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Why Do Managers Under-Delegate? A Co-Productive Principal-Agent Model

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Key words : Principal-agent; Co-production; Teams; Double moral Hazard; Contracting; Delegation; Entrepreneurship
History : June 29, 2022.

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1. Introduction

Many entrepreneurs and professionals are known to under-delegate, potentially limiting the growth of their venture (Maister 1993, White and White 2011, Yoo et al. 2016). As a notable counterexample, Larry Page, after working alone on a web crawler project called BackRub, approached Sergei Brin and offered him to join in and scale up the idea, which led to the creation of Google. Would Google have been so successful or even been created if Page had worked alone or if he had completely delegated the execution of the idea to Brin?

We study the genesis of teams within a co-productive Principal-Agent (PA) context. We consider a principal ("she") who initiates a new project (e.g., an entrepreneurial venture) and contemplates whether to partner with an agent ("he") and, if so, how to share the returns. After the team formation decision is made, the workers simultaneously and non-cooperatively choose how much effort to put in, resulting in one of the following three operating modes: Single Execution (SE) if the principal works alone; Delegated Execution (DE) if the agent works alone; and Collaborative Execution (CE) if they both contribute to the project. This single-task co-productive setting is a stylized representation of the dynamics that arise in many entrepreneurial ventures (Kamm and Nurick 1993), co-productive services (Roels 2014, Bellos and Kavadias 2019), innovation (Kavadias and Sommer 2009, Singh and Fleming 2010, Chan et al. 2021), self-managed teams (Hamel 2011, Lee and Edmondson 2017), academic research, and patent development, among others. We adopt a parametric co-production function, which allows efforts to be complementary or substitutable. We also assume limited liability—and thus no bidirectional fee-transfer payments—and no uncertainty that could lead to information asymmetry or recourse decisions.

We seek to assess how the equilibrium operating modes differ from the First-Best (FB), which we take as a proxy for assessing the long-term sustainability of the venture, e.g., from an external stakeholder's (for example, an investor's) perspective. The canonical PA model ignores co-productive opportunities and implicitly assumes that an agent has been hired to proceed to DE (Holmström 1979). Conversely, the canonical team production setting assumes that a team has been formed to proceed to CE (Holmström 1982). In contrast, we assume here that the principal may not want to partner with an agent in the first place and thus operate under SE; but if she does, she may want to contribute to the execution through CE or abstain from contributing through DE. We seek to answer the following questions:

- What operating modes emerge in equilibrium and how do they differ from the FB ones?
- Does the co-productive nature of the relationship always result in value creation, relative to the canonical PA setting? Or could it result in lower value?
- What initiatives could improve efficiency, that is, increase the total payoff? Shall we let the principal commit to her effort before the agent chooses his? Should an organization deprive the principal of the decision right to form a team and set incentives?

We obtain the following results. First, principals tend to *partner too little* with agents. But when they do, they tend to *delegate too much* the project execution to them, i.e., to contribute less than what they should. We thus develop an economic model explaining why professionals and entrepreneurs tend to under-delegate, but also offer a more nuanced perspective. Delegation is a two-step process: It involves, first, partnering with a collaborator and, second, taking a passive role in the project execution. We show that principals do not do enough of the former, but they do too much of the latter. Whereas most of the PA literature has looked for ways to control the agent, we find that a more fundamental problem is to control the principal to lead her to partner and contribute whenever necessary.

Second, we find that co-production is a double-edged sword, formalizing a conceptualization of the dual character of value creation/destruction (Echeverri and Skålén 2021). On one hand, co-production may *enhance* value creation by giving access to multiple productive resources. Specifically, relative to the canonical PA setting, not only is the agent available to produce, but so is also the principal. On the other hand, it may *impede* value creation due to the workers' strategic interactions. Specifically, the co-productive nature of the work gives the principal the opportunity to work alone, even though it might be more efficient if she partnered with the agent. Because the principal cares only about her own payoff, she ignores the fact that the agent's payoff—and thus, the total value—discontinuously drops when switching from CE or DE to SE. Also, a principal may benefit from having a higher cost to more easily convince the agent to put in more effort. This is reminiscent of the "lean and hungry" strategy in industrial organization, according to which one firm underinvests in cost reduction to accommodate entry (Fudenberg and Tirole 1984). Moreover, the prospect of co-production may lead the principal to offer the agent a higher share of output than she would do without co-production (as in the canonical PA model) so that the agent does not expect her to contribute.

Third, we find that it is in general not beneficial to let the principal commit to her effort level before the agent chooses his effort, even though the principal always prefers to do so (von Stengel 2010). In fact, it may even result in further value loss if efforts are complementary. What helps improve efficiency is to deprive the principal of her decision right to choose the mode of production and set incentives. For instance, an external stakeholder could require an entrepreneur to seek a partner[1] and share equity. In this case, the reward-sharing problem has a very simple solution: Allocate 100% of the equity to the principal under SE, 100% to the agent under DE, and 50%-50% to both workers under CE. However, this solution remains insufficient for non-standard projects that require very complementary efforts. In this case, we recommend adopting a different accounting mechanism, based on full and not fractional counting, as is sometimes done when assessing researchers' publication records (Korytkowski and Kulczycki 2019).

2. Literature Review

We contribute to three interconnected streams of literature: the literature on PA models, the literature on task allocation, and the literature on co-production.

[1] As a somewhat related example, the Twitter (now X) users voted in 2022 to request Elon Musk to appoint a CEO to second him (`https://www.npr.org/2022/12/19/1144071661/twitter-musk-poll-ceo`).

Principal-Agent Models. The PA literature typically assumes a default organizational structure and execution mode. For instance, the canonical PA model (Holmström 1979) assumes that an agent has been hired to participate in DE; and the canonical team production models (Holmström 1982, Kim and Wang 1998) assume that both the agent and the principal actively collaborate in CE. These models have been enhanced to encompass multiple tasks (Holmström and Milgrom 1991), providing team members with opportunities for knowledge sharing and helping each other (Siemsen et al. 2007, Crama et al. 2019), for the project to be executed over multiple periods (Bonatti and Hörner 2011, Georgiadis et al. 2014, Georgiadis 2014, Rahmani et al. 2017), and for sequential decision making (Winter 2004). Sticking to the one-task, full-information setting with simultaneous and sequential choices of effort, we take a step back from these models and investigate the fundamental issue of team genesis, allowing the principal to work alone (SE) or partner with an agent, in which case the operating mode can be either DE or CE.

Task Allocation. Like this study, some models in the literature investigate the choice of operating mode; however, this choice is often restricted to a dichotomous selection between SE and DE. In particular, the literature on property rights (Grossman and Hart 1986) studies who, between the principal and the agent, should own a key productive asset. Similarly, the literature on delegation analyzes who should make a critical decision about a project; see, e.g., Aghion and Tirole (1997), Prendergast (2002), Bester and Krähmer (2008), Wu et al. (2008), Hutchison-Krupat and Kavadias (2015). Relatedly, the task allocation literature (Itoh 2001, Hagiu and Wright 2019) investigates how to allocate a subset of tasks between a principal and an agent. This latter literature typically assumes a reward-sharing contract, similar to us; it also assumes that at least one task needs to be allocated to the agent so that the principal never executes the project alone, in contrast to our setting. More fundamentally, all these streams of literature assume some form of *ex-ante* allocation of property or decision rights, whereas we assume here that workers are free to exert effort or not. Thus, execution modes cannot be decided *a priori* but instead must emerge in equilibrium; in particular, the principal may not be able to enforce DE if the agent expects her to collaborate. Second, these streams of literature generally ignore the possibility for the principal and the agent to collaborate on a task, i.e., the task allocation is "either-or" and not "and."

Co-Production. Besides the models of team production (Holmström 1982, Kim and Wang 1998), co-production has recently been conceptualized within the service-dominant logic (Vargo and Lusch 2004). The fact that the service literature was until recently blind to the fact that customers may be more than simple service recipients is consistent with our result that principals (service providers) tend to not engage enough in CE. Lately, this literature has also introduced the concept of value co-destruction (Echeverri and Skålén 2021). Similarly, we find that the nature of co-production is in general value-enhancing, but that may sometimes impede the value-creation process.

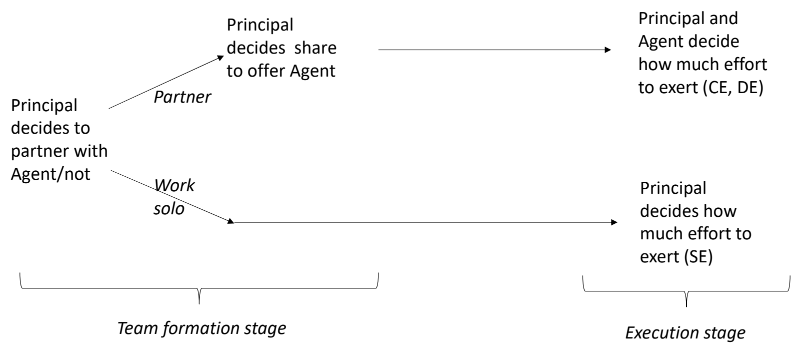
The trade-off between solo or collaborative work arises in numerous other applications such as: entrepreneurship, academic research, service co-production, innovation, team leadership (Rahmani et al. 2018), franchising (Bhattacharyya and Lafontaine 1995), and joint ventures (Sampson 2007) among others. We briefly review some of these streams of application.

In entrepreneurship, a lead entrepreneur must decide to either pursue her venture alone or seek partners to help them supply resources, such as funds (Kamm and Nurick 1993) or simply time (Yoo et al. 2016) to boost the growth of the venture at the cost of an equity share (Amit et al. 1990), potentially leading to double moral hazard (Wang and Zhou 2004). In many ventures and partnerships, an equal equity share is adopted (Bartling and von Siemens 2010, Farrell and Scotchmer 1988), as we find it is optimal to do under CE. Whether solo founders are more likely to succeed than multiple founders has been a topic of debate (Greenberg and Mollick 2018).

In co-productive service design, Karmarkar and Pitbladdo (1995) and Bellos and Kavadias (2019, 2021), utilizing a task-allocation model similar to Itoh (2001), investigate whether a service provider or its customers should perform a task. Considering a single-task model with a Constant Elasticity of Substitution (CES) co-production function, Roels (2014) characterizes the optimal execution modes from a system-wide perspective. Using an alternate model of co-production, we contribute to this service design literature by characterizing the equilibrium execution modes when the choices of effort and the setting of incentives are decentralized. We find that, while offering opportunities for co-production is always beneficial under the FB (since more productive resources are available), it is not necessarily the case under decentralized execution due to the workers' strategic interactions.

The literature on innovation and creativity has studied, from multiple angles, whether breakthrough innovations are more likely to emerge from teamwork, as argued by Wuchty et al. (2007), Singh and Fleming (2010), and Jones (2021) among others, than from working alone, as argued by Chan et al. (2021). In particular, Taylor and Greve (2006) find empirically that teams that span multiple domains of knowledge produce innovations that have more variance in value, but that individuals are more effective at combining diverse knowledge. Kavadias and Sommer (2009) find through a numerical study that teams are better suited to solve cross-functional problems whereas individuals are better suited for specialized problems. Chan et al. (2021) find that individuals are more likely to work alone than in a team for inventions that are hardly decomposable or more integral. Ahmadpoor and Jones (2019) report that team output tends toward the lower-impact member, which is explicable by the substantial complementarity in the tasks each team member performs. Our analytical results are consistent with these studies: We find that CE is optimal for projects that involve complementary efforts (e.g., non-standard projects requiring multiple individuals with different expertise) but that the execution should be carried alone by an individual (SE or DE) otherwise (e.g., standard projects, which can be efficaciously executed by one person).

Figure 1 **Timeline.**



Note. SE refers to Solo Execution, DE to Delegated Execution, and CE to Collaborative Execution.

3. The Model

We consider a principal who initiates a new *project* (e.g., an entrepreneurial venture) and is considering whether to partner with an agent, and if so, how to reward him to induce his effort.

In a *team formation stage*, the principal chooses to either work solo, in which case she captures the whole value, or to partner with an agent, in which case she also needs to decide how much output to share. Then, in an *execution stage*, efforts are exerted—only by the principal if she decided to work solo, and potentially by both if they teamed up and are incentivized to do so. The timeline is depicted in Figure 1.

3.1. Model Components

Let x and y denote the principal's and the agent's percentage of time commitment (lying between 0% and 100%) to the project—or *effort*. Assuming linear costs of effort, let $c_x > 1$ and $c_y > 1$ be their respective unit costs of effort.

The value of the project is increasing in their *joint effort* $E(x,y)$, defined as

$$E(x,y) = kx + ky + (1-2k)xy, \tag{1}$$

where $k \in [0,1]$. In this functional form, commonly used in the decision-analysis literature (Keeney and Raiffa 1976, Section 5.4.1), the joint effort is symmetric in x and y, strictly increasing in both arguments, with $E(0,0) = 0$ and $E(1,1) = 1$. In particular, $E(x,y) = xy$ when $k = 0$, $E(x,y) = (x+y)/2$ when $k = 1/2$, and $E(x,y) = x + y - xy$ when $k = 1$.

Because the cross-derivative $\frac{\partial^2 E}{\partial x \partial y}$ is negative if $k > 1/2$, zero when $k = 1/2$, and positive if $k < 1/2$, efforts are substitutes when $k > 1/2$, independent when $k = 1/2$, and complements when $k < 1/2$. Accordingly, we refer to k as the *degree of effort complementarity* (on an inverted scale). For any $0 \le x, y \le 1$, $E(x,y)$ is increasing in k; hence, efforts become less *efficacious* when they are more complementary. Typically, standard projects, which can be efficaciously executed by one person only, are associated with a low value of k, in contrast to less standard projects (Roels 2014).

We model the value of the project—or *output*—as a logarithmic function of the joint effort, i.e.,

$$V(E) = V_0 + \ln(E), \tag{2}$$

with the *base value* $V_0 > 0$. For simplicity, we do not consider participation constraints, which turn out to be immaterial if V_0 is large enough.

By default, the output is captured by the principal. If she chooses to partner with an agent, she needs to share the output to induce his effort. Such equity contracts are indeed very common in new ventures.[2] In an intra-organizational setting, output sharing can take the form of a formal project affiliation (e.g., patent co-authorship, movie credits) with more or less weight assigned depending on its prominence or recognition (e.g., first vs. second author). For simplicity, we consider a continuous output share, as is the case for patents (Kaptay 2020). Specifically, we denote by $\alpha \in [0,1]$ the principal's residual share of output; when $\alpha = 1$, the principal pursues the project alone. Accordingly, the principal's team formation decision reduces to her choice of output share, depending on whether $\alpha = 1$ (work solo) or $\alpha < 1$ (partner with an agent).

We assume no uncertainty. Uncertainty in output can be incorporated into our model as long as it keeps the analysis tractable (e.g., a multiplicative noise in (1) or an additive noise in (2)). We leave for future investigation the modeling of uncertainty that could lead to asymmetric information or trigger recourse decisions.

3.2. Game Formulation

We present the game in reverse order of its timing. First, we consider the execution stage, after the output share has been offered. For a given α, we assume that both the agent and the principal choose their effort simultaneously and non-cooperatively:

$$x^*(\alpha) \in \arg\max_{x\in[0,1]} \alpha V(E(x, y^*(\alpha))) - c_x x, \tag{3}$$

$$y^*(\alpha) \in \arg\max_{y\in[0,1]} (1-\alpha) V(E(x^*(\alpha), y)) - c_y y. \tag{4}$$

If the principal chooses to work solo in the team formation stage of the game, she sets $\alpha = 1$; then, the agent does not contribute, i.e., $y^*(1) = 0$. Hence, (3)-(4) fully describes the project execution, irrespective of whether the principal chooses to form a team or not.

Because the workers' payoffs are concave and their action sets are convex and bounded, there always *exists* a pure-strategy Nash equilibrium. Moreover, the game is submodular since

[2] Equity contracts with fixed-fee transfers turn out to be second-best optimal (Bhattacharyya and Lafontaine 1995). However, we assume limited liability, which prevents bidirectional fixed-fee transfers.

$\frac{\partial^2 V(E(x,y))}{\partial x \partial y} = -k^2/E^2 < 0$. That is, efforts are *strategic substitutes* (even if the joint effort (1) involves complementary efforts, i.e., if $k < 1/2$).

In principle, there may be *multiple* equilibria. Let $\Psi(\alpha)$ be the set of pure-strategy Nash equilibria, i.e.,

$$\Psi(\alpha) = \{(x^*(\alpha), y^*(\alpha)) \text{ s.t. (3)-(4) holds}\}.$$

In the case of multiple equilibria, we assume that, consistent with the hierarchical nature of the relationship, the principal has the decision right to select the one that gives her the highest payoff $\Pi(\alpha)$,[3] in which

$$\Pi(\alpha) \doteq \max_{(x^*(\alpha),y^*(\alpha))\in\Psi(\alpha)} \alpha V(E(x^*(\alpha), y^*(\alpha)) - c_x x^*(\alpha). \tag{5}$$

Next, we consider the team formation stage of the game. Anticipating the equilibrium effort levels, the principal sets her share of output α^* to maximize her payoff by solving:

$$\max_{\alpha\in[0,1]} \Pi(\alpha). \tag{6}$$

Whenever the principal earns the same payoff working alone or with the agent, we assume that the tie is broken in favor of working alone.

3.3. Operating Modes

For any α, the project operates under one of the following *operating modes*:

- *Single Execution* (SE) when the principal works alone, i.e., $x^*(\alpha) > 0$ and $y^*(\alpha) = 0$, and therefore no team is formed; this arises, in particular, when $\alpha = 1$;
- *Delegated Execution* (DE) when the principal fully delegates the project execution to the agent, i.e., $x^*(\alpha) = 0$ and $y^*(\alpha) > 0$, as happens in the canonical PA model (Holmström 1979); this arises, in particular, when $\alpha = 0$;
- *Collaborative Execution* (CE) when the principal teams up with the agent and they both contribute to its execution, i.e., $x^*(\alpha) > 0$ and $y^*(\alpha) > 0$, as happens in canonical team production models (Holmström 1982).

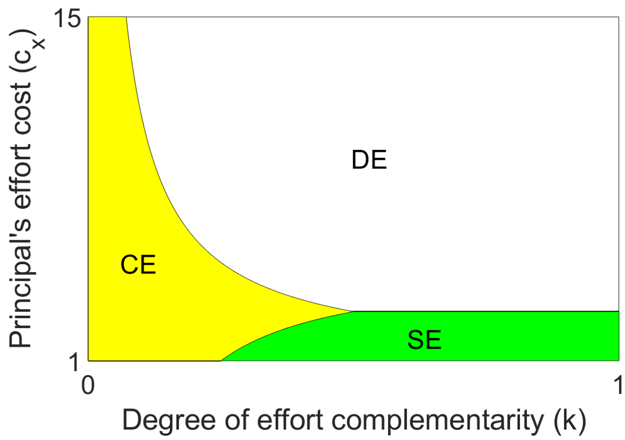
3.4. First-Best Benchmark

As a benchmark, we consider the first-best (FB) solution, which optimizes the choice of effort to maximize the total payoff, i.e., the sum of the principal's and the agent's payoffs:

$$(x^{FB}, y^{FB}) = \arg \max_{x\in[0,1], y\in[0,1]} V(E(x,y)) - c_x x - c_y y. \tag{7}$$

[3] Other equilibrium selection rules are of course possible. The region where multiple equilibria arise turns out to be relatively thin (see Figure 3), so our main insights are robust to the choice of rule.

Figure 2 FB operating modes.



Note. Here, $V_0 = 7$ and $c_y = 3$. A low value of k corresponds to a high degree of effort complementarity and *vice versa.*

The next proposition characterizes the FB operating modes. Its proof, like all other proofs and supporting results, appears in the electronic companion. To facilitate the exposition, define the following *solo* efforts: $\bar{x}_S \doteq 1/c_x$ and $\bar{y}_S \doteq 1/c_y$; and the following *duo* efforts when $k \neq 1/2$:

$$\bar{x}_D \doteq \frac{(1-2k) - 2kc_x + \sqrt{(1-2k)^2 + 4k^2 c_x c_y}}{2(1-2k)c_x}, \quad \bar{y}_D \doteq \frac{(1-2k) - 2kc_y + \sqrt{(1-2k)^2 + 4k^2 c_x c_y}}{2(1-2k)c_y}, \tag{8}$$

and when $k = 1/2$, $\bar{x}_D \doteq \frac{1}{2c_x}$ and $\bar{y}_D \doteq \frac{1}{2c_y}$.

Proposition 1. *The FB operating modes are:*

- *CE with* $x^{FB} = \bar{x}_D \in (0,1)$ *and* $y^{FB} = \bar{y}_D \in (0,1)$ *if either* $1 - 2k > k\,|c_x - c_y|$ *or* $k = 1/2$ *and* $c_x = c_y$*;*
- *DE with* $x^{FB} = 0$ *and* $y^{FB} = \bar{y}_S$ *if* $k\,(c_x - c_y) \geq \max\{0, 1-2k\}$*;*
- *SE with* $x^{FB} = \bar{x}_S$ *and* $y^{FB} = 0$ *if* $k\,(c_y - c_x) \geq \max\{0, 1-2k\}$*.*

Figure 2 depicts the FB operating modes as a function of the degree of effort complementarity k and the principal's unit cost of effort c_x. Consistent with the empirical research on innovation and analytical research on service co-production reviewed in §2, CE is optimal when efforts are complementary (i.e., non-standard projects requiring multiple individuals) and when the principal and the agent have comparable costs of effort; moreover, the more complementary the efforts, the less critical it is that their costs of effort are similar. Otherwise, the project should be executed by only one person, and the choice between SE and DE is entirely determined by the workers' relative efficiencies (costs of effort). We next analyze how the equilibrium operating modes depart from the FB when the team formation decision is made by the principal and the execution is decentralized.

4. Analysis

We solve the game by backward induction, considering first the execution stage and then the team formation stage.

4.1. Effort Choice Game

To characterize the equilibrium efforts, define the following *solo* efforts,

$$x_S(\alpha) \doteq \frac{\alpha}{c_x}, \tag{9}$$

$$y_S(\alpha) \doteq \frac{1-\alpha}{c_y}, \tag{10}$$

which respectively correspond to the principal's (resp., agent's) optimal effort solving (3) (resp., (4)) if the agent (resp., principal) is not putting in any effort, i.e., $y=0$ (resp., $x=0$). Since $\alpha \in [0,1]$ and $c_x, c_y > 1$, $0 \le x_S(\alpha), y_S(\alpha) \le 1$. Note that $x_S(1) = \bar{x}_S$ and $y_S(0) = \bar{y}_S$.

Define also the following *duo* efforts when $k \neq 1/2$:

$$x_D(\alpha) \doteq \frac{1}{2}x_S(\alpha) - \frac{k}{1-2k} + \frac{1}{2(1-2k)}\sqrt{\frac{x_S(\alpha)}{y_S(\alpha)}\left(x_S(\alpha)y_S(\alpha)(1-2k)^2 + 4k^2\right)}, \tag{11}$$

$$y_D(\alpha) \doteq \frac{1}{2}y_S(\alpha) - \frac{k}{1-2k} + \frac{1}{2(1-2k)}\sqrt{\frac{y_S(\alpha)}{x_S(\alpha)}\left(x_S(\alpha)y_S(\alpha)(1-2k)^2 + 4k^2\right)}, \tag{12}$$

and when $k = 1/2$, $x_D(\alpha) \doteq \frac{1}{2}x_S(\alpha)$ and $y_D(\alpha) \doteq \frac{1}{2}y_S(\alpha)$. Note that (11)-(12) has the same functional form as (8) after replacing $x_S(\alpha)$ with $\bar{x}_S$ and $y_S(\alpha)$ with $\bar{y}_S$.

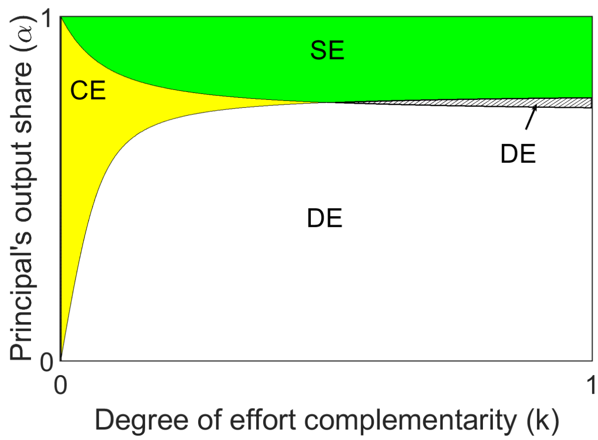
The next lemma characterizes the equilibrium operating modes for any α, keeping in mind that, when multiple equilibria exist, the principal selects the one that yields her the highest payoff.

Lemma 1. *There exist functions $\underline{\alpha}(k)$ and $\overline{\alpha}(k)$, with $\underline{\alpha}'(k) > 0$ and $\overline{\alpha}'(k) < 0$ and $\underline{\alpha}(k) \le \overline{\alpha}(k)$ if and only if $k \le 1/2$, such that, for any α, the equilibrium operating mode solving* (5) *is:*

- *DE with $x^*(\alpha) = 0$ and $y^*(\alpha) = y_S(\alpha)$ if $\alpha \le \underline{\alpha}(k)$;*
- *CE with $x^*(\alpha) = x_D(\alpha)$ and $y^*(\alpha) = y_D(\alpha)$ if $\underline{\alpha}(k) < \alpha < \overline{\alpha}(k)$;*
- *SE with $x^*(\alpha) = x_S(\alpha)$ and $y^*(\alpha) = 0$ when $\alpha \ge \overline{\alpha}(k)$ and $\alpha > \underline{\alpha}(k)$;*

The lemma is illustrated in Figure 3. As k increases, the equilibrium operating mode transitions from CE to either DE or SE since collaboration is less valuable when efforts are more substitutable; and when α increases, it transitions from DE to potentially CE, and then finally to SE. When $k \ge 1/2$, there is a region (depicted with stripes) where all three operating modes are equilibria; in this case, the principal's payoff is higher under DE (Lemma EC.3), which will then be the selected equilibrium in (5). Outside that region, the equilibrium is unique. Within each operating mode, the characterization of the equilibrium efforts mirrors that of the FB efforts, but with less intensity since, when $\alpha \in (0,1)$, $x_S(\alpha) < \bar{x}_S$ and $y_S(\alpha) < \bar{y}_S$, consistent with Holmström (1982).

Figure 3 Equilibrium operating modes.



Note. Here, $V_0 = 7$, $c_x = 9$, and $c_y = 3$. In the striped region, all three operating modes are equilibria, out of which the principal selects DE. The function $\underline{\alpha}(k)$ in Lemma 1 is the increasing upper boundary of the DE region (including the striped region). The function $\overline{\alpha}(k)$ is the decreasing boundary between the CE and SE regions when $k \le 1/2$ and the decreasing lower boundary of the striped region when $k > 1/2$. A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

4.2. Team Formation

We next characterize the optimal output share α^*, solving (6). After plugging the equilibrium efforts into the principal's payoff function, we can express the principal's payoff under CE, DE, and SE as follows:

$$\Pi_{CE}(\alpha) \doteq \alpha V(E(x_D(\alpha), y_D(\alpha))) - c_x x_D(\alpha), \tag{13}$$

$$\Pi_{DE}(\alpha) \doteq \alpha V(E(0, y_S(\alpha))), \tag{14}$$

$$\Pi_{SE}(\alpha) \doteq \alpha V(E(x_S(\alpha), 0)) - c_x x_S(\alpha). \tag{15}$$

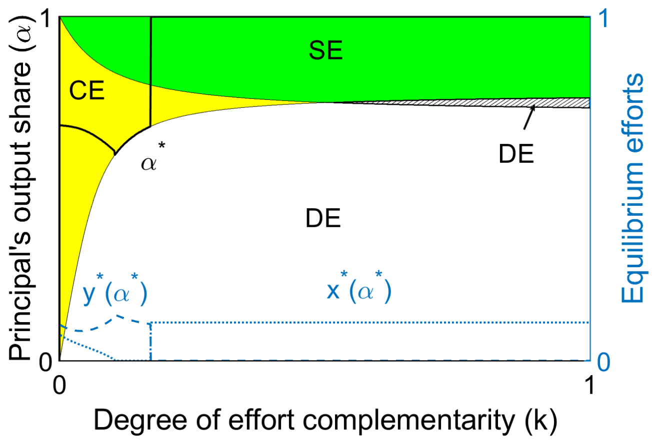
Using Lemma 1 to determine when each operating mode arises in equilibrium and removing, for simplicity, the argument from $\underline{\alpha}(k)$ and $\overline{\alpha}(k)$, we can then express the principal's problem (6) as:

$$\alpha^* = \arg\max \mathbb{1}_{[\alpha \in [0,\underline{\alpha}]]} \Pi_{DE}(\alpha) + \mathbb{1}_{[\alpha \in (\underline{\alpha},\overline{\alpha})]} \Pi_{CE}(\alpha) + \mathbb{1}_{[\alpha \in [\overline{\alpha},1] \cap (\underline{\alpha},1]]} \Pi_{SE}(\alpha), \tag{16}$$

in which $\mathbb{1}_{[X]}$ is the indicator function, equal to 1 if X is true, and zero otherwise. Although some constraint sets are not closed, the maximization problem is well defined (Lemma EC.4).

When V_0 is sufficiently large, both workers earn nonnegative profit in equilibrium (and thus, modeling their participation constraints is irrelevant) and $\alpha^* > 1/2$ (see Lemma EC.6). That is, the principal always seeks to capture at least 50% of the output. While it is efficient (from a FB perspective) for the principal to capture a large share under SE (indeed, $x^*(\alpha^*) = x^*(1) = \bar{x}_S = x^{FB}$), it is clearly detrimental under DE (since $y^*(\alpha^*) \le y_S(0) = \bar{y}_S = y^{FB}$). It is also detrimental

Figure 4 Equilibrium operating modes, optimal output share α^*, and equilibrium efforts.



Note. Here, $V_0 = 7$, $c_x = 9$, and $c_y = 3$. A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

under CE since, as we will show in §5.2, the total payoff is maximized under CE when the output is evenly shared with the agent, i.e., when $\alpha = 1/2$.

Equation (16) indicates that, for any given operating mode, the principal is constrained in her optimizing the output share. Although it does not really matter for SE, since it is optimal in that case for the principal to set $\alpha = 1$, it may constrain her under DE or CE. In particular, in canonical PA models, which ignore opportunities for co-production and implicitly assume DE, the principal is usually free to optimize her output share on $[0,1]$, while here she is restricted to optimize it on $[0, \underline{\alpha}]$. In fact, $\underline{\alpha}$ may happen to be a local maximum of $\Pi(\alpha)$. Hence, if $\arg\max_{[0,1]} \Pi_{DE}(\alpha) > \underline{\alpha}$, a canonical PA model may prescribe giving the principal a higher share of output under DE than what is truly feasible under co-production. The reason for this infeasibility is that the agent does not exert $y_S(\alpha)$ when $\alpha > \underline{\alpha}$ (as would be assumed in a canonical PA model), because he expects the principal to contribute to the project execution (accordingly, the agent exerts $y_D(\alpha)$).

This reveals a potential flip side to co-production: While it is in principle beneficial to have access to more operating modes (CE, DE, SE) than just DE, as more value can be created with more productive resources, the agent's anticipation of CE also constrains the principal's value capture optimization under DE. That is, under DE, a principal may be worse off because there exists an opportunity for co-production. To highlight when this happens, we distinguish two cases under the DE operating mode, depending on whether the principal's optimization is unconstrained, i.e., $\alpha^* < \underline{\alpha}$, which we denote as $\mathrm{DE_U}$, or constrained, i.e., $\alpha^* = \underline{\alpha}$, which we denote as $\mathrm{DE_C}$.

4.3. Equilibrium Operating Modes

We now characterize the equilibrium operating modes. To convey some preliminary intuition, consider Figure 4, which superimposes on Figure 3 the optimal output share α^*, solving (16) as a function k. The regions traversed by α^* indicate the equilibrium operating modes at α^* as a function of k. Specifically, as k increases from 0 to 1, the equilibrium operating mode starts in CE, then switches to $\mathrm{DE_C}$ (since it lies at the boundary between DE and CE), and finally jumps to SE. The equilibrium efforts, depicted in blue using the right vertical scale, are both positive when k is small (under CE). In this region, the agent's effort is convex in k, whereas the principal's effort is decreasing in k. What explains this behavior is that it is optimal for the principal to give more output share to the agent as k increases within this region (that is, α^* decreases in k), while workers become more efficacious as k increases (i.e., the joint effort function $E(x,y)$, for any given (x,y), increases in k, as their efforts become less complementary). These two drivers are aligned to induce the principal to exert less effort as k increases, but are conflicting for the agent, resulting in a non-monotone behavior of his effort with respect to k. Once the principal's effort hits zero, it stays at zero as k keeps increasing, which corresponds to the $\mathrm{DE_C}$ operating mode. Within that region, the agent's equilibrium effort, equal to $y_S(\alpha^*)$, is decreasing in k because $\alpha^* = \underline{\alpha}(k)$ is increasing in k. Finally, once the equilibrium operating mode transitions to SE, the agent's equilibrium effort drops to zero, whereas the principal's equilibrium effort jumps to $x_S(1)$, which is independent of k and thus remains constant as k keeps increasing until it reaches 1.

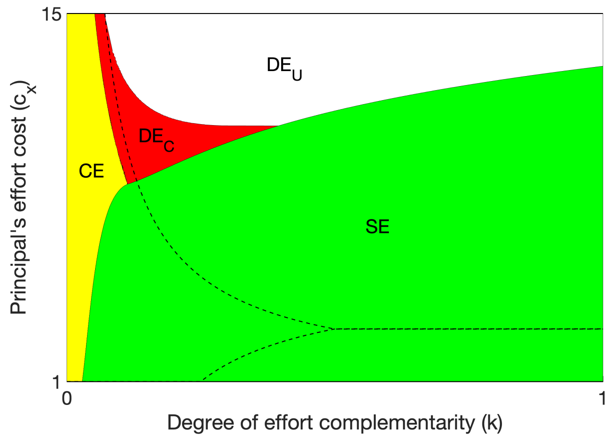
As k increases, the transition from CE to DE or SE is consistent with the FB solution (Figure 2). The next transition, from DE to SE, contrasts with the FB solution since the choice between DE and SE is driven not only by the workers' relative costs (as in the FB solution) but also by their degree of effort complementarity (unlike the FB solution). This indicates that the decentralization of the decision making (in both the team formation decision and the project execution) leads to not only less intense efforts than the FB efforts (Lemma 1), but also different trade-offs in the choice of operating modes.

Moving beyond this particular example, the next proposition characterizes the equilibrium operating modes in terms of k and c_x. The proposition is illustrated in Figure 5.

Proposition 2. *There exist thresholds $1 \le c_1 \le c_2 \le c_3 < \infty$ and continuously differentiable functions $k_1(c_x) > 0$ and $k_3(c_x) > k_2(c_x) > 0$ with $k_1'(c_x) \ge 0$, $k_2'(c_x) \le 0$, and $k_3'(c_x) \le 0$, $k_1(c_1) = k_2(c_1)$, and $k_1(c_2) = k_3(c_2)$, such that the equilibrium operating modes at α^* are as follows.*

- *If $c_x < c_1$, CE if $k < k_1(c_x)$ and SE if $k \ge k_1(c_x)$;*
- *If $c_1 \le c_x < c_2$, CE if $k < k_2(c_x)$, DE_C if $k_2(c_x) \le k < k_1(c_x)$, and SE if $k \ge k_1(c_x)$;*
- *If $c_2 \le c_x \le c_3$, CE if $k < k_2(c_x)$, DE_C if $k_2(c_x) \le k \le k_3(c_x)$, DE_U if $k_3(c_x) < k < k_1(c_x)$, and SE if $k \ge k_1(c_x)$;*
- *If $c_x > c_3$, CE if $k < k_2(c_x)$, DE_C if $k_2(c_x) \le k \le k_3(c_x)$, and DE_U if $k_3(c_x) < k$.*

Figure 5 **Equilibrium operating modes at the optimal output share α^*.**



Note. Here, $V_0 = 7$ and $c_y = 3$. The function $k_1(c_x)$ in Proposition 2 is the increasing upper boundary of the SE region. Define c_3 such that $k_1(c_3) = 1$. The function $k_2(c_x)$ is the decreasing upper boundary of the CE region and thus meets $k_1(c_x)$ at some c_1. The function $k_3(c_x)$ is the decreasing upper boundary of the $\mathrm{DE_C}$ region, and thus meets $k_1(c_x)$ at some c_2. The black dashed line represents the boundaries of the FB operating modes (Figure 2). A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

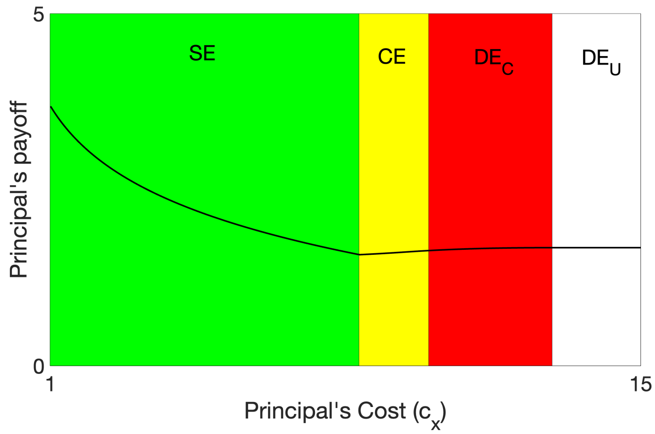
Because there is no agency issue under SE (since $x^*(\alpha^*) = x^*(1) = x^{FB}$), when SE is the FB operating mode, it also arises in equilibrium. The principal has indeed no incentive to operate differently since the amount of value created is maximized (given that it is FB) and since she is able to capture all of it. As a result, SE is more prevalent in equilibrium than in the FB. Figure 5 confirms that the region where SE arises in equilibrium (green) is larger than the one in the FB solution (as depicted by the dashed line). We conclude that principals tend to operate too often alone, without forming a team, so as to retain the full ownership of the venture.

When the principal chooses to team up, we observe in Figure 5 that DE may arise in equilibrium when it would in fact be optimal to operate under CE, but the converse does not hold. This is because CE is particularly inefficient under decentralized execution due to double moral hazard, whereas DE only suffers from single moral hazard. Moreover, CE requires that the principal put in some effort—and is thus associated with a direct cost for the principal—unlike DE. Hence, when the principal chooses to partner with an agent (which happens too infrequently), she tends to over-delegate and under-contribute.

In sum, we obtain that, when there are opportunities for co-production, the canonical DE setting of the PA model often does not arise. Paradoxically, this is not because the two workers end up co-producing—in fact, we see less CE in equilibrium than in the FB. But it is because the principal prefers not to partner with an agent in the first place and work solo instead (SE).

As noted earlier, co-production may constrain the principal in her choice of output share under DE, forcing her to give the agent a higher share $1 - \underline{\alpha}$ than she would if co-production were not an

Figure 6 Principal's payoff



Note. Here, $V_0 = 7$, $k = 0.1$ and $c_y = 3$.

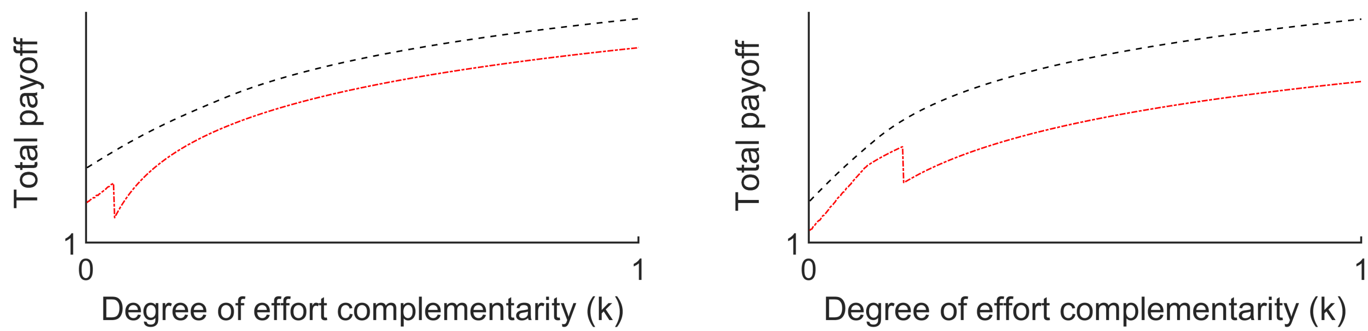
option (as is assumed in the canonical PA model). In other words, under $\mathrm{DE_C}$ a principal benefits from having a higher cost of effort so that the adoption of CE becomes less valuable, forcing the agent to work alone. Figure 6 confirms that the principal's payoff $\Pi(\alpha^*)$ is indeed increasing in c_x under $\mathrm{DE_C}$. The same behavior may also arise under CE because a less efficient principal is less likely to be asked to put in a large effort under CE and can therefore rely more on the agent. Hence, a higher cost generates a higher payoff, as in Fudenberg and Tirole (1984). (Naturally, the principal's payoff is decreasing in c_x under SE and constant with respect to c_x under $\mathrm{DE_U}$.)

Figure 7 compares the equilibrium total payoff (dash-dotted red curve), i.e., $V(x^*(\alpha^*), y^*(\alpha^*)) - c_x x^*(\alpha^*) - c_y y^*(\alpha^*)$, to the FB total payoff (solid black curve), i.e., $V(x^{FB}, y^{FB}) - c_x x^{FB} - c_y y^{FB}$, in the top panel and the corresponding (equilibrium or optimal) operating modes in the bottom panel. We make this comparison for two different values of c_x (left and right panels). We observe that, while the equilibrium total payoff is generally increasing in k (as is the FB total payoff, which is natural given that efforts become more efficacious as k increases), it exhibits a downward jump whenever the equilibrium operating mode switches from CE or DE to SE. What explains this jump is that the principal cares only about her own payoff when choosing to partner or not. While her payoff evolves continuously as she decides to no longer partner with the agent when k increases, the agent's payoff discontinuously drops to zero, resulting in a discontinuous drop in the total payoff. (This discontinuity in the total payoff only arises when the equilibrium operating mode switches from CE or DE to SE because the optimal share α^* is continuous in k when the equilibrium operating mode switches from DE to CE (or vice versa), resulting in a continuous evolution (as k changes) of both the principal's and the agent's payoffs within these regions.)

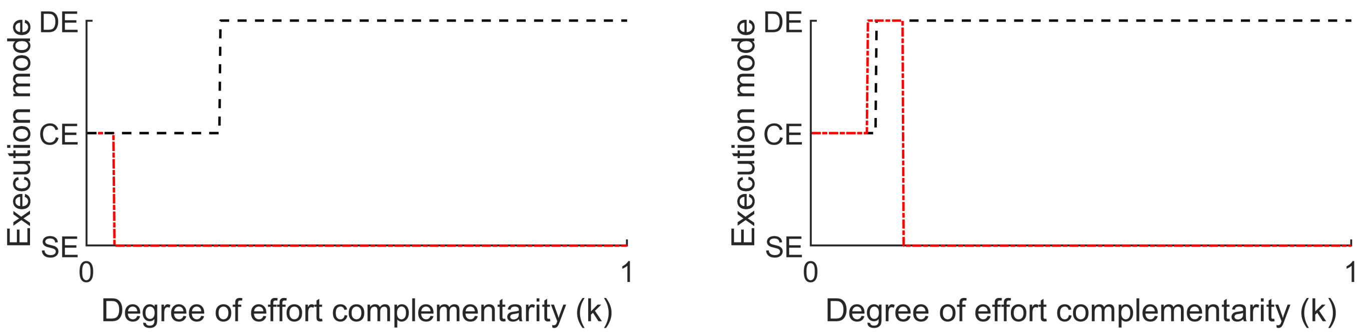
From the top panels in the figure, it appears that the optimality loss of operating under SE rather than under CE or DE, if they are the FB operating modes, can be significant and it is

Figure 7 **Total payoff and operating modes under the FB (dashed black) and in equilibrium under the optimal output share α^* (dash-dotted red).**

(a) Total Payoff



(b) Operating Modes



Note. Here, $c_x = 5$ (left) and $c_x = 9$ (right), $V_0 = 7$, and $c_y = 3$. A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

particularly severe at the point of discontinuity. At this point, the principal is effectively destroying value through her decision to no longer partner with the agent and work alone. In sum, our analysis suggests that, while most of the PA literature has focused on controlling the agent, it can be even more important to find ways to control the principal as we explore next.

5. Toward Improving Efficiency

How do we improve efficiency, i.e., increase the total payoff? We consider three mechanisms. In §5.1, we change the sequence of effort choice in the execution stage by letting the principal commit to her effort before letting the agent choose his. A principal always finds it beneficial to move first (von Stengel 2010), but does it result in a higher total payoff? In §5.2 we discuss the effect of taking away from the principal the team formation and reward-splitting decision rights while keeping the project execution decentralized. In §5.3 we briefly discuss moving from fractional to full counting (Holmström 1982, Korytkowski and Kulczycki 2019).

5.1. Principal's Effort Precommitment

In this section, we consider a situation where the principal is able to choose her effort before the agent and make it observable to him. That is, in the execution stage, (3)-(4) is replaced with

$$\hat{x}^*(\alpha) \in \arg\max_{x\in[0,1]} \alpha V(E(x,\hat{y}^*(\alpha,x))) - c_x x, \tag{17}$$

$$\hat{y}^*(\alpha,x) \in \arg\max_{y\in[0,1]} (1-\alpha)V(E(x,y)) - c_y y. \tag{18}$$

We denote with a 'hat' the quantities that are specific to this sequence of events, to distinguish them from the base case. Although the equilibrium turns out to be unique in this case, let $\hat{\Psi}(\alpha)$ denote the set of equilibria so that (5) can be adapted.

As in the base case, we solve the game backward. We first consider the agent's choice of effort, then the principal's, next the resulting equilibrium operating mode for any given α, and finally the optimization of the output share by the principal. We still ignore participation constraints by assuming that V_0 is large enough (Lemma EC.9). To simplify the exposition, we assume that $k>0$.

5.1.1. Effort Choice Game. For any given output share $\alpha\in[0,1]$ and principal's effort $x\in[0,1)$, the agent's equilibrium effort (18) is equal to:

$$\hat{y}^*(\alpha,x) = \max\left(y_S(\alpha) - \frac{kx}{k+(1-2k)x}, 0\right). \tag{19}$$

Hence, when $y_S(\alpha) < k/(1-k)$, the agent has no incentive to exert effort if the principal's effort exceeds a particular level x_0, defined as:

$$x_0(\alpha) \doteq \frac{k y_S(\alpha)}{k y_S(\alpha) + \max\{0, k - y_S(\alpha)(1-k)\}}. \tag{20}$$

In other words, a high level of effort by the principal demotivates the agent to put in any effort.

We next characterize the principal's choice of effort (17), anticipating the agent's best response (19). On one hand, if the principal exerts an effort greater than $x_0(\alpha)$, she anticipates that the agent will then exert no effort and, therefore, that she will work solo, making it optimal for her to exert $x_S(\alpha)$. On the other hand, if the principal exerts an effort smaller than $x_0(\alpha)$, she anticipates that the agent will put in some effort, and therefore, that there is an opportunity for co-production, making her exert the following duo effort:

$$\hat{x}_D(\alpha) \doteq \begin{cases} x_S(\alpha) - \frac{k}{1-2k} & \text{if } k < \frac{x_S(\alpha)}{2x_S(\alpha)+1}; \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Correspondingly, let $\hat{y}_D(\alpha) \doteq y^*(\alpha, \hat{x}_D(\alpha))$ denote the agent's duo effort. Note that when $k \geq \frac{x_S(\alpha)}{2x_S(\alpha)+1}$, which happens in particular when $k \geq 1/2$, the principal is not willing to collaborate with the agent.

In sum, under SE, $(\hat{x}^*, \hat{y}^*) = (x_S(\alpha), 0)$; under DE, $(\hat{x}^*, \hat{y}^*) = (0, y_S(\alpha))$; and under CE, $(\hat{x}^*, \hat{y}^*) = (\hat{x}_D(\alpha), \hat{y}_D(\alpha))$. To make CE feasible, we need $\hat{x}_D(\alpha) > 0$ (otherwise CE degenerates into DE) and $\hat{x}_D(\alpha) < x_0(\alpha)$ (otherwise, $\hat{y}_D(\alpha) = 0$ and CE degenerates into SE). Similarly, for SE to be feasible, we need $x_S(\alpha) \geq x_0(\alpha)$, because otherwise, the agent would want to exert positive effort whenever the principal exerts $x_S(\alpha)$ (i.e., $\hat{y}^*(\alpha, x_S(\alpha)) > 0$), resulting in CE. Hence, the feasibility of CE and SE is only guaranteed for certain values of α. In contrast, DE is feasible for any value of α since it can easily be enforced by setting $x = 0$. Besides feasibility, the equilibrium operating mode also needs to be optimal for the principal. Specifically, when the principal has the opportunity to execute the project alone (i.e., when $x_S(\alpha) \geq x_0(\alpha)$), she will evaluate her payoff under SE in comparison to her payoff if she teams up with the agent, under either CE or DE. The next lemma formalizes this discussion.

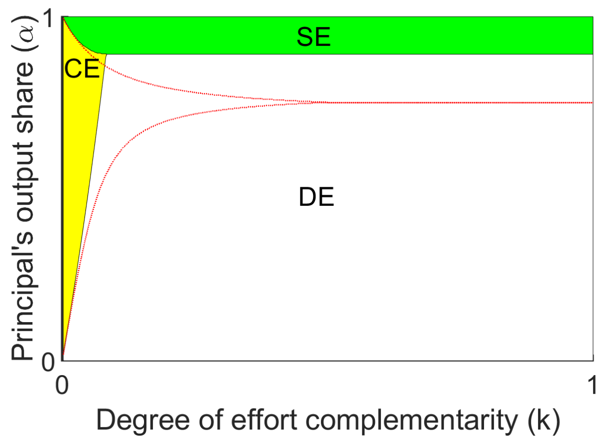
Lemma 2. *There exist functions $\underline{\hat{\alpha}}(k)$ and $\overline{\hat{\alpha}}(k)$, with $\underline{\hat{\alpha}}(k) \leq \overline{\hat{\alpha}}(k)$, $\underline{\hat{\alpha}}(k)' \geq 0$ and $\overline{\hat{\alpha}}'(k) \leq 0$, such that, for any α, the equilibrium operating mode solving* (5) *(with $\hat{\Psi}(\alpha)$ being the set of equilibria* (17)-(18)*) is:*

- *DE with $\hat{x}^*(\alpha) = 0$ and $\hat{y}^*(\alpha) = y_S(\alpha)$ if $\alpha < \underline{\hat{\alpha}}(k)$;*
- *CE with $\hat{x}^*(\alpha) = \hat{x}_D(\alpha)$ and $\hat{y}^*(\alpha) = \hat{y}_D(\alpha)$ if $\underline{\hat{\alpha}}(k) < \alpha < \overline{\hat{\alpha}}(k)$;*
- *SE with $\hat{x}^*(\alpha) = x_S(\alpha)$ and $\hat{y}^*(\alpha) = 0$ otherwise.*

Although the statement of Lemma 2 somewhat parallels that of Lemma 1, the thresholds $\underline{\hat{\alpha}}(k)$ and $\overline{\hat{\alpha}}(k)$ differ from $\underline{\alpha}(k)$ and $\overline{\alpha}(k)$. Moreover, $\hat{x}_D(\alpha) < x_D(\alpha)$ and $\hat{y}_D(\alpha) > y_D(\alpha)$, so here the balance of effort under CE shifts from the principal to the agent.

To illustrate Lemma 2, Figure 8 shows the equilibrium operating modes as a function of α and k. For comparison purposes, the limits of the equilibrium operating modes without effort precommitment (Figure 3) are also shown (with a dotted red line). Overall, it appears that with effort precommitment, the SE region shrinks while the DE region becomes larger. This is for the following two reasons. On one hand, while the equilibrium efforts under DE and SE are independent of whether the principal precommits to an effort level (Lemma 2) or not (Lemma 1), the principal can shape the balance of effort under CE to her favor through precommitment. As a result, the value of teaming increases and the principal will opt less often to work solo, i.e., the SE region shrinks. On the other hand, DE is feasible for any α: If the principal wants to implement DE, she simply needs to exert zero effort. This optionality—which was not present when efforts were chosen simultaneously—creates more opportunity for DE to arise in equilibrium, in case the principal is unhappy with the payoff she would get otherwise. Taken together, these two drivers result in a situation with a lower prevalence of SE and a greater prevalence of DE than if the principal did not precommit to an effort level, for any given α.

Figure 8 **Equilibrium operating modes when the principal precommits to her effort.**



Note. Here, $V_0 = 7$, $c_x = 9$, and $c_y = 3$. The function $\underline{\hat{\alpha}}(k)$ in Lemma 2 is defined as the (weakly) increasing upper boundary of the DE region. The function $\overline{\hat{\alpha}}(k)$ is defined as the (weakly) decreasing boundary of the SE region. The red dotted line represents the boundaries of the equilibrium operating modes without effort precommitment (Figure 3). A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

5.1.2. Team Formation. We next identify the output share $\hat{\alpha}^*$ that maximizes the principal's payoff (6), anticipating the equilibrium operating modes (17)-(18) characterized in Lemma 2. Similar to (13)-(15), define $\hat{\Pi}_{DE}(\alpha) \doteq \Pi_{DE}(\alpha)$, $\hat{\Pi}_{SE}(\alpha) \doteq \Pi_{SE}(\alpha)$, and

$$\hat{\Pi}_{CE}(\alpha) \doteq \begin{cases} \alpha V(E(\hat{x}_D(\alpha), \hat{y}_D(\alpha))) - c_x \hat{x}_D(\alpha) & \text{if } k < 1/2, \\ -\infty & \text{otherwise.} \end{cases} \tag{22}$$

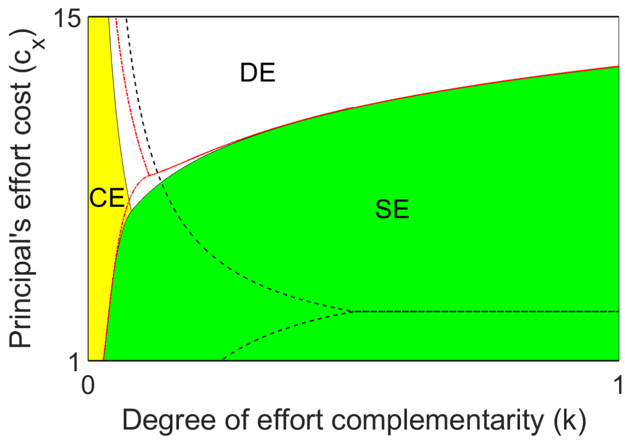
Accordingly, using Lemma 2 and ignoring the arguments from $\underline{\hat{\alpha}}(k)$ and $\overline{\hat{\alpha}}(k)$, we obtain:

$$\hat{\alpha}^* \doteq \arg\max \mathbb{1}_{[\alpha \in [0, \underline{\hat{\alpha}})]} \hat{\Pi}_{DE}(\alpha) + \mathbb{1}_{\left[\alpha \in (\underline{\hat{\alpha}}, \overline{\hat{\alpha}})\right]} \hat{\Pi}_{CE}(\alpha) + \mathbb{1}_{\left[\alpha \in \{\underline{\hat{\alpha}}\} \cup [\overline{\hat{\alpha}}, 1]\right]} \hat{\Pi}_{SE}(\alpha). \tag{23}$$

Although this formulation involves non-convex and open constraint sets, the optimization problem (23) is well-behaved (see Lemma EC.8). Moreover, the first two pieces are always maximized in the interior of their domains; this is unlike the case without effort precommitment, in which the optimal output share could lie at the boundary between the DE and CE operating modes, giving rise to a DE_C operating mode (see Figure 4). Therefore, the principal is no longer constrained in her value capture optimization under DE, similar to the canonical PA model. Moreover, it can be shown that the principal's payoff decreases in c_x under CE (see the proof of Proposition 3), unlike our base case (see Figure 6). In sum, the co-production opportunities do not impede the efficiency of CE and DE under effort precommitment.

If the base value V_0 is large enough, the optimal output share $\hat{\alpha}^*$ turns out to be greater than or equal to 1/2. Therefore, having the principal precommit does not alter the fact that she wants to capture at least 50% of the total value.

Figure 9 Equilibrium operating modes with precommitment at the optimal output share $\hat{\alpha}^*$.



Note. Here, $V_0 = 7$ and $c_y = 3$. The function $\hat{k}_1(c_x)$ in Proposition 3 is the increasing upper boundary of the SE region. Define $\hat{c}_2$ such that $\hat{k}_1(\hat{c}_2) = 1$. The function $\hat{k}_2(c_x)$ is the decreasing upper boundary of the CE region and thus meets $\hat{k}_1(c_x)$ at some $\hat{c}_1$. The red dotted line corresponds to the equilibrium operating modes without precommitment (after combining the DE_C and DE_U regions) depicted in Figure 5. The dashed black line corresponds to the FB operating modes depicted in Figure 2. A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

5.1.3. Equilibrium Operating Modes. Building on the former two subsections, we next characterize the equilibrium operating modes.

PROPOSITION 3. *There exist thresholds $1 \le \hat{c}_1 < \hat{c}_2 < \infty$ and continuously differentiable functions $\hat{k}_1(c_x) > 0$ and $\hat{k}_2(c_x) > 0$ with $\hat{k}_1'(c_x) \ge 0$ and $\hat{k}_2'(c_x) \le 0$ and $\hat{k}_1(\hat{c}_1) = \hat{k}_2(\hat{c}_1)$ such that the equilibrium operating modes at $\hat{\alpha}^*$ are as follows.*

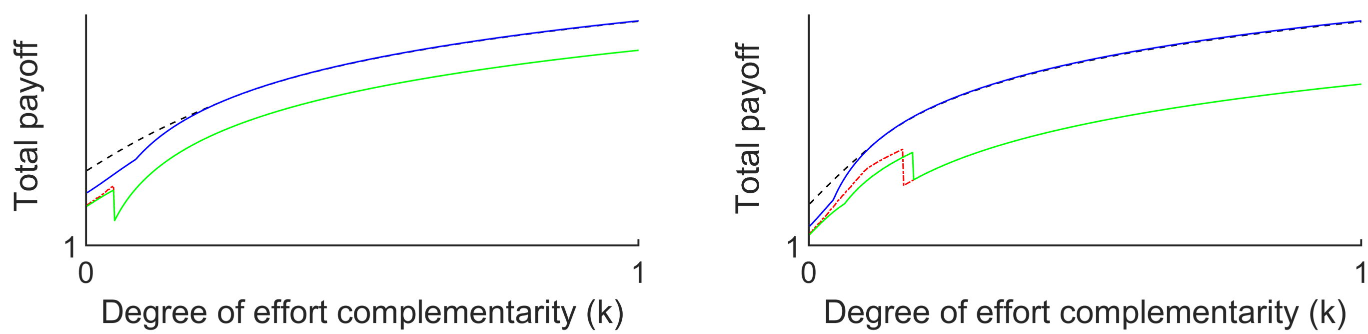
- *If $c_x < \hat{c}_1$, CE when $k < \hat{k}_1(c_x)$ and SE when $k \ge \hat{k}_1(c_x)$;*
- *If $\hat{c}_1 \le c_x < \hat{c}_2$, CE when $k < \hat{k}_2(c_x)$, DE when $\hat{k}_2(c_x) \le k < \hat{k}_1(c_x)$ and SE when $k \ge \hat{k}_1(c_x)$;*
- *If $c_x \ge \hat{c}_2$, CE when $k < \hat{k}_2(c_x)$ and DE when $k \ge \hat{k}_2(c_x)$.*

Proposition 3, illustrated in Figure 9, shows that the equilibrium operating modes are qualitatively similar to those obtained without precommitment (Figure 5)—with marginally more DE and marginally less SE. Hence the key dual insight that principals tend to team up too little and that, when they do, they tend to over-delegate and under-contribute remains valid.

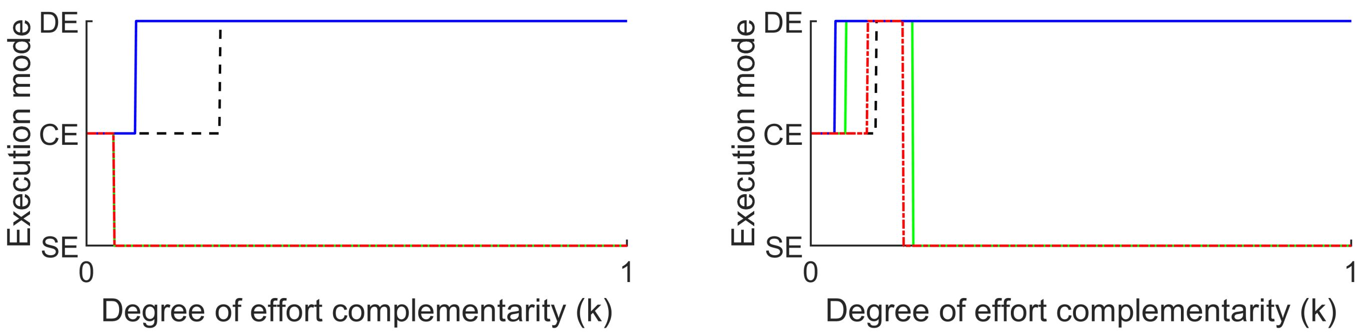
Principals always prefer to precommit (von Stengel 2010), but does precommitment increase the total payoff? Rarely. Figure 10 develops Figure 7 by comparing the total payoff when the principal precommits to an effort level (solid green), i.e., $V(\hat{x}^*(\hat{\alpha}^*), \hat{y}^*(\hat{\alpha}^*)) - c_x\hat{x}^*(\hat{\alpha}^*) - c_y\hat{y}^*(\hat{\alpha}^*)$, to the FB total payoff (dashed black line) and the total payoff when efforts are chosen simultaneously (dash-dotted red line) in the top panel and comparing the respective operating modes in the bottom panel. (The fourth curve, depicted in solid blue, will be described in the next section.) Our numerical simulations suggest that precommitment improves efficiency only when the principal's

Figure 10 **Total payoff and operating modes under the FB (dashed black) and, when efforts are chosen simultaneously with an output share $\alpha^{\circledast}$ (solid blue) or α^* (dash-dotted red) and when efforts are chosen sequentially with an output share $\hat{\alpha}^*$(solid green).**

(a) Total Payoff



(b) Operating Modes



Note. Here, $c_x = 5$ (left) and $c_x = 9$ (right), $V_0 = 7$, and $c_y = 3$. A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

cost is high (right figure) and the degree of effort complementarity lies in an intermediate (and very narrow) range of values. Otherwise, it has either a null impact (when efforts are substitutable) or a negative impact (when efforts are complementary). In particular, precommitment does not prevent the discontinuous drop in total value when the principal chooses to work alone instead of partnering with the agent. Thus, precommitment helps little toward improving efficiency. In the same vein, Smirnov and Wait (2004) find that, in an incomplete contract framework, the mere possibility of sequential (rather than simultaneous) investments can reduce the total surplus.

We conclude that, in general, it is best for organizations to ban principals to precommit to specific effort levels, even though they always prefer to do so. This is because precommitment gives the principal too much power to influence the choice of operating mode, while this decision right should in fact be taken away from her, which is what we explore next.

5.2. System-Wide Optimal Output Share

We next explore the efficiency of depriving the principal of the decision right to form a team and set the agent's incentives. For instance, an external stakeholder, caring about the total net value of the venture, could force a principal to team up with a partner and share her equity.

To assess the efficiency of this mechanism, we consider a situation where, as before, effort choice is fully decentralized, but the team formation decision—and the resulting output-share setting—is set to maximize the total payoff:

$$\alpha^{\circledast} \doteq \arg\max_{\alpha\in[0,1]} V(x^*(\alpha), y^*(\alpha)) - c_x x^*(\alpha) - c_y y^*(\alpha), \tag{24}$$

where $x^*(\alpha)$ and $y^*(\alpha)$ solve (3)-(4). (We use the circled asterisk superscript to denote the system-wide perspective in the output-share setting.)

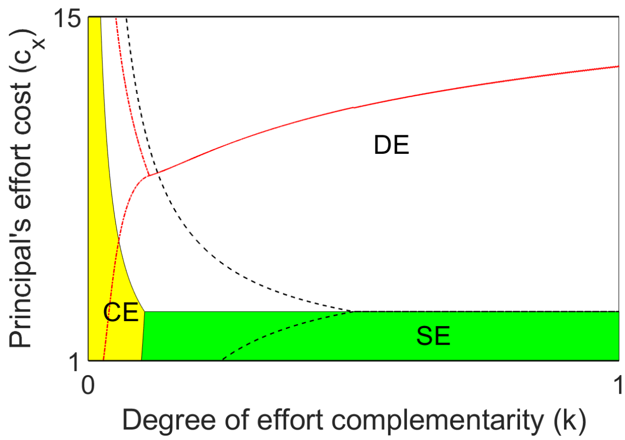
PROPOSITION 4. *The system-wide optimal output share $\alpha^{\circledast}$ is equal to 0 under DE, 1 under SE, and 1/2 under CE.*

The system-wide optimal output share is either 0%, 50%, or 100%, which facilitates the practical implementation of this mechanism. (Note that this mechanism is not equivalent to restricting the principal's choice to $\alpha \in \{0, 0.5, 1\}$ as the principal would never choose $\alpha = 0$; the team formation and reward-splitting decision right should really be taken away from the principal.) Since in the base model $\alpha^* > 1/2$ (conditional on V_0 being sufficiently large), this shows that, when the team formation and reward-splitting decisions are under the control of the principal (as in our base case), she tends to appropriate too much rent for herself. Although the total payoff naturally increases under this mechanism, the principal's payoff decreases (since she is deprived of the decision right to form a team and to choose α); hence, this mechanism will undoubtedly be met with resistance by the principal.

Figure 11 displays the equilibrium operating modes at $\alpha^{\circledast}$ and compares them to those at α^* (dash-dotted red line, identical to Figure 5) and to the FB operating modes (dashed black line, identical to Figure 2). Adopting a system-wide perspective when choosing to form a team and to set the incentives (while keeping the choice of effort decentralized) fully restores the efficiency of the choice between SE and DE, but still makes CE less prominent than it should be: While both SE at $\alpha^{\circledast} = 1$ and DE at $\alpha^{\circledast} = 0$ achieve the FB, CE at $\alpha^{\circledast}$ still suffers from double moral hazard.

To corroborate this result, Figure 10 compares the total payoff at $\alpha^{\circledast}$ (solid blue line) to the FB total payoff (dashed black line) and the equilibrium total payoff at α^* (dash-dotted red line) in the top panel and the corresponding operating modes in the bottom panel. Consistent with Figure 11, we observe that when efforts are highly substitutable (high k), as in more standard projects, full efficiency is achieved under $\alpha^{\circledast}$. However, when efforts are highly complementary (low k), as in

Figure 11 **Equilibrium operating modes at the system-wide optimal output share $\alpha^{\circledast}$.**



Note. Here, $V_0 = 7$ and $c_y = 3$. The dashed black line corresponds to the FB operating modes depicted in Figure 2. The dash-dotted red line corresponds to the equilibrium operating modes at α^* depicted in Figure 5. A low value of k corresponds to a high degree of effort complementarity and *vice versa*.

non-standard projects, some substantial inefficiency may remain with decentralized effort choice, even if the value share is set to $\alpha^{\circledast}$. This is because the inefficiency caused by CE's double moral hazard cannot be eliminated by rebalancing the reward shares (Holmström 1982), even though the optionality to switch to a different operating mode (SE or DE) mitigates the suboptimality gap. We conclude that adopting a system-wide perspective on team formation (e.g., through the involvement of an external stakeholder) always pays off, but its benefit is higher for standard projects (high k) than for non-standard ones (low k).

5.3. Full Counting

Since neither precommitment nor depriving the principal of her team formation decision right helps when efforts are complementary (low value of k), we propose in this case to change the accounting mechanism for rewarding contributions. Instead of splitting the output value among collaborators (fractional counting), each contributor should receive the full value (full counting), irrespective of whether they were involved in the project initiation or its execution. Although this solution is not feasible when the reward is financial, it is conceivable when it relates to credentials, as is sometimes done when assessing researchers' publication records (Korytkowski and Kulczycki 2019). Switching to full counting can resolve the inefficiency associated with double-moral hazard since $x_S(\alpha) < \bar{x}_S$ and $y_S(\alpha) < \bar{y}_S$ for all $\alpha \in (0,1)$; whereas, $x_S(1) = \bar{x}_S$ and $y_S(0) = \bar{y}_S$.

6. Conclusion and Discussion

In this paper, we study the genesis of teams in a co-productive PA setting. Specifically, we consider whether a principal prefers to work solo (SE) or form a team with an agent; and in the latter case, what incentives to set and whether this gives rise to DE, as in the canonical PA model, or CE. Our setup involves three sources of inefficiency: the decentralized nature of execution, the sharing of incentives, and the principal's desire to capture the value created.

We find that principals tend to work solo more than what they should, and when they partner with an agent, they tend to delegate the project execution too much to them. Hence, while most of the PA literature has implicitly assumed a DE operating mode, this operating mode is sidelined, in a co-productive setting—paradoxically, not by CE (which happens too little), but by SE because of a lack of team formation. This phenomenon is particularly salient when efforts are highly substitutable (Figure 5).

Many entrepreneurs or partners in professional firms are often reported to *under-delegate* (Maister 1993, White and White 2011). Our result is actually more subtle: When an agent is involved, principals in fact tend to *over-delegate* and not contribute enough, but the primary issue is that they tend *not to involve* an agent in the first place. Our study indicates that, although most of the PA literature has focused on controlling the agent, a more fundamental problem is to control the principal to lead her to partner and contribute whenever necessary.

We identify an intriguing duality in co-production. On one hand, co-production may *enhance* the value-creation process by enabling the two workers to join forces. On the other hand, it also *impedes* the value-creation process by giving too much freedom to the principal to influence the choice of operating mode without caring about the agent's payoff. In particular, value is destroyed when a principal switches from partnering with an agent to working alone (Figure 7). Co-production also limits the principal's choice of incentives, making her benefit from having a higher cost of effort when the agent expects her to contribute (Figure 6). Hence, having more opportunities for co-production is not necessarily a panacea once we account for the workers' strategic interactions.

To improve efficiency, we explore three mechanisms. We find that precommitment, even though it is always beneficial to the principal, has in general a non-positive effect on value creation (with the exception of when the principal has a large cost of effort and efforts are mildly complementary); see Figure 10. Depriving the principal of her team formation decision right is always beneficial, even though principals will always resist; in fact, it fully restores efficiency unless efforts are very complementary (Figure 10). Therefore, it is important for external stakeholders to take an active role in the planning stage of the project. What facilitates the implementation of this mechanism is the simplicity of the system-optimal reward split: 100% to the agent under DE, 100% to the principal under SE, and 50%-50% under CE (Proposition 4). When efforts are very complementary, as in non-standard projects, the only solution to achieve full efficiency is to shift the accounting mechanism from fractional to full counting.

Even though the parametric form of the effort function (1) is quite versatile, other production functions could in principle yield different results. We note, however, that similar FB operating modes emerge with a CES production function; see Roels (2014). Accordingly, we expect our results to be robust across various specifications of the value function.

The proposed approach can be extended in numerous ways. First, we ignored the role of uncertainty as potentially leading to information asymmetry (Radner 1993, Aghion and Tirole 1997) or recourse or affecting a risk-averse agent (Holmström 1979). Second, we could consider other functional forms of the value function (2), perhaps at the cost of analytical tractability. Third, we considered a single agent, but the model could easily be extended to account for multiple identical agents. Future research could investigate the effect of agent heterogeneity on a project's operating mode and performance. Fourth, we only considered one task, but one could generalize the approach to a multi-task setting, perhaps with different degrees of effort complementarity, some of which potentially pre-assigned to the principal and/or the agent, as in Itoh (2001), Hagiu and Wright (2019), or Bellos and Kavadias (2021). Relatedly, one could extend the model to span over multiple periods and adopt a process perspective, perhaps making the distinction between different phases of execution and characterizing the operating dynamics as the team makes progress or time goes by, in the same vein as Bonatti and Hörner (2011) or Bellos and Kavadias (2021). Fifth, one could investigate the performance of *ex-post* output share allocations, such as bargaining (Bhaskaran and Krishnan 2009) or making shares proportional to effort contributions (Moyer 2016).

Co-productive PA relationships arise in numerous settings, such as entrepreneurship, research, service co-production, team leadership, franchising, and joint ventures. We hope that our work will spur a novel interest in studying the genesis of teams and stimulate the development of mechanisms to control the principal to partner more often and contribute more.

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Proofs of Statements

Proof of Proposition 1 First, assume that $k \neq 1/2$. The system $\partial(V(E(x,y)) - c_x x - c_y y)/\partial x = 0$ and $\partial(V(E(x,y)) - c_x x - c_y y)/\partial y = 0$ has two solutions, namely, $(\bar{x}_D, \bar{y}_D)$ and $(\tilde{x}_D, \tilde{y}_D)$ in which

$$\tilde{x}_D \doteq \frac{(1-2k) - 2kc_x - \sqrt{(1-2k)^2 + 4k^2 c_x c_y}}{2(1-2k)c_x} \quad \text{and} \quad \tilde{y}_D \doteq \frac{(1-2k) - 2kc_y - \sqrt{(1-2k)^2 + 4k^2 c_x c_y}}{2(1-2k)c_y}.$$

We show next that the only feasible solution is $(\bar{x}_D, \bar{y}_D)$. When $k < 1/2$, $\tilde{x}_D \geq 0$ if and only if $(1-2k) - 2kc_x \geq \sqrt{(1-2k)^2 + 4k^2 c_x c_y}$, which holds if and only if $k(c_x - c_y) \geq 1 - 2k > 2kc_x$, implying that $-c_y > c_x$, a contradiction. When $k > 1/2$, $\tilde{x}_D \geq 0$ if and only if $(1-2k) - 2kc_x \leq \sqrt{(1-2k)^2 + 4k^2 c_x c_y}$, which holds if and only if $k(c_x - c_y) \leq 1 - 2k$. By symmetry, we find that $\tilde{y}_D \geq 0$ if and only if $k(c_y - c_x) \leq 1 - 2k$. Together, this implies that $\tilde{x}_D, \tilde{y}_D \geq 0$ if and only if $k|c_x - c_y| \leq 1 - 2k$, a contradiction since $k > 1/2$. Hence, the only potentially feasible solution to the first-order optimality conditions is $(\bar{x}_D, \bar{y}_D)$.

Note that from (8) it follows that $0 < \bar{x}_D, \bar{y}_D < 1$ if and only if $1 > k\left|\frac{c_y - c_x}{2k-1}\right|$.

We next analyze the Hessian matrix. It has negative diagonal elements. Its determinant is equal to $(1-2k)f(k,x,y)/(E(x,y))^3$, where $f(k,x,y) \doteq xy + k(x + y - 4xy) + 2k^2(1 - x - y + 2xy)$. Since $f(k, \bar{x}_D, \bar{y}_D) > 0$ for any $k \geq 0$, we obtain $(\bar{x}_D, \bar{y}_D)$ is a local maximum when $k < 1/2$ and only a saddle point when $k > 1/2$. When $k > 1/2$, the global maximum needs to be located at the boundary of the domain of definition, i.e., at either $(1/c_x, 0)$ or $(0, 1/c_y)$.

As a result, $(\bar{x}_D, \bar{y}_D)$ is feasible and a global maximum if only if $1 > k\left|\frac{c_y - c_x}{2k-1}\right|$ and $k < 1/2$, or equivalently, if and only if $1 - 2k > k|c_x - c_y|$. Otherwise, the only candidates for local maxima are $(1/c_x, 0)$ and $(0, 1/c_y)$. In case both are local maxima, comparing the total value achieved at either one reduces to comparing c_x and c_y.

If $k = 1/2$, the system $\partial(V(E(x,y)) - c_x x - c_y y)/\partial x = 0$ and $\partial(V(E(x,y)) - c_x x - c_y y)/\partial y = 0$ has no solution unless $c_x = c_y$. Hence if $c_x \neq c_y$, the search for the global maximum can be reduced to comparing $(1/c_x, 0)$ and $(0, 1/c_y)$. $\square$

LEMMA EC.1. *Suppose that $k \notin \{0, 1/2\}$ and $0 < x_S, y_S \leq 1$. If $x_S y_S > k\frac{x_S - y_S}{2k-1}$, then $(1-k)x_S y_S < \frac{y_S k^2 - x_S(1-k)^2}{2k-1}$.*

Proof. We have: $x_S y_S > k\frac{y_S - x_S}{1-2k} \Leftrightarrow \frac{k^2}{x_S(1-2k)} < k + \frac{k^2}{y_S(1-2k)}$. Since $y_S \leq 1$, $k + \frac{k^2}{y_S(1-2k)} \leq k - 1 + \frac{1}{y_S} + \frac{k^2}{y_S(1-2k)} = k - 1 + \frac{(1-k)^2}{y_S(1-2k)}$, $\frac{k^2}{x_S(1-2k)} < k - 1 + \frac{(1-k)^2}{y_S(1-2k)}$. Hence if $x_S y_S > k\frac{y_S - x_S}{1-2k}$, $\frac{k^2}{x_S(1-2k)} < k - 1 + \frac{(1-k)^2}{y_S(1-2k)}$. $\square$

Lemma EC.2. *For any $\alpha \in (0,1)$, the equilibrium operating modes solving (3)-(4) are:*

- *If $k \neq 1/2$:*
  - ⋆ *CE with $x^*(\alpha) = x_D(\alpha)$ and $y^*(\alpha) = y_D(\alpha)$ if $x_S(\alpha)y_S(\alpha) > k\left|\frac{x_S(\alpha)-y_S(\alpha)}{2k-1}\right|$;*
  - ⋆ *DE with $x^*(\alpha) = 0$ and $y^*(\alpha) = y_S(\alpha)$ if $x_S(\alpha)(k+(1-2k)y_S(\alpha)) \leq ky_S(\alpha)$;*
  - ⋆ *SE with $x^*(\alpha) = x_S(\alpha)$ and $y^*(\alpha) = 0$ if $y_S(\alpha)(k+(1-2k)x_S(\alpha)) \leq kx_S(\alpha)$.*
- *If $k = 1/2$,*
  - ⋆ *CE with $x^*(\alpha) = y^*(\alpha) = x_S(\alpha)/2$ if $x_S(\alpha) = y_S(\alpha)$;*
  - ⋆ *DE with $x^*(\alpha) = 0$ and $y^*(\alpha) = y_S(\alpha)$ if $x_S(\alpha) \leq y_S(\alpha)$;*
  - ⋆ *SE with $x^*(\alpha) = x_S(\alpha)$ and $y^*(\alpha) = 0$ if $x_S(\alpha) \geq y_S(\alpha)$.*

When $\alpha = 0$, the execution is delegated to the agent (DE), i.e., $x^*(0) = 0$ and $y^*(0) = y_S(0)$. When $\alpha = 1$, the principal executes the project alone (SE), i.e., $x^*(1) = x_S(1)$ and $y^*(1) = 0$.

Proof. In the proof, we omit the argument α from $x_D(\alpha)$, $y_D(\alpha)$, $x_S(\alpha)$, and $y_S(\alpha)$ and we assume that $\alpha \in (0,1)$. Note that the derivatives of the principal's and the agent's payoff functions are respectively proportional to $x_S(k+(1-2k)y) - E(x,y)$ and $y_S(k+(1-2k)x) - E(x,y)$. We consider the following possibilities:

- Suppose there exists an equilibrium with $x = 0$. The agent's best response to $x = 0$ is y_S since it solves $y_S k = E(0,y)$ and $y_S \in (0,1)$. The principal's best response to $y = y_S$ is equal to $x = 0$ if and only if $x_S(k+(1-2k)y_S) \leq E(0,y_S)$. Hence, $(0,y_S)$ is an equilibrium if and only if $x_S(k+(1-2k)y_S) \leq ky_S$.
- Suppose there exists an equilibrium with $x = 1$. The agent's best response to $x = 1$ is $\max\{0, y_S - \frac{k}{1-k}\}$ since $y_S - \frac{k}{1-k}$ solves $y_S(k+(1-2k)) = E(1,y)$ and $y_S < 1$. However, note that if the agent's best response were $y = 0$, the principal would respond by setting $x = x_S$, using an argument symmetric to the one above. So an equilibrium with $x = 1$ can be achieved only when $y_S > \frac{k}{1-k}$, which holds only when $k < 1/2$. The principal's best response to $y = y_S - \frac{k}{1-k}$ is equal to $x = 1$ if and only if $x_S(k^2 + (1-2k)(1-k)y_S) \geq (1-k)^2 y_S$, or equivalently (given that $k < 1$ and $x_S \in (0,1)$), if and only if $\frac{x_S k^2}{(1-k)((1-k)(1-x_S)+kx_S)} \geq y_S$. Since $\frac{k}{1-k} \geq \frac{x_S k^2}{(1-k)((1-k)(1-x_S)+kx_S)}$, this implies that $y_S > \frac{k}{1-k} \geq \frac{x_S k^2}{(1-k)((1-k)(1-x_S)+kx_S)} \geq y_S$, a contradiction. As a result, there is no equilibrium such that $x = 1$.
- Suppose there exists an equilibrium with $x \in (0,1)$ and consider the agent's best response. By using an argument similar to the one above, there exists no equilibrium such that $y = 1$. Also by symmetry, the only equilibrium with $x \in (0,1)$ and $y = 0$ is $(x_S, 0)$, and this happens if and only if $y_S(k+(1-2k)y_S) \leq kx_S$. Therefore, we hereon assume that $y \in (0,1)$. Because (x_D, y_D) is the unique solution to $x_S(k+(1-2k)y) = E(x,y)$ and $y_S(k+(1-2k)x) = E(x,y)$, the only possible equilibrium such that $x \in (0,1)$ and $y \in (0,1)$ is (x_D, y_D), and this happens if and only

if $0 < x_D < 1$ and $0 < y_D < 1$. When $k = 1/2$, this solution exists if and only if $x_S = y_S$; in that case $x_D = y_D = x_S/2 \in (0,1)$. When $k \neq 1/2$, $x_D > 0$ if and only if $x_S y_S > k\frac{x_S - y_S}{2k-1}$, $y_D > 0$ if and only if $x_S y_S > k\frac{y_S - x_S}{2k-1}$, $x_D < 1$ if and only if $(1-k)x_S y_S < \frac{x_S k^2 - y_S(1-k)^2}{(2k-1)}$, and $y_D < 1$ if and only if $(1-k)x_S y_S < \frac{y_S k^2 - x_S(1-k)^2}{(2k-1)}$. By Lemma EC.1, if $x_D > 0$, then $y_D < 1$. By symmetry, if $y_D > 0$, then $x_D < 1$. Hence, (x_D, y_D) is an equilibrium if and only if $x_D > 0$ and $y_D > 0$.

Combining all cases leads to the desired result. When $\alpha = 0$, the derivative of the principal's payoff is negative for all x, y. Hence, $x^* = 0$. As a result, $y^* = y_S$. A symmetric argument applies to the case where $\alpha = 1$. □

LEMMA EC.3. *If (3)-(4) has multiple equilibria, the principal's payoff is always higher under DE.*

Proof. In the proof, we omit the arguments α from $x_S(\alpha)$ and $y_S(\alpha)$. By Lemma EC.2, multiple equilibria happen only when $\alpha \in (0,1)$, which we assume hereon. The proof proceeds in showing the following five steps:

1. If there are multiple equilibria, then $k \geq 1/2$;
2. If $k \geq 1/2$, CE is an equilibrium if and only if both DE and SE are equilibria;
3. If $k \geq 1/2$ and CE is an equilibrium, then $x_S < 2y_S$;
4. If $x_S < 2y_S$, the principal earns more under DE than under SE;
5. If $k \geq 1/2$ and CE is an equilibrium, the principal earns more under DE than under CE.

If there are multiple equilibria, then $k \geq 1/2$. Indeed, assume that there exist multiple equilibria at $k < 1/2$. If CE is an equilibrium, then, by Lemma EC.2, $x_S y_S(1-2k) > k|x_S - y_S|$. Hence, there is no DE equilibrium (which holds when $x_S y_S(1-2k) \leq k(y_S - x_S)$ by Lemma EC.2) and no SE equilibrium (which holds when $x_S y_S(1-2k) \leq k(x_S - y_S)$). Suppose next that both DE and SE are equilibria. Summing up the necessary and sufficient conditions from Lemma EC.2, we obtain $2(1-2k)x_S y_S \leq 0$, a contradiction. Hence, if there are multiple equilibria, $k \geq 1/2$.

If $k \geq 1/2$, CE is an equilibrium if and only if both DE and SE are equilibria. If $k = 1/2$, then by Lemma EC.2, CE is an equilibrium if and only if $x_S = y_S$, i.e., if and only if both DE and SE are equilibria. Fix $k > 1/2$ and suppose that CE is an equilibrium. Then, by Lemma EC.2, $(2k-1)x_S y_S > k|x_S - y_S|$, which implies that both DE and SE are equilibria. Conversely, if both DE and SE are equilibria, then CE is an equilibrium as well.

If $k \geq 1/2$ and CE is an equilibrium, then $x_S < 2y_S$. If $k = 1/2$, CE is an equilibrium if and only if $x_S = y_S$ by Lemma EC.2, and thus $x_S < 2y_S$. Fix $k > 1/2$ and suppose the contrary, i.e., $x_S \geq 2y_S$, or equivalently $\frac{1}{y_S} \geq \frac{2}{x_S} > 1 + \frac{1}{x_S}$. When $x_S \geq y_S$, the CE condition simplifies to $(2k-1)x_S y_S > k(x_S - y_S)$, or, given that $0 < x_S, y_S < 1$, $\frac{2k-1}{k} + \frac{1}{x_S} > \frac{1}{y_S}$. Combining these two inequalities, we obtain that $\frac{2k-1}{k} + \frac{1}{x_S} > \frac{1}{y_S} > 1 + \frac{1}{x_S}$, i.e., $2k-1 > k$, a contradiction.

If $x_S < 2y_S$, the principal earns more under DE than under SE. Since $x_S < 2y_S$, $\ln(x_S/y_S) < \ln(2) < 1$. Therefore, $\alpha \ln(kx_S) - c_x x_S < \alpha \ln(ky_S)$, and so the principal earns more under DE than under SE.

If $k \geq 1/2$ and CE is an equilibrium, the principal earns more under DE than under CE. When $k = 1/2$ and CE is an equilibrium, $x_S = y_S$ by Lemma EC.2. Therefore, the principal earns $\alpha(V_0 + \ln(kx_S)) - c_x(x_S/2)$ under CE and $\alpha(V_0 + \ln(kx_S))$ under DE, and the latter is smaller. Consider now $k > 1/2$. By the logarithm inequality $\ln(\xi) \leq \xi - 1$ for $\xi > 0$. Moreover, when $k \geq 1/2$, $\frac{1}{2k}\left(\sqrt{\left(x_S^2(2k-1)^2 + 4k^2\frac{x_S}{y_S}\right)} - (2k-1)x_S\right) < 1$ if and only if $x_S y_S(2k-1) > k(x_S - y_S)$. As a result when $k > 1/2$ and CE is an equilibrium,

$$
\begin{aligned}
&\ln\left(\frac{1}{2k}\left(\sqrt{(2k-1)^2 x_S^2 + 4k^2\frac{x_S}{y_S}} - (2k-1)x_S\right)\right) \\
&+\frac{k}{x_S(2k-1)}\left(\frac{1}{2k}\left(\sqrt{x_S^2(2k-1)^2 + 4k^2\frac{x_S}{y_S}} - (2k-1)x_S\right) - 1\right) \\
\leq &\left(1 + \frac{k}{x_S(2k-1)}\right)\left(\frac{1}{2k}\left(\sqrt{x_S^2(2k-1)^2 + 4k^2\frac{x_S}{y_S}} - (2k-1)x_S\right) - 1\right) < 0.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\ln(ky_S) > \ln&\left(\frac{1}{2}\left(\sqrt{(2k-1)^2 x_S^2 y_S^2 + 4k^2 x_S y_S} - (2k-1)x_S y_S\right)\right) \\
&-\left(\frac{1}{2} + \frac{k}{x_S(2k-1)} - \frac{1}{2x_S(2k-1)}\sqrt{x_S^2(2k-1)^2 + 4k^2\frac{x_S}{y_S}}\right).
\end{aligned}
$$

That is, the principal earns more under DE than under CE. $\square$

Proof of Lemma 1. Combining Lemmas EC.2 and EC.3, we obtain that the equilibrium operating modes selected by the principal are, when $\alpha \in (0,1)$:

- CE with $x^*(\alpha) = x_D(\alpha)$ and $y^*(\alpha) = y_D(\alpha)$ if $1 - 2k > k\left|\frac{1}{x_S(\alpha)} - \frac{1}{y_S(\alpha)}\right|$;
- DE with $x^*(\alpha) = 0$ and $y^*(\alpha) = y_S(\alpha)$ if $1 - 2k \leq k\left(\frac{1}{x_S(\alpha)} - \frac{1}{y_S(\alpha)}\right)$;
- SE with $x^*(\alpha) = x_S(\alpha)$ and $y^*(\alpha) = 0$ otherwise.

When $\alpha = 0$, the equilibrium operating mode is DE with $x^*(0) = 0$ and $y^*(0) = y_S(0)$. When $\alpha = 1$, the equilibrium operating mode is SE with $x^*(1) = x_S(1)$ and $y^*(1) = 0$.

Define:

$$\underline{\alpha}(k) \doteq \begin{cases} \frac{1}{2} + \frac{k(c_x+c_y)}{2(1-2k)} - \frac{\sqrt{(1-2k)^2+k^2(c_x+c_y)^2-2k(c_x-c_y)(1-2k)}}{2(1-2k)} & \text{if } k \neq 1/2, \\ \frac{c_x}{c_x+c_y} & \text{if } k = 1/2, \end{cases} \quad \text{and} \tag{EC.1}$$

$$\overline{\alpha}(k) \doteq \begin{cases} \frac{1}{2} - \frac{k(c_x+c_y)}{2(1-2k)} + \frac{\sqrt{(1-2k)^2+k^2(c_x+c_y)^2-2k(c_y-c_x)(1-2k)}}{2(1-2k)} & \text{if } k \neq 1/2, \\ \frac{c_x}{c_x+c_y} & \text{if } k = 1/2. \end{cases} \tag{EC.2}$$

From (EC.1) and (EC.2), $\underline{\alpha}(k) \in [0,1]$, $\overline{\alpha}(k) \in [0,1]$, and $\overline{\alpha}(k) \geq \underline{\alpha}(k)$ if and only if $k \leq 1/2$. Moreover, $1-2k \leq k\left(\frac{1}{x_S(\alpha)} - \frac{1}{y_S(\alpha)}\right)$ if and only if $\alpha \leq \underline{\alpha}(k)$; and $1-2k \leq k\left(\frac{1}{y_S(\alpha)} - \frac{1}{x_S(\alpha)}\right)$ if and only if $\alpha \geq \overline{\alpha}(k)$. Hence, when $k < 1/2$, the equilibrium operating mode selected by the principal is DE if $\alpha \leq \underline{\alpha}(k)$, CE if $\underline{\alpha}(k) < \alpha < \overline{\alpha}(k)$, and SE if $\alpha \geq \overline{\alpha}(k)$. And when $k \geq 1/2$, it is DE if $\alpha \leq \underline{\alpha}(k)$ and SE if $\alpha > \underline{\alpha}(k)$. Finally from (EC.1) and (EC.2), $\underline{\alpha}'(k) > 0$ and $\overline{\alpha}'(k) < 0$. $\square$

Lemma EC.4. *The maximization problem* (16) *is well defined, i.e., its maximum exists.*

Proof. First, suppose that $k < 1/2$, i.e., $\underline{\alpha} < \overline{\alpha}$. Since $\Pi_{DE}(\underline{\alpha}) = \Pi_{CE}(\underline{\alpha})$ and $\Pi_{SE}(\overline{\alpha}) = \Pi_{CE}(\overline{\alpha})$, $\Pi(\alpha)$ is continuous. By Weierstrass's extreme value theorem, $\Pi(\alpha)$ attains its maximum on $[0,1]$.

Second, suppose that $k \geq 1/2$, i.e., $\underline{\alpha} \geq \overline{\alpha}$. Although $\Pi(\alpha)$ may experience a discontinuity at $\underline{\alpha}$, $\Pi_{SE}(\alpha)$ is increasing in α (if the principal works solo, she is better off capturing the entire value). Hence, (16) is equivalent to:

$$\alpha^* = \arg\max \mathbb{1}_{[\alpha\in[0,\underline{\alpha}]]}\Pi_{DE}(\alpha) + \mathbb{1}_{[\alpha=1]}\Pi_{SE}(\alpha),$$

which is well defined by Weierstrass's extreme value theorem. $\square$

Lemma EC.5. *For any $\xi \in (0,1/4)$ and $k \in (0,1/2)$, the function*

$$F(\xi,k) \doteq \ln\left(\frac{1}{2}\left((1-2k)\xi + \sqrt{(1-2k)^2\xi^2 + 4k^2\xi}\right)\right) - \frac{(1-2k)\xi + \sqrt{(1-2k)^2\xi^2+4k^2\xi}}{2\xi(1-2k)}$$

is decreasing in k.

Proof. Because

$$\begin{aligned} \frac{\partial F(\xi,k)}{\partial k} = &-\frac{2k}{(1-2k)^2\sqrt{(1-2k)^2\xi^2+4k^2\xi}} - \frac{2\xi}{(1-2k)\xi + \sqrt{(1-2k)^2\xi^2+4k^2\xi}} \\ &+ \frac{2\xi(2k-(1-2k)\xi)}{\sqrt{(1-2k)^2\xi^2+4k^2\xi}\left((1-2k)\xi + \sqrt{(1-2k)^2\xi^2+4k^2\xi}\right)}, \end{aligned}$$

$F(\xi,k)$ is decreasing in k if and only if

$$\sqrt{(1-2k)^2\xi^2+4k^2\xi}\left(\frac{k}{(1-2k)^2} + \xi\right) \geq \xi\frac{k-4k^2-(1-2k)\xi}{1-2k}.$$

If $k-4k^2-(1-2k)\xi \le 0$, the result holds. Suppose then that $k-4k^2-(1-2k)\xi > 0$. In that case, $F(\xi,k)$ is decreasing in k if and only if

$$(1-2k)^2\xi^2+4k^2\xi \ge \left(\frac{\xi\frac{k-4k^2-(1-2k)\xi}{1-2k}}{\frac{k}{(1-2k)^2}+\xi}\right)^2 \Leftrightarrow G(\xi,k)\le 0,$$

where $G(\xi,k) \doteq \xi k^2(1-2k)^2(1-4k+2\xi)^2-(k+\xi-4(1-k)k\xi)^2(\xi+4k^2-4k(1-k)\xi)$. Since $\xi\in(0,1/4)$ and $k\in(0,1/2)$,

$$\begin{aligned}\frac{\partial G(\xi,k)}{\partial \xi} &= -4k(1-2k)^2\left(\xi(1-k)(1-2k)+2k^2\right)\left(2(1-k)-3\xi(1-2k)\right)\\ &< -4k(1-2k)^2\left(\xi(1-k)(1-2k)+2k^2\right)\left(2(1-k)-\frac{3}{4}(1-2k)\right)<0.\end{aligned}$$

Thus, $G(\xi,k)<G(0,k)=-4k^4<0$. $\square$

Lemma EC.6. *Suppose that $c_x \ne c_y$. When $V_0 \ge \ln\left(4\min\{c_x,c_y\}\left(1+|c_y-c_x|\right)\right)+\frac{1}{1-\min\left\{\frac{c_x}{c_y},\frac{c_y}{c_x}\right\}}$, the principal's payoff is positive, the agent's payoff is nonnegative, and $\alpha^*>1/2$.*

Proof. We consider two cases, depending on whether k is smaller than $1/2$ or not. In both cases, we show that $\alpha^*>1/2$ and $\Pi(\alpha^*)>0$.

Case 1: $k<1/2$. Using (16), let $\alpha_{DE}\doteq \arg\max_{\alpha\in[0,\underline{\alpha}]}\Pi_{DE}(\alpha)$, $\alpha_{CE}\doteq \arg\max_{\alpha\in(\underline{\alpha},\overline{\alpha})}\Pi_{CE}(\alpha)$, and $\alpha_{SE}\doteq \arg\max_{\alpha\in[\overline{\alpha},1]}\Pi_{SE}(\alpha)$. Thus, $\alpha^*\in\{\alpha_{DE},\alpha_{CE},\alpha_{SE}\}$. Let us first state the first-order optimality conditions: $\Pi_{DE}(\alpha)$ is concave with $\Pi'_{DE}(\alpha)=V(0,y_S(\alpha))-\frac{\alpha}{1-\alpha}$; $\Pi_{SE}(\alpha)$ is convex with $\Pi'_{SE}(\alpha)=V(x_S(\alpha),0)$. Applying the envelope theorem and the equilibrium condition $E(x_D(\alpha),y_D(\alpha))=y_S(\alpha)(k+(1-2k)x_D(\alpha))$, we obtain:

$$\Pi'_{CE}(\alpha)=V(E(x_D(\alpha),y_D(\alpha)))+c_y\frac{\alpha}{1-\alpha}y'_D(\alpha),$$

where

$$y'_D(\alpha)=-\frac{1}{2c_y}\left(1+\frac{(1-2k)y_S(\alpha)+\frac{2k^2}{\alpha x_S(\alpha)(1-2k)}}{\sqrt{(1-2k)^2(y_S(\alpha))^2+4k^2\frac{y_S(\alpha)}{x_S(\alpha)}}}\right).$$

Since $y'_D(\alpha)<-\frac{1}{c_y}$ when $k<1/2$, $\Pi'_{CE}(\alpha)<\Pi'_{DE}(\alpha)$, i.e., $\underline{\alpha}$ could be a local maximum of $\Pi(\alpha)$, and $\Pi'_{CE}(\alpha)<\Pi'_{SE}(\alpha)$, i.e., $\overline{\alpha}$ cannot be local maximum of $\Pi(\alpha)$.

We next show the result by considering three cases, depending on whether $\underline{\alpha}$ and $\overline{\alpha}$ are smaller or greater than $1/2$:

- When $\underline{\alpha} \geq 1/2$, i.e., $k \geq \frac{1}{2(1+c_x-c_y)}$ (which implies that $c_x > c_y$ given that $k < 1/2$), and when $V_0 \geq \ln(4\min\{c_x, c_y\}(1+|c_y-c_x|)) + \frac{1}{1-\min\left\{\frac{c_x}{c_y}, \frac{c_y}{c_x}\right\}}$,

$$\Pi'_{DE}(1/2) = V(E(0, y_S(1/2)) - \frac{\frac{1}{2}}{1-\frac{1}{2}} = V_0 + \ln(ky_S(1/2))) - 1$$
$$\geq V_0 + \ln\left(\frac{1}{4c_y(1+c_x-c_y)}\right) - 1 \geq \frac{1}{1-\frac{c_y}{c_x}} - 1 > 0.$$

Because $\Pi_{DE}(\alpha)$ is concave, $\alpha_{DE} > \frac{1}{2}$. Since $\alpha_{CE} \geq \alpha_{DE}$, $\alpha_{CE} \geq 1/2$, and the same applies to α_{SE}. Therefore, $\alpha^* > 1/2$. Since $V(E(0, y_S(1/2)) > 1$ and $\Pi_{DE}(\alpha_{DE}) \geq \Pi_{DE}(1/2)$, $\Pi(\alpha^*) \geq \Pi_{DE}(\alpha_{DE}) > 1/2 > 0$.

- When $\underline{\alpha} < 1/2 < \overline{\alpha}$, i.e., when (i) either $c_x \leq c_y$ or $k < \frac{1}{2(1+c_x-c_y)}$ and (ii) either $c_y \leq c_x$ or $k < \frac{1}{2(1+c_y-c_x)}$, which can be combined into requiring that $k < \frac{1}{2(1+|c_x-c_y|)}$, and when $V_0 \geq \ln(4\min\{c_x, c_y\}(1+|c_y-c_x|)) + \frac{1}{1-\min\left\{\frac{c_x}{c_y}, \frac{c_y}{c_x}\right\}}$,

$$\begin{aligned}
\Pi'_{CE}(1/2) &= V(E(x_D(1/2), y_D(1/2))) + c_y \frac{\frac{1}{2}}{1-\frac{1}{2}} y'_D(1/2) \\
&= V_0 + \ln\left(\frac{1}{2} f(k; x_S(1/2) y_S(1/2))\right) - \frac{f(k; x_S(1/2) y_S(1/2))}{2x_S(1/2) y_S(1/2)(1-2k)} \\
&> V_0 + \ln\left(\frac{1}{2} f\left(\frac{1}{2(1+|c_x-c_y|)}; x_S(1/2) y_S(1/2)\right)\right) - \frac{f\left(\frac{1}{2(1+|c_x-c_y|)}; x_S(1/2) y_S(1/2)\right)}{2x_S(1/2) y_S(1/2)\left(1-\frac{1}{1+|c_x-c_y|}\right)} \\
&= V_0 - \ln(4\min\{c_x, c_y\}(1+|c_y-c_x|)) - \frac{1}{1-\min\left\{\frac{c_x}{c_y}, \frac{c_y}{c_x}\right\}} \geq 0,
\end{aligned}$$

where $f(k;\xi) \doteq \sqrt{(1-2k)^2\xi^2 + 4k^2\xi} + (1-2k)\xi$ and the inequality follows from Lemma EC.5. Using (13) and (11)-(12), $\Pi_{CE}(\alpha)$ can be reexpressed as $\Pi_{CE}(\alpha) = \alpha F(\alpha) + \frac{c_x k}{1-2k}$, where

$$F(\alpha) \doteq V_0 + \ln\left(\frac{1}{2} f(k; x_S(\alpha) y_S(\alpha))\right) - \frac{f(k; x_S(\alpha) y_S(\alpha))}{2x_S(\alpha) y_S(\alpha)(1-2k)}.$$

Note that $F(\alpha) = F(1-\alpha)$. Since $\Pi_{CE}(\alpha_{CE}) \geq \Pi_{CE}(1/2)$ by optimality of α_{CE} and since $F(1/2) > 0$ when $k < \frac{1}{2(1+|c_x-c_y|)}$ and $V_0 \geq \ln(4\min\{c_x, c_y\}(1+|c_y-c_x|)) + \frac{1}{1-\min\left\{\frac{c_x}{c_y}, \frac{c_y}{c_x}\right\}}$, $F(\alpha_{CE}) > 0$. As a result, $\alpha_{CE} \geq 1/2$ as otherwise $\Pi_{CE}(1-\alpha_{CE}) - \Pi_{CE}(\alpha_{CE}) = (1-2\alpha_{CE})F(\alpha_{CE}) > 0$, a contradiction. Furthermore, $\alpha_{CE} > 1/2$ since $\Pi'_{CE}(1/2) > 0$; as $\Pi_{DE}(\alpha)$ is concave, $\Pi'_{DE}(\alpha) > \Pi'_{CE}(\alpha)$, and $\Pi'_{CE}(1/2) > 0$, we also have that $\alpha_{DE} > 1/2$. Finally, because $\overline{\alpha} > 1/2$, we obtain that $\alpha_{SE} > 1/2$. As a result, $\alpha^* > 1/2$. Since $F(1/2) > 0$ and $\Pi_{CE}(\alpha_{CE}) \geq \Pi_{CE}(1/2)$, $\Pi(\alpha^*) \geq \Pi_{CE}(\alpha_{CE}) > \frac{c_x k}{1-2k} > 0$.

- When $\overline{\alpha} \leq 1/2$, i.e., $k \geq \frac{1}{2(1+c_y-c_x)}$ (which implies that $c_y > c_x$ given that $k < 1/2$), and when $V_0 \geq \ln(4\min\{c_x, c_y\}(1+|c_y-c_x|)) + \frac{1}{1-\min\left\{\frac{c_x}{c_y}, \frac{c_y}{c_x}\right\}}$,

$$\Pi_{SE}(\alpha_{SE}) \geq \Pi_{SE}(1) = V(E(x_S(1), 0) - 1$$
$$\geq V_0 + \ln\left(\frac{1}{2c_x(1+c_y-c_x)}\right) - 1 \geq \frac{c_x}{c_y-c_x} + \ln(2) > 1/2.$$

Let $\hat{\alpha}_{CE} \doteq \arg\max_{\alpha\in[0,\overline{\alpha}]} \Pi_{CE}(\alpha)$. Since $[0,\overline{\alpha}] \subset (\underline{\alpha},\overline{\alpha})$, $\Pi_{CE}(\alpha_{CE}) \leq \Pi_{CE}(\hat{\alpha}_{CE})$. Since $\Pi_{DE}(\underline{\alpha}) = \Pi_{CE}(\underline{\alpha})$ and $\Pi'_{CE}(\alpha) < \Pi'_{DE}(\alpha)$ for any α, $\Pi_{DE}(\alpha_{DE}) \leq \Pi_{CE}(\hat{\alpha}_{CE})$. If $\hat{\alpha}_{CE} = 0$, $\Pi_{CE}(\hat{\alpha}_{CE}) = 0 < \Pi_{SE}(\alpha_{SE})$. If $\hat{\alpha}_{CE}$ solves $\Pi'_{CE}(\alpha) = 0$, then $V(x_D(\hat{\alpha}_{CE}), y_D(\hat{\alpha}_{CE})) < \frac{\hat{\alpha}_{CE}}{1-\hat{\alpha}_{CE}}$, and therefore, $\Pi_{CE}(\hat{\alpha}_{CE}) < \frac{\hat{\alpha}_{CE}^2}{1-\hat{\alpha}_{CE}} < \frac{1}{2} < \Pi_{SE}(\alpha_{SE})$. Finally, if $\hat{\alpha}_{CE} = \overline{\alpha}$, $\Pi_{CE}(\hat{\alpha}_{CE}) = \Pi_{SE}(\hat{\alpha}_{CE}) \leq \max\{\Pi_{SE}(0), \Pi_{SE}(1)\} = \Pi_{SE}(1)$. Hence, $\alpha^* = \alpha_{SE} = 1 > 1/2$ and $\Pi(\alpha^*) = \Pi_{SE}(1) > 1/2 > 0$.

Case 2: $k \geq 1/2$. We consider two cases, depending on whether $\underline{\alpha}$ is smaller or greater than 1/2.

- When $\underline{\alpha} \geq 1/2$, i.e., $k \geq \frac{1}{2(1+c_x-c_y)}$, and when $V_0 \geq \ln(4\min\{c_x,c_y\}(1+|c_y-c_x|)) + \frac{1}{1-\min\left\{\frac{c_x}{c_y},\frac{c_y}{c_x}\right\}}$, similar to the argument above when $k<1/2$, we obtain that $\Pi'_{DE}(1/2) > 0$. Because $\Pi_{DE}(\alpha)$ is concave, we obtain that $\alpha_{DE} > \frac{1}{2}$. Since $\alpha_{SE} \geq \alpha_{DE}$, $\alpha_{SE} \geq 1/2$. Therefore, $\alpha^* > 1/2$. Since $V(E(0, y_S(1/2)) > 1$ and $\Pi_{DE}(\alpha_{DE}) \geq \Pi_{DE}(1/2)$, $\Pi(\alpha^*) \geq \Pi_{DE}(\alpha_{DE}) > 1/2 > 0$.
- When $\underline{\alpha} < 1/2$, i.e., $k < \frac{1}{2(1+c_x-c_y)}$ (which implies that $c_y > c_x$ given that $k \geq 1/2$) and when $V_0 \geq \ln(4\min\{c_x,c_y\}(1+|c_y-c_x|)) + \frac{1}{1-\min\left\{\frac{c_x}{c_y},\frac{c_y}{c_x}\right\}}$, we obtain that $\Pi_{SE}(\alpha_{SE}) \geq \Pi_{SE}(1) = V_0 - \ln(2c_x) - 1 \geq \ln(2(1+c_y-c_x)) + \frac{c_x}{c_y-c_x} > \ln(2) > \frac{1}{2}$. Similar to the case with $k<1/2$ above, we obtain that $\alpha^* = \alpha_{SE} = 1 > 1/2$ and $\Pi(\alpha^*) = \Pi_{SE}(\alpha_{SE}) > 1/2 > 0$.

Finally, we show that the agent earns nonnegative payoff at α^*. If $\alpha^* = \alpha_{SE}$, $\alpha^* = 1$, and therefore the agent earns zero. If $\alpha^* = \alpha_{DE}$, the agent earns $(1-\alpha_{DE})(V(0, y_S(\alpha_{DE})) - 1)$. Since $\alpha_{DE} > 1/2$, α_{DE} is such that $\Pi'_{DE}(\alpha_{DE}) \geq 0$, i.e., $V(0, y_S(\alpha_{DE})) \geq \frac{\alpha_{DE}}{1-\alpha_{DE}}$. Therefore, the agent's payoff is bounded from below by $(1-\alpha_{DE})(V(0,y_S(\alpha_{DE})) - 1) \geq 2\alpha_{DE} - 1 > 0$. If $\alpha^* = \alpha_{CE}$, which happens only when $k<1/2$, the agent earns $(1-\alpha_{CE})V(x_D(\alpha_{CE}), y_D(\alpha_{CE})) - c_y y_D(\alpha_{CE})$. Since $\alpha_{CE} \in (\underline{\alpha},\overline{\alpha})$, α_{CE} solves $\Pi'_{CE}(\alpha_{CE}) = 0$, which implies that $V(x_D(\alpha_{CE}), y_D(\alpha_{CE})) > \alpha_{CE}/(1-\alpha_{CE})$. Moreover, $y_D(\alpha) \leq y_S(\alpha)$ for any $\alpha \in (\underline{\alpha},\overline{\alpha})$. Hence, the agent's payoff is bounded from below by $(1-\alpha_{CE})V(x_D(\alpha_{CE}), y_D(\alpha_{CE})) - c_y y_D(\alpha_{CE}) > \alpha_{CE} - c_y y_S(\alpha_{CE}) = 2\alpha_{CE} - 1 > 0$, where the last inequality holds because $\alpha_{CE} > 1/2$. $\square$

Proof of Proposition 2. The first part consists in building Figure 5 by looking at it vertically, for any given k, and showing that, in the most general case, we observe the following transitions as c_x increases: SE, then CE, then $\mathrm{DE_C}$ (i.e., with $\alpha^* = \underline{\alpha}$), and finally $\mathrm{DE_U}$ (i.e., with $\alpha^* < \underline{\alpha}$), with no direct transition from CE to $\mathrm{DE_U}$. The second part looks at the figure horizontally, by building the functions $k_1(c_x)$, $k_2(c_x)$, and $k_3(c_x)$, and showing that they are monotone, as specified in the statement.

Part A: Vertical perspective. Fix k. This part proceeds in two steps: First, we characterize the derivatives of the principal's payoff with respect to c_x within each equilibrium operating mode (SE, CE, $\mathrm{DE_C}$, and $\mathrm{DE_U}$). Second, we show that, at the boundary between any two regions (e.g., between SE and CE), operating one way dominates the other as c_x increases. This establishes that there is at most one crossing between any two regions.

A.1 Derivatives of the principal's payoff within each operating mode. We extend the definitions of $\Pi_{CE}(\alpha)$, $\Pi_{DE}(\alpha)$, and $\Pi_{SE}(\alpha)$ in (13)-(15) by formally introducing a dependence on c_x. We consider in turn the total derivatives of each of these three functions with respect to c_x, assuming that the corresponding operating mode arises in equilibrium.

Using (16), denote $\alpha_{CE} \doteq \arg\sup_{\alpha\in(\underline{\alpha},\overline{\alpha})} \Pi_{CE}(\alpha)$ and $\alpha_{DE} \doteq \arg\max_{\alpha\in[0,\underline{\alpha}]} \Pi_{DE}(\alpha)$.

First, suppose that SE is the equilibrium operating mode, i.e., $\alpha^* = 1$. Then,

$$\frac{d\Pi_{SE}(\alpha^*(c_x),c_x)}{dc_x}\Big|_{\alpha^*=1} = \frac{d}{dc_x}\left(V_0 + \ln\left(kx_S(1)\right) - c_x x_S(1)\right) = -\frac{1}{c_x}. \tag{EC.3}$$

Second, suppose that CE is the equilibrium operating mode (which can only happen when $k < 1/2$), i.e., $\alpha^* = \alpha_{CE} \in (\underline{\alpha},\overline{\alpha})$. Then, $\Pi'_{CE}(\alpha^*) = 0$. Moreover, by (3)-(4), $\partial\left(\alpha^* V(x_D,y_D) - c_x x_D\right)/\partial x = 0$ and $\partial\left((1-\alpha^*)V(x_D,y_D) - c_y y_D\right)/\partial y = 0$. Using these results and applying the law of total derivative to (13), we have:

$$\begin{aligned}\frac{d\Pi_{CE}(\alpha^*(c_x),c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}} &= \frac{\partial}{\partial c_x}\left(\alpha^* V(E(x_D(\alpha^*),y_D(\alpha^*)) - c_x x_D(\alpha^*)\right)\\ &= c_y\frac{\alpha^*}{1-\alpha^*}y'_D(c_x) - x_D(\alpha^*).\end{aligned}$$

Next, we establish both a lower and an upper bounds. For the lower bound, first note that inspecting (12) reveals that $y_D(\alpha)$ decreases in $x_S(\alpha)$. Accordingly, since $y_S(\alpha)$ does not depend on c_x and $x_S(\alpha)$ decreases in c_x by (9), $y'_D(c_x) > 0$. Hence,

$$\frac{d\Pi_{CE}(\alpha^*(c_x),c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}} > -x_D(\alpha^*) > -x_S(\alpha^*) > -1/c_x. \tag{EC.4}$$

For the upper bound, first note that

$$\begin{aligned}\frac{d\Pi_{CE}(\alpha^*(c_x),c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}} &= c_y\frac{\alpha^*}{1-\alpha^*}y'_D(c_x) - x_D(\alpha^*)\\ &= -\left(\frac{1}{2}x_S(\alpha^*) - \frac{k}{1-2k}\right) - \frac{1}{2(1-2k)}\frac{x_S(\alpha^*)y_S(\alpha^*)(1-2k)^2 + 2k^2}{\sqrt{(y_S(\alpha^*))^2(1-2k)^2 + 4k^2\frac{y_S(\alpha^*)}{x_S(\alpha^*)}}},\end{aligned}$$

and the right-hand side is decreasing in $x_S(\alpha^*)$. Moreover, $1/x_S(\alpha^*) < (1-2k)/k + 1/y_S(\alpha^*)$ by Lemma 1. After replacing $x_S(\alpha^*)$ with $1/\left((1-2k)/k + 1/y_S(\alpha^*)\right)$ in the right-hand side of the last equality, we obtain

$$\begin{aligned}\frac{d\Pi_{CE}(\alpha^*(c_x),c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}} &= -\left(\frac{1}{2}x_S(\alpha^*) - \frac{k}{1-2k}\right) - \frac{1}{2(1-2k)}\frac{x_S(\alpha^*)y_S(\alpha^*)(1-2k)^2 + 2k^2}{\sqrt{(y_S(\alpha^*))^2(1-2k)^2 + 4k^2\frac{y_S(\alpha^*)}{x_S(\alpha^*)}}}\\ &< \frac{k^2}{(1-2k)((1-2k)y_S(\alpha^*) + 2k)}.\end{aligned} \tag{EC.5}$$

Third, suppose that DE is the equilibrium operating mode, i.e., $\alpha^* = \alpha_{DE}$ by (16). Since $\Pi_{DE}(\alpha)$ is independent of c_x, $\arg\max_{\alpha\in[0,1]} \Pi_{DE}(\alpha)$ is independent of c_x. On the other hand, $\underline{\alpha}'(c_x) \geq 0$ by (EC.1). Hence, when $\alpha^* = \alpha_{DE}$, there exists a threshold value $\hat{c}_x$ such that $\alpha_{DE}(c_x) = \underline{\alpha}(c_x)$ for $c_x \leq \hat{c}_x$ and $\alpha_{DE}(c_x) < \underline{\alpha}(c_x)$ for $c_x > \hat{c}_x$. Accordingly, when $c_x > \hat{c}_x$, i.e., under $\mathrm{DE_U}$,

$$\frac{d\Pi_{DE}(\alpha^*(c_x), c_x)}{dc_x}\bigg|_{\alpha^*=\alpha_{DE}} = 0, \tag{EC.6}$$

whereas when $c_x \leq \hat{c}_x$, i.e., under $\mathrm{DE_C}$,

$$\frac{d\Pi_{DE}(\alpha^*(c_x), c_x)}{dc_x}\bigg|_{\alpha^*=\alpha_{DE}} = \frac{\partial \Pi_{DE}(\underline{\alpha}(c_x), c_x)}{\partial \alpha}\frac{d\underline{\alpha}(c_x)}{dc_x} \geq 0. \tag{EC.7}$$

A.2 Crossings between equilibrium operating modes. Consider the boundaries between regions where different operating modes are equilibria.

- CE-SE: Suppose that (when $k < 1/2$), for some $\tilde{c}_x$, $\alpha^*(\tilde{c}_x) \in \{1, \alpha_{CE}(\tilde{c}_x)\}$, $\Pi_{SE}(1, \tilde{c}_x) = \Pi_{CE}(\alpha_{CE}(\tilde{c}_x), \tilde{c}_x)$, and $\partial \Pi_{CE}(\alpha_{CE}, \tilde{c}_x))/\partial\alpha = 0$. By (EC.3) and (EC.4), $\frac{d\Pi_{SE}(\alpha_{SE}(c_x),c_x)}{dc_x}\Big|_{\alpha^*=1} = -1/c_x < \frac{d\Pi_{CE}(\alpha_{CE}(c_x),c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}}$. Then, for any $\epsilon > 0$, there exists a $\delta > 0$ such that $|\alpha^*(\tilde{c}_x - \epsilon) - \alpha_{CE}(\tilde{c}_x)| > \delta$ and $|\alpha^*(\tilde{c}_x + \epsilon) - 1| > \delta$. That is, a transition from SE to CE is possible as c_x increases, but not the opposite.
- DE-SE: Suppose that for some $\tilde{c}_x$, $\alpha^*(\tilde{c}_x) \in \{1, \alpha_{DE}(\tilde{c}_x)\}$ $\Pi_{SE}(1, \tilde{c}_x) = \Pi_{DE}(\alpha_{DE}(\tilde{c}_x), \tilde{c}_x)$. by (EC.3), (EC.6), and (EC.7), $\frac{d\Pi_{SE}(\alpha_{SE}(c_x),c_x)}{dc_x}\Big|_{\alpha^*=1} = -1/c_x < 0 \leq \frac{d\Pi_{DE}(\alpha_{DE}(c_x),c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{DE}}$. Then, for any $\epsilon > 0$, there exists a $\delta > 0$ such that $|\alpha^*(\tilde{c}_x - \epsilon) - \alpha_{DE}(\tilde{c}_x)| > \delta$ and $|\alpha^*(\tilde{c}_x + \epsilon) - 1| > \delta$. That is, a transition from SE to DE is possible as c_x increases, but not the opposite.
- CE-DE: Suppose that (when $k < 1/2$), for some $\tilde{c}_x$, $\alpha^*(\tilde{c}_x) \in \{\alpha_{DE}(\tilde{c}_x), \alpha_{CE}(\tilde{c}_x)\}$, $\Pi_{DE}(\alpha_{DE}(\tilde{c}_x), \tilde{c}_x) = \Pi_{CE}(\alpha_{CE}(\tilde{c}_x), \tilde{c}_x)$, and $\partial \Pi_{CE}(\alpha_{CE}, \tilde{c}_x))/\partial\alpha = 0$. Since $\Pi_{DE}(\alpha)$ is concave and $\Pi'_{CE}(\alpha) < \Pi'_{DE}(\alpha)$ (see the proof of Lemma EC.6) and $\Pi_{CE}(\underline{\alpha}(\tilde{c}_x)) = \Pi_{DE}(\underline{\alpha}(\tilde{c}_x))$, either $\alpha_{CE}(\tilde{c}_x) = \underline{\alpha}(\tilde{c}_x)$ or $\alpha_{DE}(\tilde{c}_x) = \underline{\alpha}(\tilde{c}_x)$ (or both equalities are true). Hence, if $\Pi_{CE}(\alpha_{CE}(\tilde{c}_x), \tilde{c}_x) = \Pi_{DE}(\alpha_{DE}(\tilde{c}_x), \tilde{c}_x)$, we must have $\alpha_{CE}(\tilde{c}_x) = \alpha_{DE}(\tilde{c}_x) = \underline{\alpha}(\tilde{c}_x)$, i.e., there is no direct transition between CE and $\mathrm{DE_U}$. Hence, this case is about the boundary between CE and $\mathrm{DE_C}$. Moreover, CE cannot be an equilibrium since $\alpha_{CE}(\tilde{c}_x) \notin (\underline{\alpha}(\tilde{c}_x), \overline{\alpha}(\tilde{c}_x))$. Hence, unlike the cases of CE-SE and DE-SE, in which two operating modes could simultaneously be equilibria, in this case, either CE is an equilibrium or $\mathrm{DE_C}$ is an equilibrium, but not both at the same time.

  Fix c_x and suppose that CE is an equilibrium; i.e., $\Pi(\alpha^*(c_x), c_x) = \Pi_{CE}(\alpha_{CE}(c_x), c_x) > \Pi_{DE}(\underline{\alpha}(c_x), c_x)$. Then, applying the implicit function theorem to $1/x_S(\underline{\alpha}) = (1-2k)/k +$

$1/y_S(\underline{\alpha})$, using the fact that $\Pi'_{CE}(\underline{\alpha}) \geq 0$ whenever CE is the equilibrium operating mode, i.e., $V_0 + \ln(ky_S(\underline{\alpha})) + \underline{\alpha} y'_D(\underline{\alpha})/y_S(\underline{\alpha}) \geq 0$, and using (12), (EC.1), and (EC.5), we obtain

$$\begin{aligned}
\frac{d\Pi_{DE}(\underline{\alpha}(c_x), c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}>\underline{\alpha}} &= \frac{\partial \Pi_{DE}(\underline{\alpha}(c_x), c_x)}{\partial \alpha} \frac{d\underline{\alpha}(c_x)}{dc_x} \\
&= \left(V_0 + \ln(ky_S(\underline{\alpha})) - \frac{\underline{\alpha}}{1-\underline{\alpha}}\right) \left(\frac{1}{\underline{\alpha}\left(\frac{1}{c_x(x_S(\underline{\alpha}))^2} + \frac{1}{c_y(y_S(\underline{\alpha}))^2}\right)}\right) \\
&\geq \left(-\frac{\underline{\alpha} y'_D(\underline{\alpha})}{y_S(\underline{\alpha})} - \frac{\underline{\alpha}}{1-\underline{\alpha}}\right) \left(\frac{1}{\underline{\alpha}\left(\frac{1}{c_x(x_S(\underline{\alpha}))^2} + \frac{1}{c_y(y_S(\underline{\alpha}))^2}\right)}\right) \\
&= \frac{k^2}{(1-2k)((1-2k)y_S(\underline{\alpha}) + 2k)} > \frac{d\Pi_{CE}(\alpha^*(c_x), c_x)}{dc_x}\Big|_{\alpha^*=\alpha_{CE}}.
\end{aligned}$$

Accordingly, a transition from CE to DE_C is possible as c_x increases, but not the opposite.

- DE_C-DE_U: As discussed above, there exists a threshold value $\hat{c}_x$ such that $\alpha_{DE} = \underline{\alpha}$ for $c_x \leq \hat{c}_x$ and $\alpha_{DE} < \underline{\alpha}$ for $c_x > \hat{c}_x$. Hence as c_x increases, a transition from DE_C to DE_U is possible, but not the opposite.

Hence, for any given k, the most general transition pattern (assuming no empty region), as c_x increases, is first SE, then CE, then DE_C (i.e., with $\alpha^* = \underline{\alpha}$), and finally DE_U (i.e., with $\alpha^* < \underline{\alpha}$), with no direct transition from CE to DE_U.

Part B. Horizontal perspective. Next, we vary k and show that the boundaries between regions are continuous and monotone. We again proceed in multiple steps. First, we show the boundaries between any pair of regions (whenever they exist) are continuous and monotone using the implicit function theorem. Next, we show whether these are increasing or decreasing, focusing first on $k_2(c_x)$ and $k_3(c_x)$, and then on $k_1(c_x)$. Finally, we define the thresholds c_1, c_2, and c_3.

B.1 Continuity and monotonicity. To do so, we further extend the definitions $\Pi_{CE}(\alpha)$, $\Pi_{DE}(\alpha)$, $\Pi_{SE}(\alpha)$ in (13)-(15) by formally introducing a dependence on c_x and k. For any $\hat{c}_x$, suppose that there exists a value of $\hat{k}$ that solves $\Pi_{CE}(\alpha_{CE}(\hat{c}_x, \hat{k}), \hat{c}_x, \hat{k}) = \Pi_{SE}(1, \hat{c}_x, \hat{k})$. We study the behavior of this solution when c_x changes locally around $\hat{c}_x$. Because both $\Pi_{SE}(1, c_x, k)$ and $\Pi_{CE}(\alpha_{CE}(c_x, k), c_x, k)$ are continuously differentiable in c_x and k, the implicit function theorem yields that whenever $\Pi_{CE}(\alpha_{CE}(\hat{c}_x, \hat{k}), \hat{c}_x, \hat{k}) = \Pi_{SE}(1, \hat{c}_x, \hat{k})$, there exists a continuously differentiable function $k_{SE-CE}(c_x)$ such that $k_{SE-CE}(\hat{c}_x) = \hat{k}$. Moreover, $k_{SE-CE}(c_x)$ is monotone since, as shown above, the equilibrium operating mode transitions as c_x increases from SE to CE, but not the other way around. A similar argument can be applied to the transitions between SE and DE_C; SE and DE_U; CE and DE_C; and DE_U and DE_C.

B.2 Functions $k_2(c_x)$ and $k_3(c_x)$ are decreasing. Let $k_2(c_x) \doteq k_{DE_U-DE_C}(c_x)$ and $k_3(c_x) \doteq k_{CE-DE_C}(c_x)$. When $k=0$, CE is the only possible equilibrium operating mode since $\Pi_{CE}(\alpha_{CE}(c_x,0),c_x,0) = \alpha_{CE}(V_0 + \ln x_S(\alpha_{CE})y_S(\alpha_{CE}) - 1) > -\infty = \Pi_{SE}(1,c_x,0) = \Pi_{DE}(\alpha,c_x,0)$ for all α and c_x. Hence, $k_{CE-DE_C}(c_x)$ must be decreasing (on its domain); otherwise, since the equilibrium operating mode transitions from CE to $\mathrm{DE_C}$ as c_x increases (but not the opposite), there would exist a value $\hat{c}_x$ such that $\Pi_{DE}(\underline{\alpha}(c_x,0),c_x,0) > \Pi_{CE}(\alpha_{CE}(c_x,0),c_x,0)$ for all $c_x \geq \hat{c}_x$, a contradiction.

Consequently, $k_{DE_U-DE_C}(c_x)$ is also decreasing on its domain. Otherwise, there would exist a direct transition from CE to $\mathrm{DE_U}$, which is also a contradiction.

Accordingly, $k_2'(c_x) \leq 0$ and $k_3'(c_x) \leq 0$.

Moreover, $k_{DE_U-DE_C}(c_x)$ and $k_{CE-DE_C}(c_x)$ cannot cross each other for otherwise there would be a direct boundary between CE and $\mathrm{DE_U}$. Since CE is the only equilibrium at $k=0$ and since $k_{DE_U-DE_C}(c_x)$ and $k_{CE-DE_C}(c_x)$ do not cross each other, $k_3(c_x) > k_2(c_x) > 0$ for all $c_x \geq 1$.

B.3 Function $k_1(c_x)$ is increasing. Let $k_1(c_x)$ be the boundary of SE with all other equilibrium operating modes. Since $k_{DE_U-DE_C}(c_x)$ and $k_{CE-DE_C}(c_x)$ do not cross, $k_{SE-CE}(c_x)$ turns, as c_x increases, first into $k_{SE-DE_C}(c_x)$ and then $k_{SE-DE_U}(c_x)$.

Using a similar argument to the one characterizing $k_{CE-DE_C}(c_x)$, we obtain that $k_{CE-SE}(c_x)$ is increasing on its domain. Otherwise (since the equilibrium operating mode transitions from SE to CE as c_x increases, but not the opposite) there would exist a value $\hat{c}_x$ such that $\Pi_{SE}(\underline{\alpha}(c_x,0),c_x,0) \geq \Pi_{CE}(\alpha_{CE}(c_x,0),c_x,0)$ for all $c_x \leq \hat{c}_x$, a contradiction.

We next show that $k_{SE-DE_U}(c_x)$ is increasing on its domain. By definition, $\Pi_{SE}(1,c_x,k_{SE-DE_U}(c_x)) = \Pi_{DE}(\alpha_{DE}(k_{SE-DE_U}(c_x)),c_x,k_{SE-DE_U}(c_x))$, in which $\alpha_{DE}(k_{SE-DE_U}(c_x))$ solves $\Pi_{DE}'(\alpha_{DE},c_x,k_{SE-DE_U}(c_x)) = 0$, i.e., $V(0,y_S(\alpha_{DE})) + \alpha_{DE}(\partial V(0,y_S(\alpha_{DE}))/\partial y)y_S'(\alpha) = 0$, or equivalently, $V(0,y_S(\alpha_{DE})) - \alpha_{DE}/(1-\alpha_{DE}) = 0$. Therefore, $V_0 + \ln(k_{SE-DE_U}(c_x)/c_x) - 1 = \alpha_{DE}^2/(1-\alpha_{DE})$; i.e., $k_{SE-DE_U}(c_x) = c_x \exp(1 + \alpha_{DE}^2/(1-\alpha_{DE}) - V_0)$. Since α_{DE} is independent of c_x, we obtain that $k_{SE-DE_U}'(c_x) = \exp(1 + \alpha_{DE}^2/(1-\alpha_{DE}) - V_0) > 0$.

Hence, both $k_{CE-SE}(c_x)$ and $k_{SE-DE_U}(c_x)$ are increasing. Therefore, $k_{SE-DE_C}(c_x)$ must also be increasing; otherwise, $k_{CE-SE}(c_x)$ and $k_{SE-DE_U}(c_x)$ would not be differentiable when they intersect with $k_{SE-DE_C}(c_x)$, a contradiction.

In summary, $k_1'(c_x) \geq 0$. Since CE is the only equilibrium at $k=0$, $k_1(c_x) > 0$ for all $c_x \geq 1$.

B.4 Thresholds. Finally, define the thresholds c_1 as the intersection between $k_1(c_x)$ and $k_2(c_x)$, c_2 as the intersection between $k_1(c_x)$ and $k_3(c_x)$, and c_3 as the interesection between $k_1(c_x)$ and 1.

□

Lemma EC.7. *For any given output share $\alpha \in [0,1]$ and principal's effort $x \in [0,1)$, the agent's equilibrium effort* (18) *equals*

$$\hat{y}^*(\alpha, x) = y_S(\alpha) - \frac{kx}{k+(1-2k)x} \tag{EC.8}$$

if $x < x_0(\alpha)$, and zero otherwise.

Proof. After relaxing the constraint that $y \leq 1$, (18) simplifies to $\max_{y\geq 0}(1-\alpha)\ln(E(x,y)) - c_y y$. The first-order optimality conditions are:

$$\frac{(1-\alpha)(k+(1-2k)x)}{E(x,y)} - c_y \leq 0, \;\; y \geq 0, \;\; \left[\frac{(1-\alpha)(k+(1-2k)x)}{E(x,y)} - c_y\right] y = 0. \tag{EC.9}$$

Because the objective function is strictly concave and the constraint is linear, Conditions (EC.9) are necessary and sufficient. The solution is unique and can be expressed, using (1) and (10), as $\hat{y}^* = \max\left\{0, y_S(\alpha) - \frac{kx}{k+(1-2k)x}\right\}$. Because $\alpha \in [0,1]$ and $c_y > 1$, $y_S(\alpha) < 1$, and thus $\hat{y}^* < 1$. Hence $\hat{y}^*$ is also optimal when the constraint $y \leq 1$ is explicitly stated. Because $\frac{d}{dx}\left(y_S(\alpha) - \frac{kx}{k+(1-2k)x}\right) = \frac{-k^2}{(k+(1-2k)x)^2} < 0$, $d\hat{y}^*/dx < 0$. If $y_S(\alpha) \geq \frac{k}{1-k}$, $\hat{y}^* \geq 0$ for all $x \in [0,1]$; otherwise, there exists a value $x_0 \in [0,1)$ such that $\hat{y}^* > 0$ if $x < x_0$ and $\hat{y}^* = 0$ if $x \geq x_0$. Combining both cases leads to (20) so that, when $x < 1$, $\hat{y}^* = y_S(\alpha) - \frac{kx}{k+(1-2k)x}$ if $x < x_0(\alpha)$ and $\hat{y}^* = 0$ if $x \geq x_0(\alpha)$. $\square$

Proof of Lemma 2. Throughout the proof, we omit the arguments α and from $\hat{y}^*(\alpha, x)$, $\hat{x}^*(\alpha)$, $x_S(\alpha)$, $y_S(\alpha)$, $\hat{x}_D(\alpha)$, and $x_0(\alpha)$. By Lemma EC.7, $\hat{y}^*(x) = y_S - \frac{kx}{k+(1-2k)x}$ when $x \leq x_0$ and $\hat{y}^*(x) = 0$ otherwise. Define $[a,b] \doteq \emptyset$ if $a > b$ and $(a,b] = [a,b) \doteq \emptyset$ if $a \geq b$. Accordingly, the principal's effort choice problem in (6) simplifies to

$$\hat{x}^* = \arg\max\left\{\max_{x\in[0,x_0)} \alpha V\left(E\left(x, y_S - \frac{kx}{k+(1-2k)x}\right)\right) - c_x x, \max_{x\in[x_0,1]} \alpha V(E(x,0)) - c_x x\right\}.$$

Because $y_S - \frac{kx}{k+(1-2k)x} \geq 0$ if and only if $x \leq x_0$ and $V(E(x,y))$ is increasing in y, $\alpha V\left(E\left(x, y_S - \frac{kx}{k+(1-2k)x}\right)\right) - c_x x \geq \alpha V(E(x,0)) - c_x x$ if and only if $x \leq x_0$.

When $\alpha = 0$, $\hat{x}^* = 0$. By Lemma EC.7, $\hat{y}^*(x) = y_S$. When $\alpha = 1$, $y_S = 0$; therefore, $x_0 = 0$. Hence, $\hat{x}^* = \arg\max_{x\in[0,1]} \alpha V(E(x,0)) - c_x x = x_S$.

Finally, suppose that $\alpha \in (0,1)$. The function $\alpha V(E(x,0)) - c_x x$ is strictly concave in $x \in [0,1]$ and maximized at x_S. Because $\alpha \in (0,1)$ and $c_x > 1$, $x_S \in (0,1)$. The function $\alpha V\left(E\left(x, y_S - \frac{kx}{k+(1-2k)x}\right)\right) - c_x x$ is (convexly) decreasing in $x \in [0,1]$ if $k \geq 1/2$ and strictly concave in $x \in [0,1]$ otherwise. In either case, it is maximized at $\hat{x}_D$ as defined by (21), and $\hat{x}_D \leq x_S$.

1. When $\hat{x}_D = 0$:
   (a) If $x_S < x_0$, $\hat{x}^* = 0$.
   (b) If $x_S \geq x_0$, then $\hat{x}^* = x_S$ if $\alpha V(E(x_S, 0)) - c_x x_S \geq \alpha V(E(0, y_S))$ and $\hat{x}^* = 0$ otherwise.

2. When $0 < \hat{x}_D < x_0$:
   (a) If $x_S < x_0$, $\hat{x}^* = \hat{x}_D$.
   (b) If $x_S \geq x_0$, $\hat{x}^* = x_S$ if $\alpha V(E(x_S,0)) - c_x x_S \geq \alpha V\left(E\left(\hat{x}_D, y_S - \frac{k\hat{x}_D}{k+(1-2k)\hat{x}_D}\right)\right) - c_x\hat{x}_D$ and $\hat{x}^* = \hat{x}_D$ otherwise.
3. When $\hat{x}_D \geq x_0$, then $\hat{x}^* = x_S$.

Accordingly, DE (i.e., $\hat{x}^* = 0$) arises when $\hat{x}_D = 0$ and either $x_S < x_0$ or $\alpha V(E(x_S,0)) - c_x x_S < \alpha V(E(0,y_S))$, i.e., $\ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right) < 1$. Note that, since $\alpha V\left(E\left(x, y_S - \frac{kx}{k+(1-2k)x}\right)\right) - c_x x \geq \alpha V(E(x,0)) - c_x x$ if and only if $x \leq x_0$, $x_S < x_0$ implies that $\alpha V(E(x_S,0)) - c_x x_S < \alpha V\left(E\left(x_S, y_S - \frac{kx_S}{k+(1-2k)x_S}\right)\right) - c_x x_S \leq \max_{x\in[0,x_0)} \alpha V\left(E\left(x, y_S - \frac{kx}{k+(1-2k)x}\right)\right) - c_x x = \alpha V(E(0,y_S))$. Hence, DE arises when $\hat{x}_D = 0$ and $\alpha V(E(x_S,0)) - c_x x_S < \alpha V(E(0,y_S))$, i.e., $\ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right) < 1$.

Similarly, we obtain that CE (i.e., $\hat{x}^* = \hat{x}_D > 0$ and $\hat{y}^* > 0$) arises when $0 < \hat{x}_D < x_0$ and either $x_S < x_0$ or $\alpha V(E(x_S,0)) - c_x x_S < \alpha V\left(E\left(\hat{x}_D, y_S - \frac{k\hat{x}_D}{k+(1-2k)\hat{x}_D}\right)\right) - c_x\hat{x}_D$, i.e., $\ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) < \frac{k}{(1-2k)x_S(\alpha)}$. Note that, since $\alpha V\left(E\left(x, y_S - \frac{kx}{k+(1-2k)x}\right)\right) - c_x x \geq \alpha V(E(x,0)) - c_x x$ if and only if $x \leq x_0$, $x_S < x_0$ implies that $\alpha V(E(x_S,0)) - c_x x_S < \alpha V\left(E\left(x_S, y_S - \frac{kx_S}{k+(1-2k)x_S}\right)\right) - c_x x_S \leq \alpha V\left(E\left(\hat{x}_D, y_S - \frac{k\hat{x}_D}{k+(1-2k)\hat{x}_D}\right)\right) - c_x\hat{x}_D$. Hence, CE arises when $0 < \hat{x}_D < x_0$ and $\alpha V(E(x_S,0)) - c_x x_S < \alpha V\left(E\left(\hat{x}_D, y_S - \frac{k\hat{x}_D}{k+(1-2k)\hat{x}_D}\right)\right) - c_x\hat{x}_D$. However, the requirement that $\hat{x}_D < x_0$ is redundant since when $\hat{x}_D \geq x_0$, $\alpha V(E(x_S,0)) - c_x x_S \geq \alpha V\left(E\left(\hat{x}_D, y_S - \frac{k\hat{x}_D}{k+(1-2k)\hat{x}_D}\right)\right) - c_x\hat{x}_D$. In sum, CE arises when $0 < \hat{x}_D$ and $\ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) < \frac{k}{(1-2k)x_S(\alpha)}$.

In all other cases, SE (i.e., $\hat{x}^* = x_S$) is the equilibrium.

Denote

$$\underline{\hat{\alpha}}(k) \doteq \sup \mathcal{D}(k) \doteq \left\{\alpha : \hat{x}_D(\alpha) = 0 \text{ and } \ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right) < 1\right\}.$$

This function is well defined for all k since $\hat{x}_D(0) = 0$ and $\ln\left(\frac{x_S(0)}{y_S(0)}\right) < 1$, i.e., $\mathcal{D}(k) \neq \emptyset$. Moreover, since $\ln\left(\frac{x_S(1)}{y_S(1)}\right) > 1$, $\mathcal{D}(k)$ is bounded from above. Accordingly, $0 \leq \underline{\hat{\alpha}}(k) < 1$.

Since $\hat{x}'_D(\alpha) \geq 0$ and $\left(\ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right)\right)' > 0$, $\alpha_1 < \underline{\hat{\alpha}} \Rightarrow \alpha_2 < \underline{\hat{\alpha}}$ for any $\alpha_2 \leq \alpha_1$. Hence, $\mathcal{D}(k)$ is convex. Accordingly, DE is an equilibrium for all $\alpha < \underline{\hat{\alpha}}(k)$ and not an equilibrium otherwise (i.e., the equilibrium operating mode is then either CE or SE).

We next show that $\underline{\hat{\alpha}}'(k) \geq 0$. Suppose, for some k_1, $\ln\left(\frac{x_S(\alpha_1)}{y_S(\alpha_1)}\right) < 1$ with $\alpha_1 \doteq \underline{\hat{\alpha}}(k_1)$. Because $\hat{x}_D(\alpha)$ is weakly decreasing in k and $\left(\ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right)\right)$ is independent of k, $\underline{\hat{\alpha}}(k) \geq \alpha_1$ for all $k \geq k_1$. Since $\ln\left(\frac{x_S(0)}{y_S(0)}\right) < 1$ and given that $\underline{\hat{\alpha}}(0) = 0$, $\underline{\hat{\alpha}}(k)$ is thus increasing for all k.

Next, denote

$$\overline{\hat{\alpha}}(k) \doteq \sup \mathcal{C}(k) \doteq \left\{\alpha : \hat{x}_D(\alpha) > 0 \text{ and } \ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) < \frac{k}{(1-2k)x_S(\alpha)}\right\}.$$

Unlike $\underline{\hat{\alpha}}(k)$, $\overline{\hat{\alpha}}(k)$ is defined only on a restricted set of values of k. In particular, $\mathcal{C}(k) = \emptyset$ when $k \geq 1/2$ by (21). When $\mathcal{C}(k) \neq \emptyset$, since $\hat{x}_D(0) = 0$ for any k, $\overline{\hat{\alpha}}(k) > 0$; and since $\ln\left(\frac{k}{(1-2k)y_S(1)}\right) > \frac{k}{(1-2k)x_S(1)}$, $\overline{\hat{\alpha}}(k) \leq 1$.

Next, we show that CE is an equilibrium if and only if $\underline{\hat{\alpha}}(k) < \alpha < \overline{\hat{\alpha}}(k)$. We do so in two parts, by first showing that $\mathcal{C}(k)$ is convex and then showing that $\underline{\hat{\alpha}}(k) = \inf \mathcal{C}(k)$ whenever $\mathcal{C}(k) \neq \emptyset$. For the first part, note that $\hat{x}'_D(\alpha) \geq 0$ and $\ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) - \frac{k}{(1-2k)x_S(\alpha)}$ is increasing. Hence, $\alpha_1 < \overline{\hat{\alpha}}$ and $\alpha_2 < \overline{\hat{\alpha}} \Rightarrow \lambda\alpha_1 + (1-\lambda)\alpha_2 < \overline{\hat{\alpha}}$ for any $\lambda \in [0,1]$. For the second part, note that $\ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right) < 1 \Rightarrow \ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) < \frac{k}{(1-2k)x_S(\alpha)}$ (since both $\ln\left(\frac{x_S(\alpha)}{y_S(\alpha)}\right)$ and $\ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) - \frac{k}{(1-2k)x_S(\alpha)}$ are increasing in α and $\ln\left(\frac{k}{(1-2k)y_S(\tilde{\alpha})}\right) < \frac{k}{(1-2k)x_S(\tilde{\alpha})}$ at $\tilde{\alpha} = c_x e/(c_x e + c_y)$). That is, $\alpha \leq \underline{\hat{\alpha}}(k) \Rightarrow \ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) < \frac{k}{(1-2k)x_S(\alpha)}$. This result combined with the fact that $\hat{x}'_D(\alpha) \geq 0$ completes the proof.

Finally, because $\hat{x}_D(\alpha)$ is weakly decreasing in k and $\ln\left(\frac{k}{(1-2k)y_S(\alpha)}\right) - \frac{k}{(1-2k)x_S(\alpha)}$ is increasing in k whenever $\hat{x}_D > 0$, $\mathcal{C}(k)$ shrinks in k, i.e., $\overline{\hat{\alpha}}'(k) \leq 0$. Since $\overline{\hat{\alpha}}(0) = 1$, the function $\overline{\hat{\alpha}}(k)$ is well defined over $[0, \hat{k}]$ for some $\hat{k} < 1/2$. To extend the domain of definition, one can define $\overline{\hat{\alpha}}(k)$ as equal to $\underline{\hat{\alpha}}(k)$ whenever $\mathcal{C}(k)$ is empty. $\square$

LEMMA EC.8. *When V_0 is sufficiently large, the maximization problem* (23) *is well defined, i.e., its maximum exists.*

Proof. First, when V_0 is large enough, $\hat{\Pi}_{SE}(\alpha)$ is increasing; therefore, if SE emerges as the equilibrium operating mode, $\hat{\alpha}^* = 1$. Accordingly, the team formation decision reduces to comparing $\hat{\Pi}_{SE}(1)$ to the payoff the principal could obtain under CE or DE by setting $\alpha^* < 1$.

Second, as shown in the proof of Lemma 2, DE is feasible if $\hat{x}_D(\alpha) = 0$ and CE is feasible if $\hat{x}_D(\alpha) > 0$, so the choice between DE and CE reduces to comparing α to a threshold $\hat{\alpha} \doteq \min\{1, kc_x/(1-2k)\}$ if $k < 1/2$ and $\hat{\alpha} \doteq 1$ otherwise. Accordingly, (23) is equivalent to

$$\hat{\alpha}^* = \arg\max \mathbb{1}_{[\alpha \in [0,\hat{\alpha}] \cap [0,1)]} \hat{\Pi}_{DE}(\alpha) + \mathbb{1}_{[\alpha \in (\hat{\alpha},1)]} \hat{\Pi}_{CE}(\alpha) + \mathbb{1}_{[\alpha = 1]} \hat{\Pi}_{SE}(\alpha). \tag{EC.10}$$

Although some feasible sets remain half-open, note that $\hat{\Pi}_{DE}(\hat{\alpha}) = \hat{\Pi}_{CE}(\hat{\alpha})$, $\hat{\Pi}_{SE}(1) > \max\{\hat{\Pi}_{DE}(1), \hat{\Pi}_{CE}(1)\}$, and $\lim_{\alpha \to 1} \hat{\Pi}'_{DE}(\alpha) < 0$ and $\lim_{\alpha \to 1} \hat{\Pi}'_{CE}(\alpha) < 0$; so they can be closed without loss of generality, i.e., (EC.10) is equivalent to

$$\hat{\alpha}^* = \arg\max \mathbb{1}_{[\alpha \in [0,\hat{\alpha}]]} \hat{\Pi}_{DE}(\alpha) + \mathbb{1}_{[\alpha \in [\hat{\alpha},1]]} \hat{\Pi}_{CE}(\alpha) + \mathbb{1}_{[\alpha = 1]} \hat{\Pi}_{SE}(\alpha).$$

By Weierstrass' extreme value theorem, each piece is well defined. $\square$

LEMMA EC.9. *When $V_0 \geq 1+\ln(4c_y(1+c_x))$, the principal's payoff is positive, the agent's payoff is nonnegative, and $\alpha^* \geq 1/2$.*

Proof. Consider (EC.10). Since $\hat{\Pi}''_{DE}(\alpha) = -(2-\alpha)/(1-\alpha)^2 < 0$, $\hat{\Pi}_{DE}(\alpha)$ is strictly concave. Since $\hat{\Pi}''_{CE}(\alpha) \leq 0$ if and only if $2\alpha^2 - 4\alpha + 1 \leq 0$, $\hat{\Pi}_{CE}(\alpha)$ is convex for all $\alpha \in [0, 1-\sqrt{2}/2]$ and concave for all $\alpha \in [1-\sqrt{2}/2, 1]$. Moreover, $\lim_{\alpha\to 1} \hat{\Pi}'_{DE}(\alpha) < 0$ and $\lim_{\alpha \to 1} \hat{\Pi}'_{CE}(\alpha) < 0$. When $\hat{\alpha} < 1$, $\hat{\Pi}_{CE}(\hat{\alpha}) = \hat{\Pi}_{DE}(\hat{\alpha})$ and $\hat{\Pi}'_{CE}(\alpha) \leq \hat{\Pi}'_{DE}(\alpha)$ if and only if $\alpha \leq \hat{\alpha}$. This is because

$$\hat{\Pi}'_{DE}(\alpha) = V_0 + \ln(ky_S(\alpha)) - \frac{\alpha}{1-\alpha}, \quad \hat{\Pi}'_{CE}(\alpha) = V_0 + \ln((1-2k)x_S(\alpha)y_S(\alpha)) - \frac{\alpha}{1-\alpha}$$

and $k \geq (1-2k)x_S(\alpha)$ if and only if $\alpha \leq \hat{\alpha}$. Hence, $\hat{\Pi}(\alpha)$ is continuously differentiable over $[0,1)$. Therefore, when $1-\sqrt{2}/2 > \hat{\alpha}$, $\hat{\alpha}^* \notin (\hat{\alpha}, 1-\sqrt{2}/2]$. As a result,

$$\hat{\alpha}^* \in \{\hat{\alpha}_{DE}, \hat{\alpha}_{CE}, 1\} \tag{EC.11}$$

with

$$\hat{\alpha}_{DE} \doteq \arg\max_{0\leq\alpha\leq\hat{\alpha}} \hat{\Pi}_{DE}(\alpha) \text{ and } \hat{\alpha}_{CE} \doteq \arg \max_{\max\left\{\hat{\alpha}, 1-\frac{\sqrt{2}}{2}\right\}<\alpha\leq 1} \hat{\Pi}_{CE}(\alpha). \tag{EC.12}$$

Because $\hat{\Pi}_{DE}(\alpha)$ and $\hat{\Pi}_{CE}(\alpha)$ are concave over their respective domains of interest, the first-order optimality conditions are necessary and sufficient to identify $\hat{\alpha}_{DE}$ and $\hat{\alpha}_{CE}$.

We show that $\hat{\alpha}^* \geq 1/2$ and $\hat{\Pi}(\hat{\alpha}^*) > 0$ by considering two cases, depending on the value of $\hat{\alpha}$.

- Suppose first that $\hat{\alpha} \geq 1/2$, i.e., $k \geq \frac{1}{2(c_x+1)}$. When $V_0 \geq 1 + \ln(4c_y(c_x+1))$,

$$\hat{\Pi}'_{DE}(1/2) = V_0 + \ln(ky_S(1/2)) - \frac{\frac{1}{2}}{1-\frac{1}{2}} \geq V_0 + \ln\left(\frac{y_S(1/2)}{2(c_x+1)}\right) - 1 \geq \ln(1) = 0.$$

  Therefore, $\hat{\alpha}_{DE} \geq 1/2$ by concavity of $\hat{\Pi}_{DE}(\alpha)$. By definition, $\hat{\alpha}_{CE} \geq \hat{\alpha} \geq 1/2$. Hence, $\hat{\alpha}^* \geq 1/2$. Finally, because $\hat{\Pi}_{DE}(\hat{\alpha}_{DE}) \geq \hat{\Pi}_{DE}(1/2) = (1/2)\left(V_0 + \ln(ky_S(1/2))\right) \geq 1/2 > 0$, $\hat{\Pi}(\hat{\alpha}^*) \geq \hat{\Pi}_{DE}(\hat{\alpha}_{DE}) > 0$.
- Suppose next that $\hat{\alpha} < 1/2$, i.e., $k < \frac{1}{2(c_x+1)}$. When $V_0 \geq 1 + \ln(4c_y(c_x+1))$,

$$\begin{aligned}\hat{\Pi}'_{CE}(1/2) &= V_0 + \ln(x_S(1/2)y_S(1/2)(1-2k)) - \frac{\frac{1}{2}}{1-\frac{1}{2}} \\ &> V_0 + \ln\left(x_S(1/2)y_S(1/2)\left(1-\frac{1}{c_x+1}\right)\right) - 1 \geq 0.\end{aligned}$$

  Therefore, $\hat{\alpha}_{CE} > 1/2$ by concavity of $\hat{\Pi}_{CE}(\alpha)$ over $[1-\sqrt{2}/2, 1]$. We next show that $\hat{\alpha}^* \neq \hat{\alpha}_{DE}$ by considering two cases.

  - ⋆ When $\hat{\Pi}'_{DE}(\hat{\alpha}) > 0$, $\hat{\Pi}'_{DE}(\alpha) > 0$ for all $\alpha \leq \hat{\alpha}$ since $\hat{\Pi}_{DE}(\alpha)$ is concave; hence, $\hat{\alpha}_{DE} = \hat{\alpha}$. Therefore, $\hat{\Pi}_{CE}(\hat{\alpha}_{CE}) \geq \hat{\Pi}_{CE}(\hat{\alpha}) = \hat{\Pi}_{DE}(\hat{\alpha}) = \hat{\Pi}_{DE}(\hat{\alpha}_{DE})$, i.e., $\hat{\alpha}^* \in \{\hat{\alpha}_{CE}, 1\}$.

- ⋆ When $\hat{\Pi}'_{DE}(\hat{\alpha}) \leq 0$, either $\hat{\alpha}_{DE}$ solves $\hat{\Pi}'_{DE}(\alpha) = 0$ or $\hat{\alpha}_{DE} = 0$; in either case, $V_0 + \ln(ky_S(\hat{\alpha}_{DE})) \leq \hat{\alpha}_{DE}/(1-\hat{\alpha}_{DE})$. Because $\hat{\alpha}_{DE} \leq \hat{\alpha} < 1/2$, $V_0 + \ln(ky_S(\hat{\alpha}_{DE})) < 1$. Hence, $\hat{\Pi}_{DE}(\hat{\alpha}_{DE}) = \hat{\alpha}_{DE}\,(V_0 + \ln(ky_S(\hat{\alpha}_{DE}))) < \hat{\alpha}_{DE} \leq \hat{\alpha}$. On the other hand, $\hat{\Pi}_{CE}(\hat{\alpha}_{CE}) \geq \hat{\Pi}_{CE}(1/2) = (1/2) \times (V_0 + \ln((1-2k)/(4c_xc_y)) - 1) + c_xk/(1-2k) > \hat{\alpha}$, where the second inequality holds because $k < \frac{1}{2(c_x+1)}$ and $V_0 \geq 1 + \ln(4c_y(c_x+1))$. As a result, $\hat{\Pi}_{DE}(\hat{\alpha}_{DE}) < \hat{\Pi}_{CE}(\hat{\alpha}_{CE})$ and $\hat{\alpha}^* \in \{\hat{\alpha}_{CE}, 1\}$.

Hence, $\hat{\alpha}^* \in \{\hat{\alpha}_{CE}, 1\}$ and therefore $\hat{\alpha}^* > 1/2$. Because $\hat{\Pi}_{CE}(\hat{\alpha}_{CE}) > \hat{\Pi}_{CE}(1/2) = (1/2)\,(V_0 + \ln(x_S(1/2)y_S(1/2)(1-2k)) - 1) + c_x\frac{k}{1-2k} > c_x\frac{k}{1-2k} \geq 0$, $\hat{\Pi}(\hat{\alpha}^*) \geq \hat{\Pi}_{CE}(\hat{\alpha}_{CE}) > 0$.

In summary, we obtain that $\hat{\alpha}^* \geq 1/2$ and $\hat{\Pi}(\hat{\alpha}^*) > 0$. We next show that the agent's equilibrium payoff is nonnegative. When $\hat{\alpha}^* < 1$, $\hat{\alpha}^*$ solves the first-order optimality conditions $(V\,(E(\hat{x}^*, \hat{y}^*(\hat{\alpha}^*, \hat{x}^*))) = \frac{\hat{\alpha}^*}{1-\hat{\alpha}^*})$ and $\hat{y}^*(\alpha, x) = y_S(\alpha) - kx/(k + (1-2k)x)$; accordingly, the agent's equilibrium payoff equals $(1-\hat{\alpha}^*)V\,(E(\hat{x}^*, \hat{y}^*(\hat{\alpha}^*, \hat{x}^*))) - c_y\hat{y}^*(\hat{\alpha}^*, \hat{x}^*) = \hat{\alpha}^* - c_y\hat{y}^*(\hat{\alpha}^*, \hat{x}^*) = 2\hat{\alpha}^* - 1 + c_y\frac{k\hat{x}^*}{k+(1-2k)\hat{x}^*} \geq 0$. When $\hat{\alpha}^* = 1$, the agent's payoff equals zero. □

Proof of Proposition 3. The proof is similar to the proof of Proposition 2, and we omit the details. Let $\hat{\alpha}_{CE} \doteq \arg\max_{\alpha \in (\underline{\hat{\alpha}}, \overline{\hat{\alpha}})} \hat{\Pi}_{CE}(\alpha)$ and $\hat{\alpha}_{DE} \doteq \arg\max_{\alpha \in [0, \underline{\hat{\alpha}})} \hat{\Pi}_{DE}(\alpha)$. We extend the definitions of $\hat{\Pi}_{DE}(\alpha)$, $\hat{\Pi}_{CE}(\alpha)$, and $\hat{\Pi}_{SE}(\alpha)$ to formally introduce a dependence on c_x. Suppose first that CE is the equilibrium operating mode, which can only happen when $k < 1/2$, i.e., $\alpha^* \in (\underline{\hat{\alpha}}, \overline{\hat{\alpha}})$. By the envelope theorem, we obtain that $\partial\hat{\Pi}_{CE}(\alpha^*(c_x), c_x)/\partial c_x = k/(1-2k) - x_S$, which is strictly negative since $\hat{x}_D(\alpha^*) > 0$, and larger than $-1/c_x$. Suppose next that DE is the equilibrium operating mode, i.e., $\alpha^* \in [0, \underline{\hat{\alpha}})$. Then, $\partial\hat{\Pi}_{DE}(\alpha^*(c_x), c_x)/\partial c_x = 0$. Finally, suppose that SE is the equilibrium operating mode, i.e., $\alpha^* = 1$. Then, $\partial\hat{\Pi}_{SE}(\alpha^*(c_x), c_x)/\partial c_x = -1/c_x$. Combining these results, we obtain that, for any given k, the most general transition pattern (assuming no empty region), as c_x increases, is first SE, then CE, and finally DE.

Similar to the proof of Proposition 2, we can then show that the boundaries between these regions are monotone, using the implicit function theorem. Note that the boundary between the SE and the DE regions, defined as the set of solutions to $\hat{\Pi}_{SE}(1, c_x, k) = \hat{\Pi}_{DE}(\hat{\alpha}_{DE}(k, c_x), k, c_x)$ for any c_x, and denoted as $\hat{k}_{DE-SE}(c_x)$, is increasing in c_x. Also, similar to the proof of Proposition 2, we find that when $k = 0$, CE is the only possible equilibrium operating mode as the profit associated with the other regions tends to $-\infty$. Hence, the boundary $\hat{k}_2(c_x)$ between the CE and DE region must be decreasing in c_x while the boundary $\hat{k}_1(c_x)$ between the CE and the SE regions must be increasing in c_x. Combining these results completes the proof. □

Proof of Proposition 4. The cases when the equilibrium operating modes are SE and DE are trivial. Suppose the equilibrium operating mode is CE. Plugging (11)-(12) into (24), we obtain:

$$V_0 + \frac{1}{2} + \frac{k(c_x + c_y)}{1-2k} + \ln\left(\frac{2k^2}{\sqrt{(1-2k)^2 + \frac{4k^2c_xc_y}{\alpha(1-\alpha)}} - (1-2k)}\right) - \frac{\sqrt{(1-2k)^2 + \frac{4k^2c_xc_y}{\alpha(1-\alpha)}}}{2(1-2k)}.$$

Taking the derivative with respect to α yields:

$$\frac{(1-2\alpha)c_x c_y k^2\left(1-2k+\sqrt{(1-2k)^2+\frac{4k^2c_xc_y}{\alpha(1-\alpha)}}\right)}{(1-\alpha)^2\alpha^2(1-2k)\sqrt{(1-2k)^2+\frac{4k^2c_xc_y}{\alpha(1-\alpha)}}\left(\sqrt{(1-2k)^2+\frac{4k^2c_xc_y}{\alpha(1-\alpha)}}-(1-2k)\right)},$$

which is positive when $\alpha < 1/2$, zero at $\alpha = 1/2$, and negative when $\alpha > 1/2$. Hence setting $\alpha^{\circledast} = 1/2$ is optimal under CE. $\square$