $\pi(e_1^0, e_2^0, \Phi') > \pi(e_1^f, e_2^f, \Phi)$.*

Proof. By Lemma EC.3 (point 2), for any Φ with equilibirum efforts (e_1^f, e_2^f), there always exists $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ such that $\max\{\mathbb{B}_i(e_{-i}^f; \Phi')\} \geq e_i^f$ for $i = 1,2$. (Here, $\mathbb{B}_i(\cdot)$ is the set of best-responses in case of multiple equilibria.)

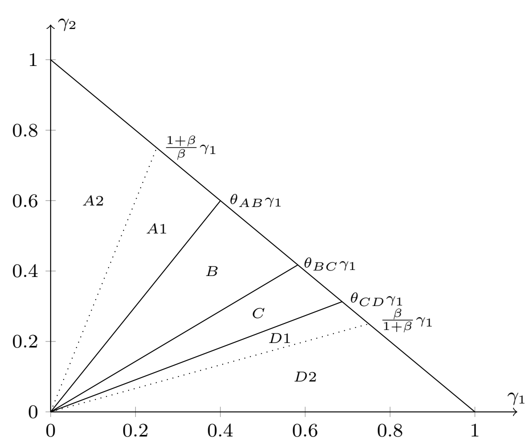
Let $s_i^0 := \max\{\mathbb{B}_i(e_{-i}^f; \Phi')\}$. Since $s_2^0 \geq e_2^f$, and by the supermodularity property proven in Lemma EC.4, we have $s_1^1 := \max\{\mathbb{B}_1(s_2^0; \Phi')\} \geq e_1^f = s_1^0$. Similarly, since $s_1^1 \geq e_1^f$, we have $s_2^1 := \max\{B_2(s_1^1; \Phi')\} \geq \max\{B_2(s_1^0; \Phi')\} = s_2^0$. Iterating the process by letting $s_1^{k+1} := \max\{\mathbb{B}_1(s_2^k; \Phi')\}$ and $s_2^{k+1} := \max\{\mathbb{B}_2(s_2^{k+1}; \Phi')\}$. Then, we have $s_i^{k+1} \geq s_i^k$, $k = 0,1,2,...$ for $i = 1,2$ by induction. Since the choice of efforts are bounded, both sequences, $\{s_i^k\}_k$ for $i = 1,2$, converge by the monotone convergence theorem. Let e_i^0 denote the limit of $\{s_i^k\}_k$ for $i = 1,2$. That is, there exists a pair of equilibirum efforts $e_1^0 \geq e_1^f$ and $e_2^0 \geq e_2^f$.

Since when the efforts are (e_1^f, e_2^f) and contract is $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, and by Lemma EC.3 (point 1), the principal's payoff is bounded by

$$\pi(e_1^f, e_2^f; \Phi) \leq \pi(e_1^f, e_2^f; \Phi') = (1 - \gamma_1' - \gamma_2')(e_1^f + k e_2^f) \leq (1 - \gamma_1' - \gamma_2')(e_1^0 + k e_2^0) = \pi(e_1^0, e_2^0; \Phi').$$

By Lemma EC.3, when at least one of f_1 and f_2 is strictly positive, $\pi(e_1^f, e_2^f; \Phi) < \pi(e_1^0, e_2^0; \Phi')$. $\square$

Figure EC.1 Regions A, B, C, and D in the proof of Theorem 1.



Note. Here, $\beta = 0.5$, $\alpha = 0.25$, $k = 0.7$.

Proof of Theorem 1. By Lemma EC.5, the optimal contract for (5) can be found by optimizing (γ_1, γ_2) with $f_1 = f_2 = 0$ and utilities (EC.8). Based on Lemma 1, we consider four sub-problems in Regions A-D; see Figure EC.1 for visualization.

Region A: $\gamma_2/\gamma_1 \geq \theta_{AB}$. We divide Region A into A1 and A2, depending on whether Agent 1's equilibrium effort is 0.

(A1): By Lemma 1, the equilibrium efforts are (e_1^β, e_2^α). Therefore, the principal's problem (5) within (A1) is

$$
\begin{aligned}
&\max_{\gamma_1,\gamma_2} (1-\gamma_1-\gamma_2)\left(e_1^\beta + k e_2^\alpha\right) \\
&\text{s.t. } \gamma_2 \geq \theta_{AB}\gamma_1 && \text{(A1.1)} \\
&\qquad \gamma_2 \leq \frac{1+\beta}{\beta}\gamma_1 && \text{(A1.2)} \\
&\qquad \gamma_1+\gamma_2 \leq 1 && \text{(A1.3)}
\end{aligned}
$$

Applying the Karush–Kuhn–Tucker (KKT) conditions, we find that because i) there is no interior solution and ii) if (A1.2) were binding, then its corresponding Lagrange multiplier would be negative, the optimal value is such that either (A1.1) or (A1.3) is binding. If (A1.3) is binding, then $\gamma_2 = 1-\gamma_1$ and therefore the objective function's value is 0, which is strictly dominated. If (A1.1) is binding, then such a contract also belongs to Region B, and will be taken into consideration there.

(A2): By Lemma 1, the equilibrium efforts are $(0, e_2^\alpha)$. Therefore, the principal's problem (5) in (A2) is

$$
\begin{aligned}
\phi^*_{A2^a} = &\max_{\gamma_1,\gamma_2} (1-\gamma_1-\gamma_2)\, k e_2^\alpha \\
&\text{s.t. } \gamma_1 \geq 0 && \text{(A2.1)} \\
&\qquad \gamma_2 \geq \frac{1+\beta}{\beta}\gamma_1 && \text{(A2.2)} \\
&\qquad \gamma_1+\gamma_2 \leq 1. && \text{(A2.3)}
\end{aligned}
$$

Applying the KKT conditions, the optimal solution turns out to make (A2.1) binding: $\gamma_1 = 0, \gamma_2 = 1/2$. The corresponding value of the objective function is $\phi^*_{A2^a} = \frac{k^2}{4}$.

Region B: For this region, we show that the directional derivative of the principal's objective function with respect to (γ_1,γ_2) along the direction $(1,-1)$ is positive and therefore the optimal solution lies on $\gamma_2 = \theta_{BC}\gamma_1$. Since $\Theta_{AB} \leq 0 \leq \Theta_{BC}$ in (B34), $\gamma_2/(1+\beta) \leq \sqrt{\frac{\gamma_1^2 k^2}{(1-\alpha)^2} + (1-k^2)(\gamma_1-\gamma_2)^2} \leq \gamma_2/(1-\alpha)$. Applying this inequality, the directional derivative of the principal's objective function with respect to (γ_1,γ_2) along the direction $(1,-1)$ is

$$
\begin{aligned}
&-\tfrac{\sqrt{2}}{2(1-\alpha)}\tfrac{1-\gamma_1-\gamma_2}{\gamma_2}\left(\frac{-2\gamma_1(1-\alpha)+\gamma_1\left(2-2\alpha-\frac{1}{1-\alpha}\right)k^2+2\gamma_2(1-\alpha)\left(1-k^2\right)}{\sqrt{\frac{\gamma_1^2k^2}{(1-\alpha)^2}+(1-k^2)(\gamma_1-\gamma_2)^2}} - (1-\alpha)(1-k^2)\right) \\
\geq\ & -\tfrac{\sqrt{2}}{2(1-\alpha)}(1-\gamma_1-\gamma_2)\max\Big\{-(1-\alpha)(1-k^2) + \tfrac{(\beta+1)\left(2\gamma_1(\alpha-1)+\gamma_1\left(-2\alpha+\frac{1}{\alpha-1}+2\right)k^2+2\gamma_2(\alpha-1)\left(k^2-1\right)\right)}{\gamma_2}, \\
&\qquad \tfrac{-2\gamma_1(1-\alpha)^2+\gamma_1(2(\alpha-2)\alpha+1)k^2-\gamma_2(\alpha-1)(2\alpha-1)(k-1)(k+1)}{\gamma_2}\Big\} \\
\geq\ & 0.
\end{aligned}
$$

The last inequality holds because in the curly bracket, i) the first expression is positive only when $\gamma_2 > \frac{(\beta+1)\left((2(\alpha-2)\alpha+1)k^2-2(1-\alpha)^2\right)}{(1-\alpha)^2(2\beta+1)(k^2-1)}\gamma_1 > \theta_{AB}\gamma_1$, which is not possible since $\gamma_2 \leq \theta_{AB}\gamma_1$ in Region (B34); ii) the second expression is positive only when $\gamma_2 > \frac{\left((2(\alpha-2)\alpha+1)k^2-2(1-\alpha)^2\right)}{(\alpha-1)(2\alpha-1)(k^2-1)}\gamma_1 > \theta_{AB}\gamma_1$, which is again infeasible. Thus, the maximum lies on the constraint $\gamma_2 = \theta_{BC}\gamma_1$. Substituting this binding constraint into the objective function and solving the first-order optimality condition yields the optimal shares to be given by (7).

Region C: Similar to Region B, the directional derivative along $(-1,1)$ is positive. Therefore, the optimal can be found on the line $\alpha_2 = \theta_{AB}\alpha_1$. Thus, the optimal shares are identical as the solution in region B.

Region D: Similar to Region A, we divide Region D into two regions depending on whether Agent 2's equilibrium effort is 0. Optimizing in both regions, one can find the optimal shares to be strictly dominated.

Comparing the payoff given by the optimal shares in Regions A-D, we find the optimal shares to be (7). □

LEMMA EC.6. *When $k > \kappa(\alpha)$, the optimal output is jointly convex in k and α.*

Proof. It follows directly computation from the optimal contract described in Theorem 1. □

EC.2. Reward Fairness

Proof of Lemma 4. When $f_1 = f_2 = 0$, (2) simplifies to

$$u_i(e_i; e_{-i}, \gamma_i, \gamma_{-i}, 0, 0) = v_i(e_i, e_{-i}, \gamma_i, 0) - \alpha\left[\gamma_i(e_1 + ke_2) - (\gamma_i + \gamma_{-i})k_i e_i\right]^+ - \beta\left[(\gamma_i + \gamma_{-i})k_i e_i - \gamma_i(e_1 + ke_2)\right], \tag{EC.13}$$

where $k_1 = 1$ and $k_2 = k$. The proof is similar to that of Lemma 1. □

Proof of Lemma 5. The proof is similar to that of Lemma 2. □

Finally, we consider the optimal contract. When $f_1 = f_2 = 0$, Agents' utilities (2) simplify to (EC.13). To prove Theorem 2, we first show that for any optimal output-sharing contract, $f_1 = f_2 = 0$; and then find the optimal contract solving (5) with $f_1 = f_2 = 0$.

Similar to the proof of Theorem 1, we show that $f_1 = f_2 = 0$ in all optimal output-sharing contracts in three steps: First, given a Nash equilibrium (e_1^f, e_2^f) to the effort game (4) under Φ, we propose in Lemma EC.7 a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ with no fixed-transfer payments that leads to both higher best effort responses and higher payoff at (e_1^f, e_2^f). Second, we show in Lemma EC.8 that any effort game with no fixed-transfer payments maximizing (EC.13) is supermodular. Finally, based on the supermodularity of the effort game under Φ' (Lemma EC.8) and the elevated best responses at (e_1^f, e_2^f) (Lemma EC.7, point 2), we show in Lemma EC.9 that there exists an equilibrium (e_1^0, e_2^0) such that $e_1^0 \geq e_1^f$ and $e_2^0 \geq e_2^f$, and thus $\pi(e_1^0, e_2^0, \Phi') \geq \pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$ (Lemma EC.7, point 1). As a result, we consider share-only contracts in the proof of Theorem 2.

LEMMA EC.7. *Under reward fairness, given any contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ and a Nash equilibrium (e_1^f, e_2^f) to the corresponding effort game (4), there exists a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ such that*

1. *Φ' leads to greater payoff when efforts are (e_1^f, e_2^f), i.e., $\pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$.*
2. *Agent i's best response to e_{-i}^f under Φ', denoted by $B_i(e_{-i}^f; \Phi')$, is greater than e_i^f, i.e., $B_i(e_{-i}^f; \Phi') \geq e_i^f$, for $i = 1, 2$.*

When at least one of f_1 and f_2 is strictly positive and $\alpha > 0$, at least one of the three inequalities (i.e., $\pi(e_1^f, e_2^f, \Phi') \geq \pi(e_1^f, e_2^f, \Phi)$, and $B_i(e_{-i}^f; \Phi') \geq e_i^f$ for $i = 1, 2$) are strict

Proof. The proof is similar to that of Lemma EC.3. □

LEMMA EC.8. *Under reward fairness, given any contract $\Phi = (\gamma_1, \gamma_2, 0, 0)$ with no fixed-transfer payments, the effort game (4) maximizing (EC.13) is supermodular.*

Proof. The proof is similar to that of Lemma EC.4. □

LEMMA EC.9. *Under reward fairness, for any output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ with non-zero fixed-transfer payments and equilibrium efforts (e_1^f, e_2^f), there exists a Nash equilibrium (e_1^0, e_2^0) to effort game (4) under the contract in Lemma EC.7 such that $e_i^0 \geq e_i^f$ for $i = 1, 2$, and thus, $\pi(e_1^0, e_2^0, \Phi') > \pi(e_1^f, e_2^f, \Phi)$.*

Proof. The proof is similar to that of Lemma EC.5. □

Proof of Theorem 2. The proof is similar to that of Theorem 1. □

EC.3. Income Fairness

Proof of Lemma 6. First, we analyze the best responses, and then, the Nash equilibrium in the effort provision game. We present the analysis for $\gamma_1 \geq \gamma_2$ only; the analysis for contracts such that $\gamma_1 < \gamma_2$ follows a similar argument, which is omitted for brevity. Under income fairness, their utilities are given by (3).

Similar to the proof of Lemma EC.1, the agents' best responses consist of three pieces:

$$e_1^*(e_2) = \begin{cases} \gamma_1 + \beta(\gamma_1 - \gamma_2), & e_2 < \frac{1}{k}\left(-\gamma_1(1+\beta) + \gamma_2\beta + \frac{f_1 - f_2}{\gamma_2 - \gamma_1}\right), \\ \frac{f_2 - f_1}{\gamma_1 - \gamma_2} - ke_2, & \frac{1}{k}\left(-\gamma_1(1+\beta) + \gamma_2\beta + \frac{f_1 - f_2}{\gamma_2 - \gamma_1}\right) \leq e_2 < \frac{1}{k}\left(-(1-\alpha)\gamma_1 - \alpha\gamma_2 + \frac{f_1 - f_2}{\gamma_2 - \gamma_1}\right), \\ \gamma_1 - \alpha(\gamma_1 - \gamma_2), & e_2 \geq \frac{1}{k}\left(-(1-\alpha)\gamma_1 - \alpha\gamma_2 + \frac{f_1 - f_2}{\gamma_2 - \gamma_1}\right), \end{cases}$$
$$e_2^*(e_1) = \begin{cases} k(\gamma_2 + \alpha(\gamma_1 - \gamma_2)), & e_2 < \frac{f_2 - f_1}{\gamma_1 - \gamma_2} - k^2(\gamma_1\alpha + \gamma_2(1-\alpha)), \\ \frac{1}{k}\left(\frac{f_2 - f_1}{\gamma_1 - \gamma_2} - e_1\right), & \frac{f_2 - f_1}{\gamma_1 - \gamma_2} - k^2(\gamma_1\alpha + \gamma_2(1-\alpha)) \leq e_2 < k^2(\beta\gamma_1 - (1+\beta)\gamma_2) + \frac{f_2 - f_1}{\gamma_1 - \gamma_2}, \\ [k(\gamma_2 - \beta(\gamma_1 - \gamma_2))]^+, & e_2 \geq \min\{k^2(\beta\gamma_1 - (1+\beta)\gamma_2) + \frac{f_2 - f_1}{\gamma_1 - \gamma_2}, \frac{f_2 - f_1}{\gamma_1 - \gamma_2}\}. \end{cases} \tag{EC.14}$$

The best responses are continuous. The middle pieces, linearly decreasing, are inverse functions of each other. These functions turn out to have three types of crossing points, i.e., there are three types of equilibria, namely:

(A) When $f_1 - f_2 \leq \zeta_{AB}(\gamma_1, \gamma_2)$, $(e_1^*, e_2^*) = (\gamma_1 + \beta(\gamma_1 - \gamma_2), k(\gamma_2 + \alpha(\gamma_1 - \gamma_2)))$;

(B) When $\zeta_{AB}(\gamma_1, \gamma_2) \leq f_1 - f_2 \leq \zeta_{BC}(\gamma_1, \gamma_2)$, (e_1^*, e_2^*) satisfies $e_1^* = \frac{f_2 - f_1}{\gamma_1 - \gamma_2} - ke_2^*$ or equivalently, $e_2^* = \frac{e_1^*(\gamma_2 - \gamma_1) - f_1 + f_2}{k(\gamma_1 - \gamma_2)}$;

(C) When $\zeta_{BC}(\gamma_1, \gamma_2) \leq f_1 - f_2 \leq \zeta_{CD}(\gamma_1, \gamma_2)$, (e_1^*, e_2^*) satisfies $e_1^* = \frac{f_2 - f_1}{\gamma_1 - \gamma_2} - ke_2^*$ or equivalently, $e_2^* = \frac{e_1^*(\gamma_2 - \gamma_1) - f_1 + f_2}{k(\gamma_1 - \gamma_2)}$;

(D) When $f_1 - f_2 \geq \zeta_{CD}(\gamma_1, \gamma_2)$, $(e_1^*, e_2^*) = (\gamma_1 - \alpha(\gamma_1 - \gamma_2), [k(\gamma_2 - \beta(\gamma_1 - \gamma_2))]^+)$;

where

$$\zeta_{AB}(\gamma_1, \gamma_2) := (\gamma_1 - \gamma_2)\left(\gamma_1\left(\alpha + \beta k^2 - 1\right) - \gamma_2\left(\alpha + (\beta+1)k^2\right)\right);$$
$$\zeta_{CD}(\gamma_1, \gamma_2) := \begin{cases} (\gamma_1 - \gamma_2)\left(\gamma_1\left(\alpha + \beta k^2 - 1\right) - \gamma_2\left(\alpha + (\beta+1)k^2\right)\right), & \gamma_2 \geq \beta/(1+\beta)\gamma_1, \\ (\gamma_1 - \gamma_2)(\gamma_1(\alpha - 1) - \gamma_2\alpha), & \gamma_2 \leq \beta/(1+\beta)\gamma_1. \end{cases}$$

□

Proof of Theorem 3. We solve for the optimal contract by sequentially finding first the optimal payments and then optimal shares within Regions A-D:

$$\begin{aligned} \max_{R\in\{A,B,C,D\}} \max_{(\gamma_1,\gamma_2)} \max_{(\gamma_1,\gamma_2,f_1,f_2)\in \text{Region}R} \quad & (1 - \gamma_1 - \gamma_2)(e_1^* + ke_2^*) - f_1 - f_2 \\ & \gamma_i \geq 0,\ \gamma_1 + \gamma_2 \leq 1, \qquad i = 1, 2. \\ & f_i \geq 0, \qquad i = 1, 2. \end{aligned} \tag{EC.15}$$

First, we solve the optimal fixed-transfer payments and potential optimal shares in each case.

- In Region A, since $\zeta_{AB}(\gamma_1,\gamma_2) \le 0$ and the equilibrium efforts are independent of f_1 and f_2, the objective function in (EC.15) is linear in f_1 and f_2. Since $f_1 - f_2 \le \zeta_{AB}$, the optimal fixed-transfer payments are $(f_1, f_2) = (0, -\zeta_{AB})$. The principal's payoff is correspondingly $(1 - 2\gamma_1)\left(\gamma_1\left(\beta + \alpha k^2 + 1\right) - \gamma_2\left(\beta + (\alpha - 1)k^2\right)\right)$. Optimizing this payoff subject to $\gamma_1 \ge 0$, $\gamma_2 \ge 0$ and $\gamma_1 + \gamma_2 \le 1$, we obtain the following optimal shares:
  —If $k \ge \sqrt{\frac{\beta}{1-\alpha}}$, $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(\frac{1}{4}, \frac{1}{4}, 0, 0\right)$, resulting in payoff $\pi^* = \frac{1+k^2}{8}$.
  —If $k < \sqrt{\frac{\beta}{1-\alpha}}$, $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(\frac{1}{4}, 0, 0, \frac{1+\beta+\alpha k^2}{16}\right)$, resulting in payoff $\pi^* = \frac{1+\beta+\alpha k^2}{8}$.
- In Regions B and C, the objective function in (EC.15), after substituting the equilibrium efforts, simplifies to $\frac{(2\gamma_2-1)f_1-(2\gamma_1-1)f_2}{\gamma_1-\gamma_2}$. It is straightforward to verify that the optimal contract is identical to that in Region A.
- In Region D,
  —If $\gamma_2 \ge \beta/(1+\beta)\gamma_1$, the objective simplifies to $(2\gamma_1 - 1)\left(\gamma_1\left(\alpha + \beta k^2 - 1\right) - \gamma_2\left(\alpha + (\beta+1)k^2\right)\right)$. The optimal contract is $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(\frac{1}{4}, \frac{1}{4}, 0, 0\right)$, leading to payoff $\pi^* = \frac{1+k^2}{8}$.
  —If $\gamma_2 < \beta/(1+\beta)\gamma_1$, the objective function simplifies to $(1-\gamma_1-\gamma_2)((1-\alpha)\gamma_1 - \alpha\gamma_2)$, which is maximized when $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(\frac{1}{2}, 0, 0, 0\right)$. The payoff is $\pi^* = \frac{1-\alpha}{4}$.

Selecting, across all regions, the maximum within-region principal's payoff yields (10). The payoffs are correspondingly

$$\pi^* = \begin{cases} \frac{1+k^2}{8}, & \max\left\{\sqrt{1-2\alpha}, \sqrt{\frac{\beta}{1-\alpha}}\right\} < k \le 1, \\ \frac{1+\beta+\alpha k^2}{8}, & \max\left\{\sqrt{\frac{1-2\alpha-\beta}{\alpha}}, 0\right\} \le k \le \sqrt{\frac{\beta}{1-\alpha}}, \\ \frac{1-\alpha}{4}, & 0 \le k \le \min\left\{\sqrt{1-2\alpha}, \sqrt{\frac{1-2\alpha-\beta}{\alpha}}\right\}. \end{cases}$$

Since, $\pi^N = 1/4$ when agents are inequality-neutral, we obtain that $\pi^* \le \pi^N$ for any α, β, k. □