Pay Transparency in Heterogeneous Teams: Could Some Fairness Concerns Hurt Productivity?

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Problem Definition: In homogeneous teams, pay transparency, which induces agents to compare themselves to others, leads to a higher output (and thus, a higher payoff for the principal managing them), results in no inequalities in equilibrium (i.e., agents find the outcome fair), and is inclusive (i.e., all agents are engaged in production). This is true irrespective of the basis of social comparisons: income, utility, or reward. How about heterogeneous teams? Could pay transparency backfire under some types of fairness concerns?

Methodology/Results: We consider a model of a principal and two agents of different abilities who are averse to inequalities between each other under pay transparency. We study three bases of comparison: income, utility, and reward. For each type of fairness concern, we characterize when pay transparency leads to a higher or lower payoff for the principal, results or not in inequalities between agents, and involves only one or both agents in production. We find that income fairness never benefits the principal and might be perceived as unfair and non-inclusive, in contrast to reward fairness, which is always beneficial on all three fronts. Utility fairness lies somewhere in between, offering benefits on all three fronts when agents are not so heterogeneous, but hurting fairness and inclusivity without changing the principal’s payoff otherwise.

Managerial Implications: Although pay transparency can help close the gender pay gap, anchoring agents to income fairness by disclosing only their incomes can hurt their productivity while creating feelings of unfairness and lack of inclusivity. In contrast, anchoring them to reward or utility fairness by contextualizing their incomes with their individual contributions or costs may be beneficial on all three fronts.

Keywords: Fairness; Principal-Agent; Teams; Moral Hazard; Inequality Aversion; Heterogeneity; Pay Transparency

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Key words: fairness, principal-agent, teams, moral hazard, inequality aversion, heterogeneity, pay transparency

1. Introduction

Many organizations have experimented with greater pay transparency (Blanes i Vidal and Nossol 2011, Song et al. 2018), not only in an effort to close the gender pay gap1 or make salary negotiations less political, but also because it often improves performance by eliciting social comparisons (Roels and Su 2014, Long and Nasiry 2020, Cullen and Pakzad-Hurson 2023). Although it is commonly believed that greater transparency always leads to better outcomes (Buell 2019, Guda et al. 2023), the fact that workers may engage in salary comparison may hurt their productivity. In fact, Obloj and Zenger (2023) report that pay transparency can lead to lower employee productivity, depending on what it reveals and to whom, and Turkoglu and Tucker (2022) find that it could be demotivating. This is

especially true in heterogeneous teams, in which workers might be expected to contribute differently, potentially leading to feelings of unfairness or non-inclusivity: “If you overwork your high performers, you will lose them because they start to resent the fact that they’re doing more. If you’re taking away work from people who are slower, [they will lose interest]” (Rebecca 2016). Could pay transparency in light of fairness concerns hurt productivity?

Pay transparency is often advocated to promote a fair and inclusive work environment (Fan et al. 2023). However, fairness is sometimes considered to be “a fuzzier idea than equality” and often “lies in the eye of the beholder.”2 When facing pay transparency, workers inevitably compare themselves to each other, but on what basis? In a monograph on distributive justice, Moulin (2004) distinguishes the following four fairness principles. First, the exogenous rights principle centers around “equality ex-ante, in the sense that we have an equal claim to resources regardless of the way they affect our welfare or that of others” (Moulin 2004, p. 22). We translate this principle in our context as workers caring about inequalities in income. Second, the compensation principle aims to ensure equality ex-post by equalizing individual utilities. For instance, Kropp et al. (2022) question whether it is fair to pay employees who work remotely from a lower cost-of-living location the same as employees who work from a higher cost-of-living location. We translate this principle in our context as workers caring about inequalities in utilities. Third, the reward principle entitles individuals to the “fruit of their own labor” (Moulin 2004, p. 22). We translate this principle in our context as workers caring about inequalities in reward, comparing their incomes to their “deserved” share of the total income, in which their notion of desert is measured as their relative contribution to the total output. Fourth, the fitness principle aims at maximizing the total utility. Given that it is not relational, it cannot be used as a basis of comparison among agents and is therefore irrelevant to our analysis.

To illustrate these dimensions of income, utility, and reward fairness, consider in Table 1 a scenario where two agents contribute differently to a common project (middle row of left panel), leading to a total output of 5 units, with different costs for their respective effort contributions (bottom row of left panel). What could be a fair split?

- Under income fairness, a split of (2.5, 2.5) would be fair because it equates the agents’ incomes.

2 https://www.economist.com/lexingtons-notebook/2011/12/07/fairness-or-equality
• Under utility fairness, a split of \((2, 3)\) would be fair because it equates Agent 1’s utility (revenue-cost), equal to \(2 - 1\), to Agent 2’s, equal to \(3 - 2\).

• Under reward fairness, a split of \((3, 2)\) would be fair because it equates Agent 1’s reward, i.e., income-to-effort ratio, equal to \(3/3\), to Agent 2’s, equal to \(2/2\).

In practice, which type of fairness consideration prevails under pay transparency depends on whether income information is contextualized or not. For instance, to anchor agents on utility comparison rather than on plain income comparison, a multinational organization paying different incomes in different jurisdictions could provide some cost-of-living information. To anchor them on reward comparison instead, the organization could report their individual output contributions (or proxies thereof) to entice them to compare themselves on the share they deserve rather than on their nominal incomes.\(^3\)

To illustrate this notion of contextualization of salary information, consider Buffer, a software company, which has devised a “salary system” that attempts to tie salaries “to the value that works brings to the business in direct monetary terms.”\(^4\) Given that work value is difficult to quantify, especially for some roles, they rely instead on market rates to determine compensations, while “maintaining flexibility in how they use it rather than taking market data and using it directly for salaries.” Figure 1 illustrates the type of information publicly reported for each individual working at Buffer: their salary, their level (from 1 to 11) and step within their level (1 or 2), a market benchmark calibrated on their level and step, and their cost-of-living adjustment rate (high or global) depending on their location. Given Buffer’s pay transparency, individuals are more likely to compare themselves to other individuals in similar or potentially different roles, which may create feelings of unfairness. Ignoring any contextual information, they may base their comparison

\(^3\) Pay transparency experts have debated the value of contextualizing income disclosure with information on individual output contributions. The International Labour Organization (ILO), a United Nations agency, has indeed advocated for the adoption of standards of “work of equal value” when comparing incomes (https://www.ilo.org/sites/default/files/wcms5/groups/public/@dgreports/@gender/documents/briefingnote/wcms_410196.pdf).

\(^4\) https://buffer.com/resources/salary-system/
solely on their income. However, if they account for their differences in level and step and for the market data, they may instead compare their salary to what they deserve, i.e., their reward. Alternatively, the adjustment for their costs of living might anchor them on their costs (and thus, their utility), instead of their contribution.

**Model.** We consider a principal overseeing a team of two agents with heterogeneous abilities (Long and Nasiry 2020) who work on a common task. (They might also work independently on individual tasks, which are not subject to social comparison.) The agents are heterogeneous in the sense that they may have different costs and different contributions per unit of effort. As is common in the literature on team production (Holmström 1982) and in many intra-organizational settings (e.g., profit-sharing agreements), agents receive a share of output and a fixed payment. Unlike the output, their efforts are unobservable. We assume their efforts are perfect substitutes to disentangle the issue of fairness from co-production, but our approach can be extended to more general settings. Under pay transparency, the agents are inequality-averse—specifically, subject to feelings of envy and guilt as in Fehr and Schmidt (1999)—after comparing their payoff (namely, their income or utility) to a reference point (namely, the other agent’s income, payoff or the amount they consider to deserve). Without pay transparency, agents are assumed to be inequality-neutral, as in Long and Nasiry (2020), given that they have no tangible information to form such comparisons.

The timeline is as follows: Before the game starts, the context is set, i.e., it is decided whether agents are inequality-neutral (under no pay transparency) or inequality-averse (under pay transparency); and in the latter case, which fairness concern they exhibit:
income, utility, or reward. In the first stage of the game, the principal offers a contract. In the second stage, agents choose their effort levels, simultaneously and noncooperatively, to maximize their individual utility.

**Research Questions.** If the team were homogeneous, the principal’s contracting strategy would be straightforward: an equal-sharing rule is beneficial to the principal because it results in a (weakly) higher output (and thus, a (weakly) higher payoff for the principal), fair on all dimensions because it results in no inequalities (agents’ incomes, utilities, and rewards are all equal) in equilibrium, and inclusive because all agents are involved in production (Bartling and von Siemens 2010a, Gill and Stone 2015). (We offer a formal proof of this result in Appendix B.) Thus, pay transparency wins on all three fronts, irrespective of the basis of the agents’ fairness concerns.

How about heterogeneous teams, in which team members have different abilities and respond differently to incentives? Do pay transparency and, therefore, inequality aversion still benefit the principal while resulting in fair and inclusive outcomes? Under which dimension of fairness could they backfire?

**Results.** Table 2 summarizes the effects of pay transparency as a function of the object of comparison (income, utility, or reward) and the degree of agent heterogeneity on the following three dimensions: (i) the principal’s payoff, (ii) whether the outcome results in no inequalities in equilibrium and is, therefore, considered to be fair by agents, and (iii) whether the optimal contract involves both agents in production and is, therefore, inclusive.

<table>
<thead>
<tr>
<th>Object of comparison</th>
<th>Agent heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mild</td>
</tr>
<tr>
<td>Income</td>
<td>×, ✓, ✓</td>
</tr>
<tr>
<td>Utility</td>
<td>✓, ✓, ✓</td>
</tr>
<tr>
<td>Reward</td>
<td>✓, ✓, ✓</td>
</tr>
</tbody>
</table>

The triplet of symbols in each cell denotes whether pay transparency (i) increases the principal’s payoff, (ii) results in a fair equilibrium outcome, and (iii) is inclusive, in this order. A “✓” indicates yes, a “×” indicates no, and a “−” indicates no change. When agents are homogeneous, under income fairness, the principal’s payoff does not change with inequality aversion, so the “×” degenerates into a “−” in this case.

When agents compare their *incomes*, inequality aversion always leads to a (weakly) lower payoff for the principal. Moreover, it leads to an unfair allocation and might exclude the least able agent from production when agents are quite heterogeneous.
When agents compare their utilities, the result depends on the degree of agent heterogeneity. On one hand, when the team is quite heterogeneous, inequality aversion does not change the principal’s payoff, but it gives rise to inequalities between agents and excludes the least able agent from production, who receives nothing. In this case, the ablest agent suffers from guilt, whereas the least able one suffers from envy. On the other hand, when the team is only moderately heterogeneous, inequality aversion benefits the principal without creating inequalities and while involving both agents in production.

When workers compare their rewards, inequality aversion always increases the payoff of the principal, results in a fair outcome, and involves both agents in production under the best affine output-sharing contract.

**Mathematical Intuition.** The difference in outcomes across the different types of fairness considerations stems from differences in the game structure: In the case of utility or reward fairness, the agents’ effort choice can be formulated as a supermodular game; in contrast, under income fairness, the game is submodular. Because efforts are strategic complements in supermodular games, agents put in efforts that exceed their inequality-neutral levels in equilibrium, thus achieving a higher output (and thus, a higher payoff for the principal). In contrast, in submodular games, efforts are strategic substitutes, rendering the previous mechanism ineffective.

**Managerial Implications.** Our research indicates that under certain types of fairness concerns, pay transparency may backfire in heterogeneous teams. We find that reward fairness tends to be uniformly beneficial, in contrast to income fairness, which tends to be uniformly detrimental; and utility fairness lies somewhere in between. Accordingly, it is important for organizations that wish to adopt pay transparency for closing the gender pay gap or making salary negotiations less political to contextualize, like Buffer does, the reporting of agents’ incomes with their relative contributions (or proxies thereof); or, if their relative contributions are too complicated to disentangle, their costs of effort (or proxies thereof), provided that they are only moderately heterogeneous. Our analytical model thus explains why, in practice, organizations that use reward-based compensation practices may see better returns than those who insist on adopting equal-pay practices (Low 2016).

### 2. Literature Review

Building on the literature on fairness principles, our study contributes to the literatures on the management of team operations and on principal-agent models with inequality-averse agents. We next review these three streams of literature.
2.1. Fairness Principles

The principles of distributive justice stem from Adams’s equity theory (Adams 1965), which argues that one’s reward should be proportional to one’s input. This theory has been extended by Deutsch (1975) and Cook and Hegtvedt (1983) to incorporate “equality” and “need”: “equality” means that people should receive the same amount irrespective of their input, whereas “need” implies the resources should be allocated to those who need it the most. Inspired by this stream of work, Moulin (2004) formalized the fairness principles to “exogenous-rights,” “compensation,” and “reward,” which serve as the theoretical basis for this paper, as discussed in §1.

2.2. Management of Team Operations

A growing body of literature in people-centric operations studies team dynamics, such as incentives to collaborate, help each other, and share knowledge (Siemsen et al. 2007, Özkan-Seely et al. 2015, Rahmani et al. 2017, Crama et al. 2019). Although fairness concerns are often not explicitly modeled in this literature, they naturally arise in teams. Of particular interest to our study is the effect of social comparison on team performance, e.g., feedback design (Roels and Su 2014, Chan et al. 2014, Song et al. 2018, Tan and Staats 2020, Long and Nasiry 2020, Niewoehner and Staats 2022, Cullen and Pakzad-Hurson 2023, Fan et al. 2023). Social comparison as a performance improvement mechanism has also been studied empirically, both in lab experiments (Charness et al. 2014, Tafkov 2013, Hannan et al. 2008) and in field experiments (Blanes i Vidal and Nossol 2011, Cowgill 2015, Song et al. 2018). Unlike previous studies, which largely promote greater transparency, we demonstrate that transparency needs careful crafting. It could backfire with heterogeneous agents under the wrong type of fairness consideration.

2.3. Principal-Agent Problems with Inequality-Averse Agents

We embed fairness considerations into a principal-agent model involving team production (Holmström 1982), building on the seminal work by Fehr and Schmidt (1999) on envy and guilt, which has received substantial empirical support (see, e.g., Bolton and Ockenfels 2000). The literature can be classified into two streams, depending on whether individual outputs are verifiable or whether it is only the team output that is so.
2.3.1. Individual Outputs. When individual outputs are verifiable, the effect of envy and guilt depends on whether efforts are verifiable or not. When efforts are not verifiable, envy is known to have two effects, resulting in an ambiguous effect on output: On one hand, envy motivates agents to increase effort (at least when agents are homogeneous); on the other hand, it might require compensation due to agents’ disutility when comparing incomes (Itoh 2004, Neilson and Stowe 2010, Kragl 2015). In contrast, guilt always has a negative impact on output: It is known to discourage agents when individual performance measures are used and agents are homogeneous (Itoh 2004); and similar to envy, guilt requires compensation due to agents’ disutility when comparing incomes (Neilson and Stowe 2010). However, these effects may vanish when at least one agent’s effort is verifiable; that is, envy (resp. guilt) may no longer motivate (resp. demotivate) agents to exert high effort (Demougin et al. 2006, Rey-Biel 2008). Further studies investigate the interplay among inequality aversion, risk aversion (Englmaier and Wambach 2010), limited liability (Bartling and von Siemens 2010b), and reference types (Bartling 2011).

2.3.2. Team Outputs. When only the team output is verifiable, inequality aversion is well known to increase team output when the team is homogeneous (Bartling and von Siemens 2010a, Gill and Stone 2015).

Considering heterogeneous agents like we do, Kölle et al. (2011) and Kölle et al. (2016) take the perspective of a social planner (or, alternatively, a principal dealing with agents that have no limited liability) offering a symmetric output-sharing contract to agents who are averse to inequalities in utility. Within their context, introducing ex-ante wealth differences is needed to increase team output. However, their proposed symmetric-share contractual arrangement is in general suboptimal for the principal. It is, in particular, inefficient when agents are inequality-neutral because it constrains the principal to involve both agents in production, whereas it is optimal to involve only the ablest one.

We contribute to this literature in the following ways. First, we do not restrict the principal’s choice of affine contract to have symmetric shares. Second, we operationalize in the context of heterogeneous teams the principle of reward fairness, which is widely prevalent in the workplace (Matuson 2022). Table A.1 in Appendix A provides a summary of the different types of fairness considerations that have been studied in this literature. As the table reveals, we contribute to this literature by comparing outcomes across all three types of fairness concerns. Similar to Cohen et al. (2022), who consider different fairness
principles in the context of pricing and reach an impossibility result, we identify a natural tension between income fairness and utility or reward fairness.

3. Model

We next present the model components, formulate the agents’ effort choice game and the principal’s contract design problem, and introduce the benchmark of inequality-neutral agents, which arises when pay is non-transparent.

3.1. Model Components

We consider a principal and two heterogeneous agents indexed by $i = 1, 2$. Agent $i$ exerts effort $e_i \geq 0$ at a cost $C_i(e_i) = \frac{1}{2}c_i e_i^2$. Their efforts are assumed to be perfect substitutes. Specifically, we assume a linear team output $P(e_1, e_2) = k_1 e_1 + k_2 e_2$, with $k_1, k_2 > 0$. Let $k := (k_2/c_2)/(k_1/c_1)$ denote Agent 2’s relative ability. We assume that Agent 1 has higher ability, i.e., $k \leq 1$. Without loss of generality, we normalize the problem by setting $c_1 = c_2 = k_1 = 1$, so that $k_2 = k$. Similar to Gill and Stone (2015) and Kölle et al. (2011, 2016), we do not consider randomness in output, but our results can be generalized when the output is subject to a random shock and agents compare their ex-ante utilities.

The team output is verifiable, unlike the agents’ individual efforts. We assume the principal offers an affine output-sharing contract parameterized by $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, in which $\gamma_i$ and $f_i$ respectively denote Agent $i$’s output share and fixed fee, with $\gamma_i \geq 0$, $f_i \geq 0$, and $\gamma_1 + \gamma_2 \leq 1$. Although affine output-sharing contracts are not necessarily optimal, they are common in practice. Accordingly, Agent $i$’s income is $I_i(e_i, e_{-i}, \gamma_i, f_i) = \gamma_i P(e_1, e_2) + f_i$, where $-i := 3 - i$ for $i = 1, 2$. Agent $i$’s nominal utility is thus $v_i(e_i, e_{-i}, \Phi) = I_i(e_i, e_{-i}, \gamma_i, f_i) - C_i(e_i)$ for $i \in \{1, 2\}$. The principal receives the residual output; hence, the principal’s payoff is $\pi(e_1, e_2, \Phi) = P(e_1, e_2) - \sum_i I_i(e_i, e_{-i}, \gamma_i, f_i) = (1 - \gamma_1 - \gamma_2)P(e_1, e_2) - f_1 - f_2$.

Under pay transparency, agents engage in social comparisons, as in Long and Nasiry (2020). In particular, they are assumed to be subject to envy and guilt as in Fehr and Schmidt (1999). Let us denote with $\alpha$ (for “ahead”) the agents’ level of guilt and $\beta$ (for “behind”) their level of envy. We consider three objects of comparison: income (‘I’), utility (superscript ‘U’), and reward (‘R’). Agents experience disutilities when they get either more (due to guilt) or less (due to envy) of the object of reference than the other agent.
Defining \([\cdot]^+ := \max\{\cdot, 0\}\), Agent \(i\)'s total utility under income, utility, or reward fairness, for \(i = 1, 2\), can respectively be expressed as follows:

\[
u_i^I(e_i; e_{-i}, \Phi) = v_i(e_i, e_{-i}, \gamma_i, f_i) - \alpha[I_i(e_i, e_{-i}, \gamma_i, f_i) - I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i})]^+
- \beta[I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i}) - I_i(e_i, e_{-i}, \gamma_i, f_i)]^+, \tag{1}
\]

\[
u_i^U(e_i; e_{-i}, \Phi) = v_i(e_i, e_{-i}, \gamma_i, f_i) - \alpha[v_i(e_i, e_{-i}, \gamma_i, f_i) - v_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i})]^+
- \beta[v_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i}) - v_i(e_i, e_{-i}, \gamma_i, f_i)]^+, \tag{2}
\]

\[
u_i^R(e_i; e_{-i}, \Phi) = v_i(e_i, e_{-i}, \gamma_i, f_i) - \alpha \left[I_i(e_i, e_{-i}, \gamma_i, f_i) - \frac{k_i e_i}{P(e_i, e_{-i})}(I_i(e_i, e_{-i}, \gamma_i, f_i) + I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i}))\right]^+
- \beta \left[\frac{k_i e_i}{P(e_i, e_{-i})}(I_i(e_i, e_{-i}, \gamma_i, f_i) + I_{-i}(e_{-i}, e_i, \gamma_{-i}, f_{-i})) - I_i(e_i, e_{-i}, \gamma_i, f_i)\right]^+, \tag{3}
\]

in which the term \((k_i e_i / P) \cdot (I_i + I_{-i})\) in (3) represents the income (more specifically, the share of total income) Agent \(i\) believes they deserve given their relative contribution \(k_i e_i / P\).

Using the example outlined in Table 1, agents would care about having their incomes equal (namely, 2.5 each) under income fairness, their utilities equal (namely, incomes of 2 and 3, respectively, for Agents 1 and 2, so that \(2 - 1 = 3 - 2\)) under utility fairness, and their incomes proportional to their effort (namely, incomes of 3 and 2, respectively, for Agents 1 and 2, so that \(3/3 = 2/2\)) under reward fairness.

We restrict \(\alpha\) to lying in \([0, 1/2]\) and \(\beta\) to lie in \([0, 1]\) like Rey-Biel (2008) and Demougin et al. (2006), assume that \(\beta \geq \alpha\) (Fehr and Schmidt 1999) given that people tend to suffer more from inequalities to their disadvantage than inequalities to their advantage (Loewenstein et al. 1989), and assume that \(\alpha + \beta \leq 1\) for simplicity of exposition. If \(\alpha < 0\), agents would be inequality-seeking, which could arise under competition but not under cooperation (McClintock et al. 1984, p. 187-188), and would contradict their concerns for fairness; and if \(\alpha > 1/2\), agents who are averse to utility or income inequality would be willing to transfer their income to the other agent. If \(\beta < 0\), agents would be inequality-seeking; and if \(\beta > 1\), an envious agent who is averse to utility inequality would gain more utility from envy than from the income itself. These parameter value ranges have also been found to be the most relevant in empirical studies (Nunnari and Pozzi 2022). Long and Nasiry (2020) consider a particular case of income fairness with \(\alpha = 0\).
The sequence of events is as follows: Before the game starts, the context is set, i.e., it is decided whether agents are inequality-neutral (under no pay transparency) or inequality-averse (under pay transparency); and in the latter case, which fairness concern they exhibit: income, utility, or reward. In the first stage of the game, the principal offers a contract. In the second stage, agents choose their effort levels, simultaneously and noncooperatively, to maximize their individual utility. For each fairness principle, we solve the problem backward by first characterizing the agents’ equilibrium efforts and then the principal’s optimal contract. Throughout our analysis, we ignore the agents’ participation constraints (i.e., agents may earn negative utility in equilibrium), but they can be incorporated at the cost of a more cumbersome analysis.

3.2. Equilibrium Efforts
Given an output-sharing contract \( \Phi = (\gamma_1, \gamma_2, f_1, f_2) \), agents maximize their utility given the other agent’s choice of effort. Under a given fairness principle \( F \in \{I, U, R\} \), in equilibrium:

\[
e^*_i = \max_{e_i \geq 0} u^F_i (e_i; e^*_\neg i, \gamma_i, f_i), \quad i \in \{1, 2\}.
\]

We next state an equilibrium selection rule based on Pareto-optimality (Mas-Colell et al. 1995, p. 313). For any given fairness principle \( F \in \{I, U, R\} \) and contract \( \Phi \), an equilibrium effort pair \((e^*_1, e^*_2)\) is said to Pareto-dominate another equilibrium effort pair \((e_1, e_2)\) if both \((e^*_1, e^*_2)\) and \((e_1, e_2)\) solve (4) and if \(u^F_i (e^*_i; e^*_\neg i, \Phi) \geq u^F_i (e_i; e^*_\neg i, \Phi)\) for \(i = 1, 2\) and \(\pi(e^*_1, e^*_2, \Phi) \geq \pi(e_1, e_2, \Phi)\), with at least one of these three inequalities holding strictly. An equilibrium is said to be Pareto-optimal if there exists no Pareto-dominating equilibrium.

**Assumption 1 (Equilibrium Selection Rule).** If there are multiple equilibria in (4), we select a Pareto-optimal equilibrium.

3.3. Optimal Contract
Anticipating the agents’ equilibrium efforts under fairness principle \( F \in \{I, U, R\} \), the principal offers an affine output-sharing contract \( \Phi = (\gamma_1, \gamma_2, f_1, f_2) \) that maximizes their payoff:

\[
\max_{\Phi = (\gamma_1, \gamma_2, f_1, f_2)} \pi(e^*_1, e^*_2, \Phi) = (1 - \gamma_1 - \gamma_2) (e^*_1 + ke^*_2) - f_1 - f_2,
\]

subject to:

\[
\begin{align*}
f_i &\geq 0, \quad i = 1, 2, \\
\gamma_1 + \gamma_2 &\leq 1, \quad 0 \leq \gamma_i \leq 1, \quad i = 1, 2, \\
e^*_i &\in \arg \max_{e_i \geq 0} u^F_i (e_i; e^*_\neg i, \gamma_i, f_i), \quad i = 1, 2.
\end{align*}
\]

(5)
The principal’s problem can be simplified under utility and reward fairness. Specifically, we show in the electronic companion that one can restrict our attention, without loss of optimality, to linear contracts without fixed fees, i.e., \( f_1 = f_2 = 0 \). We do so in three steps. First, given a Nash equilibrium to the effort game (4) under contract \( \Phi \), say, \((e^f_1, e^f_2)\), in which the superscript ‘f’ is used to denote the existence of fixed fees in \( \Phi \), we construct a linear contract \( \Phi' = (\gamma'_1, \gamma'_2, 0, 0) \) with no fixed fees that leads to a higher payoff for the principal at the effort choice \((e^f_1, e^f_2)\). Moreover, we show that, under contract \( \Phi' \), Agent \( i \)’s best response to \( e^f_{-i} \) is higher than \( e^f_i \), for \( i \in \{1, 2\} \). Second, we show that Game (4) under a linear contract that involves no fixed fees—like contract \( \Phi' \)—is supermodular. Third, we show by combining these two results that, under contract \( \Phi' \), there exists an equilibrium \((e^0_1, e^0_2)\), in which the superscript ‘0’ is used to denote the absence of fixed fees, such that \( e^0_1 \geq e^f_1 \) and \( e^0_2 \geq e^f_2 \), and thus \( \pi(e^0_1, e^0_2, \Phi') \geq \pi(e^f_1, e^f_2, \Phi') \geq \pi(e^f_1, e^f_2, \Phi) \), where at least one of the two inequalities is strict.

In contrast, under income fairness, Game (4) is submodular, so the above argument does not apply. As a result, the optimal fixed fees under income fairness are not necessarily zero.

### 3.4. Benchmark: Inequality-Neutral Agents

Under no pay transparency, agents are assumed to be inequality-neutral (superscript ‘N’), i.e, \( \alpha = \beta = 0 \), as in Long and Nasiry (2020). In this case, agents’ equilibrium efforts solving (4) are \( e^N_1 := \gamma_1 \) and \( e^N_2 := k\gamma_2 \). The next proposition, whose proof is trivial and, therefore, omitted, shows that the principal’s optimal contract allocates half of the output to the ablest agent and gives nothing to the other agent. Hence, when agents are inequality-neutral, the outcome is not inclusive given that the least able agent is excluded from production.

**Proposition 1.** When agents are inequality-neutral, the principal’s best output-sharing contract \( \Phi \) gives shares only to the ablest agent by offering \((\gamma_1, f_1) = (1/2, 0)\) and \((\gamma_2, f_2) = (0, 0)\).

In the following three sections, we present the optimal contract under income, utility, and reward fairness.

### 4. Income Fairness

Considering income fairness, we first describe the agents’ equilibrium efforts and then identify the principal’s optimal contract.
4.1. Equilibrium Efforts

Under income fairness, Game (4) is submodular: agents’ efforts act as strategic substitutes and tend to move in opposite directions.

Relative to the inequality-neutral benchmark, both envy and guilt can motivate or demotivate effort depending on the agents’ relative income and contract structure. (This result, like any other comparative statics, should be interpreted in a weak sense. That is, envy or guilt may have no effect on effort.) To see this, recall from (1) that Agent $i$’s total utility takes the form of $v_i - \alpha [I_i - I_{-i}]^+ - \beta [I_{-i} - I_i]^+$. When Agent $i$ is envious, i.e., when $I_i < I_{-i}$, Agent $i$ is enticed to put in more effort if $\gamma_i > \gamma_{-i}$ and less effort if $\gamma_i < \gamma_{-i}$ to shrink the pay difference. In contrast, when Agent $i$ is guilty, i.e., when $I_i > I_{-i}$, Agent $i$ is enticed to put in less effort if $\gamma_i > \gamma_{-i}$ and more effort if $\gamma_i < \gamma_{-i}$.

To characterize the equilibrium efforts, we define the following effort thresholds:

$$e_i^\alpha := \left[e_1^N - \alpha (\gamma_1 - \gamma_2)\right]^+, \quad e_i^\beta := e_1^N + \beta (\gamma_1 - \gamma_2),$$

$$e_i^\gamma := e_2^N + k \alpha (\gamma_1 - \gamma_2), \quad e_i^\delta := [e_2^N - k \beta (\gamma_1 - \gamma_2)]^+. \quad (6)$$

For $i \in \{1,2\}$, $e_i^\alpha$ maximizes Agent $i$’s utility when the agent is subject to guilt (which is why we use the superscript $\alpha$), i.e., when $I_i > I_{-i}$, which simplifies to $v_i - \alpha [I_i - I_{-i}]$. On the other hand, $e_i^\beta$ maximizes Agent $i$’s utility when the agent is subject to envy (which is why we use the superscript $\beta$), i.e., when $I_i < I_{-i}$, which simplifies to $v_i - \beta [I_{-i} - I_i]$.

Let also denote by $\bar{e}_i(e_{-i})$ Agent $i$’s effort level that equalizes the agents’ incomes given that Agent $-i$ exerts $e_{-i}$, i.e., $I_i(e_i(e_{-i}), e_{-i}) = I_{-i}(e_{-i}, \bar{e}_i(e_{-i}))$ for $i \in \{1,2\}$. It turns out that $\bar{e}_i(e_{-i})$ is decreasing, i.e., efforts are strategic substitutes when their incomes are equal.

The next lemma characterizes the equilibrium efforts, taking a four-region structure. Its proof, as well as any other proof or supporting result, appears in the electronic companion.

**Lemma 1.** Under income fairness, there exist thresholds $\zeta_{AB} < \zeta_{BC} < \zeta_{CD}$ such that, given an output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$,

A. If $f_1 - f_2 \leq \zeta_{AB}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: $e_1^* = e_1^\beta$, $e_2^* = e_2^\alpha$;

B. If $\zeta_{AB} \leq f_1 - f_2 \leq \zeta_{BC}$, there exists a continuum of pure-strategy Nash equilibria such that $\gamma_1(e_1^* + ke_2^*) + f_1 = \gamma_2(e_1^* + ke_2^*) + f_2$: $e_2^* \in \left[e_2^\beta, \max\{e_2^\alpha, \bar{e}_2(e_1^*)\}\right]$ and $e_1^* = \bar{e}_1(e_2^*)$;

C. If $\zeta_{BC} \leq f_1 - f_2 \leq \zeta_{CD}$, there exists a continuum of pure-strategy Nash equilibria such that $\gamma_1(e_1^* + ke_2^*) + f_1 = \gamma_2(e_1^* + ke_2^*) + f_2$: $e_1^* \in \left[\max\{e_1^\beta, \bar{e}_1(e_2^*)\}, e_1^\alpha\right]$ and $e_2^* = \bar{e}_2(e_1^*)$;
D. If \( f_1 - f_2 \geq \zeta_{CD} \), there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences guilt and Agent 2 experiences envy: \( e_1^* = c^\alpha_1 \), \( e_2^* = c^\beta_2 \).

Two types of equilibria emerge. On one hand, Cases A and D involve uneven income distributions, which leave one agent guilty and the other envious. Whenever one agent puts in more effort, the other agent reduces their effort regardless of the feelings of envy or guilt due to the submodular nature of the game. On the other hand, Cases B and C are associated with an even income distribution. In these two cases, agents’ effort choices are dominated by their fairness concerns. In particular, one agent (namely, Agent 2 in Case B and Agent 1 in Case C) puts in a “base” effort and the other chooses their effort to equalize their incomes.

Although there exists a continuum of equilibria in Cases B and C, the total output remains constant along the continuum. Thus, the agents’ incomes remain constant along the continuum of equilibria, whereas their costs vary in opposite directions, making the Pareto equilibrium selection rule (Assumption 1) inapplicable. However, an exact selection of equilibrium is irrelevant to the principal for the design of the contract, given that the equilibrium output, and therefore, the principal’s payoff, remains constant along the continuum of equilibria.

Are the equilibrium efforts under income fairness higher or lower than the inequality-neutral benchmarks? When \( f_1 = f_2 = 0 \), as it turns out to arise under some optimal contract structures (as shown in §4.2), only Case A arises. Because \( I_1 > I_2 \) if and only if \( \gamma_1 > \gamma_2 \) when \( f_1 = f_2 = 0 \), both guilt and envy demotivate effort, as outlined at the beginning of this section. In this case, the equilibrium efforts under income fairness are lower than those under inequality neutrality.

4.2. Optimal Contract

We next characterize the optimal contract and show that it depends on both the levels of envy and guilt. See Figure 2 for a graphical depiction. To simplify the exposition, let \( \bar{\pi}'(\alpha, \beta) := \max\{\sqrt{1-2\alpha}, \sqrt{\frac{\beta}{1-\alpha}}\} \) and \( \kappa'(\alpha, \beta) := \min\{\sqrt{1-2\alpha}, \sqrt{\frac{\max\{0,1-2\alpha-\beta\}}{\alpha}}\} \). Because \( \beta/(1-\alpha) \leq 1-2\alpha \) if and only if \( 1-2\alpha \leq (1-2\alpha-\beta)/\alpha \), we have that \( \bar{\pi}'(\alpha, \beta) = \sqrt{1-2\alpha} \) if and only if \( \kappa'(\alpha, \beta) = \sqrt{1-2\alpha} \).
**Proposition 2.** Under income fairness, the principal’s best output-sharing contract $\Phi$ is such that

$$
(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \begin{cases} 
\left( \frac{1}{2}, \frac{1}{2}, 0, 0 \right), & k \geq \kappa^I(\alpha, \beta), \\
\left( \frac{1}{2}, 0, 0, \frac{1 + \beta + \alpha k^2}{16} \right), & \kappa^I(\alpha, \beta) < k < \kappa^I(\alpha, \beta), \\
\left( \frac{1}{2}, 0, 0, 0 \right), & k \leq \kappa^I(\alpha, \beta).
\end{cases}
$$

Figure 2 Optimal contracts under income fairness.

Note. In both figures, an orange background indicates an unfair equilibrium outcome, while a teal background indicates a fair equilibrium outcome.

The optimal contract takes three forms depending on the agents’ level of heterogeneity and inequality aversion. When agents become more heterogeneous, i.e., when $k$ decreases, the form of the optimal contract switches from the first to the second case (if the corresponding range is nonempty), and then to the third case (if the corresponding range is nonempty); see the left panel of Figure 3 for an illustration of the optimal shares with varying $k$ (here, $\alpha = 0.15$ and $\beta = 0.5$). When agents become more inequality-averse, i.e., when $\alpha$ and $\beta$ increase, the first and second regions in Proposition 2 expand, whereas the third one shrinks.

In the first and second cases of Proposition 2, i.e., when $k > \kappa^I(\alpha, \beta)$, agents’ incomes turn out to be equal in equilibrium, and both agents are involved in production. Although this is obvious in the first case, given that both agents receive the same share $\gamma_i = 25\%$ and
Figure 3  Optimal shares and relative income under income fairness.

Note. Here, $\alpha = 0.15$, $\beta = 0.5$.

no fixed fee, the second case is more subtle. In this second case, the ablest agent (Agent 1) receives a 25%-share of the output without a fixed fee, whereas the least able agent (Agent 2) receives only a fixed fee. Despite receiving no output share, Agent 2 exerts a positive effort in equilibrium, being purely motivated by equalizing incomes. Unlike the other two cases and the inequality-neutral benchmark, the principal allocates only a quarter, and not half, of the output to the agents.

In contrast, in the third case, i.e., when $k \leq \kappa^I(\alpha, \beta)$, which happens when agents are highly heterogeneous (small $k$) and not so inequality-averse (or even not at all), the principal involves only the ablest agent in production and gives nothing to the other, resulting in income inequalities in equilibrium (and thus feelings of envy and guilt) and a non-inclusive outcome.

The optimal contract is such that greater agent heterogeneity brings greater disparity in income, though in discrete form. In particular, when agents are more heterogeneous ($k$ decreases), their shares jump discontinuously to $\gamma^*_1 = 1/2$ and $\gamma^*_2 = 0$. Generally speaking, their relative incomes track their relative abilities in a discrete manner (Figure 3, right panel). Irrespective of whether only one or both agents are involved in production, the ablest agent is given a higher share and higher income than the other agent.

The next theorem summarizes this discussion and also compares the principal’s payoff under inequality aversion (with pay transparency) to the one received under inequality neutrality (without pay transparency).
Theorem 1. Under income fairness, pay transparency leads to a lower payoff for the principal, is perceived to be unfair (i.e., giving rise to income inequalities) if and only if \( k \leq \kappa^{I}(\alpha, \beta) \), and results in a non-inclusive outcome (i.e., involving only Agent 1 in production, and not both agents) if and only if \( k \leq \kappa^{I}(\alpha, \beta) \).

In sum, pay transparency, which stimulates income fairness, always leads to a lower payoff for the principal due to the corresponding decrease in effort. And when the team is relatively heterogeneous (third case), it results in feelings of envy and guilt in equilibrium, thereby creating feelings of unfairness, and in lack of inclusivity by involving only the ablest agent in production.

5. Utility Fairness

Considering now utility fairness, we first characterize the agents’ equilibrium efforts and then the principal’s optimal contract.

5.1. Equilibrium Efforts

Under utility fairness, agents tend to choose efforts to reduce their utility gap. As discussed in §3.3, Game (4) can be transformed into a supermodular game by eliminating strictly dominated strategies. Under this reformulation, agents’ efforts are strategic complements (despite the additive nature of the production function) and tend to move in the same direction. This is in stark contrast to the case of income fairness, under which the game is submodular.

Relative to the inequality-neutral benchmark, envy demotivates effort, while guilt motivates effort. (This is also in contrast to income fairness, in which, in the presence of fixed fees, envy can both motivate and demotivate effort, and similarly for guilt; and in the absence of fixed fees, both have a demotivating effect on effort.) To see this, recall from (2) that Agent \( i \)'s total utility takes the form of \( v_i - \alpha [v_i - v_{-i}]^+ - \beta [v_{-i} - v_i]^+ \). Suppose that Agent \( i \) is envious, i.e., that \( v_i < v_{-i} \). If Agent \( i \) puts in more effort than the inequality-neutral benchmark, their nominal utility decreases, whereas the other agent’s nominal utility increases due to the larger output, enlarging the disutility caused by envy. Therefore, envy demotivates effort. In contrast, suppose that Agent \( i \) is guilty, i.e., that \( v_i > v_{-i} \). If Agent \( i \) puts in more effort than the inequality-neutral benchmark, their nominal utility decreases, whereas the other agent’s nominal utility increases, reducing the utility gap. Thus, guilt motivates effort.
To characterize the equilibrium efforts, we define the following effort thresholds:

\[
\begin{align*}
    e_1^\alpha &= e_1^N + \frac{\alpha}{1-\alpha} \gamma_2, \\
    e_1^\beta &= \left[ e_1^N - \frac{\beta}{1+\beta} \gamma_2 \right]^+, \\
    e_2^\alpha &= e_2^N + \frac{\alpha}{1-\alpha} k \gamma_1, \\
    e_2^\beta &= \left[ e_2^N - \frac{\beta}{1+\beta} k \gamma_1 \right]^+.
\end{align*}
\] (8)

Similar to (6), \(e_i^\alpha\) is the effort that maximizes Agent \(i\)'s utility when the agent suffers from guilt, i.e., when \(v_i > v_{-i}\), which simplifies to \(v_i - \alpha [v_i - v_{-i}]\). And \(e_i^\beta\) is the effort that maximizes Agent \(i\)'s utility when the agent suffers from envy, i.e., when \(v_i < v_{-i}\), which simplifies to \(v_i - \beta [v_{-i} - v_i]\). Because guilt and envy respectively lead to higher and lower effort than the inequality-neutral level, \(e_i^\alpha \geq e_i^N \geq e_i^\beta\) for \(i = 1, 2\).

Let \(\tau_i(e_{-i})\) denote Agent \(i\)'s effort level that equalizes agents’ nominal utilities given that Agent \(-i\) exerts \(e_{-i}\), i.e., \(v_i(\tau_i(e_{-i}), e_{-i}) = v_{-i}(e_{-i}, \tau_i(e_{-i}))\) for \(i = 1, 2\). Whenever it is a best response, \(\tau_i(e_{-i})\) lies between \(e_i^\beta\) and \(e_i^\alpha\), i.e., \(e_i^\alpha \geq \tau_i(e_{-i}) \geq e_i^\beta\) for \(i = 1, 2\). As a result, any effort lying outside the range \([e_i^\beta, e_i^\alpha]\) is strictly dominated. Thus, we can restrict the agents’ strategy space to these intervals without loss of generality.

In this restricted action space, the game is supermodular because \(\tau_i(e_{-i})\) is upward-sloping. As outlined in §3.3, the optimal contract, therefore, turns out to be such that \(f_1 = f_2 = 0\). Accordingly, we focus our presentation of the equilibrium efforts on such contracts. (See Lemma EC.1 for a characterization under general contracts with fixed fees.) Similar to Lemma 1, the equilibrium efforts take a four-region structure.

**Lemma 2.** Under utility fairness, there exist thresholds \(\theta_{AB} > \theta_{BC} > \theta_{CD}\) such that, given an output-sharing contract \(\Phi = (\gamma_1, \gamma_2, 0, 0)\),

A. If \(\gamma_2/\gamma_1 \geq \theta_{AB}\), there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: \(e_1^* = e_1^\beta, e_2^* = e_2^\alpha\);

B. If \(\theta_{BC} \leq \gamma_2/\gamma_1 \leq \theta_{AB}\), there exists a continuum of pure-strategy Nash equilibria such that \(v_1 = v_2\): \(e_2^* \in \left[ \max \left\{ e_2^\beta, \tau_2 \left( e_1^\beta \right) \right\}, e_2^\alpha \right]\) and \(e_1^* = \tau_1 \left( e_2^* \right)\);

C. If \(\theta_{CD} \leq \gamma_2/\gamma_1 \leq \theta_{BC}\), there exists a continuum of pure-strategy Nash equilibria such that \(v_1 = v_2\): \(e_1^* \in \left[ \max \left\{ e_1^\beta, \tau_1 \left( e_2^\beta \right) \right\}, e_1^\alpha \right]\) and \(e_2^* = \tau_2 \left( e_1^* \right)\);

D. If \(\gamma_2/\gamma_1 \leq \theta_{CD}\), there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences guilt and Agent 2 experiences envy: \(e_1^* = e_1^\alpha, e_2^* = e_2^\beta\).

Two types of equilibria emerge. On one hand, Cases A and D involve uneven utility distributions, which leave one agent guilty and the other envious. On the other hand, Cases
B and C are associated with an even utility distribution. In these two cases, agents’ effort choices are dominated by their fairness concerns. In particular, one agent (namely, Agent 2 in Case B and Agent 1 in Case C) puts in a “base” effort and the other chooses their effort to equalize their utilities.

Different from the case of income fairness where efforts are substitutes, agents’ efforts move hand in hand in Cases B and C to equalize their utility, consistent with the supermodular nature of the game. As a result, the total output changes along this continuum of equilibria, unlike the case of income fairness. Because the agent who sets the base effort effectively dictates the level of output, i.e., acts as a bottleneck for team production, the largest (resp., smallest) output among all equilibria in Cases B and C is achieved when the base effort is chosen to be $e^\alpha$ (resp., $e^\beta$), consistent with the motivating (resp., demotivating) role of guilt (resp., envy).

We next state the identification of Pareto-optimal equilibrium for the selection of equilibrium output by Assumption 1. Under the Pareto-optimal equilibrium, not only does the principal benefit from higher payoff, but agents have (strictly) higher utilities.

**Lemma 3.** When a continuum of Nash equilibria exists in Lemma 2, the equilibrium with maximum effort from both agents is Pareto-optimal.

### 5.2. Optimal Contract

The next proposition characterizes the optimal contract offered in equilibrium. To simplify the exposition, define $\hat{\gamma} := \frac{\frac{1}{2} - \frac{1}{2} (1 - \alpha)}{\sqrt{\frac{1}{2} - \alpha}}$, and $\kappa^U(\alpha) := \frac{1 - 2(1 - \alpha)}{2(1 - \alpha)^2 (1 - \alpha)}$.

**Proposition 3.** Under utility fairness, the principal’s best output-sharing contract $\Phi$ is to offer fixed fees $f_1^* = f_2^* = 0$ and output shares

$$
(\gamma_1^*, \gamma_2^*) = \begin{cases} 
(\frac{1}{2}, 0) & \text{if } k \leq \kappa^U(\alpha), \\
(\hat{\gamma}, \frac{1}{2} - \hat{\gamma}) & \text{if } k > \kappa^U(\alpha).
\end{cases}
$$

Unlike the case of income fairness, the total output shares given by the principal to the two agents always satisfy $\gamma_1^* + \gamma_2^* = 1/2$, as if they were inequality-neutral. Hence, the agents’ inequality aversion does not require additional compensation from the principal. Moreover, unlike income fairness, the optimal contract under utility fairness depends on $\alpha$ but not $\beta$. This is because guilt motivates effort while envy demotivates effort under utility
fairness as discussed before Lemma 2. Because of our choice of equilibrium selection rule (Assumption 1), the optimal contract lies in a region where guilt dominates.

Figure 4 Optimal contracts under utility fairness.

Note. An orange background indicates an unfair equilibrium outcome, while a teal background indicates a fair equilibrium outcome.

By Proposition 3, the principal’s payoff is maximized by allocating shares to either one agent (when \(\gamma_1 = \frac{1}{2}\) and \(\gamma_2 = 0\)) or both agents depending on their level of heterogeneity and their level of inequality aversion; see Figure 4. When agents are mildly heterogeneous and highly inequality-averse, i.e., when \(k > \kappa^U(\alpha)\), the principal involves both agents in production. By attempting to reduce their inequalities in utilities, agents exert higher effort, thus mitigating team moral hazard and boosting output. Otherwise, i.e., when \(k \leq \kappa^U(\alpha)\), the principal involves only the ablest agent as in the inequality-neutral case, rendering the same output as in the inequality-neutral case. Hence, the outcome is inclusive if and only if \(k > \kappa^U(\alpha)\).

We next analyze whether the principal benefits from pay transparency and whether the outcome is perceived to be fair in these two regimes, considering them in turn. When \(k > \kappa^U(\alpha)\), the optimal contract satisfies \(\gamma_1^* / \gamma_2^* = \theta_{BC}\), in which the threshold \(\theta_{BC}\) is defined in Lemma 2. That is, the equilibrium effort lies at the boundary between Cases B and C in Lemma 2. At this level, no agent is attempting to equalize their utility to the other and, therefore, no agent acts as a bottleneck on the other. Moreover, given our equilibrium selection rule (Assumption 1) and Lemma 3, both agents exert the maximum possible effort \(e_i^\alpha\) for \(i = 1, 2\). This equilibrium, which maximizes the agents’ utilities \(v_i - \alpha[v_i - v_{-i}]\), is as if each were ahead of the other. Because the agents’ utilities are equal in equilibrium,
they do not have any mental burden associated with their inequality aversion. Therefore, this optimal contract is considered to be fair by the agents given that its outcome is both envy- and guilt-free.

In addition, when $k > \kappa U(\alpha)$, the principal benefits from pay transparency: Because $e_i^\alpha$ increases in $\alpha$ for $i = 1, 2$, the more guilt, the higher the output, and thus, the higher the principal’s payoff. In the limit, when $\alpha = 1/2$, the equilibrium efforts are $e_1^* = \gamma_1 + \gamma_2$ and $e_2^* = k(\gamma_1 + \gamma_2)$; that is, the team moral hazard is completely eliminated at the highest possible level of guilt.

In contrast, when $k \leq \kappa U(\alpha)$, the ablest agent (who is the only one who is rewarded) experiences guilt in equilibrium, the other agent experiences envy, and the principal’s payoff remains identical to the one obtained with inequality-neutral agents. In particular, when $\gamma_1 > 0$ and $\gamma_2 = 0$, i.e., Agent 1 exerts effort $e_1^N$ and Agent 2 exerts zero effort despite feelings of envy and guilt. In this case, pay transparency (which stimulates inequality aversion) does not benefit the principal, but results in an outcome that is considered by the agents to be unfair and is non-inclusive.

**Figure 5** Optimal contract and relative income under utility fairness.

*Note.* Here, $\alpha = 0.25$.

The optimal contract is such that greater agent heterogeneity brings greater disparity in output shares. In particular, when agents are more heterogeneous ($k$ decreases), $\gamma_1^*$ increases and $\gamma_2^*$ decreases, to discontinuously jump to $\gamma_1^* = 1$ when only the ablest agent is involved in production (Figure 5, left panel). In fact, when $k > \kappa U(\alpha)$ so that both agents
are involved, their relative shares (or, equivalently, relative incomes) closely track their relative abilities (Figure 5, right panel). Irrespective of whether only one or both agents are involved in production, the ablest agent is given a higher share and a higher income than the other agent.

The next theorem summarizes this discussion.

**Theorem 2.** Under utility fairness, pay transparency leads to a higher payoff for the principal when $k > \kappa^U(\alpha)$ and equal otherwise, is perceived to be fair (i.e., engendering no utility inequalities) if and only if $k > \kappa^U(\alpha)$, and results in an inclusive outcome (i.e., involving both agents in production, and not only Agent 1) if and only if $k > \kappa^U(\alpha)$.

Does utility fairness conflict with income fairness? Although one might conjecture that a higher level of aversion to utility inequality induces a more even payoff division (in the spirit of the principle of income fairness, see §4), we find that it is actually the opposite that holds under utility fairness. Specifically, when $k > \kappa^U(\alpha)$, if $\alpha$ increases, $\gamma^*_1(\alpha)$ increases and $\gamma^*_2(\alpha)$ decreases. Thus, fair pay (in this utility context) is far from being the same as equal pay (Low 2016): The more averse agents are to inequalities in utility, the higher their discrepancy in incomes. As discussed in §3.3, the structures of the games are fundamentally different: supermodular under utility fairness and submodular under income fairness. Hence, in heterogeneous teams, there is no silver bullet: any initiative that stimulates utility fairness will result in income inequalities.

### 6. Reward Fairness

Considering last reward-concerned agents, we first describe their equilibrium efforts and then identify the principal’s optimal contract. The discussion is more succinct given that the characterization of the game follows similar lines to the case of utility fairness.

#### 6.1. Equilibrium Efforts

Under reward fairness, the effort choice game (4) is supermodular when the fixed fees are null (which is optimal by Lemma EC.7), across the entire agents’ action space and no longer over their set of non-dominated strategies like it was the case for utility fairness (Lemma EC.6). Thus, the equilibrium structure and the optimal contract exhibit similarities to those derived under utility fairness.
Following the same reasoning discussed in utility fairness, guilt motivates effort, whereas envy demotivates effort in the context of reward fairness. To characterize the equilibrium efforts, we define the following effort thresholds:

\[
e_i^\alpha := e_i^N + \alpha \gamma_i, \quad e_i^\beta := \left[e_i^N - \beta \gamma_i\right]^+, \\
e_i^\gamma := e_i^N + k_i \gamma_i, \quad e_i^\delta := \left[e_i^N - k_i \gamma_i\right]^+.
\]  

(10)

Similar to (6) and (8), \(e_i^\alpha\) maximizes Agent \(i\)’s utility when the agent suffers from guilt, i.e., when \(I_i > (k_i e_i / P) (I_1 + I_2)\), and \(e_i^\beta\) maximizes Agent \(i\)’s utility when the agent suffers from envy, i.e., when \(I_i < (k_i e_i / P) (I_1 + I_2)\).

Additionally, let \(\tau_i(e_{-i})\) denote Agent \(i\)’s effort level that guarantees no inequality in rewards, i.e., that agents’ incomes are proportional to their contributions, given that Agent \(-i\) exerts \(e_{-i}\), i.e., \(I_i(\tau_i(e_{-i}), e_i, \gamma_i, f_i) = k_i e_i / e_{-i}\) for \(i = 1, 2\).

Compared with the four-region structure in income and utility fairness, the equilibrium efforts take a three-region structure, which corresponds to regions A-C in Lemmas 1 and 2. As discussed in §3.3, and similar to the case of utility fairness, we focus on the contracts with no fixed fees. When \(f_1 = f_2 = 0\), the condition for Agent \(i\) to suffer from guilt simplifies to \(\gamma_i P > k_i e_i (\gamma_1 + \gamma_2)\), or equivalently, to \(\gamma_i e_i / (1 - k_i e_i) > k_i e_i / (1 - k_i e_i)\); and conversely for the case of envy.

**Lemma 4.** Under reward fairness, there exist thresholds \(\bar{\eta}_{AB} > \eta_{AB} > \eta_{BC}\) such that, given an output-sharing contract \(\Phi = (\gamma_1, \gamma_2, 0, 0)\),

A. If \(\eta_{AB} \leq \gamma_2 / \gamma_1 \leq \bar{\eta}_{AB}\), there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: \(e_1^* = e_1^\beta, e_2^* = e_2^\alpha\);

B. If \(\eta_{BC} \leq \gamma_2 / \gamma_1 \leq \eta_{AB}\) or \(\gamma_2 / \gamma_1 \geq \bar{\eta}_{AB}\), there exists a continuum of pure-strategy Nash equilibria such that \(\frac{e_1^*}{k_1 e_1^2} = 2n_2 \gamma_2: e_2^* \in \left[\max \left\{e_2^\alpha, \bar{e}_2 \left(e_1^\beta\right)\right\}, e_2^\delta\right]\) and \(e_1^* = \bar{e}_1 \left(e_2^*\right)\);

C. If \(\gamma_2 / \gamma_1 \leq \eta_{BC}\), there exists a continuum of pure-strategy Nash equilibria such that \(\frac{e_1^*}{k_1 e_1^2} = 2n_2 \gamma_2: e_1^* \in \left[\max \left\{e_1^\beta, \bar{e}_1 \left(e_2^\alpha\right)\right\}, e_1^\delta\right]\) and \(e_2^* = \bar{e}_2 \left(e_1^*\right)\).

The structure of equilibrium efforts is similar to that of Lemma 2, with a few notable exceptions. First, Case D, in which Agent 1 experiences guilt and Agent 2 experiences envy in equilibrium, does not arise. This case, which arose under utility fairness when \(\gamma_2 / \gamma_1\) is very small, is in fact subsumed, under reward fairness, into Case C, which allows Agent 2 to achieve reward fairness after Agent 1 puts in a “base” effort. Second, Case A, in which Agent 1 experiences envy and Agent 2 experiences guilt in equilibrium, arises only over a
bounded range of values for $\gamma_2/\gamma_1$ under reward fairness, whereas it arises for arbitrarily large values of $\gamma_2/\gamma_1$ under utility fairness. As a result of the truncation of the region of applicability of Case A, Case B arises on two disconnected ranges of values of $\gamma_2/\gamma_1$, namely, when $\eta_{BC} \leq \gamma_2/\gamma_1 \leq \eta_{AB}$ or $\gamma_2/\gamma_1 \geq \eta_{AB}$, for some thresholds $\eta_{AB} > \eta_{AB} > \eta_{BC}$, unlike the equilibrium efforts under utility fairness, in which this case only arose in one range (namely, when $\theta_{BC} \leq \gamma_2/\gamma_1 \leq \theta_{AB}$, for some thresholds $\theta_{AB} > \theta_{BC}$). These differences are because reward fairness is easier to achieve than utility fairness. Specifically, even when Agent $i$ receives a very low share, i.e., when $\gamma_i \ll \gamma_{-i}$, Agent $i$ finds it feasible to avoid experiencing envy and achieve reward fairness, i.e., to have $\gamma_i/\gamma_{-i} = k_i e_i/(k_{-i} e_{-i})$, by exerting a very low effort, even though it may be impossible to achieve utility fairness, i.e., to have $(\gamma_{-i} - \gamma_i)(e_1 + ke_2) = e_2 - e_1$, at any effort $e_i$. In fact, it is so easy for Agent 1 to achieve reward fairness given their higher ability that Agent 1 can completely avoid experiencing guilt.

As a result, the equilibrium is of two types. Either Agent 1 experiences envy and Agent 2 experiences guilt (Case A). Or there is no inequality in equilibrium given that one agent can respond to the other agent’s “base” effort by adjusting their effort accordingly and ensuring reward fairness (Cases B and C).

As with utility fairness, the agents’ efforts are strategic complements in Cases B and C. Accordingly, the total output varies along the continuum of equilibria in these cases. The Pareto-optimal equilibrium is similar to that under utility fairness and thus mirrors Lemma 3. Under the Pareto-optimal equilibrium, not only does the principal benefit from higher payoff, but agents have (strictly) higher utilities.

**Lemma 5.** When a continuum of Nash equilibria exists in Lemma 4, the equilibrium with maximum effort from both agents is Pareto-optimal.

### 6.2. Optimal Contract

We next characterize the principal’s optimal contract. Like the utility-fairness case, the contract depends on the level of guilt $\alpha$; moreover, both agents always receive an output share, as illustrated in Figure 6. To simplify the exposition, define $\tilde{\gamma} := \frac{\alpha}{\sqrt{4\alpha^2 k^2 + (1-k^2)^2 - 1 + 2\alpha + k^2}}$.

**Proposition 4.** Under reward fairness, the principal’s best output-sharing contract is to offer fixed fees $f_1^* = f_2^* = 0$ and output shares

$$ (\gamma_1^*, \gamma_2^*) = \left( \tilde{\gamma}, \frac{1}{2} - \tilde{\gamma} \right). $$
Similar to Proposition 3, the principal always gives half the output to the two agents. Moreover, the optimal contract under reward fairness depends on \( \alpha \) but not \( \beta \) because guilt similarly motivates effort.

**Figure 6** Optimal contract under reward fairness.

![Graph of optimal contract under reward fairness](image)

*Note.* A teal background indicates a fair equilibrium outcome.

**Figure 7** Optimal contract and relative income under reward fairness.

![Graph of optimal contract and relative income under reward fairness](image)

*Note.* Here, \( \alpha = 0.25 \).

Like utility fairness, greater agent heterogeneity, i.e., a lower value of \( k \), brings greater disparity in output shares; see the left panel of Figure 7. However, unlike utility fairness, the optimal contract becomes less asymmetric when \( \alpha \) increases. This happens because, contrary to utility fairness, \( \eta_{BC} \) increases in \( \alpha \) (as opposed to \( \theta_{BC} \) in Lemma 2, which decreases in \( \alpha \)). As a result, when \( \alpha \) increases, Region B (resp. Region C) characterized in Lemma 4 (resp. Lemma 2) shrinks. The equilibrium efforts are thus most likely to fall
into Case C when \( \alpha \) increases, shifting the role of the agent that chooses the “base” effort (the bottleneck) from Agent 1 to Agent 2. As a result, when \( \alpha \) increases, it makes sense to allocate a greater output share to Agent 2 to induce them to achieve a higher output (and thus, a higher payoff for the principal).

Similar to utility fairness, reward fairness helps mitigate team moral hazard. This is because agents are more likely to put in effort when they see the other agent doing the same (due to the supermodular nature of the game), ensuring that their incomes align with their contributions. Moreover, the optimal contract involves both agents in production, i.e., is inclusive, and results in no inequalities in equilibrium, i.e., is perceived to be fair.

The next theorem summarizes this discussion.

**Theorem 3.** Under reward fairness, pay transparency leads to a higher payoff for the principal, is perceived to be fair (i.e., engendering no reward inequalities), and results in an inclusive outcome (i.e., involving both agents in production, and not only Agent 1).

In contrast to utility fairness, under which pay transparency may raise fairness issues without increasing the principal’s payoff when agents are very heterogeneous (namely, when \( k \leq \kappa^U(\alpha) \), see Proposition 3), pay transparency (which stimulates inequality aversion) is always beneficial for the principal under reward fairness while being perceived to be fair and leading to an inclusive outcome (under the optimal contract).

### 7. Conclusion

Fairness concerns often arise in the management of heterogeneous teams: although overusing the abler agents seems more efficient, such uneven involvement might demotivate other agents whenever they are inequality-averse. In homogeneous teams, pay transparency results in a fair and inclusive outcome for the agents, and it benefits the principal under any form of fairness concerns: income, utility, and reward.

However, this result is not robust when we consider heterogeneous teams. In fact, pay transparency backfires when agents are concerned about inequalities in income. In this case, the principal earns less payoff, the contract may be perceived to be unfair if the agents are highly heterogeneous, resulting in feelings of guilt and envy in equilibrium, and it may not be inclusive given that it might involve only the ablest agent in production. It is only when agents care about inequalities in reward or when they care about inequalities in utility and they are only moderately heterogeneous, that pay transparency can result...
in a higher payoff for the principal, while being considered fair by the agents and leading to an inclusive outcome.

In practice, fairness concerns can be shaped by the information that is provided. Sharing only income information will undoubtedly anchor workers on income fairness. But contextualizing this income information with the agents’ relative contributions or costs of effort (if somewhat measurable) could help shift the nature of the comparison toward reward or utility fairness. Pay transparency is far from being a binary decision, its contextualization needs to be engineered to help frame the nature of comparison and drive the organization to becoming a productive, fair, and inclusive environment. If fairness is indeed in the eye of the beholder, it is crucial for principals to fully understand the organizational culture of their teams and contextualization their reporting information accordingly before jumping on the bandwagon of greater pay transparency—or being mandated by law to do so.

Our stylized model can be extended in multiple ways. First, one could study fairness concerns in larger teams, with more than two agents. Second, we consider an additive production function to isolate the effect of fairness concerns, but in many settings, team output could be nonlinear and involve complementarities (coproduction). While these two extensions can be easily accommodated in the context of homogeneous teams (Gill and Stone 2015), their treatment in heterogeneous teams quickly becomes intractable. Based on preliminary numerical evidence, we conjecture that the effect of inequality aversion to mitigate team moral hazard would persist with nonlinear production functions. Third, one could apply the model to explore the impact of fairness concerns on other operational aspects, for example, on the agents’ willingness to collaborate or help each other (Siemsen et al. 2007) and how to best manage them. Fourth, we assumed that pay transparency would induce inequality aversion, operationalized as guilt and envy (Fehr and Schmidt 1999), but agents could adopt other, non-fair behaviors, such as seeking to accentuate inequalities if they are competitive.

In contrast to some popular posts that have unequivocally advocated for greater pay transparency to reduce inequalities in the workplace,⁵ our study identifies important boundary conditions, namely, the degree of team heterogeneity and the type of fairness consideration. Although pay transparency certainly offers many benefits to close the gender

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⁵https://www.linkedin.com/pulse/impact-pay-transparency-business-david-weaver-cca-v0xce/
pay gap or make salary negotiations less political, it could also lead to lower productivity if workers structure their comparisons on incomes and not on rewards. We hope that our results will invite managers to reflect on the potential downsides of social comparison and contextualize their reporting information before committing to greater pay transparency.

References
Deutsch M (1975) Equity, equality, and need: What determines which value will be used as the basis of distributive justice? *J Soc Issues* 31(3):137–149.


Chen, Désir, and Roels: Pay Transparency in Heterogeneous Teams
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Appendix

A. Fairness Concern Types in the Literature

<table>
<thead>
<tr>
<th>Table A.1 Fairness concern types in the literature.</th>
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<tbody>
<tr>
<td>Income Inequality</td>
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<td>Individual Outputs</td>
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<td>Fehr and Schmidt (1999)</td>
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<td>Team Outputs</td>
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<td>Bartling and von Siemens (2010a)</td>
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<td>Gill and Stone (2015)</td>
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<td>Avcı et al. (2014)</td>
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An “X” in merged cells of “income inequality” and “utility inequality” refers to settings with no costs of effort, which make utility and income equal.

B. Homogeneous Teams

When agents are homogeneous \((k=1)\), pay transparency leads to a higher payoff for the principal and a fair and inclusive equilibrium outcome for the agents, irrespective of whether the object of reference is their incomes (Hellmann and Wasserman 2017), utility, or reward (Gill and Stone 2015). Although the result is well established, the next theorem formalizes it, building on Theorems 1-3 given that it is a special case.

**Theorem B.1.** Under all dimensions of fairness, when agents are homogeneous \((k=1)\), pay transparency leads to a (weakly) higher payoff for the principal, is perceived to be fair (i.e., engendering no inequalities), and results in an inclusive outcome (i.e., involving both agents in production, and not only Agent 1).

This is because the principal’s contracting strategy—an equal-sharing rule—is fair on all dimensions: agents’ incomes are equal, their utilities are equal, and their rewards are proportional to their efforts. Hence, agents have in equilibrium no “mental burden” associated with any dimension of inequality aversion, despite their sensitivity to it.
Electronic Companion: Proofs and Supplementary Results

In the electronic companion, we prove results concerning inequality-averse agents who compare their incomes (§EC.1), their utilities (§EC.2), or their incomes relative to their contributions (rewards) (§EC.3). We then derive the case with homogeneous agents (Theorem B.1) as a corollary to these other results (§EC.4).

EC.1. Income Fairness

Proof of Lemma 1. We analyze first the best responses and then the Nash equilibrium in the effort choice game. We present the analysis for \( \gamma_1 \geq \gamma_2 \) only; the analysis for contracts such that \( \gamma_1 < \gamma_2 \) follows a similar argument, which is omitted for brevity.

Under income fairness, agents' utilities are given by (1). The agents' best responses can be expressed as the following piecewise continuous functions:

\[
e_1^\beta(e_2) = \begin{cases} 
  e_1^\beta, & e_2 \leq \frac{1}{k} \left( -\gamma_1 (1 + \beta) + \gamma_2 \beta + \frac{f_1}{\gamma_1 - \gamma_2} \right), \\
  e_1^\alpha(e_2), & e_2 \leq \frac{1}{k} \left( -\gamma_1 (1 + \beta) + \gamma_2 \beta + \frac{f_1}{\gamma_1 - \gamma_2} \right) \leq e_2 \leq \frac{1}{k} \left( -(1 - \alpha) \gamma_1 - \alpha \gamma_2 + \frac{f_1}{\gamma_1 - \gamma_2} \right), \\
  e_1^\gamma, & e_2 \geq \frac{1}{k} \left( -(1 - \alpha) \gamma_1 - \alpha \gamma_2 + \frac{f_1}{\gamma_1 - \gamma_2} \right), \\
\end{cases}
\]

where \( e_1^\alpha, e_1^\beta, \) and \( \tau_i(e_{-i}) \), for \( i = 1, 2 \), are defined in (6). For both agents, the first and last segments of the best-response function are constant with respect to the other agent’s effort. The middle segments, which are linearly decreasing, are inverse functions of each other.

It turns out that the best-response functions intersect or overlap at least at one point. Specifically, the equilibrium efforts can take one of four forms, depending on the contract \( \Phi = (\gamma_1, \gamma_2, f_1, f_2) \). For notational simplicity,

(A) When \( f_1 - f_2 \leq \zeta_{AB}(\gamma_1, \gamma_2) \), \( (e_1^\alpha, e_2^\alpha) = (e_1^\beta, e_2^\beta) \);

(B) When \( \zeta_{AB}(\gamma_1, \gamma_2) \leq f_1 - f_2 \leq \zeta_{BC}(\gamma_1, \gamma_2) \), \( (e_1^\gamma, e_2^\gamma) \) satisfies \( e_2^\gamma \in \left[ e_2^\beta, \max \{ e_2^\beta, \tau_2(e_1^\gamma) \} \right] \) and \( e_1^\gamma = \tau_1(e_2^\gamma) \);

(C) When \( \zeta_{BC}(\gamma_1, \gamma_2) \leq f_1 - f_2 \leq \zeta_{CD}(\gamma_1, \gamma_2) \), \( (e_1^\alpha, e_2^\alpha) \) satisfies \( e_1^\alpha \in \left[ \max \{ e_1^\alpha, \tau_1(e_2^\beta) \} \right] \) and \( e_2^\alpha = \tau_2(e_1^\alpha) \);

(D) When \( f_1 - f_2 \geq \zeta_{CD}(\gamma_1, \gamma_2) \), \( (e_1^\alpha, e_2^\alpha) = (e_1^\alpha, e_2^\alpha) \).

where

\[
\zeta_{AB}(\gamma_1, \gamma_2) := (\gamma_1 - \gamma_2) (\gamma_1 (1 + \beta k^2 - 1) - \gamma_2 (\alpha + (\beta + 1) k^2)), \\
\zeta_{BC}(\gamma_1, \gamma_2) := \begin{cases} 
  (\gamma_1 - \gamma_2) (\gamma_1 (1 + \beta k^2 - 1) - \gamma_2 (\alpha + (\beta + 1) k^2)), & \text{if } \gamma_2 \geq \beta/(1 + \beta) \gamma_1, \\
  (\gamma_1 - \gamma_2) (\gamma_1 (\alpha - 1) - \gamma_2 \alpha), & \text{if } \gamma_2 \leq \beta/(1 + \beta) \gamma_1, 
\end{cases}
\]

and the expression of \( \zeta_{BC} \) is omitted for brevity, as the second and third cases will be jointly discussed in the analysis of the optimal contract.

Proof of Proposition 2. We identify the optimal contract by finding first the optimal fixed fees and then the optimal output shares within Regions A-D identified in Lemma 1:

\[
\max_{R \in \{A, B, C, D\}} \max_{(\gamma_1, \gamma_2) \in (\gamma_1, \gamma_2, f_1, f_2) \in \text{Region}\ R} \max_{\gamma_i \geq 0, \ g_i + \gamma_2 \leq 1, \ \gamma_i \geq 0, \ i = 1, 2} (1 - \gamma_1 - \gamma_2)(e_1^\gamma + ke_2^\gamma) - f_1 - f_2 \tag{EC.2}
\]
In Region A, given that $\zeta_{AB}(\gamma_1, \gamma_2) \leq 0$ for all $\alpha \leq 1/2$ and $\beta \leq 1$ and that the equilibrium efforts are independent of $f_1$ and $f_2$, the objective function in (EC.2) is linear in $f_1$ and $f_2$. Because $f_1 - f_2 \leq \zeta_{AB}$, the optimal fixed fees are $(f_1, f_2) = (0, -\zeta_{AB})$. The principal’s payoff is correspondingly $(1 - 2\gamma_1)(\gamma_1(\beta + \alpha k^2 + 1) - \gamma_2(\beta + (\alpha - 1)k^2))$. Optimizing this payoff subject to $\gamma_1 \geq 0$, $\gamma_2 \geq 0$ and $\gamma_1 + \gamma_2 \leq 1$, we obtain the following optimal shares:

- If $k \geq \frac{\beta}{1 - \alpha}$, $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(0, \frac{1}{4}, 1, 0\right)$, resulting in payoff $\pi^* = \frac{1 + k^2}{8}$.
- If $k < \frac{\beta}{1 - \alpha}$, $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(0, 0, 0, 1 + \frac{\beta + \alpha k^2}{16}\right)$, resulting in payoff $\pi^* = \frac{1 + \beta + \alpha k^2}{8}$.

In Regions B and C, the objective function in (EC.2), after substituting the equilibrium efforts, simplifies to

$$\frac{f_1}{\gamma_1 - \gamma_2}.$$

It is straightforward to verify that the optimal contract is identical to that in Region A.

In Region D,

- If $\gamma_2 \geq \beta / (1 + \beta) \gamma_1$, the objective function simplifies to $(2\gamma_1 - 1)(\gamma_1(\alpha + \beta k^2 - 1) - \gamma_2(\alpha + (\beta + 1)k^2))$. The optimal contract is $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(\frac{1}{4}, 0, 0, 0\right)$, leading to payoff $\pi^* = \frac{1 + k^2}{8}$.

- If $\gamma_2 < \beta / (1 + \beta) \gamma_1$, the objective function simplifies to $(1 - \gamma_1 - \gamma_2)((1 - \alpha)\gamma_1 - \alpha \gamma_2)$, which is maximized when $(\gamma_1^*, \gamma_2^*, f_1^*, f_2^*) = \left(0, 0, 0, 0\right)$. The payoff is $\pi^* = \frac{1 - \alpha}{4}$.

Selecting the maximum principal’s payoff across all regions yields (7).

**Proof of Theorem 1.** By Proposition 2, the principal’s payoff is correspondingly

$$\pi^* = \begin{cases} \frac{1 + k^2}{8}, & \max \left\{ \sqrt{1 - 2\alpha}, \sqrt{\frac{\beta}{1 - \alpha}} \right\} < k, \\ \frac{1 + k^2 + \alpha k^2}{8}, & \max \left\{ \sqrt{1 - 2\alpha - \beta}, \frac{\beta}{1 - \alpha} \right\} \leq k \leq \sqrt{\frac{\beta}{1 - \alpha}}, \\ \frac{1 - \alpha}{4}, & k \leq \min \left\{ \sqrt{1 - 2\alpha}, \sqrt{\frac{1 - 2\alpha - \beta}{\alpha}} \right\}. \end{cases}$$

Because $\pi^N = 1/4$ when agents are inequality-neutral, we obtain that $\pi^* \leq \pi^N$ for any $\alpha$, $\beta$, $k$. The rest of the theorem follows directly from Proposition 2.

**EC.2. Utility Fairness.**

We first characterize the equilibrium efforts given an affine output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$. Lemma 2 is a special case when $f_1 = f_2 = 0$.

**Lemma EC.1.** Under utility fairness, there exist thresholds $\Theta_{AB}(\gamma_1, \gamma_2) \leq \Theta_{BC}(\gamma_1, \gamma_2) \leq \Theta_{CD}(\gamma_1, \gamma_2)$ such that given any output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$,

(A) If $f_1 - f_2 \leq \Theta_{AB}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences envy and Agent 2 experiences guilt: $e_1^* = e_1^\beta$, $e_2^* = e_2^\alpha$;

(B) If $\Theta_{AB} \leq f_1 - f_2 \leq \Theta_{BC}$, there exists a continuum of pure-strategy Nash equilibria such that $v_1 = v_2$:

$$e_2^* \in \left\{ e_2^\beta, \tau_2(e_1^\alpha) \right\}, e_2^* = \tau_1(e_2^\beta);$$

(C) If $\Theta_{BC} \leq f_1 - f_2 \leq \Theta_{CD}$, there exists a continuum of pure-strategy Nash equilibria such that $v_1 = v_2$:

$$e_1^* \in \left\{ e_1^\alpha, \tau_1(e_2^\beta) \right\}, e_1^* = \tau_2(e_1^\alpha);$$

(D) If $f_1 - f_2 \geq \Theta_{CD}$, there is a unique pure-strategy Nash equilibrium such that Agent 1 experiences guilt and Agent 2 experiences envy: $e_1^* = e_1^\alpha$, $e_2^* = e_2^\beta$. 
Proof. The proof is organized as follows. We first simplify the agents’ utilities (2); then, we analyze the agents’ best responses solving (4); finally, we identify the Nash equilibrium.

Utilities. First, consider Agent 1. In (2), the equation $v_2 - v_1 = (\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_1^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2) = 0$ (convex in $e_1$) has two roots in $e_1$, denoted as $r_-$ and $r_+$ with $r_- \leq r_+$:

$$r_{+, -} = \gamma_1 - \gamma_2 \pm \sqrt{(\gamma_1 - \gamma_2 + e_2)^2 - 2(1-k)e_2(\gamma_1 - \gamma_2) + 2(f_1 - f_2)}.$$ 

Thus,

$$u_1(e_1; e_2, \gamma_1, \gamma_2, f_1, f_2) = \begin{cases} 
\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2 - \beta((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2)), & e_1 \leq r_- , \\
\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2 - \alpha((\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2) - (\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2)), & r_- \leq e_1 \leq r_+ , \\
\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2 - \beta((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2)), & e_1 \geq r_+. 
\end{cases}$$

Since the first branch $\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2 - \beta((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2))$ (concave in $e_1$) is maximized at $\frac{\gamma_2 + \gamma_1 - 2\gamma_2^2}{1 + \beta}$, which is greater than $r_-$, any effort level $e_1 < r_-$ is strictly dominated. Furthermore, the second branch $\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2 - \alpha((\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2) - (\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2))$ is maximized at $\frac{\gamma_1(1-\alpha)^{1/\gamma_2}}{1-\alpha}$, which is also greater than $r_-$. Thus, the following utility function leads to the same best responses as the original utility function for Agent 1.

$$u_1(e_1; e_2, \gamma_1, \gamma_2, f_1, f_2) = \begin{cases} 
\bar{u}_1 := \gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2 - \alpha((\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2) - (\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2)), & e_1 \leq \bar{e}_1(e_2), \\
\tilde{u}_1 := \gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2 - \beta((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2)), & e_1 \geq \tilde{e}_1(e_2), 
\end{cases}$$

where $\bar{e}_1(e_2) := r_+$, which is the only value of $e_1$ such that $v_1 - v_2 = 0$ and $e_1 \geq (r_+ + r_-)/2 = (\gamma_1 - \gamma_2)$.

Similarly, Agent 2’s utility can be simplified, without loss of optimality, to:

$$u_2(e_1; e_2, \gamma_1, \gamma_2, f_1, f_2) = \begin{cases} 
\bar{u}_2 := \gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2 - \alpha((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2)), & e_2 \leq \bar{e}_2(e_1), \\
\tilde{u}_2 := \gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2 - \beta((\gamma_2(e_1 + ke_2) + f_2 - \frac{1}{2}e_2^2) - (\gamma_1(e_1 + ke_2) + f_1 - \frac{1}{2}e_1^2)), & e_2 \geq \tilde{e}_2(e_1), 
\end{cases}$$

where $\bar{e}_2(e_1)$ is the only value of $e_2$ such that $v_1 - v_2 = 0$ and $e_2 \geq k(\gamma_2 - \gamma_1)$. Moreover, $\bar{e}_2(e_1)$ and $\tilde{e}_2(e_1)$ are inverse functions to each other.

Best responses. Given a contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, agents choose their efforts to maximize their utility by solving (4). Define the following constants such that their counterparts, i.e., $t_{ij}, t'_{ij}, \tilde{t}_{ij}$, and $\tilde{t}_{ij}$ for $j = 1, 2$ are defined by symmetry.

$$t_{21} := -k(\gamma_1 - \gamma_2) + \sqrt{\frac{\gamma_2}{1+\alpha} - 2(f_1 - f_2) - (1-k^2)(\gamma_1 - \gamma_2)^2};$$
$$t'_{21} := -k(\gamma_1 - \gamma_2) - \frac{\gamma_2}{1+\alpha} - 2(f_1 - f_2) - (1-k^2)(\gamma_1 - \gamma_2)^2;$$
$$t_{22} := -k(\gamma_1 - \gamma_2) + \sqrt{\frac{\gamma_2}{1+\alpha} - 2(f_1 - f_2) - (1-k^2)(\gamma_1 - \gamma_2)^2};$$
$$t'_{22} := -k(\gamma_1 - \gamma_2) - \frac{\gamma_2}{1+\alpha} - 2(f_1 - f_2) - (1-k^2)(\gamma_1 - \gamma_2)^2;$$
$$\tilde{t}_{21} := -k(\gamma_1 - \gamma_2) + \sqrt{-2(f_1 - f_2) + k^2(\gamma_1 - \gamma_2)^2};$$
$$\tilde{t}_{21} := -k(\gamma_1 - \gamma_2) - \sqrt{-2(f_1 - f_2) + k^2(\gamma_1 - \gamma_2)^2};$$
here, the first digit $i$ in the double subscript refers to Agent $i$; the second digit refers to the order of absolute difference from $k(\gamma_2 - \gamma_1)$, i.e., $t_{i_1} \leq t_{i_2}$ for $i = 1, 2$; the group of values denoted by an apostrophe is smaller than the group without it: $t'_{i_2} \leq t'_{i_1} \leq t_{i_1} \leq t_{i_2}$; and the $\tilde{t}_{i_j}$ refers to an alternative threshold instead of $t_{i_j}$ in some boundary cases.

Because Agent 1’s simplified utility function (EC.4) is piece-wise quadratic and $\partial u_1^\alpha/\partial e_1 \big|_{e_1 = \pi_1(e_2)} < \partial u_1^\beta/\partial e_1 \big|_{e_1 = \pi_1(e_2)}$, Agent 1’s maximum utility is attained either at the optima of these quadratic functions or at the breakpoint. We define the values of $e_1$ maximizing $u_1^\alpha$ and $u_1^\beta$ by the following:

$$
e_i^\alpha := \gamma_1 + \frac{\alpha}{1 - \alpha} \gamma_2, \quad e_i^\beta := \gamma_1 - \frac{\beta}{1 + \beta} \gamma_2. \tag{EC.5}$$

Unlike (8), $e_i^\alpha$ and $e_i^\beta$ defined by (EC.5) are not restricted to be nonnegative. Agent 1’s best responses can be summarized into three cases, depending on, first, the value of $\gamma_1/\gamma_2$, and then, the slopes of $u_1^\alpha$ and $u_1^\beta$ at the threshold $e_i = \pi_1(e_2)$:

- **If** $\gamma_1/\gamma_2 \geq \beta/(1 + \beta)$,
  - **When** $t_{11} < e_2 < t_{21}$, we have $\frac{\partial u_1^\alpha}{\partial e_1} \big|_{e_1 = \pi_1(e_2)} > \frac{\partial u_1^\beta}{\partial e_1} \big|_{e_1 = \pi_1(e_2)} > 0$. Thus, Agent 1’s utility is maximized when $e_1 > \pi_1(e_2)$, i.e., on $u^\alpha(e_1)$. Solving the first-order optimality condition, the optimal effort level is $e_i^\alpha$.
  - **When** $t_{22} < e_2 < t_{21} < t_{12} < e_2 < t_{22}$, we have $\frac{\partial u_1^\alpha}{\partial e_1} \big|_{e_1 = \pi_1(e_2)} < 0 < \frac{\partial u_1^\beta}{\partial e_1} \big|_{e_1 = \pi_1(e_2)}$. Correspondingly, Agent 1’s utility is maximized at the threshold $\bar{\pi}_2(e_1)$.
  - **When** $e_2 < t'_{22}$ or $e_2 > t_{22}$, we have $\frac{\partial u_1^\alpha}{\partial e_1} \big|_{e_1 = \pi_1(e_2)} < \frac{\partial u_1^\beta}{\partial e_1} \big|_{e_1 = \pi_1(e_2)} < 0$. Thus, Agent 1’s utility is maximized when $e_1 < \pi_1(e_2)$, i.e., on $u^\alpha(e_1)$. Solving the first-order optimality condition, the optimal effort level is $e_i^\alpha$.

- **If** $\gamma_1/\gamma_2 \leq \beta/(1 + \beta)$, $e_i^\beta \leq 0$; therefore, $u^\beta(e_1)$ is decreasing for all $e_1 \geq 0$. Considering that efforts must be non-negative, Agent 1’s best-response functions have the same structure as the best-responses when $\gamma_1/\gamma_2 \geq \beta/(1 + \beta)$:
  - **When** $t_{21} < e_2 < \tilde{t}_{21}$, we have $\bar{\pi}_1(e_2) < 0$. Therefore, Agent 1’s utility is monotonically decreasing for any $e_1 \geq 0$, and the optimal effort level is 0.
  - **When** $t_{22} < e_2 < \tilde{t}_{21} < t_{22}$, we have $\bar{\pi}_1(e_2) > 0$ and $\frac{\partial u_1^\alpha}{\partial e_1} \big|_{e_1 = \pi_1(e_2)} > 0 > \frac{\partial u_1^\beta}{\partial e_1} \big|_{e_1 = \pi_1(e_2)}$. Correspondingly, Agent 1’s utility is maximized at the threshold $\bar{\pi}_2(e_1)$.

  - When $e_2 < t'_{22}$ or $e_2 > t_{22}$, Agent 1’s utility is maximized at $e_i^\alpha$.

In summary,

$$
e_1^\alpha(e_2) = \begin{cases} e^\alpha_1, & t_{21} \leq e_2 \leq t_{22}, \\ \pi_1(e_2), & t_{22} \leq e_2 \leq t_{21} \text{ or } t_{21} \leq e_2 \leq t_{22}, \\ e^\alpha_1, & e_2 \leq t'_{22} \text{ or } e_2 \geq t_{22}, \end{cases} \quad e_1^\beta(e_2) = \begin{cases} 0, & t_{21} \leq e_2 \leq \tilde{t}_{21}, \\ \pi_1(e_2), & t_{22} \leq e_2 \leq \tilde{t}_{21} \text{ or } \tilde{t}_{21} \leq e_2 \leq t_{22}, \\ e^\beta_1, & e_2 \leq t'_{22} \text{ or } e_2 \geq t_{22}. \end{cases}$$

Similarly, Agent 2’s best response is

$$
e_2^\alpha(e_1) = \begin{cases} e^\alpha_2, & t_{11} \leq e_1 \leq t_{12}, \\ \pi_2(e_1), & t_{12} \leq e_1 \leq t_{11} \text{ or } t_{11} \leq e_1 \leq t_{12}, \\ e^\alpha_2, & e_1 \leq t'_{12} \text{ or } e_1 \geq t_{12}, \end{cases} \quad e_2^\beta(e_1) = \begin{cases} 0, & \tilde{t}_{11} \leq e_1 \leq \tilde{t}_{11}, \\ \pi_2(e_1), & t_{12} \leq e_1 \leq \tilde{t}_{11} \text{ or } \tilde{t}_{11} \leq e_1 \leq t_{12}, \\ e^\beta_2, & e_1 \leq t'_{12} \text{ or } e_1 \geq t_{12}. \end{cases}$$
Equilibrium. From the best-response functions, we obtain that Agent $i$’s best response lies between $\max\{e_i^\beta, 0\}$ and $e_i^\alpha$. That is, any effort level $e_i < \max\{e_i^\beta, 0\}$, for $i = 1, 2$, is strictly dominated. Additionally, it is easy to see that: 1) $t_{12} < t_{11} < e_1^\beta$ and $t_{22} < t_{21} < e_2^\beta$; 2) when $\gamma_1/\gamma_2 \leq \beta/(1 + \beta)$, $t_{1i} \leq 0$; 3) when $\gamma_1/\gamma_2 > \beta/(1 + \beta)$. Consequently, to identify the equilibrium, we only need to consider the best responses to non-dominated strategies listed below.

- If $\beta/(1 + \beta) \leq \gamma_1/\gamma_2 \leq (1 + \beta)/\beta$, 
  
  
  \[
  e_i^\ast (e_2) = \begin{cases}
  e_1^\beta, & e_2^\beta \leq e_2 \leq t_{12}, \\
  e_1^\alpha, & e_2 \geq t_{12},
  \end{cases} \quad \begin{cases}
  e_2^\beta, & e_1^\beta \leq e_1 \leq t_{11}, \\
  e_2^\alpha, & e_1 \geq t_{12}.
  \end{cases}
  \]

- If $\gamma_1/\gamma_2 \leq \beta/(1 + \beta)$,
  
  \[
  e_i^\ast (e_2) = \begin{cases}
  0, & e_2^\beta \leq e_2 \leq \tilde{t}_{12}, \\
  e_i^\alpha, & e_2 \geq \tilde{t}_{12},
  \end{cases} \quad \begin{cases}
  e_2^\beta, & e_1^\beta \leq e_1 \leq \tilde{t}_{11}, \\
  e_2^\alpha, & e_1 \geq \tilde{t}_{12}.
  \end{cases}
  \]

- If $\gamma_1/\gamma_2 \geq (1 + \beta)/\beta$,
  
  \[
  e_i^\ast (e_2) = \begin{cases}
  e_1^\beta, & 0 \leq e_2 \leq t_{21}, \\
  e_1^\alpha, & e_2 \geq t_{21},
  \end{cases} \quad \begin{cases}
  0, & e_1^\beta \leq e_1 \leq \tilde{t}_{11}, \\
  e_1^\alpha, & e_1 \geq \tilde{t}_{12}.
  \end{cases}
  \]

Combining the best responses, since $\bar{e}_1(\cdot)$ and $\bar{e}_2(\cdot)$ are inverse functions to each other, there are only three ways the best responses can intersect. In order to characterize these crossing points in terms of $f_1 - f_2$, define

\[
\Theta_{AB} := \frac{1}{2} \left( \frac{\gamma_1^2}{(1 + \beta)^2} - \frac{\beta}{1 - \beta} \right) - (1 - k^2) (\gamma_1 - \gamma_2)^2, \\
\Theta_{BC} := \frac{1}{2} \left( \frac{\gamma_2^2}{(1 + \alpha)^2} - \frac{\beta}{1 - \beta} \right) - (1 - k^2) (\gamma_1 - \gamma_2)^2, \\
\Theta_{CD} := -\frac{1}{2} \left( \frac{\gamma_2^2}{(1 + \alpha)^2} - \frac{\beta}{1 - \beta} \right), \\
\Theta_{AB} := \frac{1}{2} \left( (\gamma_1/\gamma_2)^2 - \frac{\beta}{1 - \beta} \right), \\
\tilde{\Theta}_{CD} := \frac{1}{2} \left( (\gamma_1 - \gamma_2)^2 - \frac{\beta}{1 - \beta} \right). \\
\]

such that $\Theta_{AB} \leq \Theta_{BC} \leq \Theta_{CD}$ and $\tilde{\Theta}_{AB} \leq \Theta_{BC} \leq \tilde{\Theta}_{CD}$. The effort equilibrium can be summarized as follows:

- When $\beta/(1 + \beta) \leq \gamma_1/\gamma_2 \leq (1 + \beta)/\beta$,
  - If $f_1 - f_2 \leq \Theta_{AB}$, then $e_2^\beta \leq t_{21}$ and $e_1^\beta \geq t_{12}$. The corresponding equilibrium efforts are $(e_1^\ast, e_2^\ast) = (e_1^\beta, e_2^\beta)$.
  - If $\Theta_{AB} \leq f_1 - f_2 \leq \Theta_{BC}$, then $e_2 \geq t_{21}$ and $e_1 \geq t_{11}$. There is a continuum of equilibrium efforts: $e_2^\ast \in [\max\{e_2^\beta, \bar{e}_2(e_2^\beta)\}, e_2^\alpha]$ and $e_1^\ast = \bar{e}_1(e_2^\beta)$;
  - If $\Theta_{BC} \leq f_1 - f_2 \leq \Theta_{CD}$. There is a continuum of equilibrium efforts: $e_1^\ast \in [\max\{e_1^\beta, \bar{e}_1(e_2^\beta)\}, e_1^\alpha]$ and $e_2^\ast = \bar{e}_2(e_1^\alpha)$;
  - If $f_1 - f_2 \geq \Theta_{CD}$, then the best-response functions cross only once at $(e_1^\ast, e_2^\ast) = (e_1^\beta, e_2^\beta)$.

- When $\gamma_1/\gamma_2 \leq \beta/(1 + \beta)$, the analysis is similar, the results above hold after replacing $e_1^\beta$ by 0 and $\Theta_{AB}$ by $\tilde{\Theta}_{AB}$.

- When $\gamma_1/\gamma_2 \leq (1 + \beta)/\beta$, the results in the first bullet point hold after replacing $e_2^\beta$ by 0 and $\Theta_{CD}$ by $\tilde{\Theta}_{CD}$. 

We re-define $e_i^\beta := \max \{ e_i^\beta, 0 \}$ and $e_i^\alpha := e_i^\alpha$ for $i = 1, 2$; and re-define
\[
\Theta_{AB} := \begin{cases} 
\Theta_{AB}, & \gamma_1/\gamma_2 \geq \beta/(1 + \beta), \\
\Theta_{AB}^\prime, & \gamma_1/\gamma_2 \leq \beta/(1 + \beta), \end{cases} \quad \Theta_{BC} := \Theta_{BC}, \quad \Theta_{CD} := \begin{cases} 
\Theta_{CD}, & \gamma_1/\gamma_2 \leq (1 + \beta)/\beta, \\
\Theta_{CD}^\prime, & \gamma_1/\gamma_2 \geq (1 + \beta)/\beta. \end{cases}
\]
Summarizing all three cases yields the statement of Lemma EC.1. □

**Proof of Lemma 2.** Define the following constants, where $\theta_{ij}$ is the positive root to $\Theta_{ij}$ (defined in (EC.6)) with respect to $\gamma_2/\gamma_1$ for $ij \in \{AB, BC, CD\}$.
\[
\theta_{AB} := \frac{(1 + \beta)(1 - \alpha(1 + \beta)(1 - k))^2 - \sqrt{1 - (1 - \alpha)(1 + \beta)(1 + k)^2(1 - k)^2(1 + \beta)^2}}{(1 - \alpha(1 + \beta)^2(1 - k)^2)}, \\
\theta_{BC} := \frac{-\alpha(1 + \beta)(1 - k^2) + (1 - \alpha)^2(1 + \beta)^2k^2 - (1 + 2\alpha - 4\alpha^2)k^2}{(1 - \alpha)(2k^2 - (\alpha - 2)\alpha)}, \quad \theta_{CD} := \frac{(1 - \beta)(\beta(1 - \gamma) + \gamma(1 - \beta)(1 + \gamma)k^2 + \sqrt{(1 + \gamma)^2 - k^2(\beta - 2\beta + (\gamma + 1)^2) + (\beta - 2\beta)^2k^4 + 1}}{(\gamma + 1)(2k^2 + (2 - \beta)\beta)}.
\]
With these constants, Lemma 2 follows directly from Lemma EC.1. □

**Proof of Lemma 3.** Since all points at $(e_1^*, e_2^*) = (\bar{e}_1(e_2^*), e_2^*)$ or $(e_1^*, \bar{e}_2(e_1^*))$ lead to equal nominal utilities for Agents 1 and 2, by (2), the sum of agents’ utilities is in equilibrium
\[
u_T := u_1 + u_2 = (\gamma_1 + \gamma_2)(e_1 + ke_2) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2.
\]
Since $u_T$ increases in $e_1$ for $e_1 \leq \gamma_1 + \gamma_2$ and $u_T$ increases in $e_2$ for $e_2 \leq k(\gamma_1 + \gamma_2)$, the sum of agents’ utilities—and therefore their individual utilities (given that $u_1 = u_2 = u_T/2$)—increases in $e_1$ and $e_2$ because, by Lemma 2, $e_1^* \leq e_1^* = \frac{\gamma_1(1 - \alpha) + 2\gamma_2}{1 - \alpha} \leq \gamma_1 + \gamma_2$ and $e_2^* \leq e_2^* = k(\gamma_1(1 - \alpha) + \gamma_2) \leq k(\gamma_1 + \gamma_2)$ when $\alpha \leq 1/2$. The principal’s payoff increases because it is half the equilibrium output. □

Before proving Proposition 3, we present auxiliary Lemmas EC.2-EC.4. By the proof of Lemma 2, we consider only action spaces $e_i \in [e_i^\beta, e_i^\alpha]$ for $i = 1, 2$. When $f_1 = f_2 = 0$, (2) simplifies to
\[
u_i(e_i; e_{-i}, \gamma_i, \gamma_{-i}, 0, 0) = \gamma_i(e_1 + ke_2) - ce_i^2 - \alpha(\gamma_i(e_1 + ke_2) - ce_i^2) - (\gamma_{-i}(e_1 + ke_2) - ce_{-i}^2)) + \left(1 \right) - \beta((\gamma_{-i}(e_1 + ke_2) - ce_{-i}^2) - (\gamma_i(e_1 + ke_2) - ce_i^2))^+,
\]
where $e_i \in [e_i^\beta, e_i^\alpha]$. To prove Proposition 3, we first show that for any optimal output-sharing contract, $f_1 = f_2 = 0$; and then find the optimal contract solving (5) with $f_1 = f_2 = 0$.

We show that $f_1 = f_2 = 0$ in all optimal output-sharing contracts in three steps. First, given a Nash equilibrium $(e_1^*, e_2^*)$ to the effort game (4) under contract $\Phi$, we propose in Lemma EC.2 a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ with no fixed fees that leads to both higher best effort responses and higher payoff at $(e_1^*, e_2^*)$. Second, we show in Lemma EC.3 that any effort game (4) with no fixed fees maximizing (EC.8) is supermodular. Finally, based on the supermodularity of the effort game under $\Phi'$ (Lemma EC.3) and the elevated best responses at $(e_1^*, e_2^*)$ (Lemma EC.2, point 2), we show in Lemma EC.4 that there exists an equilibrium $(e_1^0, e_2^0)$ such that $e_1^0 \geq e_1^*$ and $e_2^0 \geq e_2^*$, and thus $\pi(e_1^0, e_2^0, \Phi') \geq \pi(e_1^0, e_2^0, \Phi)$ (Lemma EC.2, point 1). As a result, we consider share-only contracts in the proof of Proposition 3.

**Lemma EC.2.** Under utility fairness, given any contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ and a Nash equilibrium $(e_1^*, e_2^*)$ to the effort game (4), there exists a contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ such that
1. $\Phi'$ leads to greater payoff when efforts are $(e_1', e_2')$, i.e., $\pi(e_1', e_2'; \Phi') \geq \pi(e_1', e_2'; \Phi)$.

2. Agent $i$’s best response to $e_{i-1}'$ under $\Phi'$, denoted by $B_i(e_{i-1}'; \Phi')$, is greater than $e_i'$, i.e., $B_i(e_{i-1}'; \Phi') \geq e_i'$, for $i = 1, 2$.

When either $f_1 > 0$ or $f_2 > 0$ and when $\alpha > 0$, at least one of the three inequalities (i.e., $\pi(e_1', e_2'; \Phi') \geq \pi(e_1', e_2', \Phi)$, and given that Agent 1’s utility function (EC.8) under $\Phi'$ and $B_i(e_{i-1}'; \Phi') \geq e_i'$ for $i = 1, 2$) are strict.

Proof. Given contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$, we construct the desired contract $\Phi' = (\gamma_1', \gamma_2', 0, 0)$ in three cases, depending on the relative values of $v_1$ and $v_2$ when efforts are $(e_1', e_2')$ under $\Phi$.

- Case 1: $v_1 < v_2$, i.e., $(e_1', e_2')$ is in the interior of $(u_1^0, u_2^0)$ under $\Phi$, where $(u_1^0, u_2^0)$ are defined by (EC.3)-(EC.4). Depending on the values of $f_1$ and $f_2$, we consider three subcases.
  a) When $\frac{1+\beta}{1+\beta} f_2 \geq f_1 \geq \frac{1+\beta}{1+\beta} f_2$, let

\[ (\gamma_1', \gamma_2') := \left(\frac{f_1}{e_1' + ke_2'}, \frac{f_2}{e_1' + ke_2'}\right). \]

Since the function values and utility gaps $(v_1 - v_2)$ at $(e_1', e_2')$ are identical in (2) or (EC.8), i.e., under $\Phi$ or $\Phi'$, by construction, $(e_1', e_2')$ also lies in $(u_1^0, u_2^0)$ under $\Phi'$. For Agent 1, given that

\[ \frac{du_1^0(e_1'; e_2', \Phi')}{de_1} = (1 + \beta)(\gamma_1' - 2ce_1) - \beta\gamma_2' \]

plugging in (2)

\[ \geq (1 + \beta)(\gamma_1' - 2ce_1) - \beta\gamma_2' \quad \text{by (EC.9) and } f_1 \geq \beta/(1 + \beta)f_2 \]

\[ = \frac{du_1^0(e_1'; e_2', \Phi)}{de_1} = 0 \quad \text{plugging in (EC.8)} \]

and given that Agent 1’s utility function (EC.8) under $\Phi'$ is quasi-concave and piece-wise quadratic by definition, it follows that $B_1(e_2'; \Phi') \geq e_1'$.

As for Agent 2, similarly, we have

\[ \frac{du_2^0(e_2'; e_1', \Phi')}{de_2} \Big|_{e_2 = e_2'} = (1 - \alpha)(k\gamma_2' - 2ce_2) + k\alpha\gamma_1' \]

plugging in (2)

\[ > (1 - \alpha)(k\gamma_2 - 2ce_2) + k\alpha\gamma_1 \quad \text{since } \alpha > 0 \text{ and } f_i > 0 \text{ for at least one } i \]

\[ = \frac{du_2^0(e_2'; e_1', \Phi)}{de_2} \Big|_{e_2 = e_2'} = 0 \quad \text{plugging in (EC.8)} \]

Because Agent 2’s utility function is quasi-concave and piece-wise quadratic, it follows that $B_2(e_1'; \Phi') \geq e_2'$.

Finally, by (EC.9), $\pi(e_1', e_2', \Phi') = \pi(e_1', e_2', \Phi)$.

b) When $f_1 < \frac{\alpha}{1+\beta} f_2$, let

\[ \gamma_1' := \frac{f_1}{e_1' + ke_2'} + \gamma_1; \quad \gamma_2' := \frac{1 + \beta}{\beta(\gamma_1' - \gamma_1)} + \gamma_2. \]

We consider two subsubcases depending on whether the contract $(\gamma_1', \gamma_2', 0, 0)$ changes the relative envy or guilt of the two agents under efforts $(e_1', e_2')$ compared to the given contract $\Phi$.

- If $\gamma_2'(e_1' + ke_2') - ce_2'^2 - \gamma_1'(e_1' + ke_2') + ce_1'^2 \geq 0$, let $\Phi' = (\gamma_1', \gamma_2', 0, 0)$. Then, $(e_1', e_2')$ is on $(u_1^0, u_2^0)$ under $\Phi'$ because the fairness gap $(v_1 - v_2)$ in (EC.8) is

\[ \gamma_2'(e_1' + ke_2') - ce_2'^2 - \gamma_1'(e_1' + ke_2') + ce_1'^2 \geq 0. \]

Following similar arguments to a), we have $B_1(e_2'; \Phi') \geq e_1', B_2(e_1'; \Phi') \geq e_2'$. By (EC.10) and $f_1 < \frac{\alpha}{1+\beta} f_2$, $\pi(e_1', e_2', \Phi') > \pi(e_1', e_2', \Phi)$.
If $\gamma_2'(e_1' + ke_2') - ce_1'^2 - \gamma_1'(e_1' + ke_2') + ce_1'^2 < 0$, consider the contract where $\gamma_1' > \gamma_1$ and $\gamma_2' \geq \gamma_2$ jointly solve the fairness condition $\gamma_2'(e_1' + ke_2') - ce_1'^2 - \gamma_1'(e_1' + ke_2') + ce_1'^2 + f_2 - f_1 = 0$ and the budget-balancing condition $(\gamma_1' - \gamma_1 - \gamma_2' - \gamma_2')(e_1' + ke_2') = f_1 + f_2$:

$$
\gamma_1' = \frac{\gamma_2(e_1' + ke_2') - ce_1^2 - \gamma_1'(e_1' + ke_2') + ce_1'^2 + f_1 + f_2 + \gamma_1}{2(e_1' + ke_2')}
$$

$$
\gamma_2' = \frac{\gamma_1(e_1' + ke_2') - ce_1'^2 - \gamma_2(e_1' + ke_2') + ce_1'^2 + f_1 + f_2 + \gamma_2}{2(e_1' + ke_2')}
$$

(11.12)

Given that $(e_1', e_2')$ is on the interior of $(u_1^0, u_2^0)$ in Case 1, by (2), the utility gap $(v_1 - v_2)$ equals

$$
\gamma_2(e_1' + ke_2') - ce_1'^2 - \gamma_1(e_1' + ke_2') + ce_1'^2 + f_2 - f_1 > 0,
$$

and given that by assumption

$$
0 > \gamma_2'(e_1' + ke_2') - ce_1'^2 - \gamma_1'(e_1' + ke_2') + ce_1'^2
$$

by case assumption

$$
\geq \gamma_2(e_1' + ke_2') - ce_1'^2 - f_1 - \gamma_1(e_1' + ke_2') + ce_1'^2
$$

since $f_1 \geq 0$.

the existence and uniqueness of (11.11)-(11.12) with $\gamma_1' \geq \gamma_1$ and $\gamma_2' \geq \gamma_2$ are guaranteed.

Compared with $\Phi$, Agent 1’s best response under $\Phi$’ satisfies $B_1(e_2'; \Phi') \geq e_1'$ because $(e_1', e_2')$ is on the boundary of $u_1^\Phi(e_1'; e_2', \Phi')$ and $u_1^\Phi(e_1'; e_2', \Phi')$, and

$$
\frac{du_1(e_1'; e_2', \Phi')}{de_1} \bigg|_{e_1=e_1'} = \frac{du_1^\Phi(e_1; e_2', \Phi')}{de_1} \bigg|_{e_1=e_1'}
$$

by (11.8)

$$
= (1 - \alpha)(\gamma_1' - 2ce_1' + \alpha \gamma_2')
$$

by (11.8)

$$
> (1 - \alpha)(\gamma_1 - 2ce_1' + \alpha \gamma_2)
$$

since $\gamma_1' > \gamma_1$, $\gamma_2' \geq \gamma_2$

$$
= (1 + \beta)(\gamma_1 - 2ce_1' - \beta \gamma_2 + (\alpha + \beta)(2ce_1' - \gamma_1 + \gamma_2)
$$

rearranging

$$
\geq (1 + \beta)(\gamma_1 - 2ce_1' - \beta \gamma_2)
$$

since $e_1' \geq e_1^0$

$$
= \frac{du_1^0(e_1; e_2', \Phi)}{de_1} \bigg|_{e_1=e_1'} = 0
$$

by (2).

As for Agent 2, as $(e_1', e_2')$ is on the boundary of $u_2^\Phi$ and $u_2^\Phi$ under $\Phi'$, we consider the left-derivative of $u_1$:

$$
\frac{du_2(e_2; e_1', \Phi')}{de_2} \bigg|_{e_2=e_2'} = \frac{du_2^0(e_2; e_1', \Phi')}{de_2} \bigg|_{e_2=e_2'}
$$

by (11.8)

$$
= (1 - \alpha)(k\gamma_1' - 2ce_2' + \alpha k \gamma_2')
$$

by (11.8)

$$
= \frac{du_2^0(e_2; e_1', \Phi)}{de_2} \bigg|_{e_2=e_2'} + k(\alpha(\gamma_1' - \gamma_1) + (1 - \alpha)(\gamma_2' - \gamma_2))
$$

by (11.11)-(11.12)

$$
\geq \frac{du_2^0(e_2; e_1', \Phi)}{de_2} \bigg|_{e_2=e_2'} = 0
$$

since $\gamma_1' \geq \gamma_1$.

Thus, $B_1(e_2'; \Phi') > e_1'$, $B_2(e_1'; \Phi') \geq e_2'$. Moreover, $\pi(e_1', e_2', \Phi') = \pi(e_1', e_2', \Phi)$ due to the budget-balancing constraint.

When $f_2 < \frac{\alpha}{1-\beta}f_1$, then let $\gamma_2' = f_2/(e_1' + ke_2') + \gamma_2$ and $\gamma_4' = \frac{1+\beta}{\beta}(\gamma_2' - \gamma_2) + \gamma_1$. Similar to b), when $\gamma_2'(e_1' + ke_2') - ce_1'^2 - \gamma_4'(e_1' + ke_2') + ce_1'^2 \geq 0$, let $\Phi' = (\gamma_1', \gamma_2', 0, 0)$; otherwise, let $(\gamma_1', \gamma_2')$ defined by (11.11)-(11.12). Following the same arguments to b), we have $B_i(e_1', \Phi') \geq e_i'$ for $i = 1, 2$, and $\pi(e_1', e_2', \Phi') \geq \pi(e_1', e_2', \Phi)$, and at least one of the three inequalities are strict.
• Case 2: $v_1 > v_2$, i.e., $(e_1^i, e_2^i)$ is on $(u_1^i, u_2^i)$ under $\Phi$ defined by (EC.3)-(EC.4): following a similar argument to Case 1, we can construct the desired contract $\Phi'$.

• Case 3: $v_1 = v_2$, i.e., $(e_1^i, e_2^i)$ is on the boundary of $(u_1^i, u_2^i)$ and $(u_1^i, u_2^i)$ defined by (EC.3)-(EC.4). Consider $\Phi' = (\gamma_1, \gamma_2, 0, 0)$ defined by

$$(\gamma_1', \gamma_2') := (f_i/(e_1^i + ke_2^i) + \gamma_i - \epsilon, f_{-i}/(e_1 + ke_2^i) + \gamma_{-i}),$$

where $\epsilon > 0$ is arbitrarily small. (EC.9). Then for both agents $u_i^\alpha$ has a higher slope at $(e_1^i, e_2^i)$ under $\Phi'$ than $\Phi$. For example, for Agent 1,

$$\frac{du_i^\alpha(e_1^i; e_2^i, \Phi')}{de_1} \big|_{e_1 = e_1^i} = (1 - \alpha)(1 - 2ce_1^i) + \alpha \gamma_2$$

$$\frac{du_i^\alpha(e_1^i; e_2^i, \Phi')}{de_1} \big|_{e_1 = e_1^i} > (1 - \alpha)(\gamma_1 - 2ce_1^i) + \alpha \gamma_2$$

by (EC.8)

$$\frac{du_i^\alpha(e_1^i; e_2^i, \Phi')}{de_1} \big|_{e_1 = e_1^i} = \frac{du_i^\alpha(e_1^i; e_2^i, \Phi)}{de_1} \big|_{e_1 = e_1^i}$$

by (2).

Since $(e_1^i, e_2^i)$ is a Nash equilibrium under $\Phi$, $\frac{du_i^\alpha(e_1^i; e_2^i, \Phi)}{de_1} \big|_{e_1 = e_1^i} \geq 0$. Thus, $\frac{du_i^\alpha(e_1^i; e_2^i, \Phi)}{de_1} \big|_{e_1 = e_1^i} \geq 0$. Since $u_i^\alpha$ is on the left side of $u_i^\beta$ by (EC.8), the best responses satisfy $B_i(e_{-i}) \geq e_1^i$. Moreover, by (EC.9), the principal’s payoff $\pi(e_1^i, e_2^i, \Phi') = (1 - \gamma_1 - \gamma_2)(e_1^i + ke_2^i) > (1 - \gamma_1 - \gamma_2)(e_1^i + ke_2^i) + f_1 + f_2 = \pi(e_1^i, e_2^i, \Phi)$.

Summarizing Cases 1-3 above, there always exists a contract $\Phi'$ that satisfies the two criteria in the lemma. □

**Lemma EC.3.** Under utility fairness, given any contract $\Phi = (\gamma_1, \gamma_2, 0, 0)$ with no fixed fees, the corresponding effort game (4) maximizing (EC.8) is supermodular given $e_i \in [e_i^\alpha, e_i^\beta]$ for $i = 1, 2$.

**Proof.** In the proof, we omit the contract arguments in $u_i(e_i; e_{-i}, \gamma_i, \gamma_{-i}, 0, 0)$ and write instead $u_i(e_i, e_{-i})$. Given that the action spaces $e_i \in [e_i^\alpha, e_i^\beta]$ are compact and $u_i(e_i, e_{-i})$ are continuous in $e_i$ and $e_{-i}$ by definition, we need to show that $u_i(e_i, e_{-i})$ has increasing differences. Because the utility difference $(\gamma_1(e_1 + ke_2) - ce_1^2) - (\gamma_{-i}(e_1 + ke_2) - ce_1^2)$ is increasing in $e_{-i}$ when $e_{-i} \in [e_{-i}^\alpha, e_{-i}^\beta]$, $u_i$ moves from $u_i^\beta$ to $u_i^\alpha$ (defined by (EC.3)-(EC.4)) when $e_{-i}$ increases. Thus, it is sufficient to show $\partial u_i(e_i, e_{-i})/\partial e_i$ is increasing in $e_{-i}$ including the kink (which is still continuous). When $u_i(e_i, e_{-i}) = u_i^\alpha(e_i, e_{-i})$ (when $e_{-i}$ is small), the derivative is

$$\partial u_i(e_i, e_{-i})/\partial e_i = \partial u_i^\beta(e_i, e_{-i})/\partial e_i = (1 + \beta)(\gamma_i - 2ce_i) - \beta \gamma_{-i} k_i,$$

which is constant in $e_{-i}$. Similarly, when $u_i(e_i, e_{-i}) = u_i^\alpha(e_i, e_{-i})$ (when $e_{-i}$ is big), the derivative is

$$\partial u_i(e_i, e_{-i})/\partial e_i = \partial u_i^\alpha(e_i, e_{-i})/\partial e_i = (1 - \alpha)(\gamma_i - 2ce_i) + \alpha \gamma_{-i} k_i,$$

which is also constant in $e_{-i}$. Hence, the derivative $\partial u_i(e_i, e_{-i})/\partial e_i$ is piecewise constant. Furthermore,

$$\partial u_i^\alpha(e_i, e_{-i})/\partial e_i = (1 + \beta)(\gamma_i - 2ce_i) - \beta \gamma_{-i} k_i - (\alpha + \beta)(\gamma_i - \gamma_{-i}) k_i - 2ce_i)$$

rearranging

$$\geq \frac{\partial u_i^\beta(e_i, e_{-i})}{\partial e_i}$$

since $e_i \geq e_i^\beta$. Therefore, $\partial u_i(e_i, e_{-i})/\partial e_i$ is piecewise constant in $e_{-i}$ with a positive jump at the transition point, when $u_i(e_i, e_{-i}) = u_i^\alpha(e_i, e_{-i}) = u_i^\beta(e_i, e_{-i})$. As a result, $\partial u_i(e_i, e_{-i})/\partial e_i$ is weakly increasing in $e_{-i}$, i.e., $u_i(e_i, e_{-i})$ has increasing differences when $e_i \in [e_i^\alpha, e_i^\beta]$, and the game of interest is supermodular. □
**Lemma EC.4.** Under utility fairness, for any output-sharing contract \( \Phi = (\gamma_1, \gamma_2, f_1, f_2) \) with non-zero fixed fees and equilibrium efforts \((e_1', e_2')\), there exists a Nash equilibrium \((e_1^0, e_2^0)\) to effort game (4) under the contract in Lemma EC.2 such that \(e_i^0 \geq e_i'\) for \(i = 1, 2\), and thus, \(\pi(e_1^0, e_2^0, \Phi') > \pi(e_1', e_2', \Phi)\).

**Proof.** By Lemma EC.2 (point 2), for any \( \Phi \) with equilibrium efforts \((e_1', e_2')\), there always exists \( \Phi' = (\gamma_1', \gamma_2', 0, 0) \) such that \(\max\{B_i(e_i'; \Phi')\} \geq e_i'\) for \(i = 1, 2\). (Here, \(B_i(\cdot)\) is the set of best-responses in case of multiple equilibria.

Let \(s_0^i := \max\{B_i(e_i'; \Phi')\}\). Given that \(s_0^i \geq e_i'\), and by the supermodularity property proven in Lemma EC.3, we have \(s_1^i := \max\{B_1(s_0^i; \Phi')\} \geq s_1^i = s_0^i\). Similarly, since \(s_1^i \geq e_i'\), we have \(s_2^i := \max\{B_2(s_0^i; \Phi')\} \geq \max\{B_2(s_1^i; \Phi')\} = s_0^i\). Iterating the process by letting \(s_{k+1}^i := \max\{B_2(s_k^i; \Phi')\}\) and \(s_{k+1}^i := \max\{B_2(s_k^i; \Phi')\}\). Then, we have \(s_{k+1}^i \geq s_k^i\), \(k = 0, 1, 2, \ldots\) for \(i = 1, 2\) by induction. Because efforts are bounded, both sequences, \(\{s_k^i\}_k\) for \(i = 1, 2\), converge by the monotone convergence theorem. Let \(e_i^0\) denote the limit of \(\{s_k^i\}_k\) for \(i = 1, 2\). That is, there exists a pair of equilibrium efforts \(e_1^0 \geq e_1'\) and \(e_2^0 \geq e_2'\).

When the efforts are \((e_1', e_2')\) and the contract is \(\Phi = (\gamma_1, \gamma_2, f_1, f_2)\), by Lemma EC.2 (point 1), the principal’s payoff is bounded by

\[
\pi(e_1', e_2'; \Phi) \leq \pi(e_1^0, e_2^0; \Phi') = (1 - \gamma_1' - \gamma_2')(e_1'^0 + ke_2'^0) \leq (1 - \gamma_1' - \gamma_2')(e_1^0 + ke_2^0) = \pi(e_1^0, e_2^0; \Phi').
\]

By Lemma EC.2, when at least one of \(f_1\) and \(f_2\) is strictly positive, \(\pi(e_1', e_2'; \Phi) < \pi(e_1^0, e_2^0; \Phi').\) \(\square\)

**Figure EC.1** Regions A, B, C, and D in the proof of Proposition 3.

![Figure EC.1](image)

*Note. Here, \(\beta = 0.5\), \(\alpha = 0.25\), \(k = 0.7\).*

**Proof of Proposition 3.** By Lemma EC.4, the optimal contract for (5) can be found by optimizing \((\gamma_1, \gamma_2)\) with \(f_1 = f_2 = 0\) and utilities (EC.8). Based on Lemma 2, we consider four sub-problems in Regions A-D; see Figure EC.1 for visualization.

**Region A:** \(\gamma_2/\gamma_1 \geq \theta_{AB} \). We divide Region A into two subregions, denoted as A1 and A2, depending on whether Agent 1’s equilibrium effort is 0.
(A1): By Lemma 2, the equilibrium efforts are \((e_1^*, e_2^*)\). Therefore, the principal’s problem (5) within (A1) is

\[
\begin{align*}
\max_{\gamma_1, \gamma_2} (1 - \gamma_1 - \gamma_2) (e_1^* + ke_2^*) \\
\text{s.t. } &\gamma_2 \geq \theta_{AB}\gamma_1 \\
&\gamma_2 \leq \frac{1 + \beta}{\beta} - \gamma_1 \\
&\gamma_1 + \gamma_2 \leq 1
\end{align*}
\]

Applying the Karush–Kuhn–Tucker (KKT) conditions, we find that because i) there is no interior solution and ii) if (A1.2) were binding, then its corresponding Lagrange multiplier would be negative, the optimal value is such that either (A1.1) or (A1.3) is binding. If (A1.3) is binding, then \(\gamma_2 = 1 - \gamma_1\) and therefore the objective function’s value is 0, which is strictly dominated. If (A1.1) is binding, then such a contract also belongs to Region B, and will be taken into consideration there.

(A2): By Lemma 2, the equilibrium efforts are \((0, e_2^*)\). Therefore, the principal’s problem (5) in (A2) is

\[
\begin{align*}
\phi_{A2s}^* &= \max_{\gamma_1, \gamma_2} (1 - \gamma_1 - \gamma_2) ke_2^* \\
\text{s.t. } &\gamma_1 \geq 0 \\
&\gamma_2 \geq \frac{1 + \beta}{\beta} - \gamma_1 \\
&\gamma_1 + \gamma_2 \leq 1
\end{align*}
\]

Applying the KKT conditions, the optimal solution turns out to make (A2.1) binding: \(\gamma_1 = 0, \gamma_2 = 1/2\). The corresponding value of the objective function is \(\phi_{A2s}^* = \frac{k^2}{1}\).

**Region B:** For this region, we show that the directional derivative of the principal’s objective function with respect to \((\gamma_1, \gamma_2)\) along the direction \((1, -1)\) is positive and therefore the optimal solution lies on \(\gamma_2 = \Theta_{BC}\gamma_1\). Since \(\Theta_{AB} \leq 0 \leq \Theta_{BC}\) in region B, \(\gamma_2 / (1 + \beta) \leq \sqrt{\frac{\beta k^2}{(1-\alpha)^2} + (1-k^2) (\gamma_1 - \gamma_2)^2} \leq \gamma_2 / (1 - \alpha)\). Applying this inequality, the directional derivative of the principal’s objective function with respect to \((\gamma_1, \gamma_2)\) along the direction \((1, -1)\) is

\[
-\frac{\sqrt{2}}{2(1-\alpha)} \gamma_2 \left( -\frac{2\gamma_1 (1-\alpha) + \gamma_1 (2-2\alpha - \frac{1}{\gamma_2}) k^2 + 2\gamma_2 (1-\alpha) (1-k^2)}{\sqrt{\frac{\beta k^2}{(1-\alpha)^2} + (1-k^2)(\gamma_1 - \gamma_2)^2}} - (1-\alpha)(1-k^2) \right) \\
\geq -\frac{\sqrt{2}}{2(1-\alpha)} (1 - \gamma_1 - \gamma_2) \max \left\{ - (1-\alpha)(1-k^2) + \left(\frac{(\beta+1)(2\gamma_1 (1-\alpha) + \gamma_1 (2-2\alpha - \frac{1}{\gamma_2}) k^2 + 2\gamma_2 (1-\alpha) (1-k^2))}{\gamma_2} \right)^{-1} \right\},
\]

\[
\geq 0.
\]

The last inequality holds because in the curly bracket, i) the first expression is positive only when \(\gamma_2 > \frac{(\beta+1)(2\alpha - 2\gamma_1 + \gamma_1 k^2 - 2(1-\alpha)^2)}{(1-\alpha)^2 (2\beta + 1) (k^2 - 1)}\gamma_1 > \theta_{AB}\gamma_1\), which is not possible since \(\gamma_2 \leq \theta_{AB}\gamma_1\) in Region (B34): ii) the second expression is positive only when \(\gamma_2 > \frac{(2\alpha - 2\gamma_1 + \gamma_1 k^2 - 2(1-\alpha)^2)}{(1-\alpha)^2 (2\alpha - 1) (k^2 - 1)}\gamma_1 > \theta_{AB}\gamma_1\), which is again infeasible. Thus, the maximum lies on the constraint \(\gamma_2 = \Theta_{BC}\gamma_1\). Substituting this binding constraint into the objective function and solving the first-order optimality condition yields the optimal shares to be given by (9).

**Region C:** Similar to Region B, the directional derivative along \((-1, 1)\) is positive. Therefore, the optimal can be found on the line \(\alpha_2 = \theta_{AB}\alpha_1\). Thus, the optimal shares are identical as the solution in region B.
**Region D:** Similar to Region A, we divide Region D into two regions depending on whether Agent 2’s equilibrium effort is 0. Optimizing in both regions, one can find the optimal shares to be strictly dominated.

Comparing the payoff given by the optimal shares in Regions A-D, we find the optimal shares to be (9). □

**Proof of Theorem 2.** The results follow from Proposition 3. □

**EC.3. Reward Fairness**

Proof of Lemma 4. When $f_1 = f_2 = 0$, (3) simplifies to

$$u_i(e_i; e_{-i}, \gamma_i, \gamma_{-i}, 0, 0) = v_i(e_i, e_{-i}, \gamma_i, 0) - \alpha [\gamma_i (e_1 + ke_2) - (\gamma_i + \gamma_{-i})ke_i] + \beta [(\gamma_i + \gamma_{-i})ke_i - \gamma_i (e_1 + ke_2)],$$

(EC.13)

where $k_1 = 1$ and $k_2 = k$. The proof is similar to that of Lemma 2. □

Proof of Lemma 5. The proof is similar to that of Lemma 3. □

To prove Proposition 4, we first show that for any optimal output-sharing contract, $f_1 = f_2 = 0$; and then find the optimal contract solving (5) with $f_1 = f_2 = 0$.

Similar to the proof of Proposition 3, we show that $f_1 = f_2 = 0$ in all optimal output-sharing contracts in three steps: First, given a Nash equilibrium $(e'_1, e'_2)$ to the effort game (4) under $\Phi$, we propose in Lemma EC.5 a contract $\Phi' = (\gamma'_1, \gamma'_2, 0, 0)$ with no fixed fees that leads to both higher best effort responses and higher payoff at $(e'_1, e'_2)$. Second, we show in Lemma EC.6 that any effort game with no fixed fees maximizing (EC.13) is supermodular. Finally, based on the supermodularity of the effort game under $\Phi'$ (Lemma EC.6) and the elevated best responses at $(e'_1, e'_2)$ (Lemma EC.5, point 2), we show in Lemma EC.7 that there exists an equilibrium $(e^0_1, e^0_2)$ such that $e^0_1 \geq e'_1$ and $e^0_2 \geq e'_2$, and thus $\pi(e^0_1, e^0_2; \Phi') \geq \pi(e'_1, e'_2; \Phi') \geq \pi(e'_1, e'_2; \Phi)$ (Lemma EC.5, point 1). As a result, we consider share-only contracts in the proof of Proposition 4.

**Lemma EC.5.** Under reward fairness, given any contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ and a Nash equilibrium $(e'_1, e'_2)$ to the corresponding effort game (4), there exists a contract $\Phi' = (\gamma'_1, \gamma'_2, 0, 0)$ such that

1. $\Phi'$ leads to greater payoff when efforts are $(e'_1, e'_2)$, i.e., $\pi(e'_1, e'_2; \Phi') \geq \pi(e'_1, e'_2; \Phi)$.

2. Agent $i$’s best response to $e'_i$ under $\Phi'$, denoted by $B_i(e'_i; \Phi')$, is greater than $e'_i$, i.e., $B_i(e'_i; \Phi') \geq e'_i$, for $i = 1, 2$.

When either $f_1 > 0$ or $f_2 > 0$ and when $\alpha > 0$, at least one of the three inequalities (i.e., $\pi(e'_1, e'_2; \Phi') \geq \pi(e^0_1, e^0_2; \Phi)$, and $B_i(e'_i; \Phi') \geq e'_i$ for $i = 1, 2$) are strict

Proof. The proof is similar to that of Lemma EC.2. □

**Lemma EC.6.** Under reward fairness, given any contract $\Phi = (\gamma_1, \gamma_2, 0, 0)$ with no fixed fees, the effort choice game (4) maximizing (EC.13) is supermodular.

Proof. The proof is similar to that of Lemma EC.3. □
Lemma EC.7. Under reward fairness, for any output-sharing contract $\Phi = (\gamma_1, \gamma_2, f_1, f_2)$ with non-zero fixed fees and equilibrium efforts $(e_1^f, e_2^f)$, there exists a Nash equilibrium $(e_1^0, e_2^0)$ to effort game (4) under the contract in Lemma EC.5 such that $e_i^0 \geq e_i^f$ for $i = 1, 2$, and thus, $\pi(e_1^0, e_2^0, \Phi') > \pi(e_1^f, e_2^f, \Phi)$.

Proof. The proof is similar to that of Lemma EC.4. □

Proof of Proposition 4. The proof is similar to that of Proposition 3. □

Proof of Theorem 3. The results follow from Proposition 4. □

EC.4. Homogeneous Agents

Proof of Theorem B.1. The result is a corollary to Theorems 1, 2, and 3. We only detail the case of income fairness to show that the principal’s payoff is in fact equal with or without pay transparency. By Proposition 2, the principal sets $\gamma_1^* = \gamma_2^* = 1/4$ and $f_1^* = f_2^* = 0$. Hence, the agents’ incomes are identical, there is no inequality in equilibrium and both agents are involved in production. By Case A in Lemma 1 and (6), $e_1^* = e_1^N$ and $e_2^* = e_2^N$, i.e., the output is identical to the inequality-neutral case. Because the principal’s total allocation of output $\gamma_1^* + \gamma_2^*$ is identical to the total allocation of output under inequality neutrality (Proposition 1), the principal’s payoff remains identical under pay transparency. □