Smallholder-Farmer Selection into Yield-Improvement Programs: Is It Really Doing Any Good?

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Managerial Implications: Our study refutes the myth that farmers uniformly benefit from improved yields, especially when we consider the entire farming ecosystem. On a more optimistic note, we show that a small degree of altruism on the part of the buyer can significantly improve farmers’ welfare.

Keywords: Socially Responsible Operations; Smallholder Farmers; Cournot Competition; Game Theory

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1. Introduction

Although small-scale farming of agricultural commodities (e.g., hazelnuts, cocoa, coffee) is essential to many food supply chains, many smallholder farmers live below the poverty level (World Bank 2016). One of the major contributors to their poor living conditions is the low and variable yields of their crops, which can be as much as 90% lower than those of larger, commercial farms (Porter et al. 2017), due to adverse weather, poor soil conditions, and pest infestation.

To improve the livelihoods of smallholder farmers and ensure a stable supply of the commodity, some processors and manufacturers—the main buyers of these commodities—have committed to yield-improvement programs, to increase farmers’ average yields (see Ferrero 2018a, p. 180 or Ferrero 2018b, p. 160) and, according to our interviews with industry experts, reduce their variability. Yield
improvement is, in fact, an important pillar of various sustainability certifications (e.g., Starbucks’ C.A.F.E. Practices and Unilever’s Sustainable Agriculture Code), which are increasingly managed by the buyers themselves (Thorlakson 2018). For instance, Ferrero, a large chocolate manufacturer, conducts field trips to hazelnut growing areas, introduces farmers to modern farming techniques, and provides them with equipment for mechanized farming and drying stations under the scope of the Ferrero Farming Values (FFV) program. Similarly, Barry Callebaut, a major cocoa processor, provides farmers with a productivity package that includes training on tree pruning techniques and the use of fertilizer.

Even though yield improvement is often framed by buyers as contributing to the United Nations’ Sustainable Development Goals (Barry Callebaut 2023, p. 9), increased production could lead to a supply glut in the market and, therefore, to a price drop; see, e.g., Waarts and Kiewisch (2021, pp. 40-41), Hurtz-Adams (2022, p. 5), and Oxfam Belgïe/Belgique (2024, p. 37). In fact, Roozen (2021) warns us that “it is too simplistic to assume that higher yield will result in the end in increased income.” Among other reasons, this may explain why some yield-improvement practices, like grafting cocoa trees, are prohibited by some governments presumably to avoid a sudden decrease in price and drive farmers out of business (Smith et al. 2014). Yield improvement may also result in inequalities. Through extensive interviews of hazelnut growers (see Appendix B), we have indeed found out that many farmers are quite skeptical about the benefit of the FFV program: Enrolled farmers certainly enjoy improved yields, but these improved yields increase the available crop in the market and drive down market prices. Could yield-improvement programs hurt smallholder farmers’ well-being, despite what is often claimed by buyers?

Yield-improvement programs indeed rarely reach every farmer due to their high cost (associated with field visits, coaching sessions, and provision of equipment). They must, therefore, be selective. In the Black Sea region of Turkey (which produces more than 70% of the global hazelnut), only about 20,000 farmers are registered into the FFV program per season (Ferrero 2018b) out of a population of 440,000 farmers (ZMO 2018). Besides deciding how many farmers to enroll in the yield-improvement program, a buyer needs to decide which farmers to select. Farmers are indeed quite heterogeneous due to differences in the available land area, access to technology, know-how, and traits of farmlands. For instance, in the cocoa industry, production tends to be concentrated

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1 Although enrolled farmers are often considered to be partners, they are typically not bound to any contractual obligation towards the buyer (unlike contract farming); see, e.g., Whewell (2019) for FFV, or Nestlé (2024) for NESCAFÉ plan. For instance, our interviews revealed that it is not uncommon for cocoa farmers in Western Africa to sell some of their crop to third parties to cover short-term financial needs.

2 For instance, Ferrero participates in CocoaAction, which aims to “[offer] a profitable way of life for professionalized and economically empowered cocoa farmers and their families, while providing a significantly improved quality of life for cocoa-growing communities” (Ferrero 2018b, p. 132).
at a relatively small share of the population: In an experiment conducted by the Cocoa Challenge
Fund, 20% of the largest producers in the sample were responsible for around 50% of the total cocoa
production (Kuit et al. 2021, p. 70). Overall, organizations that lead yield-improvement programs
are often biased in their farmer selection processes. For instance, in the Philippines, large buyers'
programs and the Fair Trade program tend to select the most accessible farms, frequently overlooking
remotely located, disadvantaged farms (Üçel 2022). One of our interviewees also indicated that
large cocoa buyers generally partner with farmers who possess a minimum farm size, as it would
be otherwise impossible to take them out of poverty. This raises the question: to what extent is a
buyer’s farmer selection strategy effective in improving farmers’ living conditions?

Research Questions. By modeling the local market dynamics between heterogeneous farmers—
who exhibit different efficiencies (average yield per cost of planting effort)—and a buyer who
helps some farmers improve their yield (both average and variability), we investigate the following
research questions:

• What is the impact of a buyer’s yield-improvement program on the farmers’ total welfare?
• For any fixed enrollment size of a yield-improvement program, to what extent does the buyer’s
  optimal farmer selection decision deviate from the maximization of the farmers’ total welfare?

Model. To answer these questions, we consider a stylized Cournot competition model where farm-
ers produce a homogeneous crop and compete in the face of uncertain yields (Alizamir et al. 2019).
The sequence of events is as follows: First, the buyer decides the number and selection of farmers
to enroll in the yield-improvement program, which aims to increase their average yields and/or
reduce their yield variability, without affecting the yields of the non-enrolled farmers. Second, farm-
ers independently and simultaneously choose their planting efforts. Third, the yields are realized.
Fourth, the transaction takes place at the market-clearing price, matching supply (i.e., the farmers’
aggregate production) with demand (i.e., the buyer’s needs).

We ignore quality differences among farmers (besides those that directly translate into yield
improvements), price hedging in global financial markets, governmental price-setting intervention,
and supply chain intermediaries (e.g., governmental export authorities) given the operational (i.e.,
non-strategic) scope of the farmer selection decision. Although farmers are assumed to be risk-neutral
and to have no limited liability, their smallholder nature is intrinsic to the buyer’s initiative. This
is because larger farmers typically have limited potential for yield improvement, and the primary
concern is enhancing the welfare of smaller, more vulnerable farmers.

Results. We find that an increase in the total available crop due to the improved yields pushes
market prices down (as expected given the Cournot market dynamics). This hurts not only non-
enrolled farmers, but also potentially some enrolled farmers. Consequently, the often-heard claims
by buyers that yield-improvement initiatives are intended to alleviate farmer poverty become questionable, once we account for all farmers in their ecosystem. Even though some enrolled farmers may be worse off due to the program, they are still in a better position than if the buyer had chosen to enroll other farmers instead—a clear indication of the vicious cycle they are trapped in. As a result of these intricate dynamics, the farmers’ total welfare is typically maximized when the yield-improvement program targets only a limited number of farmers. In contrast, the buyer always benefits from improved yields and only refrains from enrolling all of them because of the cost associated with their enrollment.

We next show that the buyer’s farmer selection decision is often misaligned with the selection decision that would maximize the farmers’ total welfare. To establish this result, we proceed in two steps. First, we express the buyer’s utility as the sum of the square of the mean production and the production variance, revealing an inherent tension between these two objectives in the farmer selection decision. Specifically, when the program focuses more on increasing the average yields than on reducing their variability, selecting the most efficient farmers maximizes the variance of the production, whereas selecting the least efficient farmers (among those that remain in business) maximizes the mean production. Conversely, when the program focuses on reducing yield variability over increasing average yields, this relationship is reversed. Despite this inherent tension, our calibrated numerical experiments indicate that the buyer’s utility tends to be dominated by the mean production rather than the production variance. Second, we demonstrate that the farmer selection strategy that maximizes the farmers’ total welfare tends to be quite aligned with the strategy that maximizes the production variance. Together, these two findings suggest that the buyer primarily focuses on maximizing mean production (and thus securing the lowest price) while only slightly considering the improvement of farmers’ well-being (through maximizing production variance).

Comparing the farmer selection strategies, we show that the buyer’s objective tends to be more aligned with the goal of improving the farmers’ living conditions only when the program aims to substantially reduce yield variability. Otherwise, our numerical experiments reveal that the opportunity loss in welfare can be significant. However, the buyer seems to incur minimal losses when deviating from their optimal farmer selection strategy, whereas farmers could potentially derive substantial benefits from a different selection. Therefore, we recommend that buyers, if genuinely committed to improving the welfare of farmers through yield-improvement initiatives, should consider slightly compromising their individual interests when selecting farmers for these programs. This adjustment could significantly enhance the overall benefits for farmers.

**Outline.** After positioning our work relative to the literature in §2, we present our model in §3. Then, in §4, we characterize the effect of a yield-improvement program on the production quantities
and we identify the trade-offs involved, while verifying the farmers’ and buyer’s willingness to participate in such a program. We investigate the farmer selection strategy that aims at maximizing the buyer’s utility and the farmers’ total welfare in §5. In §6, we calibrate our model with hazelnut industry data and assess the welfare opportunity loss induced by the buyer’s farmer selection decision. We conclude our study and outline future research directions in §7. Appendices A, B, and C present, respectively, the sources of our model calibration, details on the interviews, and numerical experiments calibrated on the cocoa and coffee data. All proofs appear in an electronic companion.

2. Literature Review

Our research contributes to the growing body of work on socially responsible operations (Tang and Zhou 2012). For conciseness, we limit our review to the operations management literature on smallholder farmers in emerging markets; see Boyabatlı et al. (2022) for an overview. We categorize the literature along two dimensions, namely, the scope of analysis and the type of assistance provided. In terms of scope, we distinguish studies that assume exogenous market prices (i.e., farmers cannot influence prices, individually or collectively) from those that consider endogenous price formation. In terms of assistance, we restrict our review to three types of assistance, namely: access to market information, yield improvement, and financial help. Table 1 presents this classification.

<table>
<thead>
<tr>
<th>Type of Assistance</th>
<th>Exogenous Price</th>
<th>Identical Farmers</th>
<th>Endogenous Price</th>
<th>Non-Identical Farmers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Improvement</td>
<td>de Zegher et al. (2019), Tang et al. (2015), An et al. (2015), Xiao et al. (2019)</td>
<td>This Study</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

First, we review the works that assume exogenous prices. Farmers can be provided with different types of assistance, including information to reduce price dispersion and improve market access (Parker et al. 2016, Levi et al. 2020); yield improvement that is enabled by contracts and new sourcing channels (de Zegher et al. 2019); or financial assistance, such as subsidies (Akkaya 2017) or reward policies (de Zegher et al. 2018). Zhang and Swaminathan (2020) and Boyabatlı et al. (2019) propose an operations research model to optimize, respectively, planting schedule and crop rotation and, thus, to improve yields. We differ from these works by considering not only the subset of farmers who have access to the assistance, but also the other farmers in the ecosystem. Contrary to this literature, we show that operational improvements might hurt farmers’ well-being once we account for their negative externality on the market prices.

Next, we review the stream of literature that considers an endogenous price formation. Most of this literature, which we review next, assumes identical farmers. An early reference on the use...
of Cournot competition under yield uncertainty to guide operational decisions—in the context of vaccine production—is Deo and Corbett (2009). Tang et al. (2015) study farmers’ decision to adopt market information (which is free) and yield-improvement advice (which involves a fixed cost, as discussed in 1). They find that market information is beneficial, but yield-improvement advice may not be welfare-maximizing if its cost is too high. Similarly, we find that enrolled farmers may not always benefit from yield-improvement advice; in our study, however, this happens even if it is free to them. Chen and Tang (2015) characterize the value of public and private market information. They find that although both types of information stabilize prices, public information might not always benefit farmers. We identify similar trade-offs between the buyer’s utility and the farmers’ total welfare. An et al. (2015) assess the benefits and downsides of aggregation of smallholder farmers in terms of yield improvement (increasing/stabilizing process yield) and financial benefit (reducing planting costs), among others. Very much like aggregation, yield-improvement programs create two classes of farmers (enrolled vs. non-enrolled) and might not always be beneficial to them. Finally, Alizamir et al. (2019) and Guda et al. (2021) inform policy-makers on the implications of farming subsidy and guaranteed-support price schemes.

As noted above, there is limited literature that specifically accounts for farmer heterogeneity, with the following exceptions. Liao et al. (2019) investigate the impact of information provision policies on farmer welfare considering heterogenous farmers in terms of the inherent preference for selling in a certain market. Zhou et al. (2021) consider heterogeneity in access to market information (private signals) and find that providing assistance (in the form of information provision) may be detrimental to some farmers. Tang et al. (2018) investigate the impact of input- and output-based farm subsidies on both farmer aggregate profit and income inequality when farmers have different yields. Xiao et al. (2020) model heterogeneous farmers in their farming knowledge and find that specific reward mechanism should be put in place to ensure efficient to maximize farmer welfare. Last, Chintapalli and Tang (2021, 2022a,b) consider heterogeneous production costs and study the impact of minimum support prices. Similar to these works, we identify a misalignment between objectives, namely, the buyer’s utility and the farmers’ total welfare. In contrast to these works, we consider the case when the assistance takes the form of yield improvement and we care not only about how many, but also which farmers to assist.

In sum, our work differs from the literature on Cournot competition in small-scale farming on two dimensions. First, we assume that farmers are heterogeneous in terms of both their efficiency and their access to the assistance program. Second, we consider the buyer as the primary decision-maker. Research in sustainability commonly adopts a multi-objective approach (Pacini et al. 2004, Falconer and Hodge 2001, De Koeijer et al. 2002), but often takes a societal perspective, unlike this study, which explicitly associates the different objectives with the supply chain’s stakeholders (namely, the buyer and the farmers).
3. Model

We consider a local market for a particular crop (e.g., hazelnuts, cocoa beans, coffee beans) over a time horizon of a few harvest seasons, allowing farmers adequate time to adjust their planting efforts in response to a buyer’s yield-improvement program. Although these commodities are frequently traded on a global scale, this is often for hedging purposes. Buyers, who have established facilities and supply chains in the growing areas, are typically influenced by local market conditions.

On the demand side, we consider a single buyer (a monopsony), consistent with the highly concentrated market structure that characterizes such commodity markets, dominated by a few players (e.g., Ferrero, Cargill, Olam, Barry Callebaut). On the supply side, we consider a Cournot oligopolistic market, as is common in agricultural economics (Mas-Colell et al. 1995, Section 12.C) and operations (An et al. 2015, Tang et al. 2015, Alizamir et al. 2019), noting that it tends to a perfectly competitive market as the number of farmers becomes large.

We model first the buyer side of the market, which gives rise to an inverse demand function, and then the supply side of the market, consisting of the farmers. We then detail the impact of the yield-improvement program on the farmers’ yields.

3.1. Buyer

We assume that the buyer’s overall expenditure on the commodity is small relative to other expenditures (e.g., other commodities, labor, facilities, packaging), which are treated as a single composite commodity, called the numeraire. Moreover, the buyer’s utility is assumed to be separable in the two goods and linear in the numeraire, which allows us to proceed to a partial equilibrium analysis (Mas-Colell et al. 1995, Section 10.C or Vives 1999, Chapter 3). Similar to Singh and Vives (1984), we assume that the buyer’s utility is quadratic in the purchased quantity of the crop $q$. Specifically, at a crop’s market price $p$, the buyer derives utility from the crop $u(q, p) = \alpha q - \beta q^2/2 - pq$ with $\alpha, \beta > 0$; accordingly, the buyer requests quantity $q(p) = \arg \max_q u(q, p) = (\alpha - p)/\beta$. The market-clearing price matches supply (the farmers’ production) with demand (the buyers’ needs), yielding the following inverse demand: $p(q) = \alpha - \beta q$. With a slight abuse of notation, let

$$u(q) = u(q, p(q)) = \beta q^2/2$$

(1)

denote the buyer’s equilibrium utility when $q$ units of supply are available.

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3 Although we do not model the role of governments in setting prices, the horizon should also be sufficiently long to give them the opportunity to adjust their farm-gate price, if it is under their control.

4 Oxfam België/Belgique (2024, p. 24) notes that a small group of companies is buying most of the harvested cocoa, grinding and processing it, and even producing read-made chocolate mixtures for the chocolate manufacturers.

5 See, e.g., Oxfam België/Belgique (2024, p. 9).
3.2. Farmers

On the supply side, we consider a set $S$ of $n$ farmers who are heterogeneous in both their costs of planting effort and their yields. For instance, hazelnuts can be harvested with vacuuming harvester machines in coastal farms, whereas they can only be harvested by manual labor in mountainous farms, which is significantly less efficient; see also Kuit et al. (2021). In addition to heterogeneity in farm size and soil and weather conditions, farmers differ in terms of their education and/or willingness to make upfront investments, resulting in effort misallocation.

For any $i \in S$, let $\kappa_i$ be farmer $i$’s cost of planting effort and $\theta_i$ be their yield. We assume multiplicative yields, as is common in the agricultural operations literature (Yano and Lee 1995, Kazaz 2004, Alizamir et al. 2019, Zhou et al. 2021). Yields are random and assumed to be independent of each other, with mean $E[\theta_i]$, equal to $\mu_i$ prior to the implementation of the yield-improvement program, variance $V[\theta_i]$, and coefficient of variation $\psi_i \equiv \sqrt{V[\theta_i]/E[\theta_i]}$. Given the local nature of our market, farmers are generally subject to similar exogenous shocks; accordingly, we assume that, prior to the implementation of the yield-improvement program, all farmers have the same coefficient of variation. Let $c_i = \kappa_i/E[\theta_i]$ denote farmer $i$’s inverse mean efficiency. Without loss of generality, we assume that farmers are labeled in decreasing order of mean efficiency prior to the program, i.e., $\kappa_1/\mu_1 \leq \kappa_2/\mu_2 \leq \ldots \leq \kappa_n/\mu_n$.

In this Cournot market, farmers choose their planting effort anticipating the market clearing price $p(q) = \alpha - \beta q$. That is, farmer $i$ chooses their planting effort $x_i$ to maximize their expected profit $E[p(\sum_j \theta_j x_j)\theta_i x_i] - \kappa_i x_i$, taking into account the other farmers’ planting efforts. Changing variables, farmers’ profits can equivalently be expressed in terms of their expected production (i.e., harvested crops), denoted as $q_i \equiv E[\theta_i x_i]$, as follows: for $i \in S$,

$$
\pi_i(q_i; q_{-i}) = E \left[ p \left( \sum_j \frac{\theta_j q_j}{E[\theta_j]} \cdot \frac{\theta_i q_i}{E[\theta_i]} \right) \cdot \frac{\theta_i q_i}{E[\theta_i]} - c_i q_i \right] = E \left[ \left( \alpha - \beta \left( \sum_j \frac{\theta_j q_j}{E[\theta_j]} \right) \right) \cdot \frac{\theta_i q_i}{E[\theta_i]} \right] - c_i q_i, \quad \text{(2)}
$$

with $q_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_N)$. Thus, $q$ is a market equilibrium if it solves

$$
q_i = \arg \max_{q_i} \pi_i(q_i; q_{-i}) \quad \forall i. \quad \text{(3)}
$$

3.3. Yield-Improvement Program

Let $Y \subseteq S$ denote the set of enrolled farmers and $N = S \setminus Y$ denote the set of non-enrolled farmers.

The buyer’s yield-improvement program increases the average yield of the enrolled farmers by a factor $\gamma \geq 1$ and reduces their coefficient of variation by a factor $\psi_Y/\psi_N \leq 1$, without changing

\footnote{A former executive at a large cocoa processor told us that farmers should ideally spend 2/3 of their efforts pre-production and 1/3 post-production, but many tend to do the opposite. A less educated farmer might thus have a higher cost of investing the “right” effort.}
the yields of the non-enrolled farmers. Accordingly, after the yield-improvement program has been implemented, $E[\theta_i] = \mu_i$ and $\psi_i = \psi_N$ for all $i \in N$ and $E[\theta_i] = \gamma \mu_i$ and $\psi_i = \psi_Y$ for all $i \in Y$. Consequently, $c_i = \kappa_i / \mu_i$ for $i \in N$ and $c_i = \kappa_i / (\gamma \mu_i)$ for $i \in Y$; thus, after the program implementation, farmers are no longer ordered in decreasing order of mean efficiency. Let us also define $C_Y = \sum_{i \in Y} c_i$, $C_N = \sum_{i \in N} c_i$, and, as another proxy for yield variability, $v_Y = 1 + 2\psi_Y^2$ and $v_N = 1 + 2\psi_N^2$.

From the buyer’s perspective, implementing a yield-improvement program entails field visits, coaching sessions, and provision of equipment, and it is, therefore, typically associated with a fixed cost per enrolled farmer. Let $k(n_Y)$ represent the cost of enrolling $n_Y = |Y|$ farmers in the program, which is assumed to be increasing. Once the size of the enrollment program has been set, the question of farmer selection becomes pertinent. We distinguish two extreme selection strategies: the most efficient (ME) selection, which includes farmers $\{1, \ldots, n_Y\}$, and the least efficient (LE) selection, which comprises farmers $\{n_N + 1, \ldots, n\}$ with $n_N = n - n_Y$.

Because the farmers’ profits depend on their yields’ means and coefficients of variation, we append a ‘$Y$’ to the set of arguments, i.e., $\pi_i(q_i; q_{-i}, Y)$. Similarly, the equilibrium quantities solving the market equilibrium conditions is denoted as $q(Y)$. The resulting farmers’ equilibrium profits are denoted as $\pi_i(Y) = \pi_i(q_i(Y); q_{-i}(Y), Y)$ and, with a slight abuse of notation, the buyer’s equilibrium utility under an enrolled set of farmers, $Y$, is denoted as $u(Y)$ and defined from after subtracting the enrollment cost as follows:

\[
\pi_i(Y) = u\left(\sum_i \frac{\theta_i q_i(Y)}{E[\theta_i]}\right) - k(n_Y) = \frac{\beta}{2} \left(\sum_i \frac{\theta_i q_i(Y)}{E[\theta_i]}\right)^2 - k(n_Y).
\]

4. Equilibrium Characterization

4.1. Equilibrium Quantities

In our analytical characterization of the model, we assume that the number of farmers planting a positive quantity remains identical before and after the implementation of the yield-improvement program. (We relax this assumption in our numerical experiments in §6.) This happens, in particular, when $\alpha$ (namely, the market potential, which comes from the buyer’s valuation of the crop) is sufficiently high. For this purpose, we define the following lower bound:

\[
\alpha(Y) = \frac{-C_Y v_N - C_N v_Y + (\kappa_n / \mu_n) (n_Y v_N + n_N v_Y + v_N v_Y)}{v_Y v_N}.
\]

We next characterize the farmers’ equilibrium production quantities (solving) and their resulting profits.

---

7 We ignore any improvement in quality stemming from the yield-improvement program, unless its impact can be translated into a yield increase (e.g., higher shell-to-nut ratio). We also ignore any price premium or subsidies on equipment or fertilizers enrolled farmers may receive, but our model can easily be generalized to incorporate these factors. The farmers’ enrollment in the program does not change their obligations to the buyer unlike contract farming or vertical integration, which are sometimes adopted for other types of crops (e.g., maize, rice, cassava; see Meemken and Bellemare 2020), i.e., they can sell their crop to other channels (typically at a lower price to fulfill short-term financial needs) and do not pre-commit to production quantities (as they are inherently subject to random shocks).
Lemma 1. For any \( Y \), if \( \alpha \geq \alpha(Y) \),
\[
q_i(Y) = \frac{C_Y v_N + C_N v_Y + \alpha v_Y v_N}{v_i \beta (n_Y v_N + n_N v_Y + v_N v_Y)} - \frac{c_i}{v_i \beta},
\]
(6)
and
\[
\pi_i(Y) = q_i(Y)^2 \beta (1 + \psi_i^2).
\]
(7)

By Lemma EC.1 in the electronic companion, \( \alpha(Y) \) is increasing in \( Y \). Accordingly, we later require that \( \alpha \geq \alpha(Y \cup \{ j \}) \) whenever we study the incremental impact of adding farmer \( j \) to an enrolled set of farmers \( Y \) and that \( \alpha \geq \max_{Y:|Y|=n_Y} \alpha(Y) \) whenever we study the optimal selection of farmers in an enrollment program of size \( n_Y \).

4.2. Trade-Off between Farmers’ Total Welfare and Buyer’s Utility

Given the increasing public awareness of the poor living conditions of the farmers and its consequences on child labor and deforestation among other things, many buyers claim that their yield-improvement programs improve farmers’ well-being. To measure this, let \( \Pi(Y) = \sum_i \pi_i(Y) \) denote the farmers’ total welfare associated with an enrolled set of farmers \( Y \). From (7), we obtain that
\[
\Pi(Y) = \beta \sum_i \frac{\mathbb{E}[\theta_i^2 q_i(Y)^2]}{\mathbb{E}[\theta_i]^2}.
\]
(8)
In contrast, the buyer’s utility is a function of \( \frac{\beta}{2} \left( \sum_i \theta_i q_i(Y)/\mathbb{E}[\theta_i] \right)^2 \) by (4). Put it differently,
\[
u(Y) = \frac{\Pi(Y)}{2} + \frac{\beta}{2} \sum_{i \neq j} q_i(Y) q_j(Y) - k(n_Y).
\]

Therefore, the buyer’s objective is, in general, not necessarily aligned with the objective of maximizing the farmers’ total welfare.

To explore this tension further, we break down the buyer’s utility (9) as follows:
\[
u(Y) = \frac{\beta}{2} \mathbb{E} \left[ \sum_i \frac{\theta_i q_i(Y)}{\mathbb{E}[\theta_i]} \right]^2 + \frac{\beta}{2} \mathbb{V} \left[ \sum_i \frac{\theta_i q_i(Y)}{\mathbb{E}[\theta_i]} \right] - k(n_Y).
\]
(9)
Hence, the buyer is concerned with maximizing the mean production (the first term in (9)) and—for a given mean production—maximizing its variance (the second term in (9)). Alternatively, given that price is a linear decreasing function of the production, the buyer aims to minimize the mean price and—for a given mean price—maximizes its variance.

Note the similarity between the structure of the variance of price, i.e., \( \mathbb{V} \left[ \sum_i \theta_i q_i(Y)/\mathbb{E}[\theta_i] \right] \), and farmers’ total welfare, i.e., \( \beta \sum_i \mathbb{E}[\theta_i^2 q_i(Y)^2]/\mathbb{E}[\theta_i]^2 \). If the yield-improvement program has no impact on the yield coefficient of variation, i.e., if \( v_N = v_Y = v \), the variance of the production simplifies to \( \frac{v-1}{2} \sum_i q_i(Y)^2 = \Pi(Y) \frac{(v-1)}{\beta(v+1)} \). Therefore, maximizing the farmers’ total welfare is equivalent to
maximizing the production variance when \( v_Y = v_N \). In other words, the buyer cares not only about maximizing the mean production (or getting the lowest price), but also about the farmers’ well-being, although indirectly, through the variance of the production. Some buyers indeed appear to be genuinely concerned by the farmers’ well-being, even if motivated by pragmatic business reasons; e.g., “no beans means no chocolate” (Smith et al. 2014, p. 10).

4.3. Farmers’ Willingness to Participate

We next check that farmers are willing to participate in the yield-improvement program:

**Proposition 1.** For any \( j \) and \( Y \subseteq S \setminus \{j\} \), when \( \alpha \geq \alpha(Y \cup \{j\}) \), improving the yield of farmer \( j \) results in an increase in the profit of farmer \( j \) and a decrease in the profit of each farmer \( i \neq j \).

Proposition 1 reveals that a farmer always prefers to join a yield-improvement program than not. To understand why, consider the following dynamics: Enrolled farmers benefit from improved yields, so for any planting effort, their production quantity is higher. However, this increased production quantity results in a decrease in the market price. Even if all farmers adjust their planting effort anticipating this lower price, the total production is higher, as shown in Proposition EC.1 in the electronic companion, leading to a drop in the market price. As a result, each additional enrollment in the yield-improvement program hurts all farmers but the one that is newly enrolled.

This implies that all non-enrolled farmers are worse off. Moreover, enrolled farmers, even though they benefit from their own enrollment, are hurt by the enrollment of any other farmer into the program; as a result, the net effect of the yield-improvement program on their profit might be positive or negative. The following proposition formalizes this intuition.

**Proposition 2.** For any set \( Y \), when \( \alpha \geq \alpha(Y) \), as a result of the yield-improvement program,

i) the profit of each non-enrolled farmer decreases;

ii) there exists a threshold \( \hat{c}(Y) \) on the farmer inverse efficiency such that the profit of enrolled farmer \( i \) increases if and only if

\[
(v_N \sqrt{1 + \psi_Y^2} - v_Y \gamma \sqrt{1 + \psi_N^2}) (\hat{c}(Y) - c_i) \geq 0.
\]

(10)

Even though many buyers present their yield-improvement programs as initiatives to improve the well-being of the farmers, Proposition 2 shows that such programs can have negative consequences not only for the non-enrolled farmers but also for some enrolled farmers. Nevertheless, once a buyer has committed to a certain program size \( n_Y \), farmers are better off joining the program than not, even if participation leaves them worse off than if the program did not exist. This paradox epitomizes the vicious cycle in which poor farmers are often caught in.

Could a yield-improvement program avoid harming any farmer? This would be possible only if it included all farmers and ensured that \((v_N \sqrt{1 + \psi_Y^2} - v_Y \gamma \sqrt{1 + \psi_N^2})(\hat{c}(S) - c_i) \geq 0, \forall i \). Unfortunately,
neither of these conditions is likely to hold in practice. First, the current farmer enrollment is often below 50 percent (Meier et al. 2020). Second, the second condition cannot hold if some farmers have their inverse mean efficiency $c_i$ below $\hat{c}(S)$, whereas others have theirs above it, i.e., when farmers are highly heterogeneous.

In practice, yield-improvement programs are often implemented incrementally, enrolling different groups of farmers over time. Although newly-enrolled farmers are initially better off, their benefits may be short-lived: Once the program becomes larger, they are hurt by the addition of every new farmer and could eventually be worse off. As a result, the farmers’ total welfare may be maximized when the yield-improvement program is limited to some, but not all, farmers, as indicated by the next proposition.

**Proposition 3.** Assume identical ex-ante efficiencies, i.e., $\kappa_1/\mu_1 = \kappa_2/\mu_2 = ... = \kappa_n/\mu_n$. Then, the farmers’ total welfare is quasi-concave in the number of enrolled farmers.

In summary, farmers are willing to enroll in the yield-improvement program, but their benefits are often short-lived. Therefore, their motivation for enrollment is primarily driven by the desire to avoid being left behind.

### 4.4. Buyer’s Willingness to Improve Farmers’ Yields

We next investigate the buyer’s motivation for the yield-improvement program. Specifically, we analyze how the buyer’s utility changes as more farmers are enrolled, i.e., as $n_Y$ increases. To decouple this question from the selection decision discussed in §5, we consider identical ex-ante efficiencies, i.e., $\kappa_1/\mu_1 = \kappa_2/\mu_2 = ... = \kappa_n/\mu_n$. We separately consider the effects of an increase in the average yield, assuming no change in the coefficient of variation ($\psi_Y = \psi_N$), and the effects of a reduction in yield variability, assuming no change in the average yield ($\gamma = 1$).

**Proposition 4.** Assume $n_Y \geq 1$, $n \geq 2$, $k(n_Y) = 0$, and identical ex-ante efficiencies, i.e., $\kappa_1/\mu_1 = \kappa_2/\mu_2 = ... = \kappa_n/\mu_n$. Then the buyer’s utility is concavely increasing in the number of enrolled farmers if

a) the coefficients of variation do not improve, i.e., $\psi_Y = \psi_N$;

b) the average yields do not improve, i.e., $\gamma = 1$.

Proposition 4 thus shows that, absent enrollment costs $k(n_Y)$, the buyer benefits from the yield-improvement program, but at a decreasing marginal rate. Accordingly, in the presence of a convex enrollment cost $k(n_Y)$, the buyer might enroll only a selected subset of farmers, until their marginal

---

8 In general, when the yield-improvement program impacts both the average yield and its variability, the buyer’s utility might not always be increasing in the number of enrolled farmers, even when $k(n_Y) = 0$. For instance, when $\nu_Y = 1.5, \nu_N = 5, \alpha = 1, \beta = 1, \gamma = 1.2, \kappa = 1, \mu = 1, n = 100$, the buyer’s utility peaks at $n_Y = 48$ farmers.
return on enrollment equates their marginal cost. Combining Propositions 3 and 4 shows that both
the buyer and the farmers would like to limit the scope of the yield-improvement program, but for
different reasons. The question, then, becomes which farmers to select in the program for a given
size $n_Y$, which we investigate next.

5. Optimal Farmer Selection

In this section, we first characterize the farmer selections that maximize the mean production, the
variance of the production, and the farmers’ total welfare, and then summarize the findings to
determine when the buyer’s farmer selection decision, made to maximize the mean and variance of
the production (9), may conflict with the selection that would maximize the farmers’ total welfare,
for a given program size $n_Y$.

5.1. Mean Production

The following proposition characterizes the farmer selection that maximizes the mean production,
which, by (6), is equal to

$$\sum_i q_i(Y) = \frac{\alpha(v_Nn_Y + v_Yn_N) - (C_Yv_N + C_Nv_Y)}{\beta(n_Yv_N + n_Nv_Y + v_Nv_Y)}.$$  

**Proposition 5.** For any $n_Y$, $1 \leq n_Y \leq n - 1$, suppose that $\alpha \geq \max_{|Y|=n_Y} \alpha(Y)$. Then, the
mean production is maximized under the ME selection if $\gamma \leq \frac{v_N}{v_Y}$ and under the LE selection if $\gamma \geq \frac{v_N}{v_Y}$.

Therefore, when the increase in the average yield is greater than the reduction in its variability,
i.e., $\gamma \geq \frac{v_N}{v_Y}$, enrolling the LE farmers is optimal. In the opposite case, i.e., when $\gamma \leq \frac{v_N}{v_Y}$, enrolling
the ME farmers is optimal.

To get some intuition into Proposition 5, note that, in (11), the inverse efficiency of farmer $i$, $\kappa_i/\mu_i$,
is multiplied by $v_Y$ if $i \in N$ and by $v_N/\gamma$ if $i \in Y$. Because $\sum_i q_i(Y)$ is maximized by associating
the largest inverse efficiencies with the smallest coefficients, it is optimal to enroll the ME farmers
(lowest values of $\kappa_i/\mu_i$ if $v_N/\gamma \geq v_N$, and enroll the LE farmers otherwise.

From Proposition 5, when $\gamma = \frac{v_N}{v_Y}$, the mean production turns out to be independent of the
farmer selection strategy. Hence, the selection that maximizes the buyer’s utility is the same as the
one that maximizes the production variance, which we study next.

5.2. Variance of the Production

In contrast to the mean production, which is maximized under the ME selection (resp., LE selection)
when $\gamma$ is small (resp., large), the next proposition shows that the variance of the total production
tends to be maximized under the ME selection (resp., LE selection) when $\gamma$ is large (resp., small).
The second part of the proposition involves a threshold $\overline{\sigma}(Y)$, defined in [EC.10], which is greater
than $\alpha(Y)$. 
Proposition 6. For any \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y \mid |Y| = n_Y} \alpha(Y) \). If \( v_Y \leq v_N \frac{2-(n_Y-2)v_N+\sqrt{(1+n_Y+8v_N+(2+n_Y)^2v_N^2)}}{2(1+v_N+2v_N^2)} \), there exist thresholds \( \gamma_L^v \leq \gamma_M^v \) such that the variance of the total production is maximized under the LE selection if \( \gamma \leq \gamma_L^v \) and under the ME selection if \( \gamma \geq \gamma_M^v \). If \( \alpha \geq \max_{Y \mid |Y| = n_Y} \bar{\pi}(Y) \), \( \gamma_L^v \geq v_N/v_Y \).

To get some intuition into this result, note that the variance of the total production, \( \mathbb{V}[\sum_i \theta_iq_i(Y)/\mathbb{E}[\theta_i]] \), tends to be maximized when the largest mean production quantities \( q_i \) are associated with the largest yield variances \( \mathbb{V}[\theta_i] \). Accordingly, when the program has little impact on the average yield, i.e., when \( \gamma \) is small, so that the lower-indexed farmers remain the most efficient irrespective of who is enrolled, it is optimal to enroll the LE farmers so as to make the ME farmers (yielding the largest production quantities) subject to highest yield variability. However, when \( \gamma \) is large, the higher-indexed farmers, if enrolled, may end up becoming more efficient than the lower-indexed farmers, and their production quantities tend to be more similar. Because the variance of the total production tends to be larger when the individual production quantities are the most differentiated, it is better in this case to select the ME farmers.

Proposition 6 is only valid under the assumption that \( v_Y \) is smaller than a threshold depicted in the left panel of Figure 1, i.e., when the yield-improvement program substantially reduces the yield coefficient of variation. Although this condition seems restrictive, especially at large values of \( n_Y \), we further explore the case when \( v_Y \) is greater than this threshold in Proposition EC.2 in the electronic companion and show that the region in which the LE selection maximizes the variance of the production may be disconnected. Specifically, we show that there exist thresholds \( \gamma_L^v \leq \gamma_M^v \leq \tau_M \leq \tau_L^v \) such that the variance of the production is maximized under the ME selection when \( \gamma \in \left[ \gamma_L^v, \tau_M \right] \) and under the LE selection when \( \gamma \in \left[ 1, \gamma_L^v \right] \cup [\tau_L^v, \infty) \).

Finally, when \( \alpha \) is large (and under the other conditions in Proposition 6), the LE selection is optimal when \( \gamma = v_N/v_Y \), which is the threshold at which the selection strategy that maximizes the mean production switches (Proposition 5). Accordingly, the objectives of maximizing the mean production and maximizing the production variance lead to different farmer selections when either \( \gamma \leq v_N/v_Y \) or \( \gamma \geq \gamma_M^v \), and are aligned when \( \gamma \in [v_N/v_Y, \gamma_L^v] \).
5.3. Farmers’ Total Welfare

We next investigate the farmer selection that maximizes the farmers’ total welfare. To generate intuition into our main result, we first characterize how individual farmers’ profits are impacted by the farmer selection strategy. We find that farmers prefer to have, among the selected farmers, a more efficient farmer than a less efficient one if and only if \( \gamma \geq \frac{v_N}{v_Y} \).

**Lemma 2.** For any \( Y \) and any \( j \in Y \) and \( k \in N \), if \( \alpha \geq \max\{\alpha(Y), \alpha(\{k\} \cup Y \setminus \{j\})\} \), then \( \pi_i(Y) \geq \pi_i(\{k\} \cup Y \setminus \{j\}) \) for any \( i \neq j, k \) if and only if farmer \( j \) is less efficient than farmer \( k \) whenever \( \gamma \leq \frac{v_N}{v_Y} \) and more efficient otherwise.

From Proposition 5, we know that the mean production is minimized by enrolling the LE farmers when \( \gamma \leq \frac{v_N}{v_Y} \), and the ME farmers otherwise. Doing so also maximizes the expected price, which farmers appreciate because they get a higher return per planted area. Hence, there appears to be a natural tension between the goal of maximizing the mean production (Proposition 5) and the goal of maximizing the farmers’ total welfare. In particular, Lemma 2 suggests that the LE selection tends to be optimal when \( \gamma \) is small and the ME selection tends to be so when \( \gamma \) is large, similar to the objective of maximizing the variance of the production (Proposition 6).

**Proposition 7.** For any \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{|Y|=n_Y} \alpha(Y) \), one of the conditions in Lemma EC.18 holds, i.e., \( n_N \geq \hat{n}_N \) for some \( \hat{n}_N \), and \( v_Y \leq \frac{\sqrt{n_Y^2 + 4v_Nn_Y(v_N+1)n_Y} - n_Y}{2(v_N+1)} \), \( \Phi(1/\gamma) \). There exist thresholds \( \gamma^w_L \leq \gamma^w_M \leq \frac{v_N}{v_Y} \) such that the farmers’ total welfare is maximized under the LE selection if \( \gamma \leq \gamma^w_L \) and under the ME selection if \( \gamma \geq \gamma^w_M \).

Similar to Proposition 6, Proposition 7 requires some conditions to hold. First, it requires that the number of non-enrolled farmers \( n_N \) be larger than a threshold, which reduces to 1 when \( n_Y \) is itself above a threshold; see Lemma EC.18 in the electronic companion. Second, it requires that \( v_Y \) be no larger than an upper bound, depicted in the right panel of Figure 1, which becomes looser as \( n_Y \) increases. Proposition EC.3 in the electronic companion relaxes this latter condition and shows that the region in which the LE selection maximizes the farmers’ total welfare may be disconnected. Specifically, there exist thresholds \( \gamma^w_L \leq \gamma^w_M \leq \gamma^w_M \leq \gamma^w_L \) such that the farmers’ total welfare is maximized under the ME selection when \( \gamma \in \left[ \gamma^w_M, \gamma^w_M \right] \) and under the LE selection if \( \gamma \in \left[ 1, \gamma^w_L \right] \cup [\gamma^w_L, \infty) \).

Recall from §4.2 that when \( \frac{v_N}{v_Y} = 1 \), maximizing the farmers’ total welfare is equivalent to maximizing the production variance. Accordingly, when the yield-improvement program does not change the yields’ coefficient of variation, the buyer cares about both the mean production and the farmers’ total welfare. These two components of the buyer’s objective lead to polar prescriptions: Whereas the LE selection maximizes the mean production (Proposition 5), the ME selection maximizes the farmers’ total welfare (Proposition 7). Which of the two effects dominates and dictates the buyer’s optimal strategy is \textit{a priori} unclear, which is what we explore numerically in §6.
Table 2: Summary of the optimal farmer selection policies depending on the objective

<table>
<thead>
<tr>
<th>Objective</th>
<th>ME</th>
<th>ME</th>
<th>ME</th>
<th>LE</th>
<th>LE</th>
<th>LE</th>
<th>ME</th>
<th>ME</th>
<th>ME</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Production (Proposition 5)</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td></td>
</tr>
<tr>
<td>Variance of Production (Proposition 6)</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td></td>
</tr>
<tr>
<td>Farmers’ Total Welfare (Proposition 7)</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td></td>
</tr>
</tbody>
</table>

Assuming that \( \alpha \geq \max_{Y:\|Y\|=n_{Y}} \bar{\sigma}(Y) \), one of the conditions in Lemma EC.18 holds, i.e., \( n_{N} \geq \hat{n}_{N} \) for some \( \hat{n}_{N} \), and \( v_{Y} \leq \min \left\{ \frac{v_{N}^{2} + \frac{4v_{N}v_{Y}(v_{N} + 1) - \alpha_{N}}{2(v_{N} + 1)} - \gamma_{L}}{2(1 + v_{N} + 2\alpha_{N})}, \frac{\sqrt{\frac{4v_{N}v_{Y}(v_{N} + 1) - \alpha_{N}}{2(v_{N} + 1)} + \frac{4v_{N}v_{Y}(v_{N} + 1) - \alpha_{N}}{2(1 + v_{N} + 2\alpha_{N})}}}{2(1 + v_{N} + 2\alpha_{N})} \right\} \).  

5.4. Summary

Table 2 summarizes the results derived in Propositions 5, 6, and 7 assuming their most stringent conditions hold. Unlike Proposition 5, which offers a complete characterization of the optimal selection strategy that maximizes the mean production, Propositions 6 and 7 offer only sufficient conditions for optimality, leaving an indeterminate region denoted with a question mark. The table relies on the requirement that \( \alpha \geq \max_{Y:\|Y\|=n_{Y}} \bar{\sigma}(Y) \), which ensures that \( \gamma_{vL}^{w} \geq v_{N}/v_{Y} \) by Proposition 6 whereas Proposition 7 guarantees that \( \gamma_{vM}^{w} \leq v_{N}/v_{Y} \).

Table 2 shows that the ME selection maximizes the mean production at low values of \( \gamma \) and both the variance of the production and the farmers’ total welfare at high values of \( \gamma \), and vice versa for the LE selection. Although the objective of maximizing the farmers’ total welfare is in general more aligned with the objective of maximizing the variance of the production than with the objective of maximizing the mean production, Table 2 indicates that they provably result in the same selection strategy only when \( \gamma \) is very small (less than \( \gamma_{vL}^{w} \)) or very large (greater than \( \gamma_{vM}^{w} \)). Along the same vein, the buyer, whose objective consists of the sum of the square of the mean and the variance of the production, only manages to maximize both simultaneously when \( \gamma \in [v_{N}/v_{Y}, \gamma_{vL}^{w}] \) and needs to trade off one for the other otherwise.

6. Calibrated Numerical Experiments

To get a better sense of the tension between the buyer’s utility and the farmers’ total welfare, we resort to calibrated numerical experiments, focusing on the hazelnut commodity. (We also consider cocoa and coffee in Appendix C.) We also test the robustness of our analytical results by relaxing the requirement that \( \alpha \geq \alpha(Y) \) (defined in Equation 5), which allows farmers to produce a null quantity, i.e., to get out of business.

Following the procedure outlined in Appendix A, we estimate the average yield as \( \mathbb{E}[\theta_{i}] = \mu_{i} = 1.21 \) tons per hectare (ha) and its standard deviation as \( \sqrt{\mathbb{V}[\theta_{i}]} = 0.23 \) ton per ha, both assumed to be identical for all farmers; accordingly, \( \psi_{i} = 0.23/1.21 \) for all \( i \in S \) prior to the yield-improvement program. The global demand parameters are estimated to be \( \alpha =$5,448 per ton and \( \beta =$1.95 per ton². Given the global scope of our numerical experiments, an atomic farmer should be interpreted...
Table 3 Farmer selection strategies that maximize the mean production (top left), variance of the production (top right), buyer’s utility (bottom left), and farmers’ total welfare (bottom right) of hazelnuts

<table>
<thead>
<tr>
<th></th>
<th>Maximizing the Mean of the Production</th>
<th>Maximizing the Variance of the Production</th>
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<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$ = 2</td>
<td>1</td>
<td>LE</td>
</tr>
<tr>
<td></td>
<td>1.125</td>
<td>ME</td>
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<td></td>
<td>1.25</td>
<td>ME</td>
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<tr>
<td></td>
<td>1.375</td>
<td>ME</td>
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<td></td>
<td>1.5</td>
<td>ME</td>
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Maximizing the Buyer’s Utility

<table>
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<tr>
<th></th>
<th>Maximizing the Farmers’ Total Welfare</th>
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<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
</tr>
<tr>
<td>$\gamma$ = 2</td>
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<td>1.125</td>
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<td>1.25</td>
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<td></td>
<td>1.375</td>
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<td></td>
<td>1.5</td>
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</table>

Note: ‘LE’ corresponds to the least efficient selection; ‘ME’ corresponds to the most efficient selection; ‘ME+’ corresponds to any selection that does not include the farmers that would go out of business under the ME selection (namely, the least efficient ones).

as a whole farming region, and not an individual smallholder farmer. In line with current practice (Meier et al. 2020), we consider a 30% enrollment rate, i.e., $n_Y/n = 0.3$. For tractability, we set $n = 10$, which results in $\binom{10}{3} = 120$ possible selections. This small number of farmers is a way to capture various market frictions prevalent in practice, which prevent the market from functioning as a perfectly competitive one; see Appendix A.4. We also assume that farmers’ costs of planting effort are evenly spaced on $[3, 280, 3, 662]$, centered around the estimated average of $3,471$ per ha per year, which ensures that all farmers produce a positive quantity before the implementation of the yield-improvement program. Given that a 125%-improvement in yield is commonly reported (Ferrero 2018b), we consider different scenarios of increases in the average yield around this estimate, namely, $\gamma \in \{1, 1.125, 1.25, 1.375, 1.5\}$, and different scenarios in variability reduction, namely, $v_N/v_Y \in \{1, 1.125, 1.25, 1.375, 1.5\}$, keeping $v_Y$ constant at $1 + 2\psi_1^2 = 1 + 2(0.23/1.21)^2 = 1.07$ and changing $v_N$. (We change $v_N$, and not $v_Y$, to consider a broad range of variability.)

Table 3 reports the farmer selection strategies that maximize the buyer’s utility (bottom left), decomposed as the mean production (top left) and the variance of the production (top right), and the farmers’ total welfare (bottom right). In all cases, we derive the optimal farmer selections through complete enumeration: We evaluate all possible selections, allowing, for each case, farmers to go out of business. We then identify the selections that generate the maximum value.

The optimal farmer selection strategies reported in Table 3 are by and large consistent with Propositions 5, 6, and 7—with some subtle differences. Consider first the farmer selections that maximize the mean production (Table 3 top left). When $v_N/v_Y \geq \gamma$, the ME selection is optimal in line with Proposition 5. In contrast, when $v_N/v_Y \leq \gamma$, the optimal selection appears to deviate from
LE selection, but this is because we allow farmers to go out of business (since we do not require that \( \alpha \geq \alpha(Y) \)). For instance, when \( v_N = v_Y \) and \( \gamma = 1.25 \), the buyer anticipates that Farmers 9 and 10 might go out of business and thus prefers to improve the yield of Farmers 6, 7, and 8. Our prescription that LE farmers should be selected in the yield-improvement program thus remains valid once we restrict our attention to the surviving farmers.

Consider next the farmer selections that maximize the variance of the production (Table 3 top right). Consistent with Proposition 6, we find that the ME selection tends to be optimal for high values of \( \gamma \), and so does the LE selection for low values of \( \gamma \). We also observe that the smaller \( v_N/v_Y \), the more likely the variance is maximized under the ME selection. Outside the ranges of values for which we have characterized the optimality of the LE or ME selections, the variance may be maximized with a hybrid selection.

Third, consider the farmer selections that maximize the farmers’ total welfare (Table 3 bottom right). In this case, the ME selection is optimal across the whole range of values under consideration. Consistent with our discussion in §5.4, the region in which the ME selection is optimal tends to be wider than in the case of the maximization of the variance of the production and, as discussed in §4.2, these two objectives are aligned when \( v_N/v_Y = 1 \).

Finally, consider the farmer selection that maximizes the buyer’s utility, which combines considerations for both the mean and the variance of the production by (4), in the bottom left panel of Table 3. The buyer’s optimal selection appears to be heavily influenced by the mean production “off the diagonal,” namely: when \( \gamma \) is large and \( v_N/v_Y \) is small and when \( \gamma \) is small (but greater than 1) and \( v_N/v_Y \) is large. In the other cases, the buyer’s optimal selection strategy is a hybrid, trading off the two components of the buyer’s objective.

Comparing the farmer selections that maximize the buyer’s utility (Table 3 bottom left) to those that maximize the farmers’ total welfare (Table 3 bottom right) reveals that the two objectives are well aligned when \( v_N/v_Y \) is large and \( \gamma > 1 \), given that they both recommend an ME selection. This indicates that, for a given size of the yield-improvement program, the buyer’s farmer selection

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9 Because the conditions required by Proposition 6 are not necessarily satisfied, we refer the reader to Proposition EC.2 in the electronic companion for a more general characterization. Using the threshold notations from this proposition, we find that when \( v_N/v_Y \) takes values in \( \{1, 1.125, 1.25, 1.375, 1.5\} \), \( \gamma^w_1 \) respectively takes the values 1, 1.07, 1.11, 1.14, and 1.17, while \( \gamma^w_L \) respectively takes the values 1, 1.17, 1.26, 1.31, and 1.34 whereas we always have that \( \gamma^w_M = \gamma^w_M = \infty \). Accordingly, the regions where the production variance is provably maximized by the LE selection are \( [1, 1], [1, 1.07], [1, 1.11], [1, 1.14], \) and \( [1, 1.17] \), whereas the regions where the production variance is provably maximized by the ME selection are \( [1, \infty], [1.17, \infty], [1.26, \infty], [1.31, \infty], \) and \( [1.34, \infty] \), respectively when \( v_N/v_Y \in \{1, 1.125, 1.25, 1.375, 1.5\} \). The discrepancy between these theoretical results and the upper right panel of Table 3 occurs because we allow farmers to go out of business in our numerical experiments.

10 Because the conditions required by Proposition 7 are not necessarily satisfied, we refer the reader to Proposition EC.3 in the electronic companion for a more general characterization. Using the threshold notations from this proposition, we find that, for any \( v_N/v_Y \in \{1, 1.125, 1.25, 1.375, 1.5\} \), \( \gamma^w_L = \gamma^w_M = 1 \) and \( \gamma^w_L = \gamma^w_M = \infty \); accordingly, the farmers’ total welfare is provably always maximized by the ME selection.
is indeed identical to the one that would maximize the farmers’ total welfare when the program
targets a substantial reduction in yield variability. In such cases, the buyers’ claim that their yield-
improvement program is “doing good” is well founded.

In contrast, we observe a misalignment in the farmer selection strategies when the yield-
improvement program does not target a substantial reduction in yield variability or does not aim to
increase the average yield. In such cases, how much potential welfare is lost by adopting the buyer’s farmer selection decision? To answer this question, the left panel of Table 4 depicts the effectiveness
of the buyer’s optimal selection at maximizing the farmers’ total welfare, i.e., $\Pi(Y^u)/\Pi(Y^w)$, in
which $Y^u = \arg\max_{Y:|Y|=n_Y} u(Y)$ and $Y^w = \arg\max_{Y:|Y|=n_Y} \Pi(Y)$. We observe that the opportunity loss is the highest when $v_N/v_Y$ is the lowest (although it is not necessarily monotone as shown in the case where $\gamma = 1$) and $\gamma$ is the lowest. In particular, when $v_N/v_Y = 1$ and $\gamma = 1.125$, the buyer’s selection of the LE farmers, instead of the welfare-maximizing ME selection, results in a welfare opportunity loss of $1 - 64% = 36\%$. The opportunity loss could be even larger if the program size in the welfare-maximizing benchmark were not constrained to be equal to the buyer’s choice $n_Y$. For instance when $v_N/v_Y = 1$ and $\gamma = 1.125$, the buyer’s selection of the three LE farmers turns out to generate only 60% of the potential farmer welfare that could be attained by selecting the five ME farmers, i.e., $\Pi(Y^u)/(\max_{Y:|Y|\leq S} \Pi(Y)) = 60\%$.

Could the buyer be nudged towards making a welfare-maximizing farmer selection? To answer this question, the right panel of Table 4 depicts the effectiveness of the selection that maximizes the farmers’ total welfare at maximizing the buyer’s utility, i.e., $u(Y^w)/u(Y^u)$. Across all cases, we find that the buyer would lose only very little by switching to a selection that maximizes the farmers’ total welfare, in contrast to the farmers, who may gain tremendously from this switch, as depicted in the left panel. For instance, when $v_N/v_Y = 1$ and $\gamma = 1.125$, the farmers’ total welfare could increase by 36\% if the buyer were willing to sacrifice 0.2\% of their utility. Altogether, the two panels of Table 4 suggest that the buyer could be truly “doing good” by sacrificing a bit their personal interests in their farmer selection strategy for the greater benefit of the farmers.
Even though the farmers’ total welfare increases, irrespective of whether it maximizes the buyer’s utility (in which case, $\Pi(\mathcal{Y}^u)/\Pi(\emptyset) = 2.3$ when $\gamma = v_N/v_Y = 1.25$) or whether it maximizes the farmers’ total welfare (in which case, $\Pi(\mathcal{Y}^w)/\Pi(\emptyset) = 2.5$ when $\gamma = v_N/v_Y = 1.25$), the yield-improvement program creates disparities in outcomes among farmers. On one hand, all enrolled farmers appear to substantially benefit from the yield-improvement program when $\gamma = v_N/v_Y = 1.25^{11}$. All three enrolled farmers see their profit more than quadruple, i.e., $\pi_i(\mathcal{Y}^u)/\pi_i(\emptyset) > 452\%$ for all $i \in \mathcal{Y}^u$ and $\pi_i(\mathcal{Y}^w)/\pi_i(\emptyset) > 415\%$ for all $i \in \mathcal{Y}^w$. On the other hand, the profit of all non-enrolled substantially drops. Paradoxically, it is when the selection is welfare-maximizing that the drop is the most severe: $\pi_i(\mathcal{Y}^u)/\pi_i(\emptyset) < 32\%$ for all $i \in \mathcal{S} \setminus \mathcal{Y}^u$ such that $q_i(\mathcal{Y}^u) > 0$ and $\pi_i(\mathcal{Y}^w)/\pi_i(\emptyset) < 19\%$ for all $i \in \mathcal{S} \setminus \mathcal{Y}^w$ such that $q_i(\mathcal{Y}^w) > 0$. These statistics remind us that even though the buyer’s farmer selection ends up maximizing the farmers’ total welfare (the utilitarian solution), it may still be perceived to be unfair by accentuating inequalities \cite{bertsimas2012}. 

7. Conclusion

Several buyers of commodities (e.g., hazelnuts, cocoa, coffee) develop yield-improvement programs to improve the livelihoods of smallholder farmers, but are they really helping alleviate poverty? Modeling the local market dynamics between a monopsonistic buyer and numerous smallholder farmers competing in a Cournot fashion, we show that such programs can have disastrous consequences for some farmers—not only the farmers who are not enrolled, but also potentially some enrolled farmers as well. By improving farmers’ yields, both in terms of their average and variability, the total production tends to increase, which results in lower prices (Proposition [1]). The non-enrolled farmers end up selling the same crop at a lower price, which hurts their financial viability. Similarly, some enrolled farmers may turn out to be worse off, despite their improved yields, but still prefer to join the program rather than having another farmer join in their stead (Proposition [2]). As a result, the community of farmers may not benefit from improved yields at least beyond a certain size of the yield-improvement program (Proposition [3]), unlike the buyer, who—absent any enrollment costs—generally benefits from improved yields (Proposition [4]).

Besides deciding how many farmers to enroll in their yield-improvement programs, buyers also need to choose which farmers to select, when farmers have heterogeneous yields and costs of planting effort. We find that the buyer’s farmer selection decision, which aims at maximizing the mean production and its variance, may be quite misaligned with the selection that would maximize the farmers’ total welfare (Table [2]). Specifically, the mean production tends to be maximized by selecting the most efficient farmers when the program focuses more on reducing the farmers’ yield variability

\footnote{Inequality (10) turns out to be satisfied for all $i \in \mathcal{Y}^u$ because $\hat{c}(\mathcal{Y}^u) < 0$ and $v_N \sqrt{1 + \psi_N} < v_Y \gamma \sqrt{1 + \psi_N}$; and similarly for all $i \in \mathcal{Y}^w$.}
than on increasing their average yields, and the least efficient farmers otherwise (Proposition 5). In contrast, the variance of the production and the farmers’ total welfare tend to be maximized with the opposite pattern (Propositions 6 and 7).

In our calibrated numerical experiments, we observed that the buyer’s optimal farmer selection truly maximizes farmers’ total welfare only when the variability of the enrolled farmers is substantially reduced. Otherwise, the potential loss in farmers’ total welfare could be significant. Fortunately, it would require only a marginal decrease in buyers’ utility to change their farmer selection strategy. By slightly sacrificing their own interests, buyers could bridge the welfare gap and significantly improve the livelihoods of the farmers.

Although it is reassuring to observe that a bit of altruistic behavior can have a big impact on the farmers’ total welfare, inequalities among farmers are most likely going to be accentuated by such yield-improvement programs, potentially driving some farmers out of business. This calls for adopting other objectives than a purely utilitarian criterion to capture income inequalities (Bertsimas et al. 2012). Alternative measures to yield improvement, which would bear less negative externalities on the overall ecosystem, could also be considered, such as a shift from a transactional to a collaborative relationship with greater transparency and long-term purchasing agreements (e.g., contract farming, see Meemken and Bellemare 2020) to encourage investment, the provision of price premiums, and close work with farmers to eradicate child labor, prevent deforestation, and preserve biodiversity. A Nestlé executive indeed reached the conclusion that the problem “goes beyond good agricultural practices. You need to have an adequate social infrastructure such as medical assistance, road systems, communication, and schooling” (Porter et al. 2017, p. 13)

Properly capturing the impact of these complementary initiatives may require relaxing some of our model’s assumptions, such as formally modeling the role of governments and other supply chain intermediaries, capturing the smallholder farmers’ risk-averse behavior and limited liability, and introducing—whenever relevant—opportunities for vertical quality differentiation. We hope our research will stimulate follow-up work to study ways to break the vicious cycle many smallholder farmers have been caught in and improve their living conditions.

References


Ferrero (2018b) Sharing values to create value. URL [https://s3-eu-west-1.amazonaws.com/ferrero-static/globalcms/documenti/3733.pdf](https://s3-eu-west-1.amazonaws.com/ferrero-static/globalcms/documenti/3733.pdf)  
Nestlé (2024) Quality that goes beyond the cup. URL [https://www.nestle.com/brands/coffee/coffeecsv](https://www.nestle.com/brands/coffee/coffeecsv).


**Appendix A: Simulation Parameters**

**A.1. Cost of Planting Effort**

To estimate the cost of planting effort for each crop in $ per hectare (ha) per year, we used references for multiple countries that produce a majority of the crop and applied the following procedure.

1. We reviewed various streams of literature (listed below) to estimate the costs of planting hazelnut, cocoa, and coffee. When we encountered different estimates for the same country from different sources, we took an average to obtain one estimate per country.

2. We converted the reported estimates to USD using the currency conversion rate in the corresponding year.

3. Given that we are considering longitudinal data, we adjusted the commodity price estimates by the inflation rates using historical consumer price index data.

4. We computed the mean and standard deviation of the cost distribution as follows. Let $w_i$ be country $i$’s share of the average farming land (total quantity produced divided by the country’s average yield) during the horizon of our study (2008-2018) using [FAO](https://www.fao.org) website data and $\kappa_i$ be its estimated cost. The mean cost is computed as $\sum_i w_i \kappa_i$, and its variance as $\sum_i w_i (\kappa_i - \sum_i w_i \kappa_i)^2$.

[12] https://www.exchangerates.org.uk/

For hazelnut, we obtained data for Italy (Coppola et al. 2020) and Turkey (Demir 2018, Siray et al. 2015). The cost range that appears in these studies is [\$3,328, \$5,428] per ha per year, with a weighted average of \$3,471 per ha per year. These numbers are consistent with the cost estimates of the farmers we interviewed.

For cocoa, we used three sources: Côte d’Ivoire (Neale 2016), Ghana (Yahaya et al. 2015), and Nigeria (Oseni and Adams 2013). The cost range that appears in these studies is [\$535, \$760] per ha per year, with a weighted average equal to \$679 per ha per year.

For coffee, we used data for Honduras, Nicaragua, Brazil, Colombia, Costa Rica, Ethiopia, Kenya, Tanzania, and El Salvador (Montagnon 2017). The cost range that appears in these studies is [\$435, \$4,121] per ha per year (excluding Ethiopia, which is an outlier), with a weighted average of \$2,463 per ha per year.

**A.2. Yield Parameters**

To estimate yield parameters \(\mu\) and \(\sigma\), we used the 2008-2018 annual production data from the statistics tool of the Food and Agriculture Organization of the United Nations (FAO 2020). The yield mean, \(\mu\), is equal to \(1.21\) ton per ha for hazelnut; \(0.45\) ton per ha for cocoa; and \(0.85\) ton per ha for coffee. Similarly, the yield standard deviation, \(\sigma\), is \(0.23\) ton per ha for hazelnut; \(0.005\) ton per ha for cocoa; \(0.057\) ton per ha for coffee.

As for the yield increase, \(\gamma\), Ferrero (2018b) reports that orchards that participated in the Ferrero Farming Values yield-improvement programs saw an average of 25-30% increase in yield. In our interviews, we confirmed a yield increase of 25%. Accordingly, we used a median case of \(\gamma = 1.25\) and \(v_N/v_Y = 1.25\) in our simulations.

**A.3. Price-Demand Curve Parameters**

For the market prices, we used local commodity exchange data (OTB 2021) for hazelnut, and the data portal IndexMundi, which contains data from the International Cocoa Organization and the International Coffee Organization, for cocoa and coffee (IndexMundi 2021a, b). Similar to the cost of farming, we also adjusted the prices for inflation. Using the same production data as in our estimation of the yield parameters, we estimated the demand parameters, \(\alpha\) and \(\beta\), using linear regression by expressing the average market prices as a function of the annual production quantity. From the regression, we obtained the following estimates for \(\alpha\): \$5,448 per ton for hazelnut; \$8,106 per ton for cocoa; and \$11,647 per ton for coffee. Similarly, we estimated \(\beta\) as equal to \$1.95 per ton\(^2\) for hazelnut; \$1.09 per ton\(^2\) for cocoa; and \$0.84 per ton\(^2\) for coffee.

**A.4. Internal Consistency Check**

To test the internal consistency of our model, we compared our model’s predicted equilibrium total production quantity with \(n=10\) farmers to the average reported quantity of FAO during the time period we considered (2008-2018), using the base parameter estimates with no yield improvement. Our model predicts an equilibrium total production of hazelnuts equal to \(\sum q_i^* = 1,267,390\) tons, which is 38% higher than the estimated average hazelnut production quantity, estimated to be equal to 918,390 tons.\(^{14}\) An equilibrium production of cocoa of 5,521,610 tons, which is 12% higher than the estimated average cocoa production quantity of 4,913,550 tons; and an equilibrium production of coffee of 9,566,430 tons, which is 6% higher.

\(^{14}\) We obtained the total production quantity by multiplying the reported planting area from FAO with the average yield parameter, as explained in Appendix A.2.
than the estimated average production of 9,015,100 tons. In sum, our game-theoretic model appears to over-estimate the actual production quantity, from marginally (for coffee) to more substantially (for hazelnut), suggesting, especially in the latter case, that there could be additional market frictions not captured by the model. Given that an increase in the number of farmers \( n \) would lead to a larger equilibrium quantity, this comparative analysis suggests that one way to capture the impact of such frictions is to limit the number of farmers under consideration, therefore justifying our choice of a small value for \( n \).

**Appendix B: Details of Interviews**

We conducted ten 30-minute interviews during the summer of 2018 in Ordu, Turkey with the following stakeholders:

- Five smallholder farmers (three in person and two over the phone)
- Two managers of hazelnut processors (in person)
- One hazelnut trader (over the phone)
- One fertilizer/equipment seller (over the phone)
- The general secretary of Ordu hazelnut commodity exchange (in person)

The following questions were asked:

1. What is the cost of producing hazelnut?
2. Are you getting fair prices for your hazelnut? What do you think influences hazelnut prices?
3. Which channel do you prefer to use to sell your hazelnut?
4. Do you require financing? If so, how do you acquire it?
5. What do you think about the yield-improvement programs, such as Ferrero’s FFV program?
   (a) What is the benefit of being enrolled in a yield-improvement program?
   (b) What is the cost of being enrolled in a yield-improvement program?
6. What are other problems in the hazelnut farming sector?

Additionally, during the summer of 2022, we conducted additional extensive interviews with one former executive at a large cocoa grinder, one hazelnut trader, and one coffee farmer. The following questions were asked:

1. What is the impact of yield-improvement programs on the average yield and yield variance? What other benefits do these programs provide to the farmers?
2. How do buyers select farmers to enroll into the yield-improvement programs?
3. Who do the farmers typically sell their produce to? Are farmers who are enrolled in yield-improvement programs obliged to sell their crop to the company that runs these programs?
4. Is yield a main determinant of prices? Do government policies have an impact on them?

Interview notes are available upon request.
Table 5 Farmer selection strategies that maximize the mean production (top left), variance of the production (top right), buyer’s utility (bottom left), and farmers’ total welfare (bottom right) of cocoa

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Maximizing the Variance of the Production

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Maximizing the Buyer’s Utility

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Maximizing the Farmers’ Total Welfare

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Note: ‘LE’ corresponds to the least efficient selection; ‘ME’ corresponds to the most efficient selection; ‘ME+’ corresponds to any selection that does not include the farmers that would go out of business under the ME selection (namely, the least efficient ones).

Appendix C: Numerical Experiments for Cocoa and Coffee

C.1. Cocoa

As detailed in Appendix A, we estimate the average yield as $E[\theta_i] = \mu_i = 0.45$ tons per ha and its standard deviation as $\sqrt{E[\theta_i]} = 0.005$ ton per ha, both assumed to be identical for all farmers; accordingly, $\psi_i = 0.005/0.45$. The global demand parameters are estimated to be $\alpha = $8,106 per ton and $\beta = $1.09 per ton$^2$. We set $n$ to 10 and assume that farmers’ costs of planting effort are evenly spaced on $[638, 719]$, centered around the estimated average of $679$ per ha per year, which ensures that all farmers are producing a positive quantity before their yields are improved. In addition, we set $\nu_n/n = 30\%$. Similar to §6, we consider the following scenarios of yield improvement: $\gamma \in \{1, 1.125, 1.25, 1.375, 1.5\}$ and $\nu_n/\nu_Y \in \{1, 1.125, 1.25, 1.375, 1.5\}$, keeping $\nu_Y$ constant at $1 + 2\psi_i^2 = 1 + 2(0.005/0.45)^2 = 1.0002$ and changing $\nu_n$.

Table 5 reports the farmer selections that maximize the mean production (top left), the variance of the production (top right), the buyer’s utility (bottom left), and farmers’ total welfare (bottom right). Unlike the case of hazelnuts (§6), no farmer gets out of business as a result of the yield-improvement program. Accordingly, the farmer selection strategy that maximizes the mean production is as identified in Proposition 5; hence, the notation “ME+” means here any selection (including ME and LE). The selection that maximizes the variance of the production and the one that maximizes the farmers’ total welfare are consistent with the ones characterized in, respectively, Propositions 6 and 7. Accordingly, the objectives of maximizing the mean and the variance of the production are fully aligned with $\nu_Y < \nu_n$ and $\gamma \geq \nu_n/\nu_Y$. In the other cases, the buyer’s utility appears to be primarily dominated by the consideration of the variance of the production, except at very low values of $\gamma$.

The buyer’s utility is in general misaligned with the objective of maximizing the farmers’ total welfare, except when $\nu_n/\nu_Y$ is very large and $\gamma = 1$, which is more restrictive than in the case of hazelnuts. The left panel of Table 6 which reports the efficacy of the buyer’s optimal farmer selection at maximizing the
Table 6  Effectiveness of the buyer’s optimal farmer selection at maximizing the farmers’ total welfare (left) and effectiveness of the welfare-maximizing farmer selection at maximizing the buyer’s utility (right) for cocoa

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Note: \( Y^w = \arg \max_{Y | Y \in nY} u(Y) \) and \( Y^w = \arg \max_{Y | Y \in nY} II(Y) \). Percentages are rounded to the nearest integer unless they would round to 100%.

Table 7  Farmer selection strategies that maximize the mean production (top left), variance of the production (top right), buyer’s utility (bottom left), and farmers’ total welfare (bottom right) of coffee

Maximizing the Mean of the Production

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Maximizing the Variance of the Production

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Maximizing the Buyer’s Utility

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<td>LE</td>
<td>LE</td>
</tr>
<tr>
<td>1.375</td>
<td>{5.6}</td>
<td>LE</td>
<td>LE</td>
<td>LE</td>
</tr>
<tr>
<td>1.5</td>
<td>{2.3, 4}</td>
<td>{6.7}</td>
<td>LE</td>
<td>LE</td>
</tr>
</tbody>
</table>

Maximizing the Farmers’ Total Welfare

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \gamma )</th>
<th>( \gamma )</th>
<th>( \gamma )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.125</td>
<td>1.25</td>
<td>1.375</td>
<td>1.5</td>
</tr>
<tr>
<td>1.25</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
</tr>
<tr>
<td>1.25</td>
<td>ME</td>
<td>ME</td>
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<td>ME</td>
</tr>
<tr>
<td>1.375</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
</tr>
<tr>
<td>1.5</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
<td>ME</td>
</tr>
</tbody>
</table>

Note: ‘LE’ corresponds to the least efficient selection; ‘ME’ corresponds to the most efficient selection; ‘ME+’ corresponds to any selection that does not include the farmers that would go out of business under the ME selection (namely, the least efficient ones).

farmers’ total welfare, shows that the welfare loss remains more moderate than in the case of hazelnuts, though potentially still significant (at most 7%). More importantly, the right panel of Table 6, which reports the efficacy of the selection that maximizes the farmers’ total welfare at maximizing the buyer’s utility, indicates that, like for hazelnuts, the buyer would lose very little (at most 0.4%) from deviating from their optimal strategy to significantly enhance the farmers’ welfare (by 7%).

C.2. Coffee

As detailed in Appendix A, we estimate the average yield as \( E[\theta_i] = \mu_i = 0.85 \) tons per ha and its standard deviation as \( \sqrt{\psi^2} = 0.057 \) ton per ha, both assumed to be identical for all farmers; accordingly, \( \psi_i = 0.057/0.85 \). The global demand parameters are estimated to be \( \alpha = $11,647 \) per ton and \( \beta = $0.84 \) per ton². We set \( n \) to 10 and assume that farmers’ costs of planting effort are evenly spaced on \([$2192, $2734]\), centered around the estimated average of $2,463 per ha per year, so as to ensure that all farmers are producing a positive quantity before their yields are improved. In addition, we set \( n_Y/n = 30\% \). Similar to 8, we consider the following scenarios of yield improvement: \( \gamma \in \{1, 1.125, 1.25, 1.375, 1.5\} \) and \( v_y / v_Y \in \{1, 1.125, 1.25, 1.375, 1.5\} \), keeping \( v_Y \) constant at \( 1 + 2\psi_i^2 = 1 + 2(0.005/0.45)^2 = 1.0002 \) and changing \( v_y \).

Table 7 reports the farmer selections that maximize the mean production (top left), the variance of the production (top right), the buyer’s utility (bottom left), and farmers’ total welfare (bottom right).
Table 8 Effectiveness of the buyer’s optimal farmer selection at maximizing the farmers’ total welfare (left) and effectiveness of the welfare-maximizing farmer selection at maximizing the buyer’s utility (right) for coffee

<table>
<thead>
<tr>
<th>( \frac{\Pi(Y^w)}{\Pi(Y^u)} )</th>
<th>1</th>
<th>1.125</th>
<th>1.25</th>
<th>1.375</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\gamma}{vY} )</td>
<td>1</td>
<td>88%</td>
<td>83%</td>
<td>80%</td>
<td>79%</td>
</tr>
<tr>
<td>1.125</td>
<td>97%</td>
<td>88%</td>
<td>82%</td>
<td>80%</td>
<td>79%</td>
</tr>
<tr>
<td>1.25</td>
<td>90%</td>
<td>85%</td>
<td>82%</td>
<td>80%</td>
<td>79%</td>
</tr>
<tr>
<td>1.375</td>
<td>98%</td>
<td>89%</td>
<td>81%</td>
<td>80%</td>
<td>79%</td>
</tr>
<tr>
<td>1.5</td>
<td>99%</td>
<td>89%</td>
<td>81%</td>
<td>80%</td>
<td>79%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \frac{u(Y^w)}{u(Y^u)} )</th>
<th>1</th>
<th>1.125</th>
<th>1.25</th>
<th>1.375</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\gamma}{vY} )</td>
<td>1</td>
<td>99.9%</td>
<td>99.7%</td>
<td>99.4%</td>
<td>99.1%</td>
</tr>
<tr>
<td>1.125</td>
<td>99.9%</td>
<td>99.0%</td>
<td>99.1%</td>
<td>99.0%</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>99.9%</td>
<td>99.3%</td>
<td>99.4%</td>
<td>99.1%</td>
<td></td>
</tr>
<tr>
<td>1.375</td>
<td>99.9%</td>
<td>99.8%</td>
<td>99.5%</td>
<td>99.3%</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>99.9%</td>
<td>99.9%</td>
<td>99.6%</td>
<td>99.4%</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( Y^u = \arg \max_{Y:|Y|=nY} u(Y) \) and \( Y^w = \arg \max_{Y:|Y|=nY} \Pi(Y) \). Percentages are rounded to the nearest integer unless they would round to 100%.

Similar to the case of cocoa (§C.1), no farmer gets out of business as a result of the yield-improvement program. Accordingly, the farmer selection strategy that maximizes the mean production, the variance of the production, the resulting buyer’s utility, and the farmer’s total welfare are similar to the ones characterized in Propositions 5-7 as well as those depicted in Table 5.

In this case, the buyer’s utility is never aligned with the objective of maximizing the farmers’ total welfare. The left panel of Table 8 which reports the efficacy of the buyer’s optimal farmer selection at maximizing farmer’s total welfare, shows that the worst-case welfare loss lies somewhere between the ones found in the cases of hazelnuts (36%) and cocoa (7%), namely, 21%. Like for the other two commodities, the right panel of Table 8 which reports the efficacy of the selection that maximizes the farmers’ total welfare at maximizing the buyer’s utility, indicates that the buyer would lose very little (at most 1.1%) from deviating from their optimal strategy to significantly enhance the farmers’ welfare (by 21%).
Proofs of Statements

Proof of Lemma 4 Fix \( Y \). Expanding (2) and using the fact that \( \mathbb{E}[\theta_i^2]/\mathbb{E}[\theta_i]^2 = 1 + \psi_i^2 \) yield \( \pi_i(q_i; q_{-i}, Y) = \alpha q_i - \beta q_i \sum_{j \neq i} q_j - \beta(1 + \psi_i^2)q_i - c_i q_i \), which is continuous and strictly concave because \( \frac{\partial^2 \pi_i}{\partial q_i^2} = -2\beta(1 + \psi_i^2) < 0 \). Moreover, a farmer’s action set can be bounded by \([0, \alpha/\beta]\). By Mas-Colell et al. (1995) Proposition 8.D.3), there exists a Nash equilibrium. Solving the first-order optimality conditions while ignoring the non-negativity constraints, we obtain: (1995, Proposition 8.D.3), there exists a Nash equilibrium. Solving the first-order optimality conditions while ignoring the non-negativity constraints, we obtain:

\[
(1995, \text{Proposition } 8.D.3), \text{ there exists a Nash equilibrium.}
\]

Proof. Because \( \alpha - \beta \sum_i q_i - c_i \) for all \( i \), Solving this system of equations yields (3). From (3) \( q_i(Y) \geq 0 \forall i \). Plugging (3) into \( \pi_i(q_i(Y); q_{-i}(Y), Y) \), \( \forall i \in S \) yields (7).

Lemma EC.1. For any \( j \) and \( Y \subseteq \{1, \ldots, n\} \setminus \{j\} \), \( \alpha(Y \cup \{j\}) \geq \alpha(Y) \).

Proof. Because \( v_Y \leq v_N \) and \( \kappa_j/\mu_j \leq \kappa_n/\mu_n \), using (5), we obtain

\[
\alpha(Y \cup \{j\}) - \alpha(Y) = \frac{(\kappa_n/\mu_n)(v_N - v_Y)\gamma + (\kappa_j/\mu_j)(-v_N + v_Y\gamma)}{v_Y v_N} \geq \frac{(\kappa_j/\mu_j)(\gamma - 1)}{\gamma v_Y},
\]

which is non-negative because \( \gamma \geq 1 \).

Proof of Proposition 7 Fix \( j \) and \( Y \subseteq S \setminus \{j\} \); let \( N \subseteq S \setminus \{Y\} \). First, consider any \( i \neq j \). We seek to establish that \( \pi_i(Y) \geq \pi_i(Y \cup \{j\}) \), which, by (7) in Lemma 4 is equivalent to \( q_i(Y) - q_i(Y \cup \{j\}) \geq 0 \). If \( i \in Y \), set \( v_i = v_Y \) and define \( v_{-i} = v_N \); and vice versa. Accordingly,

\[
\frac{\partial(q_i(Y \cup \{j\}) - q_i(Y))}{\partial \alpha} = \frac{v_{-i}(v_Y - v_N)}{(n_Y v_N + v_N(1 + v_Y) - v_Y)((n_Y - 1)v_Y + v_N(1 + n_Y + v_Y))}\beta \leq 0.
\]

Therefore, \( q_i(Y \cup \{j\}) - q_i((Y) \) is maximized when \( \alpha \) takes its smallest value. Note that \( \alpha \geq \alpha(Y \cup \{j\}) \geq \alpha(Y) \) by Lemma EC.1. Replacing \( \alpha \) with \( \alpha(Y) \), we obtain

\[
q_i(Y \cup \{j\}) - q_i(Y) \leq \frac{c_i(v_N - v_Y\gamma - c_n(v_N - v_Y)\gamma}{v_i((n_N - 1)v_Y + v_N(1 + n_Y + v_Y))\beta}\gamma.
\]

The right-hand side is non-positive because \( c_i \geq c_j, v_N \geq v_Y \), and \( \gamma \geq 1 \).

Second, consider farmer \( j \). We seek to establish that \( \pi_j(Y \cup \{j\}) \geq \pi_j(Y) \), which, by (7) in Lemma 4 is equivalent to \( q_j(Y \cup \{j\}) - q_j(Y) \geq 0 \). Because

\[
\frac{\partial(q_j(Y \cup \{j\}) - q_j(Y))}{\partial \alpha} = \frac{v_N - v_Y}{(n_Y v_N + v_N(1 + v_Y) - v_Y)((n_N - 1)v_Y + v_N(1 + n_Y + v_Y))}\gamma.
\]

given that \( v_N \geq v_Y \), \( v_N \geq 1 \) (because \( j \in N \)), \( q_j(Y \cup \{j\}) - q_j(Y) \) is minimized when \( \alpha \) takes its smallest value. Note that \( \alpha \geq \alpha(Y \cup \{j\}) \geq \alpha(Y) \geq \alpha(Y \cup \{j\}) \geq \alpha(Y) \) by Lemma EC.1 and given that \( \kappa_n/\mu_n \geq \kappa_j/\mu_j \).

Replacing \( \alpha \) with the right-hand side of the last inequality, we obtain

\[
q_j(Y \cup \{j\}) - q_j(Y) \geq \frac{c_j v_Y}{v_Y(v_N - v_Y)((n_N - 1)v_Y + v_N(1 + n_Y + v_Y))}\gamma \geq 0.
\]

The right-hand side is non-negative because \( n_N \geq 1, v_N \geq 1 \), and \( \gamma \geq 1 \).
Proposition EC.1. For any $j$ and $Y \subseteq S \setminus \{j\}$, when $\alpha \geq \alpha(Y \cup \{j\})$, enrolling farmer $j$ into the yield-improvement program leads to an increase in the mean total production quantity.

Proof. Fix $j$ and $Y \subseteq S \setminus \{j\}$. Let $Q(Y) = \sum q_i(Y)$ and $Q(Y \cup \{j\}) = \sum q_i(Y \cup \{j\})$. By Lemma EC.1 $\alpha \geq \alpha(Y \cup \{j\}) \geq \alpha(Y)$. Thus, $q(Y)$ is given by (6) in Lemma 1. Accordingly,\[
Q(Y) - Q(Y \cup \{j\}) = \left(\frac{C_N + κ_μ/μγ}{β(nY + 1)v_N + (nN - 1 + v_N)v_Y} - \frac{C_N + κ_μ/μγ}{β(nY + 1)v_N + (nN + v_N)v_Y} \right)\]
Because
\[
\frac{∂(Q(Y) - Q(Y \cup \{j\}))}{∂(n)} = -\frac{v_N v_Y (v_N - v_Y)}{β(nY + 1)v_N + (nN - 1 + v_N)v_Y + (nN + v_N)v_Y} \leq 0,
\]
\[
Q(Y) - Q(Y \cup \{j\}) \leq \frac{κ_μ/μγ(v_N - v_Y)}{γβ(nN - 1) \sqrt{1 + v_N νY}} \leq 0.
\]
The right-hand side is non-positive because $κ_μ/μγ ≥ κ_μ/μγ$ by assumption on the labeling of farmers, $v_N ≥ v_Y$, and $γ ≥ 1$. Therefore, $Q(Y \cup \{j\}) ≥ Q(Y)$.

Proof of Proposition 3. Fix $Y$; let $N \equiv S \setminus \{Y\}$. First, consider any $i \in N$. Applying Proposition 1 iteratively for all farmers in $Y \equiv \{j\}$ (given that $α ≥ \alpha(Y) \geq \alpha(\{j\}) ≥ \alpha(\emptyset)$ by Lemma EC.1), we obtain $π_i(∅) ≥ π_i(\{j\}) ≥ \ldots ≥ π_i(Y)$.

Second, consider any $i \in Y$. By Lemma 1 (given that $α ≥ \alpha(Y) ≥ \ldots ≥ \alpha(∅)$ by Lemma EC.1), $π_i(∅) ≤ π_i(Y)$ if and only if
\[
K_1c_i + K_2 ≤ 0,
\]
where
\[
K_1 = (n + v_N)(nYv_N + (nN + v_N)v_Y)(v_N \sqrt{1 + ψ_i} - v_Yγ√1 + ψ_iγ)\]
\[
K_2 = (n + v_N)(nYv_N + (nN + v_N)v_Y)(γC_N + v_Nα) - (n + v_N)(nYv_N + (nN + v_N)v_Y)\]
Suppose that $K_1 ≠ 0$. Let $c(\{j\}) = -\frac{K_2}{K_1}$. Note that $K_1$ is positive if and only if $v_N \sqrt{1 + ψ_i} > v_Yγ√1 + ψ_iγ$. Thus, if $v_N \sqrt{1 + ψ_i} > v_Yγ√1 + ψ_iγ$, then $π_i(∅) ≤ π_i(Y)$ if and only if $c_i ≤ c(\{j\})$. Else, if $v_N \sqrt{1 + ψ_i} < v_Yγ√1 + ψ_iγ$, then $π_i(∅) ≤ π_i(Y)$ if and only if $c_i ≥ c(\{j\})$. Combining these two results yields that $π_i(∅) ≤ π_i(Y)$ if and only if (10) holds.

Proof of Proposition 3. Because $κ/μγ = κ/μγ$ for all $i$, $π(i)$, defined in (8), depends on $Y$ only through $n_Y$. Accordingly, let us denote it as $π(n_Y)$. Its second derivative, $π''(n_Y)$, can be expressed as $f(n_Y)/(2(n_Yv_N + (nN + v_N)v_Y))(γ^2μ^2)$ where $f(n_Y) ≡ g(n)n_Y + h(n)$, in which $g(n)$ and $h(n)$ are independent of $n_Y$. Note that the denominator is non-negative and that $f(n_Y)$ is a linear function of $n_Y$.

It turns out that $g(1) = 2(v_N - v_Y)(v_N + v_Y + v_Yv_Y)((1 + v_N - (1 + v_Y)γ)κ + (v_N + v_Y)γμ) ≥ 0$ and $g'(1) = 4(v_N - v_Y)(v_N + v_Y)(γ - 1)κ((v_N - v_Y)γμ - (1 + v_N - (1 + v_Y)γ)κ) ≥ 0$ because $γ ≥ 1$, $v_N ≥ v_Y$ and
\( \alpha \mu \geq \kappa \). Moreover, \( g''(n) = 4(v_N - v_Y)(v_N + v_Y - v_Nv_Y)(\gamma - 1)^2 \kappa^2 \geq 0 \) if and only if \( v_N + v_Y \geq v_Nv_Y \). We next consider two cases, depending on whether \( g''(n) \geq 0 \) or not.

First, suppose that \( v_N + v_Y \geq v_Nv_Y \), i.e., \( g''(n) \geq 0 \). Then \( g(n) \geq 0, \forall n \geq 1 \). Accordingly, \( f(n_Y) \) is increasing, i.e., \( \Pi(n_Y) \) may be initially concave and then becomes convex as \( n_Y \) increases. Moreover, \( \lim_{n_Y \to \infty} \Pi'(n_Y) \leq 0 \). As a result, \( \Pi(n_Y) \) has no interior local minimum.

Second, suppose that \( v_N + v_Y < v_Nv_Y \), i.e., \( g''(n) < 0 \). Then \( g(n) \) is concave and there exists a threshold value
\[
\bar{n} = -v_N(v_N + (3 + v_N)v_Y)\kappa + v_Y(v_Y + v_N(3 + v_Y))\gamma \kappa + (v_N - v_Y)(v_N + v_Y + v_Nv_Y)\alpha \gamma \mu
\]
such that \( \forall n \in [1, \bar{n}] \), \( g(n) \geq 0 \) and for all \( n > \max(1, \bar{n}) \), \( g(n) < 0 \). Hence, if \( n \in [1, \bar{n}] \), \( f(n_Y) \) is increasing and, therefore, using the same argument as above, \( \Pi(n_Y) \) has no interior local minimum.

Now, assume \( v_N + v_Y < v_Nv_Y \) and \( n > \bar{n} \), so that \( g(n) \leq 0 \). In this case, using a symmetric argument to the one above, \( f(n_Y) \) is decreasing; hence, \( \Pi(n_Y) \) may be initially convex and then becomes concave as \( n_Y \) increases. However, as we show next, \( h(n) \leq 0 \ \forall n > \bar{n} \) which ensures that \( f(n_Y) \leq 0 \) for all \( n_Y \geq 0 \), i.e., \( \Pi(n_Y) \) is concave over its entire domain; accordingly, there is no interior local minimum.

To show that \( h(n) \leq 0 \ \forall n > \bar{n} \), note that \( h(n) \) turns out to be a cubic function of \( n \) of the form of \( an^3 + bn^2 + cn + d \) where \( a, b, \) and \( d \) can be shown to be negative using \( \alpha \mu \geq \kappa, v_N \geq v_Y, \) and \( \gamma \geq 1 \). Hence, \( h''(n) \leq 0 \) when \( n \geq 0 \). Moreover, \( h(\bar{n}) \leq 0 \) and \( h'(\bar{n}) \leq 0 \); both inequalities can be shown using also \( \alpha \mu \geq \kappa, v_N \geq v_Y, \) and \( \gamma \geq 1 \). As a result, \( h(n) \leq 0 \ \forall n > \bar{n} \), which concludes the proof.

**Proof of Proposition 4** First, assume \( \psi_Y = \psi_Y \) and \( \kappa_1 / \mu_1 = \kappa_2 / \mu_2 = \ldots = \kappa_n / \mu_n = \kappa / \mu \). Then, for a given \( n \),
\[
\frac{du(Y)}{d n_Y} = \frac{(\gamma - 1)\kappa z(n_Y)}{4v_N^2(n + v_N)^2 \beta^2 \mu^2}
\]
where \( z(n_Y) = n^2(v_N - 1)(\gamma - 1)\kappa - 2n(n_Y(v_N - 1)(\gamma - 1) + v_N(v_N - 1 + \gamma + v_N\gamma))\kappa + 4n v_N^2 \alpha \gamma \mu + v_N(4v_N(\gamma - 1)v_N(v_N - (\gamma + \kappa - 2\alpha \gamma \mu))). \)

In \( z(n_Y) \), the coefficient of \( \alpha \) is equal to \( 2v_N^2(v_N + 2n - 1)\gamma \mu \). Then, replacing \( \alpha \) with \( \gamma(Y) \), \( z(n_Y) = (n + v_N(\gamma - 1)\kappa(n(v_N - 1) + (v_N - 1)v_N + 2n_Y(v_Y + 1)) \geq 0 \). Therefore, \( \frac{du(Y)}{d n_Y} \geq 0 \). Moreover, \( \frac{d^2 u(Y)}{d n_Y^2} = -\frac{\kappa(\gamma - 1)^2(n(v_N - 1) - 2n_Y)^2 \beta \mu^2}{2v_N^4(n + v_N)^2 \beta^2 \gamma^2 \mu^2} \leq 0 \) if \( n \geq 2 \).

Second, assume \( \gamma = 1 \) and \( \kappa_1 / \mu_1 = \kappa_2 / \mu_2 = \ldots = \kappa_n / \mu_n = \kappa / \mu \). Then, for a given \( n \),
\[
\frac{du(Y)}{d n_Y} = \frac{(v_N - v_Y)(\kappa - \alpha \mu)^2 h(n)}{4(v_N(v_N - v_Y) + (n + v_Nv_Y))^2 \beta^2 \mu^2}
\]
where \( h(n) = n_Y(v_N - v_Y)(v_N + v_Y + 3v_Nv_Y) + v_Y(-v_N(n + v_N) + (n + 3nv_N + (v_N - 1)v_N)v_Y)). \)

Because \( \frac{db(n)}{d n} = v_Y(v_Y + v_N(3v_Y - 1)) \) and \( h(2) = n_Y(v_N - v_Y)(v_N + v_Y + 3v_Nv_Y) + v_Y(-v_N(2 + v_N) + (2 + 2v_Y + v_N v_Y)) \geq 0 \), \( \frac{du(Y)}{d n_Y} \geq 0 \). Moreover,
\[
\frac{d^2 u(Y)}{d n_Y^2} = -\frac{(v_N - v_Y)^2(\kappa - \alpha \mu)^2 y(n)}{2(v_Y(v_N - v_Y) + (n + v_Nv_Y))^4 \beta^2 \mu^2}
\]
\( y(n) = -2v_Nv_Y(v_N + v_Y) + n_Y(v_N - v_Y)(v_N + v_Y + 3v_Nv_Y) + nv_Y(v_N + v_Y(-2 + 3v_Y)). \)

Because \( \frac{du(n)}{d n} = v_Y(v_Y + v_N(3v_Y - 2)) \geq 0 \), it is minimized when \( n \) takes the smallest value, i.e., \( n = 2 \) and \( y(2) = v_N^2(v_N + 1) + v_N^2 + v_N(v_N - 4). \) Moreover, \( y(2) \) is increasing in \( v_Y \) and in turn, minimized when \( v_Y = 1 \). Replacing \( v_Y = 1 \), the lower bound for \( y(n) \) is obtained when \( n = 2 \) and \( v_Y = 1 \), which is equal to \( 2v_N^2 - 3v_N + 1 \geq 0 \).

Therefore \( \frac{d^2 u(Y)}{d n_Y^2} \leq 0 \).
The right-hand side is nonpositive if and only if we obtain:

\[ P_{\text{Farmer}} \geq \alpha \]

\[ \frac{v_N - v_Y \gamma}{\beta(v_N n_Y + n_N v_Y + v_Y v_N)} \]

The right-hand side is nonpositive if and only if \( \frac{v_N}{v_Y} \geq \gamma \). Therefore, \( \forall Y \) such that \( |Y| = n_Y, Q(Y^{ME}) \geq Q(Y) \geq Q(Y^{LE}) \) if and only if \( \frac{v_N}{v_Y} \geq \gamma \).

**Lemma EC.2.** For any \( n_Y, 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{\theta:|Y|=n_Y} \alpha(Y) \). For any \( Y \) and \( N = S \setminus Y \) with \( |Y| = n_Y \), enrolling Farmer \( j \) rather than Farmer \( k \) generates a higher variance of the total production quantity if and only if

\[
0 \leq \left( \frac{\kappa_k}{\mu_k} - \frac{\kappa_j}{\mu_j} \right) \times \left( \frac{\kappa_j}{\mu_j} + \frac{\kappa_j}{\mu_j} \right) F(v_Y, v_N, \gamma, n_Y, n_N) - \alpha G(v_Y, v_N, \gamma, n_Y, n_N) \]

\[ -H_N(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in N \setminus S} \frac{\kappa_i}{\mu_i} - \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in Y \setminus S \setminus \{j\}} \frac{\kappa_i}{\mu_i} \]

in which

\[
F(v_Y, v_N, \gamma, n_Y, n_N) = \frac{1}{2} \left( n_Y v_Y v_N + n_N v_Y + v_Y v_N \right) \left( \frac{v_N - 1}{v_N^2} - \frac{1}{\gamma^2} \right) v_Y^2 \]

\[ + \frac{1}{2} \left( n_Y \frac{v_Y - 1}{v_N^2} + n_N \frac{v_N - 1}{v_Y^2} \right) \left( v_Y^2 - \frac{1}{\gamma^2} \right) \]

\[ - \left( n_Y v_N + n_N v_Y + v_Y v_N \right) \left( v_Y \frac{v_N - 1}{v_N^2} \right) \frac{v_N - 1}{v_N^2} v_Y - 1 \]

\[
G(v_Y, v_N, \gamma, n_Y, n_N) = v_Y n_Y \left( n_Y v_N + n_N v_Y + v_Y v_N \right) \left( \frac{v_N - 1}{v_N^2} - \frac{1}{\gamma^2} \right) v_Y^2 \]

\[ - v_Y v_N \left( v_Y - v_N \frac{1}{\gamma} \right) \left( \frac{v_N - 1}{v_N^2} + n_N \frac{v_N - 1}{v_N^2} \right) \]

\[
H_Y(v_Y, v_N, \gamma, n_Y, n_N) = v_N \left( n_Y v_N + n_N v_Y + v_Y v_N \right) \left( \frac{v_N - 1}{v_N^2} - \frac{1}{\gamma^2} \right) v_Y^2 \]

\[ + \left( v_Y - v_N \frac{1}{\gamma} \right) \left( n_N + v_N \right) \frac{v_N - 1}{v_N} - n_N \frac{v_N - 1}{v_N} \]

\[
H_N(v_Y, v_N, \gamma, n_Y, n_N) = v_Y \left( n_Y v_N + n_N v_Y + v_Y v_N \right) \left( \frac{v_N - 1}{v_N^2} - \frac{1}{\gamma^2} \right) v_Y^2 \]

\[ + \left( v_Y - v_N \frac{1}{\gamma} \right) \left( n_N + v_Y \right) \frac{v_N - 1}{v_N} - n_N \frac{v_N - 1}{v_N} \]

**Proof** The variance of the total production quantity equals \( \mathbb{V}[\sum_i q_i(Y)\theta_i / \mathbb{E}[\theta_i]] = \beta^2 \sum_i (q_i(Y))^2 \psi_i^2 \). Let \( Y^- \) and \( N^- \) denote the set of enrolled and non-enrolled farmers, excluding farmers \( j \) and \( k \). Enrolling Farmer \( j \) rather than Farmer \( k \) generates a higher variance of the total production quantity if and only if \( \sum_i (q_i(Y^- \cup \{j\}))^2 \psi_i^2 - \sum_i (q_i(Y^- \cup \{k\}))^2 \psi_i^2 \geq 0 \). After substituting \( \psi_i^2 = \frac{v_Y - 1}{v_Y^2} \) and \( \psi_i^2 = \frac{v_Y - 1}{v_Y^2} \), factoring
the difference of squares for each farmer, and grouping farmers into four sets (i.e., $N^−$, $Y^−$, \{j\}, and \{k\}), we obtain that adding farmer \(j\) to \(Y^−\) leads to a higher variance than adding farmer \(k\) if and only if:

\[
\frac{v_Y}{2} - \frac{1}{2} \sum_{i \in Y^−} [(q_i(Y^− \cup \{j\}) - q_i(Y^− \cup \{k\}))(q_i(Y^− \cup \{j\}) + q_i(Y^− \cup \{k\}))]
\]

\[
+ \frac{v_N}{2} - \frac{1}{2} \sum_{i \in \hat{n}^−} [(q_i(Y^− \cup \{j\}) - q_i(Y^− \cup \{k\}))(q_i(Y^− \cup \{j\}) + q_i(Y^− \cup \{k\}))]
\]

\[
+ \frac{v_Y}{2} - \frac{1}{2} [(q_j(Y^− \cup \{j\}) - q_k(Y^− \cup \{k\}))(q_j(Y^− \cup \{j\}) + q_k(Y^− \cup \{k\}))]
\]

\[
+ \frac{v_N}{2} - \frac{1}{2} [(q_k(Y^− \cup \{j\}) - q_j(Y^− \cup \{k\}))(q_k(Y^− \cup \{j\}) + q_j(Y^− \cup \{k\}))] \geq 0.
\]

Next, we substitute \(q_i\) with the right-hand side of (\(6\)) and rearrange terms by grouping coefficients of \((\frac{\kappa_i}{\mu_j} + \frac{\kappa_j}{\mu_i})\), \(\alpha\), \(\sum_{i \in N} \frac{\kappa_i}{\mu_i}\), and \(\sum_{i \in Y} \frac{\kappa_i}{\mu_i}\), which yields (EC.1).

**Lemma EC.3.** There exists a threshold \(\hat{\gamma} < \frac{v_N}{v_Y}\) such that \(H_N(v_Y, v_N, \gamma, n_Y, n_N) < \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N)\), in which \(H_N()\) and \(H_Y()\) are defined in Lemma EC.2, if and only if \(\gamma \in (\hat{\gamma}, \frac{v_N}{v_Y})\). If \((v_N + v_Y - v_Nv_Y)(2v_Nv_Y + n_Yv_N + n_Nv_Y)+v_Yv_Nn \geq 0, \hat{\gamma} \leq 1.\)

**Proof.** The function \(\Psi(1/\gamma) = H_N(v_Y, v_N, \gamma, n_Y, n_N) - H_Y(v_Y, v_N, \gamma, n_Y, n_N)/\gamma\) is quadratic in \(1/\gamma\). One root is \(1/\gamma = v_Y/v_N\); the other is \(1/\hat{\gamma}\), which is defined as follows:

\[
\hat{\gamma} = \frac{v_Y}{v_N} \frac{v_Y}{v_N^2} (v_Y - 1) (n_Y + 2v_Y) + v_Nv_Y n_N (v_Y - 1) - n_Nv_Y (v_Y - v_Y)
\]

It can be checked that \(\hat{\gamma} \leq v_N/v_Y\) given that \(v_N \geq v_Y\). The function \(\Psi(1/\gamma)\) turns out to be convex if and only if the numerator in the definition of \(\hat{\gamma}\) is positive.

We next consider two cases, depending on whether \(\Psi(1/\gamma)\) is concave or convex. If \(\Psi''(1/\gamma) \leq 0\), which implies that \((v_N + v_Y - v_Nv_Y)(2v_Nv_Y + n_Yv_N + n_Nv_Y) + v_Yv_Nn \geq 0, 1/\hat{\gamma} \leq 0\). Hence, for any \(1/\gamma \in [0, 1]\), \(\Psi(1/\gamma) \leq 0\) if and only if \(1/\gamma \geq v_Y/v_N\). On the other hand if \(\Psi''(1/\gamma) > 0\), \(\Psi(1/\gamma) \leq 0\) if and only if \(1/\gamma \geq v_Y/v_N\). Because

\[
\frac{1}{\hat{\gamma}} \geq 1 \iff (v_N - v_Y) ((v_N + v_Y - v_Nv_Y)(2v_Nv_Y + n_Yv_N + n_Nv_Y) + v_Yv_Nn) \geq 0,
\]

when \((v_N - v_Y) ((v_N + v_Y - v_Nv_Y)(2v_Nv_Y + n_Yv_N + n_Nv_Y) + v_Yv_Nn) \geq 0\), for any \(1/\gamma \in [0, 1]\), \(\Psi(1/\gamma) \leq 0\) if and only if \(1/\gamma \geq v_Y/v_N\). Combining the two cases leads to the result.

**Lemma EC.4.** For any \(n_Y, 1 \leq n_Y \leq n - 1\), suppose that \(\alpha \geq \max_{|Y|=n_Y} \alpha(Y)\) and \(\gamma \notin (\hat{\gamma}, \frac{v_N}{v_Y})\), in which \(\hat{\gamma}\) is defined in Lemma EC.3. For any given \(n_Y, 1 \leq n_Y \leq n - 1\), the variance of the total production quantity is maximized under the ME selection if and only if

\[
\left(\frac{\kappa_j}{\mu_j} + \frac{\kappa_k}{\mu_k}\right) F(v_Y, v_N, \gamma, n_Y, n_N)
\]

\[
\geq \alpha G(v_Y, v_N, \gamma, n_Y, n_N) + H_N(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in \hat{N} \setminus \{j, k\}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in \hat{Y} \setminus \{j, k\}} \frac{\kappa_i}{\mu_i}
\]

for \((j, k) \in \{(1, n_Y + 1), (n_Y, n_Y + 1), (n_Y, n)\}\).
Proof. The proof uses Lemmas [EC.2] and [EC.3] in the appendix. Throughout the proof, we ignore the arguments of the functions \( F, G, H_Y, \) and \( H_N \) and treat them as constants instead. Suppose the variance of the total production quantity is maximized under an ME selection. Then, any interchange must decrease the variance. By Lemma [EC.2] for any \( j \in Y^{ME} \) and \( k \in N^{ME} \),

\[
\left( \frac{\kappa_k}{\mu_k} + \frac{\kappa_j}{\mu_j} \right) F \geq \alpha G + H_N \sum_{i \in N^{ME}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y \sum_{i \in Y^{ME}} \frac{\kappa_i}{\mu_i}.
\]

In particular, this applies to any \( (j, k) \in \{(1, n_Y + 1), (n_Y, n_Y + 1), (n_Y, n)\} \), in which \( F(\cdot), G(\cdot), H_N(\cdot), \) and \( H_Y(\cdot) \) are defined in Lemma [EC.2]

For the converse, suppose that, for all \( (j, k) \in \{(1, n_Y + 1), (n_Y, n_Y + 1), (n_Y, n)\} \),

\[
\frac{\kappa_k}{\mu_k}(F + H_N) + \frac{\kappa_j}{\mu_j} \left( F + \frac{1}{\gamma} H_Y \right) \geq \alpha G + H_N \sum_{i \in N^{ME}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y \sum_{i \in Y^{ME}} \frac{\kappa_i}{\mu_i}.
\]

Because the left-hand side is linear in \( \kappa_k/\mu_k \) and \( \kappa_j/\mu_j \), and the farmers are indexed in increasing order of their productivity, and because \( H_Y/\gamma \leq H_N \) when \( \gamma \notin \left( \frac{1}{\gamma}, \frac{\gamma}{\gamma} \right) \), the inequality must also hold for any \( j \in \{1, \ldots, n_Y\} \) and \( k \in \{n_Y + 1, \ldots, n\} \), i.e.,

\[
\left( \frac{\kappa_k}{\mu_k} + \frac{\kappa_j}{\mu_j} \right) F \geq \alpha G + H_N \sum_{i \in N^{ME}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y \sum_{i \in Y^{ME}} \frac{\kappa_i}{\mu_i}.
\]

(EC.2)

To obtain a contradiction, suppose that there exists another set \( \tilde{Y} \) with \( |\tilde{Y}| = n_Y \) such that the variance of the total production quantity is maximized with \( \tilde{Y} \) and that it is strictly higher than with \( Y^{ME} \). Define \( \tilde{N} = S \setminus \tilde{Y} \). By Lemma [EC.2] there must exist an index \( \tilde{j} \in \tilde{Y} \cap N^{ME} \) and an index \( \tilde{k} \in \tilde{N} \cap Y^{ME} \) such that

\[
\left( \frac{\kappa_{\tilde{k}}}{\mu_{\tilde{k}}} + \frac{\kappa_{\tilde{j}}}{\mu_{\tilde{j}}} \right) F < \alpha G + H_N \sum_{i \in \tilde{N}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y \sum_{i \in \tilde{Y}} \frac{\kappa_i}{\mu_i},
\]

which, combined (EC.2), implies that

\[
H_N \sum_{i \in \tilde{N}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y \sum_{i \in \tilde{Y}} \frac{\kappa_i}{\mu_i} > H_N \sum_{i \in N^{ME}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y \sum_{i \in Y^{ME}} \frac{\kappa_i}{\mu_i},
\]

i.e.,

\[
\left( \frac{1}{\gamma} H_Y - H_N \right) \left( \sum_{i \in \tilde{Y}} \frac{\kappa_i}{\mu_i} - \sum_{i \in Y^{ME}} \frac{\kappa_i}{\mu_i} \right) > 0,
\]

because \( \sum_{i \in \tilde{Y} \setminus \tilde{j} \setminus \tilde{k}} \frac{\kappa_i}{\mu_i} + \sum_{i \in \tilde{N} \setminus \tilde{j} \setminus \tilde{k}} \frac{\kappa_i}{\mu_i} = \sum_{i \in Y^{ME} \setminus \tilde{j} \setminus \tilde{k}} \frac{\kappa_i}{\mu_i} + \sum_{i \in N^{ME} \setminus \tilde{j} \setminus \tilde{k}} \frac{\kappa_i}{\mu_i} \). By definition of ME, \( \sum_{i \in \tilde{Y} \setminus \tilde{j} \setminus \tilde{k}} \frac{\kappa_i}{\mu_i} \geq \sum_{i \in Y^{ME} \setminus \tilde{j} \setminus \tilde{k}} \frac{\kappa_i}{\mu_i} \). And by Lemma [EC.3] \( H_Y/\gamma \leq H_N \) when \( \gamma \notin \left( \frac{1}{\gamma}, \frac{\gamma}{\gamma} \right) \). We thus obtain a contradiction, showing that there is no such \( \tilde{Y} \) that dominates \( Y^{ME} \), i.e., \( Y^{ME} \) is the global optimum.
LEMMA EC.5. For any \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y:|Y|=n_Y} \alpha(Y) \) and \( \gamma \not\in \left( \frac{\gamma_X}{n_Y}, \frac{\gamma_Y}{n_Y} \right) \), in which \( \gamma \) is defined in Lemma [EC.3]. For any given \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), the variance of the total production quantity is maximized under the LE selection if and only if

\[
\left( \frac{\kappa_j}{\mu_j} + \frac{\kappa_k}{\mu_k} \right) F(v_Y, v_N, \gamma, n_Y, n_N) \\
\leq \alpha G(v_Y, v_N, \gamma, n_Y, n_N) + H_N(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in N \setminus \{j,k\}} \frac{K_i}{\mu_i} + \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in Y \setminus \{j,k\}} \frac{K_i}{\mu_i},
\]

for \((j, k) \in \{ (n, n), (1, n + 1), (n, n + 1) \}\) with \( n_Y = n - n_Y \), in which \( F(\cdot), G(\cdot), H_N(\cdot), \) and \( H_Y(\cdot) \) are defined in Lemma [EC.2].

Proof. The proof is symmetric to the proof of Lemma [EC.4]. \( \square \)

LEMMA EC.6. For any \( v_N \geq v_Y \geq 1 \), the function \( f(v_Y, v_N) = (v_N^2 - 1)v_Y^3(v_Y - 1) - 4v_N(v_Y - 1)(v_N - v_Y) + v_Y^3(v_N - 1)^2 \) is nonnegative.

Proof. Let \( g(v_Y) = 4 - 4v_Y - v_Y^2 + 4v_Y^3 \). Because \( g'(v_Y) = 2(2 + 3v_Y)(v_Y - 1) \geq 0 \) and \( g(1) = 5, g(v_Y) \geq 0 \). Because \( \partial f(v_Y, v_N)/\partial v_Y^2 = 2g(v_Y), f(v_Y, v_N) \) is convex in \( v_N \). Moreover, \( \partial f(v_Y, v_N)/\partial v_N = 4v_Y(1 + v_Y)(v_Y - 1)^2 \geq 0 \) when \( v_N = v_Y \) and \( f(v_Y, v_N) = v_Y^3(1 + 2v_Y)(v_Y - 1)^2 \geq 0 \) when \( v_N = v_Y \). Hence, \( f(v_Y, v_N) \geq 0 \) for all \( v_N \geq v_Y \). \( \square \)

LEMMA EC.7. For any \( Y \) with \( n_Y \geq 1 \), \( j \in Y \) and \( k \in N \), the function

\[
\Phi(1/\gamma) \doteq \left( \frac{\kappa_k}{\mu_k} + \frac{\kappa_j}{\mu_j} \right) F(v_Y, v_N, \gamma, n_Y, n_N) - \alpha G(v_Y, v_N, \gamma, n_Y, n_N) \\
-H_N(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in N \setminus \{j,k\}} \frac{K_i}{\mu_i} - \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in Y \setminus \{j,k\}} \frac{K_i}{\mu_i}
\]

is concave, in which \( F(\cdot), G(\cdot), H_N(\cdot), \) and \( H_Y(\cdot) \) are defined in Lemma [EC.2] when

\[
n_N \geq \frac{1}{2(v_Y - 1)v_Y} \left( 2 \sum_{i \in N \setminus \{j,k\}} \frac{\kappa_i}{\mu_i} \right) (v_N(v_Y - 1) - (v_N - v_Y)) \tag{EC.4}
\]

in which

\[
\Delta = 2 \left( \sum_{i \in N \setminus \{j,k\}} \frac{\kappa_i}{\mu_i} \right) (v_N(v_Y - 2) + v_Y)^2 \\
+ 4v_Y v_N(v_N - v_Y)(v_Y - 1) + v_Y^3(v_N - 1)^2 + 4(v_Y - 1)^2 v_N(1 + (v_N - 1)(v_Y + 1)),
\]

which turns out to always be nonnegative.

Proof. Let \( \Phi''(1/\gamma) = -\partial^2 \Phi(1/\gamma)/\partial (1/\gamma)^2 \). Because \( \partial^2 \Phi'(1/\gamma)/\partial (1/\gamma)^2 = -2(v_Y - 1) \left( \frac{\kappa_j}{\mu_j} + \frac{\kappa_k}{\mu_k} \right) \), \( \Phi''(1/\gamma) \) is quadratic concave in \( n_N \). Let \( \Delta = 2 \left( 2 \sum_{i \in N \setminus \{j,k\}} \frac{\kappa_i}{\mu_i} + \frac{\kappa_j}{\mu_j} + \frac{\kappa_k}{\mu_k} \right) /v_Y^2 \) be the discriminant of \( \Phi''(1/\gamma) \) with respect to \( n_N \). By Lemma [EC.6] all terms are nonnegative because \( v_N \geq v_Y \geq 1 \) and \( n_Y \geq 1 \).
Because $\Delta \geq 0$, $\Phi'(1/\gamma)$ crosses zero (at most) twice, first from below and then from above, as $n_N$ increases. The two roots are equal to
\[
2 \left( \frac{\sum_{i \in Y} \frac{\kappa_i}{\mu_i}}{2+\frac{\mu_j}{\mu_k}} \right) (v_Y(v_Y - 1) - (v_Y - v_Y) - v_Y(v_Y - v_Y) - 2(v_Y - 1)v_Y) \leq \sqrt{\Delta}.
\]
Hence, if $n_N$ is either smaller than the smallest root or larger than the largest root, $\Phi(1/\gamma)$ is concave. We next show that the lowest root is always less than 1, which simplifies the condition to requiring that $n_N$ be larger than the largest root, as specified in the lemma statement.

The lowest root is smaller than 1 if and only if
\[
2 \left( \frac{\sum_{i \in Y} \frac{\kappa_i}{\mu_i}}{2+\frac{\mu_j}{\mu_k}} \right) (v_Y(v_Y - 1) - (v_Y - v_Y)) \leq \sqrt{\Delta}.
\]
If the left-hand side is negative, this inequality holds trivially, so we hereon assume that
\[
2 \left( \frac{\sum_{i \in Y} \frac{\kappa_i}{\mu_i}}{2+\frac{\mu_j}{\mu_k}} \right) (v_Y(v_Y - 1) - (v_Y - v_Y)) \geq (v_Y - v_Y) + v_Y(v_Y - 1)(2v_Y + 2v_Y - 1) + 2(v_Y - 1)v_Y,
\]
which implies, given that the right-hand side is nonnegative, that when $v_Y \geq 2v_Y/(1 + v_Y)$. In this case, $\Phi(1/\gamma)$ can be expressed by squaring both sides, which simplifies, after some rescaling, to
\[
2 \left( \frac{\sum_{i \in Y} \frac{\kappa_i}{\mu_i}}{2+\frac{\mu_j}{\mu_k}} \right) f(v_Y, v_Y, n_Y) - g(v_Y, v_Y, n_Y) \geq 0,
\]
in which $f(v_Y, v_Y, n_Y) = n_Y v_Y^2(v_Y - 1) + v_Y(v_Y(1 + v_Y + 2v_Y^2) - 2v_Y(1 + v_Y))$ and $g(v_Y, v_Y, n_Y) = n_Y^2 v_Y^2(v_Y - 1) + n_Y v_Y(v_Y - 1)(2v_Y + 2v_Y) + v_Y((1 + v_Y)(v_Y(v_Y - 2)(v_Y - 1) + v_Y^2) - 2v_Y)$. We next show that $f(v_Y, v_Y, n_Y) \geq 0$. First, note that $f(v_Y, v_Y, n_Y)$ is increasing in $n_Y$. Second, $f(v_Y, v_Y, 0)/v_Y$ is linear increasing in $v_Y$; because $v_Y \geq 2v_Y/(1 + v_Y)$ by assumption, $f(v_Y, v_Y, 0)/v_Y \geq 2n_Y^2(v_Y - 1)/(v_Y + 1) \geq 0$. Because $v_Y \geq 1 \geq 0$, this implies that $f(v_Y, v_Y, 0) \geq 0$. Therefore, $f(v_Y, v_Y, n_Y) \geq f(v_Y, v_Y, 0) \geq 0$. As a result, the left-hand side of (EC.7) is increasing in $\sum_{i \in Y, j \in k} \frac{\kappa_i}{\mu_i}$ under the assumption that (EC.6) holds. Therefore, under (EC.6), a lower bound on the left-hand side of (EC.7) is achieved by taking $\sum_{i \in Y, j \in k} \frac{\kappa_i}{\mu_i}$ such that (EC.6) is tight. For this value, the left-hand side of (EC.5) is equal to zero, and thus, given that $\Delta \geq 0$, (EC.5) is true. As a result, the lowest root of $\Phi'(1/\gamma)$ with respect to $n_N$ is always less than 1. Hence, when $n_N \geq 1$, $\Phi'(1/\gamma) < 0$ if and only if $n_N$ is greater than its largest root.

**Lemma EC.8.** When (EC.4) holds and $v_Y \leq v_N \frac{v_N + n_N}{v_N + n_N}$, $\Phi(1/\gamma)$, defined in (EC.3), crosses zero at most once as $1/\gamma$ increases, and the crossing, if it happens, is from above.

**Proof.** By Lemma EC.7 in the appendix, $\Phi(1/\gamma)$ is quadratic concave when (EC.4) holds. If its discriminant, denoted as $\Delta, (\alpha)$, is nonpositive, then $\Phi(1/\gamma) \leq 0$ for all $1/\gamma$; so the result holds nontrivially only
when $\Delta_\gamma(\alpha) > 0$, which we assume hereon. In this case, $\Phi(1/\gamma)$ has two roots. It has at most one crossing
and the crossing is from above if and only the lowest root is negative. The lowest root can be expressed as
follows:
\[
\frac{1}{\gamma(\alpha)} = \frac{1}{-\Phi''(1/\gamma)} \left( \alpha(v_Y(v_N^2 + n_N) - v_N(v_N + n_N)) + \left( \sum_{i \in \mathcal{N} \setminus j,k} K_i \mu_i - \sum_{j \in \mathcal{Y} \setminus i \neq j,k} K_i \mu_i \right) (v_N(n_Y - 1 + 2v_Y) - n + v_Y(n_N - 1)) \right) - \sqrt{\Delta_\gamma(\alpha)}.
\]

By assumption, $\Phi''(1/\gamma) \leq 0$.

If $\alpha(v_Y(v_N^2 + n_N) - v_N(v_N + n_N)) + \left( \sum_{i \in \mathcal{N} \setminus j,k} K_i \mu_i - \sum_{j \in \mathcal{Y} \setminus i \neq j,k} K_i \mu_i \right) (v_N(n_Y - 1 + 2v_Y) - n + v_Y(n_N - 1)) \leq 0$, then $1/\gamma(\alpha) \leq 0$. Thus, in the following, we assume that
\[
\alpha(v_Y(v_N^2 + n_N) - v_N(v_N + n_N)) + \left( \sum_{i \in \mathcal{N} \setminus j,k} K_i \mu_i - \sum_{j \in \mathcal{Y} \setminus i \neq j,k} K_i \mu_i \right) (v_N(n_Y - 1 + 2v_Y) - n + v_Y(n_N - 1)) > 0. \tag{EC.8}
\]

Because $v_Y \leq v_N^{v_N^2+n_N}$, the left-hand side is decreasing in $\alpha$; therefore, assuming \tag{EC.8} is equivalent to assuming that
\[
\alpha \leq \overline{\alpha} = \left( \sum_{i \in \mathcal{N} \setminus j,k} K_i \mu_i - \sum_{j \in \mathcal{Y} \setminus i \neq j,k} K_i \mu_i \right) n - v_N(n_Y - 1) - v_Y(n_N - 1) - 2v_Nv_Y

v_Y(n_N + v_N^2) - v_N(n_N + v_N).
\]

Given that $\alpha \leq \overline{\alpha}$, $1/\gamma(\alpha) \leq 0$ if and only if
\[
\left( \alpha(v_Y(v_N^2 + n_N) - v_N(v_N + n_N)) + \left( \sum_{i \in \mathcal{N} \setminus j,k} K_i \mu_i - \sum_{j \in \mathcal{Y} \setminus i \neq j,k} K_i \mu_i \right) (v_N(n_Y - 1 + 2v_Y) - n + v_Y(n_N - 1)) \right)^2 - \Delta_\gamma(\alpha)
\leq 0.
\]

The left-hand side turns out to be linear in $\alpha$ with coefficient $-2(n_Y(v_N - v_Y) + (v_N - 1)v_Y^2)\Phi''(1/\gamma) \geq 0$. Hence, an upper bound on the left-hand side is obtained by replacing $\alpha$ with its upper bound $\overline{\alpha}$. By definition, $1/\gamma(\overline{\alpha}) = \sqrt{\Delta_\gamma(\overline{\alpha})}/\Phi''(1/\gamma) \leq 0$.

**Lemma EC.9.** When $n_Y \geq 1$,
\[
\frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2 v_N^2}}{2(1 + v_N + 2v_N^2)} \leq v_N \frac{2}{v_N + 1}.
\]

**Proof.** The inequality is equivalent to:
\[
4(1 + v_N + 2v_N^2) - 2(v_N + 1) + (n_Y - 2)v_N(v_N + 1) \geq (v_N + 1)\sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2 v_N^2}. \tag{EC.9}
\]

We first check that the left-hand side is positive. Because it is increasing in $n_Y$, a lower bound is achieved by replacing $n_Y$ with 1, i.e.,
\[
4(1 + v_N + 2v_N^2) - 2(v_N + 1) + (n_Y - 2)v_N(v_N + 1) \geq 7v_N^2 + v_N + 2 \geq 0.
\]
Accordingly (EC.9) is equivalent to:

\[
(4(1 + v_N) + 2v_N^2) - 2(v_N + 1) + (n_Y - 2)v_N(v_N + 1))^2 \geq (v_N + 1)^2 \left(4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2\right),
\]
or equivalently, \(4(v_N - 1)(n_Y + v_N(4 + n_Y))(1 + v_N + 2v_N^2) \geq 0\), which always holds.

**Lemma EC.10.** When \(n_Y \geq 1\), Condition (EC.4) always holds if

\[
v_Y \leq v_N \frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)}
\]

**Proof.** The proof uses Lemma EC.9 in appendix. The right-hand side of (EC.4) is less than or equal to 1 if and only if

\[
\sqrt{1 + 2 \frac{\sum_{i \notin \{j,k\}} n_i \mu_i}{\sum_{i \notin \{j,k\}} n_i \nu_i} \sqrt{A}} \leq -2 \left(\sum_{i \notin \{j,k\}} n_i \mu_i + \sum_{i \notin \{j,k\}} n_i \nu_i\right) (v_Y - 2) + v_Y + 2v_Y(v_Y - 1) + v_N - v_Y + v_N(v_Y - 1)(2n_Y + 2v_Y - 1),
\]

By Lemma EC.9, the first term in the right-hand side is nonnegative when \(v_Y \leq v_N \frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)}\). The second term is always nonnegative. Accordingly, this inequality can be equivalently expressed by squaring both sides. Rearranging the terms, we obtain:

\[
A(v_Y) \geq 2B(v_Y) = \frac{\sum_{i \notin \{j,k\}} n_i \mu_i v_Y^2}{\sum_{i \notin \{j,k\}} n_i \nu_i}
\]

where

\[
A(v_Y) \doteq n_Y^2 v_N^2 (v_Y - 1) + n_Y v_N(v_Y - 1)(2v_Y + v_N(2v_Y - 1)) + v_Y (2v_N(v_N + 1) - (2 + 3v_N(1 + v_N))v_Y + (1 + v_N + v_N^2 + v_N^3)v_Y^2),
\]

\[
B(v_Y) \doteq v_Y^2 (1 + v_N + 2v_N^2) + v_N v_N(n_Y v_N - 2 - 2v_N) - n_Y v_N^2.
\]

Because \(n_Y \geq 1\) and the coefficients in \(n_Y^2\) and \(n_Y\) in \(A\) are nonnegative, a lower bound on \(A(v_Y)\) is obtained by replacing \(n_Y\) with 1, i.e.,

\[
A(v_Y) \geq (v_Y - 1) + (v_N - 1) + v_N(v_Y + 1)(v_Y - 1) \geq 0.
\]

Because \(B(v_Y)\) is quadratic convex, \(B(1) = -(v_N - 1) \leq 0\) and \(B(v_N) = (v_Y - 1) v_N^2 (1 + n_Y + 2v_N) \geq 0\). \(B(v_Y) \leq 0\) if and only if \(v_Y \leq v_N \frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)}\). Hence, \(v_Y \leq \frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)}\), the right-hand side of (EC.4) is less than or equal to 1.

**Lemma EC.11.** When \(n_Y \geq 1\),

\[
\frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)} \leq v_N + n_N.
\]

**Proof.** The inequality holds if and only if

\[
\sqrt{4(n_Y + 1) + 8v_N + (n_Y + 2)^2v_N^2} \leq \frac{v_N(2 + (n_Y + 2)v_N^2 + n_N(n_Y + 4v_N))}{n_N + v_N^2}.
\]

Because both terms are positive, this inequality can be equivalently expressed by taking their squares, i.e.,

\[
\frac{4(n_Y + 1) + 8v_N + (n_Y + 2)^2v_N^2}{n_N + v_N^2} \leq \frac{v_N^2(2 + (n_Y + 2)v_N^2 + n_N(n_Y + 4v_N))^2}{(n_N + v_N^2)^2},
\]

which is equivalent to

\[
4(v_N - 1)(1 + v_N + 2v_N^2)((n_N(2 + n_Y) - 1)v_N^2 + n_N^2(1 + n_Y + 2v_N)) \geq 0.
\]

The latter condition always holds true when \(n_N \geq 1\).
Lemma EC.12. When \( n_Y \geq 1 \),
\[
\frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)} \leq \frac{v_N}{v_N - 1}.
\]

**Proof.** Because the left-hand side is decreasing in \( n_Y \), it is sufficient given that \( n_Y \geq 1 \) to check that
\[
\frac{2 + v_N + \sqrt{8 + 8v_N + 9v_N^2}}{2(1 + v_N + 2v_N^2)} \leq \frac{v_N}{v_N - 1}
\]
or equivalently that
\[
(v_N - 1)\sqrt{8 + 8v_N + 9v_N^2} \leq 4 + v_N + 3v_N^2.
\]
Because the right-hand side is nonnegative, this condition is equivalent to one where both sides are squared, yielding
\[
-8(1 + v_N)(1 + v_N + 2v_N^2) \leq 0,
\]
which is always true. \(\square\)

For any \( Y \), define
\[
\pi(Y) = -C_Yv_N - C_Nv_Y + v_Y(\kappa_i/\mu_i)(n_Yv_N + n_Nv_Y + v_Nv_Y).
\]
(EC.10)

Note that \( \pi(Y) \geq \alpha(Y) \).

Lemma EC.13. For any \( n_Y, 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y, |Y| = n_Y} \pi(Y) \), defined by (EC.10). Then \( \Phi(v_Y/v_N) \leq 0 \), in which \( \Phi(1/\gamma) \) is defined in (EC.3).

**Proof.** We have:
\[
\Phi \left( \frac{v_Y}{v_N} \right) = \left( \frac{v_N - v_Y}{2v_N^2} \right) \left( n_Yv_N + n_Nv_Y + v_Nv_Y \right) \times \left( -2 \sum_i k_i \mu_i - 2\alpha_N \gamma \left( n_Yv_N + n_Nv_Y + v_Nv_Y \right) \left( \frac{k_i}{\mu_i} + \frac{k_j}{\mu_j} \right) \right) \leq 0,
\]
when \( \alpha \geq \pi(Y) \) given that \( \frac{\alpha_k}{\mu_k} + \frac{\alpha_j}{\mu_j} \leq 2 \frac{\alpha_n}{\mu_n} \). \(\square\)

Next, we assume that \( v_Y \leq v_N \)
\[
\frac{2 - (n_Y - 2)v_N + \sqrt{4(1 + n_Y) + 8v_N + (2 + n_Y)^2v_N^2}}{2(1 + v_N + 2v_N^2)} \leq \frac{v_N}{v_N - 1}.
\]
The upper bound is decreasing in \( n_Y \) and tends to 1 as \( n_Y \to \infty \).

**Proposition EC.2.** For any \( n_Y, 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y, |Y| = n_Y} \alpha(Y) \) and \( n_N \geq \hat{n}_N \) for some \( \hat{n}_N \). There exist thresholds \( \gamma^n_L \leq \gamma^n_M \leq \gamma^n_R \) such that the variance of the total production quantity is maximized under ME if \( \gamma \in \left[ \gamma^n_M, \gamma^n_M \right] \) and under LE if \( \gamma \in \left[ 1, \gamma^n_M \right] \cup \left[ \gamma^n_M, \infty \right) \).

**Proof of Proposition EC.2.** Lemma EC.4 provides necessary and sufficient conditions for the ME selection to be optimal when \( \gamma \notin \left( \frac{\gamma^n_M}{\gamma^n_R} \right) \). Let \( [a, b] = \emptyset \) if \( a > b \). Using Lemma EC.7, when \( n_N \) is larger than a certain threshold, denoted as \( \hat{n}_N \), these conditions hold when \( \gamma \in \left[ \Gamma_M, \Gamma_M \right] \) for some thresholds \( \Gamma_M \) and \( \Gamma_M \). If \( \Gamma_M > \Gamma_M \), the interval is empty.
Lemma [EC.5] provides necessary and sufficient conditions for the LE selection to be optimal when $\gamma \notin \left(\hat{\gamma}, \frac{\infty}{v_Y}\right)$. Using Lemma [EC.7] when $n_N \geq \hat{n}_N$, these conditions hold when $\gamma \notin \left[\Gamma_L, \Gamma_L\right]$ for some thresholds $\Gamma_L \leq \Gamma_L$.

As a result, the variance of the total production quantity is maximized under ME if $\gamma \in [\Gamma_M, \min\{\hat{\gamma}, \Gamma_M\}] \cup \left[\max\left\{\frac{\infty}{v_Y}, \Gamma_M\right\}, \Gamma_M\right]$ and under LE if $\gamma \in [1, \min\{\hat{\gamma}, \Gamma_L\}] \cup \left[\max\left\{\frac{\infty}{v_Y}, \Gamma_L\right\}, \infty\right]$.

Because LE and ME cannot be both optimal at the same time, we must have that if $\Gamma_M \leq \hat{\gamma}$, then $\Gamma_L \leq \Gamma_M$; and that if $\Gamma_M \geq v_N/v_Y$, then $\Gamma_L \geq \Gamma_M$. If $\Gamma_M > \hat{\gamma}$, then $\Gamma_L$ can be redefined to be set to equal to $\hat{\gamma}$. And if $\Gamma_M < v_N/v_Y$, then $\Gamma_L$ can be redefined to be set to equal to $v_N/v_Y$. Hence, it can be assumed without loss of generality that $\Gamma_L \leq \Gamma_M \leq \Gamma_M \leq \Gamma_L$.

Next, define $\gamma^v_Y = \min\{\hat{\gamma}, \Gamma_L\}$ and $\gamma^v_M = \max\{\frac{\infty}{v_Y}, \Gamma_M\}$. If $\Gamma_M \leq \hat{\gamma}$ and $v_N/v_Y > \Gamma_M$, define $\gamma^v_M = \Gamma_M$ and $\gamma^v_Y = \min\{\hat{\gamma}, \Gamma_M\}$. If $\Gamma_M > \hat{\gamma}$ and $v_N/v_Y \leq \Gamma_M$, define $\gamma^v_M = \Gamma_M$ and $\gamma^v_Y = \Gamma_M$. If $\Gamma_M > \hat{\gamma}$ and $v_N/v_Y > \Gamma_M$, define $\gamma^v_M = \Gamma_L$ and $\gamma^v_Y = \Gamma_M$ to result in an empty interval $[\gamma^v_M, \gamma^v_Y]$. Finally, if $\Gamma_M \leq \hat{\gamma}$ and $v_N/v_Y \leq \Gamma_M$, we claim that ME is optimal for all $\gamma \in [\Gamma_M, \Gamma_M]$. For any $Y, j$, and $k$, $\Phi(1/\gamma|Y, j, k) \geq 0$ for any $1/\gamma \in [\Gamma_M, \Gamma_M]$ because $\Phi(1/\gamma|Y, j, k)$ is concave by Lemma [EC.7] $\Phi(1/\Gamma_M|Y, j, k) \geq \Phi(1/\Gamma_M|Y, j, k)$ by Lemma [EC.4] given that $\Gamma_M \leq \hat{\gamma}$, and $\Phi(1/\Gamma_M|Y, j, k) \geq \Phi(1/\Gamma_M|Y, j, k)$ by Lemma [EC.4] given that $\Gamma_M \geq v_N/v_Y$. In this case, define $\gamma^v_M = \Gamma_M$ and $\gamma^v_Y = \Gamma_M$. With these new definitions, we preserve the ordering $\gamma^v_Y \leq \gamma^v_M \leq \gamma^v_Y \leq \gamma^v_Y$.

**Proof of Proposition 2** To simplify the exposition, let us denote with $\Phi(1/\gamma|Y, j, k)$ the function (EC.3) corresponding to a particular set $Y, j \in Y$ and $k \in N$. Combining Lemmas [EC.8] and [EC.11] we obtain that, when $v_Y \leq v_N 2^{-\left(\left(\gamma - 1\right)^{-1} + \gamma^{-1}\right)}$, which implies that $v_Y \leq v_N \frac{\gamma + n_N}{\gamma + n_N}$ and that (EC.4) holds, for any $Y, j \in Y$, and $k \in N$, $\Phi(1/\gamma|Y, j, k)$ crosses zero at most once, and the crossing, if any, is from above as $1/\gamma$ increases. In particular, it does so when $Y = Y^{ME}$ and $(j, k) \in \left\{(n, n + 1), (n, n + 1), (n, n, n + 1)\right\}$. Thereby by Lemmas [EC.4] and [EC.5] the thresholds $\Gamma_M$ and $\Gamma_L$ defined in the proof of Proposition [EC.2] are infinite.

Moreover, by Lemma [EC.12] $v_Y \leq v_N \frac{\gamma + n_N}{\gamma + n_N}$ when $v_Y \leq v_N 2^{-\left(\left(\gamma - 1\right)^{-1} + \gamma^{-1}\right)}$, which implies that $(v_N + v_Y - v_N v_Y)(2v_N v_Y + n_N v_N + n_N v_Y) + v_Y v_N n \geq 0$. By Lemma [EC.3] $\gamma \leq 1$.

Given that $\gamma \leq 1$ and that $\Gamma_M = \Gamma_L = \infty$, the statement in the proof of Proposition [EC.2] simplifies the following: the variance of the total production quantity is maximized under ME if $\gamma \in \left[\max\{\frac{\infty}{v_Y}, \Gamma_M\}, \infty\right]$ and under LE if $\gamma \in [1, \Gamma_L]$. Define $\gamma^v_M = \max\{\frac{\infty}{v_Y}, \Gamma_M\}$ and $\gamma^v_Y = \min\{\gamma_M, \Gamma_L\}$.

Finally, using Lemma [EC.13] shows that $\gamma^v_Y \geq v_N/v_Y$ when $\alpha \geq \max_{1 \leq j \leq n} \pi(Y)$.

**Proof of Lemma 3** Fix $Y, j \in Y$, and $k \in N \setminus S\{j\}$. Consider any $i \neq j, k$. Using (i) in Lemma [1], which is valid given that $\alpha \geq \max_{i \in S\{j\}} \alpha_{i}(\{k\} \cup \{j\})$, after replacing $C_N$ by $\sum_{i} \kappa_i \mu_i - \gamma C_Y$, we obtain

$$q_{i}(Y) - q_{i}(\{k\} \cup \{Y\}) = \frac{v_N - v_Y \gamma}{v_Y \beta (v_N v_N + (n_N + v_Y) v_Y)} (C_Y - C_{(k) \cup j \cup \{Y\}})$$

The right-hand side is non-negative, i.e., $\pi_i(Y) \geq \pi_i(\{k\} \cup \{Y\})$ by Lemma [1] if and only if (i) either $\gamma \leq \frac{\infty}{v_Y}$ and $c_i \geq c_k$ or (ii) $\gamma \geq \frac{\infty}{v_Y}$ and $c_j \leq c_k$.

□
Lemma EC.14. For any \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y:|Y|=n_Y} \alpha(Y) \). For any \( Y \) and \( N \) with \( |Y|=n_Y \), enrolling Farmer \( j \) rather than Farmer \( k \) generates a higher farmers' total welfare if and only if
\[
0 \leq \left( \frac{k_j}{\mu_j} - \frac{k_k}{\mu_k} \right) \times \left( \frac{k_k}{\mu_k} + \frac{k_j}{\mu_j} \right) F(v_Y, v_N, \gamma, n_Y, n_N) - \alpha G(v_Y, v_N, \gamma, n_Y, n_N) - H_N(v_Y, v_N, \gamma, n_Y, n_N) - H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in Y \setminus \{j, k\}} \frac{\kappa_i}{\mu_i} - H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in Y \setminus \{j, k\}} \frac{\kappa_i}{\mu_i},
\]
in which
\[
F(v_Y, v_N, \gamma, n_Y, n_N) \doteq \frac{1}{2} (n_Y v_Y + n_N v_Y + v_Y v_N)^2 \left( \frac{v_N + 1}{v_Y^2} - \frac{1}{\gamma} \frac{v_Y + 1}{v_N^2} \right) + \frac{1}{2} n_Y \left( \frac{v_Y + 1}{v_Y^2} + n_N \frac{v_N + 1}{v_N^2} \right) \left( \frac{v_Y}{\gamma} - \frac{1}{\gamma} \frac{v_N}{v_Y} \right)
- (n_Y v_Y + n_N v_Y + v_Y v_N) \left( \frac{v_Y}{\gamma} + 1 \right) \left( \frac{v_N + 1}{v_Y^2} - \frac{1}{\gamma} \frac{v_Y + 1}{v_N} \right),
G(v_Y, v_N, \gamma, n_Y, n_N) \doteq v_Y v_N \left( n_Y v_Y + n_N v_Y + v_Y v_N \right) \left( \frac{v_Y + 1}{v_Y^2} - \frac{1}{\gamma} \frac{v_Y + 1}{v_N} \right) - v_Y v_N \left( v_Y - n_Y \frac{1}{\gamma} \right) \left( n_Y \frac{v_Y + 1}{v_Y^2} + n_N \frac{v_N + 1}{v_N^2} \right),
H_Y(v_Y, v_N, \gamma, n_Y, n_N) \doteq v_N \left( n_Y v_Y + n_N v_Y + v_Y v_N \right) \left( \frac{v_Y + 1}{v_Y^2} - \frac{1}{\gamma} \frac{v_Y + 1}{v_N} \right) + (v_Y - n_Y \frac{1}{\gamma}) \left( n_Y + v_Y \right) \left( \frac{v_Y + 1}{v_Y} - n_N \frac{v_N + 1}{v_N} \right),
H_N(v_Y, v_N, \gamma, n_Y, n_N) \doteq v_Y \left( n_Y v_Y + n_N v_Y + v_Y v_N \right) \left( \frac{v_Y + 1}{v_Y^2} - \frac{1}{\gamma} \frac{v_Y + 1}{v_N} \right) + (v_Y - n_Y \frac{1}{\gamma}) \left( n_Y + v_Y \right) \left( \frac{v_Y + 1}{v_N} - v_Y \right).
\]

Proof. The proof is similar to the proof of Lemma EC.2 and is omitted for brevity.

Lemma EC.15. There exists a threshold \( \tilde{\gamma} \) such that \( H_N(v_Y, v_N, \gamma, n_Y, n_N) \leq \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \), in which \( H_Y(.) \) and \( H_N(.) \) are defined in Lemma EC.14, if and only if \( \gamma \in \left( \frac{v_N}{v_Y}, \tilde{\gamma} \right) \).

Proof. The proof is similar to the proof of Lemma EC.3 and is omitted for brevity.

Lemma EC.16. For any \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y:|Y|=n_Y} \alpha(Y) \) and \( \gamma \notin \left( \frac{v_N}{v_Y}, \tilde{\gamma} \right) \), in which \( \tilde{\gamma} \) is defined in Lemma EC.15. For any \( n_Y \), \( 1 \leq n_Y \leq n - 1 \), the farmers' total welfare is maximized under the ME selection if and only if
\[
\left( \frac{k_j}{\mu_j} + \frac{k_k}{\mu_k} \right) F(v_Y, v_N, \gamma, n_Y, n_N) \geq \alpha G(v_Y, v_N, \gamma, n_Y, n_N) + H_N(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in N \setminus \{j, k\}} \frac{\kappa_i}{\mu_i} + \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in Y \setminus \{j, k\}} \frac{\kappa_i}{\mu_i},
\]
for \((j,k) \in \{(1,n_Y+1),(n_Y, n_Y+1), (n_Y, n_Y)\}\), in which \(F(\cdot), G(\cdot), H_N(\cdot), \) and \(H_Y(\cdot)\) are defined in Lemma [EC.14]

**Proof.** The proof is similar to the proof of Lemma [EC.14] and is omitted for brevity. □

**Lemma EC.17.** For any \(n_Y, 1 \leq n_Y \leq n-1\), suppose that \(\alpha \geq \max_{Y:|Y|=n_Y} \Omega(Y)\) and \(\gamma \notin \left(\hat{\gamma}, \frac{\alpha G}{\gamma} \right)\), in which \(\hat{\gamma}\) is defined in Lemma [EC.15]. For any given \(n_Y \leq n\), in which \(F(\cdot), G(\cdot), H_N(\cdot), \) and \(H_Y(\cdot)\) are defined in Lemma [EC.14], when one of the conditions in Lemma [EC.16] holds, i.e., \(\sum_{i \in N} H_i \geq \frac{1}{\gamma} \Phi(1/\gamma) \) with \(\Phi(1/\gamma) = \left(\frac{k_i}{\mu_1} + \frac{k_k}{\mu_k}\right) F(v_Y, v_N, \gamma, n_Y, n_N) \) \(\leq \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N)\), the farmers’ total welfare is maximized under the LE selection if and only if

\[
\left(\frac{k_i}{\mu_i} + \frac{k_k}{\mu_k}\right) F(v_Y, v_N, \gamma, n_Y, n_N) \leq \frac{1}{\gamma} H_Y(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in N} \frac{k_i}{\mu_i} \quad \text{for } (j,k) \in \{(n_Y, n), (1, n_Y+1), (n_Y, n_Y+1)\} \quad \text{with } n \leq n_Y.
\]

**Proof.** The proof is symmetric to the proof of Lemma [EC.14] □

**Lemma EC.18.** For any \(Y\) with \(n_Y \geq 1\), \(j \in Y\) and \(k \in N\), the function

\[
\Phi(1/\gamma) = \left(\frac{k_k}{\mu_k} + \frac{k_k}{\mu_j}\right) F(v_Y, v_N, \gamma, n_Y, n_N) - \alpha G(v_Y, v_N, \gamma, n_Y, n_N) - H_N(v_Y, v_N, \gamma, n_Y, n_N) \sum_{i \in N \setminus \{j,k\}} \frac{k_i}{\mu_i} \quad \text{for } (j,k) \in \{(n_Y, n), (1, n_Y+1), (n_Y, n_Y+1)\}
\]

is concave, in which \(F(\cdot), G(\cdot), H_N(\cdot), \) and \(H_Y(\cdot)\) are defined in Lemma [EC.14], when

\[
n_N \geq \frac{1}{2(v_Y+1)v_Y} \left( \frac{\sum_{i \in Y \setminus \{j,k\}} \frac{k_i}{\mu_i}}{\frac{1}{\mu_k} + \frac{1}{\mu_j}} \right) (v_N(v_Y+1) + (v_N - v_Y)) + (v_N - v_Y) - v_N(v_Y+1)(2n_Y + 2v_Y - 1) + \sqrt{1 + \frac{2}{\mu_k} + \frac{2}{\mu_j} \sqrt{\Delta}},
\]

if \(n_Y \leq n\) or \(n_Y \geq 1\) otherwise, in which

\[
\Delta = \left(\frac{1}{\mu_k} + \frac{1}{\mu_j}\right) (v_Y - v_N(v_Y+2))^2 + v_Y^2 + 2v_N v_Y (v_Y + 2(v_Y^2 - 1)) + v_N^2 (4 + 4v_Y + 5v_Y^2 + 4v_Y^3) - 4n_Y v_N (v_N - v_Y)(1 + v_Y).
\]

**Proof.** The proof is similar to the proof of Lemma [EC.7] and is omitted for brevity. □

**Lemma EC.19.** When one of the conditions in Lemma [EC.17] holds, i.e., \(n_N \geq n_N\) for some \(n_N\), holds and when \(v_Y \leq \sqrt{\frac{\alpha G}{\gamma} + \frac{1}{\gamma} \Phi(1/\gamma)}\) \(\geq \frac{1}{\gamma} \Phi(1/\gamma)\), defined in [EC.12], crosses zero at most once as \(1/\gamma\) increases, and the crossing, if it happens, is from above.
Proof. The proof is similar to the proof of Lemma EC.8 and is omitted for brevity.

Lemma EC.20. For any \( n_Y, 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y:|Y|=n_Y} \alpha(Y) \). Then \( \Phi(v_Y/v_N) \geq 0 \), in which \( \Phi(1/\gamma) \) is defined in (EC.12).

Proof. We have:

\[
\Phi\left(\frac{v_Y}{v_N}\right) = \left(\frac{v_N v_Y + n_Y v_Y + v_Y v_N}{2v_N}\right) \times \left(2 \sum_i \frac{\kappa_i}{\mu_i} + 2 v_N \alpha + (n_Y v_N + n_N v_Y + v_Y v_N) \left(\frac{\kappa_h}{\mu_h} + \frac{\kappa_f}{\mu_f}\right)\right) \\
\geq 0.
\]

Proposition EC.3. For any \( n_Y, 1 \leq n_Y \leq n - 1 \), suppose that \( \alpha \geq \max_{Y:|Y|=n_Y} \alpha(Y) \) and one of the conditions in Lemma EC.18 holds, i.e., \( n_N \geq \hat{n}_N \) for some \( \hat{n}_N \). There exist thresholds \( \gamma_L \leq \gamma_M \leq \gamma_L \) such that the farmers’ total welfare is maximized under ME if \( \gamma \in \left[\frac{\gamma_L}{\gamma_M}, \gamma_M\right] \) and under LE if \( \gamma \in \left[1, \frac{\gamma_L}{\gamma_M}\right] \cup \left[\frac{\gamma_M}{\gamma_L}, \infty\right) \).

Proof of Proposition EC.3. Lemma EC.16 provides necessary and sufficient conditions for the ME selection to be optimal when \( \gamma \notin \left(\frac{\gamma_L}{\gamma_M}, \gamma_M\right) \). Let \( [a, b] = \emptyset \) if \( a > b \). Using Lemma EC.18, when either \( \Delta < 0 \) or \( n_\hat{N} \) is larger than a certain threshold, denoted as \( \gamma_N \), these conditions hold when \( \gamma \in \left[\gamma_M, \gamma_M\right] \) for some thresholds \( \gamma_M \leq \Gamma_M \).

Lemma EC.17 provides necessary and sufficient conditions for the LE selection to be optimal when \( \gamma \notin \left(\frac{\gamma_L}{\gamma_M}, \gamma_M\right) \). Using Lemma EC.18, when either \( \Delta < 0 \) or \( n_N \geq \hat{n}_N \), these conditions hold when \( \gamma \notin \left[\Gamma_M, \Gamma_M\right] \) for some thresholds \( \Gamma_M \leq \Gamma_M \).

As a result, the welfare is maximized under ME if \( \gamma \in \left[\Gamma_M, \min\{\frac{\gamma_M}{\gamma_M}, \Gamma_M\}\right] \cup \left[\max\{\gamma_M, \Gamma_M\}, \Gamma_M\right] \) and under LE if \( \gamma \in \left[1, \min\{\frac{\gamma_M}{\gamma_M}, \Gamma_M\}\right] \cup \left[\max\{\gamma_M, \Gamma_M\}, \infty\right) \).

Because LE and ME cannot be both optimal at the same time, we must have that if \( \Gamma_M \leq \frac{\gamma_M}{\gamma_M} \), then \( \Gamma_M \leq \Gamma_M \); and that if \( \Gamma_M \geq \gamma_M \), then \( \Gamma_M \geq \Gamma_M \). If \( \Gamma_M > \frac{\gamma_M}{\gamma_M} \), then \( \Gamma_M \) can be redefined to be set to be equal to \( \frac{\gamma_M}{\gamma_M} \). If \( \Gamma_M < \gamma_M \), then \( \Gamma_M \) can be redefined to be set to be equal to \( \gamma_M \). Hence, it can be assumed without loss of generality that \( \Gamma_M \leq \gamma_M \leq \Gamma_M \leq \Gamma_M \).

Next, define \( \gamma_L^w = \min\{v_N/v_Y, \Gamma_M\} \) and \( \gamma_M^w = \max\{\gamma_M, \Gamma_M\} \). If \( \Gamma_M \leq v_N/v_Y \) and \( \gamma_M \geq \Gamma_M \), define \( \gamma_L^w = \gamma_M^w = \Gamma_M \) and \( \gamma_M^w = \min\{v_N/v_Y, \Gamma_M\} \). If \( \Gamma_M > v_N/v_Y \) and \( \gamma_M \geq \Gamma_M \), define \( \gamma_L^w = \max\{\gamma_M, \Gamma_M\} \) and \( \gamma_M^w = \Gamma_M \). If \( \Gamma_M > v_N/v_Y \) and \( \gamma_M < \Gamma_M \), define \( \gamma_L^w = \Gamma_M \) and \( \gamma_M^w = \Gamma_M \) to result in an empty interval \( \left[\gamma_L^w, \gamma_M^w\right] \). Finally, if \( \Gamma_M \leq v_N/v_Y \) and \( \gamma_M \geq \Gamma_M \), we claim that ME is optimal for all \( \gamma \in \left[\Gamma_M, \Gamma_M\right] \). For any \( Y, j \), and \( k \), \( \Phi(1/\gamma) \geq 0 \) for any \( 1/\gamma \in \left[\Gamma_M, \Gamma_M\right] \) because \( \Phi(1/\gamma) \) is concave by Lemma EC.18 given that \( \Gamma_M \leq v_N/v_Y \) and \( \Phi(1/\Gamma_M) \geq \Gamma_M \) by Lemma EC.16. Therefore, we obtain that \( \Gamma_M \geq \gamma_M^w \). In this case, define \( \gamma_L^w = \Gamma_M \) and \( \gamma_M^w = \Gamma_M \). With these new definitions, we preserve the ordering \( \gamma_L^w \leq \gamma_M^w \leq \gamma_M \).

Proof of Proposition 7. By Lemma EC.19 \( \Phi(1/\gamma) \), defined in (EC.12) crosses zero at most once and the crossing is from above. Using same argument as in the proof of Proposition EC.3 in combination with Lemmas EC.16 and EC.17, we obtain that \( \gamma_M^w = \gamma_M^w = \infty \). Using then Lemma EC.20 then shows that the ME selection is optimal when \( \gamma = v_N/v_Y \).