The Short-Termism Trap: Competition for Informed Investors under Stock-Based CEO Compensation *

James Dow†, Jungsuk Han‡ and Francesco Sangiorgi§

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Abstract

We show that stock-based CEO compensation can create a “race to the bottom” among firms that escalates short-termist pressure. More informative stock prices reduce the agency cost of using stock-based compensation to incentivize managers. Also, shortening a firm’s project maturity improves stock price informativeness by attracting informed investors, who prefer to invest in short-term assets. However, when informed trading capital is a scarce resource, competition for informed investors creates excessive short-termism that destroys shareholder value, while in equilibrium, price informativeness stays the same. More intense competition between firms sharpens incentives to shorten project maturity, deepening the “short-termism trap.”

JEL Classification: G14, G32, G38

1 Introduction

Stock based compensation allows shareholders to incentivize managers with short horizons to pursue long-term projects. Because stock prices reflect the NPV of future cash flows, they are a signal of long-term value creation. However, we show that this can backfire: incentive contracts that are optimal for individual firms can, through competition, encourage a race to the bottom in which firms all choose projects of excessively short duration.

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†London Business School, Regent’s Park, London, United Kingdom, NW14SA, E-mail: jdow@london.edu.
‡Stockholm School of Economics, Drottninggatan 98, Stockholm, Sweden, SE 11160, E-mail: jung-suk.han@hhs.se
§Frankfurt School of Finance and Management, Adickesallee 32-34, 60322, Frankfurt am Main, Germany, E-mail: F.Sangiorgi@fs.de
This race to the bottom occurs through investor constraints in the stock market. Informed investors (hedge funds, pension funds or any investor that trades based on their analysis of the stock) make stock prices more informative, so they add value by making stock-based compensation more effective. But they typically have limited capital and may need to liquidate early. Thus shorter-term projects are more attractive to informed investors. Since the total stock of informed capital is limited, competition for informed capital is counterproductive.

There is an extensive literature on theoretical aspects of short-termism. Our paper differs in that our model exhibits short-termism that is optimal for individual firms, but collectively suboptimal. Also, our model features short-termism in the context of publicly traded companies and stock-based managerial compensation. These differences are discussed in the literature section below.

We study a model in which firms' shares are traded in a stock market with privately informed investors and uninformed investors. Informed investors have limited capital, which we model by assuming they can only make one unit of investment. They also have a preference for short-term investments, modelled by assuming that each investor may receive a liquidity shock which forces them to liquidate early. This implies that price informativeness of long-term investment should be lower than that of short-term investment, to compensate for the possibility that investors leave the market before the project’s liquidation without realizing full value.

Firms choose their projects which are run by managers who are subject to moral hazard. Managers may need to leave early for exogenous reasons. Stock prices, however, are informative about managerial efforts. Therefore, fixing price informativeness, stock-based compensation reduces agency costs because managerial contracts become more efficient. This allows the firm to pursue longer term projects without impairing incentives, thereby enhancing value. In that sense, the stock market has the potential to support implementation of long term projects.

A firm can also make contracts more efficient and increase value by increasing the informativeness of its stock price. Given the project choices made by other firms, an individual firm can do this by reducing its project maturity. However, informed trading is limited, so the increase in one firm’s price informativeness is at the expense of other firms. Thus, if one firm reduces its project maturity, other firms want to do the same to regain price informativeness. In other words, project maturity choices are strategic complements. This causes a race to the bottom in which, in equilibrium, project maturity is too low: all firms would have higher value if they all coordinated to choose longer projects. Indeed, in equilibrium, projects can be even shorter duration than if there were no stock-based managerial incentives at all.

We show how the short-termism trap is affected by the degree of investor short termism and by industry characteristics. If investors’ horizons get shorter, how does this affect the

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1 We assume that production efficiency is increasing in project duration.
race to the bottom in project duration? We show that a stronger investor propensity for early exit reduces project maturity and firm value. We also examine the effects of industry competitiveness on the short-termism trap. With more firms in an industry, the competition for informed investor capital is more intense. Thus, an increase in the number of firms leads to shorter duration projects. It also reduces shareholder value but while this is a feature of standard models because of product market competition, this channel is shut down in our model and shareholder value is reduced purely as a result of firms picking shorter term projects.

In an extension, we study whether long-term investing curbs corporate short-termism. We consider the case where a mass of investors have long horizon. We show that for long-term investors to curb corporate short-termism, they must be marginal investors on all firms, i.e., their mass must be larger than a threshold. By contrast, if long-term investors are below a critical mass, they have no impact on equilibrium short-termism at all. If the mass of long-term investors is intermediate, there is a clientele equilibrium where ex-ante identical firms cater to different clienteles: some firms choose shorter duration and are held by short-term investors, and the remaining firms choose longer duration and are held by long-term investors.

Finally, we study policy implications of our model. We extend our baseline model by introducing a salary cap. Limiting CEO compensation is often proposed as a mechanism to improve the management of listed companies. In our analysis, however, we find that a salary cap may have an ambiguous effect; an introduction of salary cap may promote short-termism rather than prevent it. It is because it becomes more expensive to incentivize managers under salary cap at any level of project maturities, forcing firms to complement stock-based compensation with short-term compensation.

The paper is organized as follows: In Section 2 we connect our paper to the existing literature. In Section 3 we describe the model setup. In Section 4 we solve for the financial market equilibrium given firms’ maturity choices and we solve each firms’ optimal managerial compensation and choice of project maturities taking other firms’ behaviour as given. In Section 5 we describe the equilibrium concept and show existence and uniqueness of equilibrium. In Section 6 we characterize properties of the equilibrium. In Section 7 we study the impact of long-term investors. In Section 8 we study the salary cap. Section 9 concludes.

2 Literature

There is a large literature on short-termism. It identifies two main possible sources of short-termism. First, it could arise because shareholders have short horizons, so that they want to maximize the share price at the end of their horizon, not the value of projects that mature later. This may lead them to encourage managers to choose projects that deliver value quickly,
rather than better projects that do not demonstrate value until later.\footnote{Porter (1992) argues that companies pursue short-term share price appreciations at the expense of the long-term performance due to the pressure from shareholders’ short-term interests. For example, Bushee (1998) finds that high ownership by short-horizon investors induces firms’ myopic investment. Gaspar, Massa, and Matos (2005) find that firms with short-term shareholders tend to get lower premiums in acquisition bids. Cremers, Pareek, and Sautner (2020) also find that an increase in ownership by short-horizon investors has an incremental effect on corporate short-termism such as reducing R&D expenses.} Second, short-termism could arise when managers themselves have short horizons (or higher discount rates) and act in their own interests. If managers own stock or are compensated with a mix of stock and salary, they have an incentive to choose projects that deliver value quickly, rather than better long-term projects (Stein (1989)).\footnote{For example, empirical evidence shows that shorter CEO horizon reduces investment and lower firm value where horizons are measured by expected tenure (Antia, Pantzalis, and Park (2010)), financial reporting frequency (Kraft, Vashishtha, and Venkatachalam (2018), and option vesting periods (Ladika and Sautner (2020)).} Similar outcomes arise when incentives for such managers are designed optimally in response to contracting frictions (Edmans, Gabaix, Sadzik, and Sannikov (2012), Varas (2018)). In that case, corporate short-termism is second-best given those contracting constraints.\footnote{This is obvious, in the sense that in any model with just a principal and an agent, the optimal contract is by definition second best.}

Related papers include Bolton, Scheinkman, and Xiong (2006) who show that managerial short-termism persists when shareholders optimally induce managers to chase short-term profits to exploit market over-optimism. In Edmans (2009), blockholders’ trading on private information causes prices to reflect fundamentals, encouraging managers to invest in valuable long-run projects rather than chasing short-term profits. Thakor (forthcoming) finds that greater noise in performance assessment with long-horizon projects leads to higher agency costs and thus induces a preference for short-termism.

In this literature however, there is no welfare analysis demonstrating that stock market short-termism is value-reducing. In those papers that permit a welfare analysis, short-termism is second best. However, there are two papers demonstrating welfare suboptimal short termism via different channels from the stock market. Milbradt and Oehmke (2015) study debt financing when long term projects are more likely to default. In response, firms may shorten project maturity even at the cost of further increasing default risk, initiating a race to the bottom in which all firms choose the shortest term projects. In Thanassoulis (2013), firms may be willing to tolerate lower value short-term projects in order to reduce the cost of compensating impatient managers. Our paper differs from these papers because of the role of the stock market.

Our paper is also related to the literature on real investment under information asymmetries. Generally, prices in all markets in the economy serve to influence economic decisions, but literature in finance has specifically focused on the feedback effect between an individual firm’s investment and its own stock price (Dow and Rahi (2003), Bond, Edmans, and Goldstein (2012), Goldstein, Ozdenoren, and Yuan (2013), Sockin and Xiong (2015)). While producing
private information is helpful in guiding investment, the incentives to produce private information are not necessarily optimal. More informative prices may also either help or hinder the allocation of risk (Dow and Rahi (2003)). The market has a strong incentive to concentrate on predicting the payoffs of “no-brainer” projects that are so profitable they will surely be invested in, so the predictions have no social value (Dow, Goldstein, and Guembel (2017)). By contrast, our paper studies the real impact of competing firms with endogenous managerial contracting.

There is a stand of literature following the seminal paper by Holmstrom and Tirole (1993), that studies the effect of stock prices in motivating managers in a model of trading on private information (e.g., Baiman and Verrecchia (1995), Dow and Gorton (1997), Kang and Liu (2010), Strobl (2014), Lin, Liu, and Sun (2019)). For example, Strobl (2014) shows that the amount of information produced by the market is not necessarily optimal; shareholders may have an incentive to encourage “overinvestment” (in the first-best sense) in order to make the stock price more informative and improve the managerial agency problem. In Dow and Gorton (1997), prices combine the two roles of guiding investment and motivating managers. These papers show the benefits of stock-based compensation. In our paper, we also use an agency framework. Project maturity choice is a key variable, unlike the aforementioned papers, and crucially, we also study the effects of competition among firms for informed trading. This results in socially sub-optimal short-termism, even though each firm’s managerial contract is individually optimal. In other words if there were only one firm in our model, short-termism would be prevented by stock-based compensation. Our model shows that this result is reversed under competition for investors.

3 Setup

Consider a three-period economy \((t = 0, 1, 2)\) with a corporate sector and a financial market. In the corporate sector, there are firms with a productive technology. In the financial market, the shares of firms in the corporate sector and the risk-free asset are traded. The risk-free rate in the economy is normalized to zero.

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5There are several papers exploring the role of competition with real investment but without endogenous managerial contracting. In Fishman and Hagerty (1989) prices are useful in improving investment policy so shareholders know they will be able to sell at informative prices, inducing excessive information disclosure as firms’ compete for investor attention. Peress (2010) argues that monopolists have more informative prices because their stock prices are sensitive to information, while in competitive industries profits are so low anyway that there are only weak incentives to produce information. Foucault and Frésard (2019) argue that firms have an incentive to piggyback on information that is produced about other firms, and this induces them to prefer making products that are not differentiated.
3.1 Firms

In the initial period, \( t = 0 \), firms are established, and their shares are traded in the market. We assume that there are \( N \) firms in the economy, and denote by \( \mathcal{N} \equiv \{1, 2, ..., N\} \) the set of firms. Each firm has risk-neutral shareholders and a risk-averse manager. The shareholders are long-lived, and maximize firm value by choosing a project as well as a managerial contract with the manager.

Upon the creation of a firm (indexed by \( n \in \mathcal{N} \)), shareholders choose the maturity of its project, characterized by \( \tau^n \in [0, 1] \), where the project matures early \((t = 1)\) with probability \( 1 - \tau^n \), and late \((t = 2)\) with probability \( \tau^n \). So if \( \tau^n = 0 \) the project always matures early, while if \( \tau^n = 1 \) it always matures late.

The firm’s project generates a payoff only when it matures, and the firm distributes this as a liquidating dividend. At liquidation, the output of firm \( n \) is given by

\[
V^n \equiv f(\tau^n) + R^n \quad \text{where} \quad R^n = \begin{cases} 
\Delta V & \text{if the project is successful} \\
0 & \text{otherwise}
\end{cases},
\]

where \( \Delta V \) is a positive constant, which captures the increased output in case of the project’s success. The first component \( f(\tau^n) \) is the maturity-sensitive component and the second component \( R^n \) is the component sensitive to managerial effort. We assume that output increases as maturity is lengthened, i.e., \( f(\cdot) \) is a non-negative, increasing, concave, twice-differentiable function with \( f'(0) = \infty \) and \( f'(1) = 0 \). Throughout the paper, the term “increasing” is synonymous with “strictly increasing”, and “concave” (or “convex”) is synonymous with “strictly concave” (or “strictly convex”). Firms choose \( \tau^n \)’s simultaneously. Once all firms make their maturity choices, those choices become publicly observable.

We assume that with probability \( \delta \in [0, 1] \) each manager exits the economy early \((t = 1)\) — managers are long-lived in the special case where \( \delta = 0 \). They are subject to limited liability and an outside option. The managerial effort choice is private information, and is not verifiable. We denote by \( e^n \) the effort level of firm \( n \)’s manager which is either \( H \) (“high effort”) or \( L \) (“low effort”). Given effort level \( e^n \), the project of firm \( n \) succeeds with probability \( \rho(e^n) \), and fails with probability \( 1 - \rho(e^n) \). We assume that firms’ success is independent of each other. If the manager of firm \( n \) exerts high effort, the project is more likely to succeed, i.e.,

\[
\rho(e^n) = \begin{cases} 
\rho_H & \text{if } e^n = H \\
\rho_L & \text{if } e^n = L
\end{cases}.
\]

\(^6\)The assumption that \( f'(0) = \infty \) means the marginal benefit of lengthening maturity is infinity for an extremely short-term project (i.e., \( \tau^n = 0 \)). The assumption that \( f'(1) = 0 \) means that the marginal benefit of lengthening maturity \( \tau^n \) is zero when it is an extremely long term project (i.e., \( \tau^n = 1 \)). Concavity together with these two assumptions is assumed for simplicity to ensure a unique interior solution.
where $\Delta \rho \equiv \rho_H - \rho_L > 0$. The manager’s utility given his wage $w^n$ and his effort choice $e$ is
\[
u(w^n) - I(e^n = H)K,
\]
where $K$ is the manager’s effort cost, and $\nu$ is an increasing, concave, thrice-differentiable function with $\nu(0) = 0$. We further assume that $\nu'(0) = \infty$ and $\lim_{w \to \infty} \nu(w) = \infty$, which ensure a unique interior solution.\footnote{The assumption that $\nu'(0) = \infty$ rules out a corner solution where no compensation is given to managers even when good information arrives. The assumption that $\lim_{w \to \infty} \nu(w) = \infty$ prevents the situation where it is impossible to incentivize managers because their effort cost $K$ is too high.} As is standard in the literature, we assume that the outside option utility, denoted $\bar{\nu}$, is low enough that the participation constraint is non-binding. We restrict the parameter value of $K$ to be less than the upper bound $\bar{K} \equiv (1 - \delta)\Delta \rho u \left( \frac{\Delta \rho \Delta V}{(1 - \delta)\rho_H} \right)$, which ensures that the cost of incentivizing the manager never exceeds the benefit of enhancing firm value.\footnote{See the proof of Proposition 3.}

Shareholders in firm $n$ maximize the expectation of shareholder value by choosing maturity $\tau^n$ and state-contingent compensation $\tilde{w}^n$. Shareholder value of firm $n$ is given by
\[
V^n - \tilde{w}^n.
\]
The cost of incentivizing managers, $\tilde{w}^n$, is borne by shareholders. In contrast, we will refer to $V^n$ as “production” or “final payoff.” When we refer to “efficiency” we mean with respect to shareholder value.

3.2 The Financial Markets

In the financial market, participants trade shares of firms in the corporate sector. The shares are claims on firms’ final payoff; the stock of firm $n$ pays $V^n$ to all claim holders whenever the payoff realizes. There is no constraint on short sales of the security.

There is a unit mass of risk-neutral informed investors who either consume early ($t = 1$) with probability $\gamma$ or late ($t = 2$) with probability $1 - \gamma$. We denote by $\mathcal{I}$ the set of informed investors in the economy. Each informed investor can produce private information about one firm in the initial period, $t = 0$. All the informed investors who investigate firm $n \in \mathcal{N}$ receive an identical signal $s^n$, which is either good ($G$) or bad ($B$). High managerial effort results in a higher probability that informed investors receive a good signal. We denote by $\sigma_e$ the probability that the signal is good given effort $e \in \{H, L\}$; the signal is good with probability $\sigma_G \equiv \Pr(s^n = G | e^n = H)$ given high effort, and $\sigma_B \equiv \Pr(s^n = G | e^n = L)$ given low effort where $\Delta \sigma \equiv \sigma_G - \sigma_B > 0$.

We denote by $\nu_G$ and $\nu_B$ the posterior probability of a high payoff conditioning on a good and bad signal, respectively. For simplicity, we assume that the signal is a sufficient statistic.
for the final payoff.\(^9\) Equivalently, we assume:

\[
\rho_H = \sigma_G v_G + (1 - \sigma_G) v_B; \quad \rho_L = \sigma_B v_G + (1 - \sigma_B) v_B.
\] (3)

For simplicity, we further assume that \(\sigma_G > \rho_H\), which means the good signal has a higher frequency relative to the high final payoff under high effort, and also that

\[
\rho_H = \frac{\nu_B + \nu_G}{2},
\] (4)

which equalizes investors’ speculative profits whether private signals are good or bad.\(^{10}\)

There are long-lived, competitive, risk-neutral market makers who set prices to clear the market. There are also noise investors who trade for exogenous reasons such as liquidity needs. As in case of informed investors, noise investors also consume early or late (with probability \(\gamma\) and \(1 - \gamma\), respectively). In each period, informed investors and noise investors submit market orders to the market makers. We denote \(x_i^n(t)\) the market order of informed investor \(i\) in stock \(n\) at time \(t = 0, 1\). In the initial period, \(t = 0\), noise investors submit order flow \(z^n\) in aggregate for each stock \(n\), which follow an i.i.d. uniform distribution on \([-\bar{z}/2, \bar{z}/2]\). The parameter \(\bar{z}\) captures the intensity of noise in the financial market. Next period, at \(t = 1\), those who got liquidity shocks (i.e., \(\gamma\) fraction) reverse their orders. Consequently, they submit \(-\gamma z^n\) for each stock \(n\) at \(t = 1\). In each period \((t = 0, 1)\) market makers observe aggregate order flow for each stock \(n \in \mathcal{N}\) such that

\[
X^n(t) = \int_{i \in \mathcal{I}} x_i^n(t)di + Z^n(t),
\]

where \(Z^n(0) = z^n\) and \(Z^n(1) = -\gamma z^n\).

\(^9\)More formally, \(s^n\) is a sufficient statistic for \((s^n, R^n)\) if the posterior distribution of \(e^n\) conditional on \((s^n, R^n)\) only depends on \(s^n\) (see Chapter 9 in DeGroot (1970)). The conditions in Eq. (3) are equivalent to \(P(r(R^n|s^n, e^n)) = P(r(R^n|s^n))\) because

\[
Pr(R^n|e^n) = \sum_{s^n \in \{G, B\}} Pr(R^n|s^n, e^n)Pr(s^n|e^n) = \sum_{s^n \in \{G, B\}} Pr(R^n|s^n)Pr(s^n|e^n).
\]

Then, it is immediate that the condition \(P(r(R^n|s^n, e^n)) = P(r(R^n|s^n))\) is in turn equivalent to the condition that \(s^n\) is a sufficient statistic because Bayes’ Rule implies

\[
Pr(e^n|s^n, R^n) = \frac{Pr(R^n|s^n, e^n)Pr(e^n|s^n)}{Pr(R^n|s^n)} = Pr(e^n|s^n).
\]

The sufficient statistic assumption in agency theory is introduced in Holmstrom (1979) or Shavell (1979); for a textbook discussion with discrete signals see Tirole (2006).

\(^{10}\)The assumption that \(\sigma_G > \rho_H\) ensures a unique interior equilibrium by making the expected cost of compensation well behaved (monotone and convex). The assumption in Eq. (4) equals the difference in absolute value between the posterior and the prior whether it is the signal is good or bad, i.e., \(\nu_G - \rho_H = \rho_H - \nu_B\). We do not need this assumption to perform our analysis, but without it the algebra is considerably messier. See the proof of Lemma 2 for details.
In our model, informed trading is a scarce resource in the economy. To this end, we make the following assumptions. First, we assume that $N\bar{z}$ (the total noise trading intensity) is greater than one (the maximum possible size of the informed investors’ total order flow). This ensures that the given mass of informed investors cannot fully reveal the signal for every traded firm.\footnote{If $N\bar{z}$ is small relative to the mass of informed investors, the economy trivially degenerates to one with fully-revealing prices for every firm.}

Second, we assume that each informed investor can hold at most one unit of one stock (either a long or short position).\footnote{Because informed investors are risk-neutral, they will choose the maximum amount of trading even though they are allowed to trade less than one unit.} Informed investor $i$ in firm $n$ can submit a market order $x^n_i(0) \in \{-1, 0, 1\}$ at $t = 0$. If $x^n_i(0) \in \{-1, 1\}$ and firm $n$ liquidates late, informed investor $i$ can reverse their position in $t = 1$, or, if they consume late, hold it until $t = 2$. If $x^n_i(0) = 0$ and firm $n$ liquidates late and informed investor $i$ consumes late, they can submit an order $x^n_i(1) \in \{-1, 0, 1\}$ at $t = 1$.

In addition, we assume that $\bar{z} < \frac{1}{\gamma(N - 1)}$, which ensures that there exist enough informed investors that, for each firm, some informed investors choose its stock so that it will have positive price informativeness, regardless of other firms’ choice of maturities (see the proof of Proposition 1). Furthermore, we assume that $(N - 1)(1 - \gamma) \geq 1$,\footnote{Lemma C.7 in Appendix C shows that a sufficient condition alternative to Eq. (6) is that the manager has CRRA utility with relative risk aversion close to one.} which is a sufficient condition for establishing that firms’ maturity choices are strategic complements (see Proposition 4).

Finally, we assume that all exogenous random variables in our model are jointly independent.

### 4 Optimal Choice

#### 4.1 Investment Choice in the Financial Market

In this subsection, we derive price informativeness of stocks in the financial market by solving informed investors and market makers’ problems. For this, we assume that all managers exert effort. In equilibrium, this is true, as will be verified in the next subsection.

Market makers set prices given aggregate order flows from informed investors and noise investors as in the standard Kyle (1985) model. Because market makers are competitive and
risk neutral, the price of each security is its expected liquidation value conditional on market makers’ information: the price of stock \( n \) in each period \( (t = 0, 1) \) is given by

\[
P^n(t) = \mathbb{E}[V^n | \mathcal{F}(t)],
\]

where \( \mathcal{F}(t) \) is the market makers’ information in period \( t \).

Prices are either fully-revealing or non-revealing due to the uniformly-distributed noise trading (see, for example, Dow, Han, and Sangiorgi (2021) for a more detailed discussion of this feature). If the order flow is large enough (in absolute value, whether buy or sell) then it can only result from both informed investors and noise investors trading in the same direction, so it is fully revealing. But if the absolute value of order flow is smaller than the threshold value at which full revelation occurs, then it could have resulted from either informed investors buying and noise investors selling, or vice versa. Because noise trading is uniformly distributed, any level of the order flow is equally likely regardless of whether arbitrageurs are buying or selling, so it is non-revealing. We denote \( P^n_L \) and \( P^n_H \) to be the fully-revealing price for good or bad signal, respectively. We also denote \( P^n_\emptyset \) to be the non-revealing price. We denote \( \lambda^n \) to be the probability of information revelation for stock \( n \).

We can now show the following result:

**Lemma 1.** If \( \mu^n \) mass of informed investors trade on private information on stock \( n \in \mathcal{N} \), the price of stock \( n \) in the initial period, \( t = 0 \), is given by

\[
P^n(0) = \begin{cases} 
  P^n_L & \text{if } -\mu^n - \frac{\alpha}{2} \leq X^n(0) < \mu^n - \frac{\alpha}{2} \\
  P^n_\emptyset & \text{if } \mu^n - \frac{\alpha}{2} \leq X^n(0) \leq -\mu^n + \frac{\alpha}{2} \\
  P^n_H & \text{if } -\mu^n + \frac{\alpha}{2} < X^n(0) \leq \mu^n + \frac{\alpha}{2} 
\end{cases}
\]

where

\[
P^n_L = f(\tau^n) + \nu_B \Delta V, \quad P^n_\emptyset = f(\tau^n) + \rho_H \Delta V, \quad P^n_H = f(\tau^n) + \nu_G \Delta V;
\]

and the probability of information revelation for stock \( n \) in the initial period, \( t = 0 \), is given by

\[
\lambda^n(0) = \frac{\mu^n}{\bar{z}}.
\]

\[14\]For a general case, the notation for price informativeness should be

\[
\lambda^n(0) = \min\left(\frac{\mu^n}{\bar{z}}, 1\right).
\]

If \( \mu^n \geq \bar{z} \) (the mass of informed investors who have private information on stock \( n \) is greater than the intensity of noise trading), \( \lambda^n(0) \) is equal to one. But such case never arises in equilibrium because it would be incompatible with the incentive of informed investors, which is clear from Proposition 3. Therefore, we use the notation in Eq. (9) for convenience.
Proof. See Appendix A.

Given informed investor $i$’s choice to produce information on stock $n$, we can represent the maximization problem as follows:

$$J^n_0 \equiv \max_{x^n_i(0) \in \{-1, 0, 1\}} -E[P^n(0)|s^n]x^n_i(0) + \gamma \Gamma^n(s^n)x^n_i(0) + (1 - \gamma)E[J^n_1(x^n_i(0), P^n(0))|s^n], \quad (10)$$

where

$$\Gamma^n(s^n) \equiv (1 - \tau^n)E[V^n|s^n] + \tau^n E[P^n(1)|s^n],$$

and

$$J^n_1(x^n_i, P^n(0)) \equiv E[V^n|s^n]x^n_i + \tau^n (1 - |x^n_i|) \max_{x^n_i(1) \in \{-1, 0, 1\}} E[(V^n - P^n(1))|s^n, P^n(0)]x^n_i(1).$$

In other words, $\Gamma^n(s^n)$ is the expected value of early-liquidated one unit of position in stock $n$ conditional on $s^n$, and $J^n_1(x^n_i, P^n(0))$ is the expected continuation value at $t = 1$ for a late consumer given the position $x^n_i$ in the previous period and conditional on $s^n$ and $P^n(0)$. In case the informed investor waits one period (i.e., $x^n_i(0) = 0$), they will trade in $t = 1$ only if the firm’ project pays off late (with probability $\tau^n$) and if $P^n(0)$ is non-revealing. On the other hand, the continuation value of a non-zero position $x^n_i$ in $t = 0$ is simply $E[V^n|s^n]x^n_i$.

The next lemma shows that the problem can be greatly simplified: first, all informed investors choose to trade at $t = 0$; second, the value function reduces to a much simpler expression; and third, the price at $t = 1$ does not contain additional information because there is no further informed trading.

**Lemma 2.** Each informed investor $i$ who has signal $s^n$ on stock $n$ always finds it optimal to trade at $t = 0$, and the expected value of trading stock $n$ in Eq. (10) is equivalent to

$$J^n_0 = (1 - \lambda^n(0))(1 - \gamma \tau^n)\Delta P,$$  \hspace{1cm} (11)

where $\Delta P$ is the mispricing wedge such that

$$\Delta P = P^n_H - P^n_\emptyset = P^n_\emptyset - P^n_L = \frac{\nu_G - \nu_B}{2} \Delta V.$$

---

15 The assumption in Eq. (4) equalizes speculative profits regardless of signal realization. Therefore, we omit the dependence of $J^n_0$ and $J^n_1$ on $s^n$ to simplify notation.

16 If the firm’ project pays off late and $P^n(0)$ is non-revealing, the informed investor could close the position early in $t = 1$ instead of holding it until $t = 2$. However, the proof of Lemma 2 shows that closing the position early is never optimal.

17 Because informed investors are constrained and choose to trade at $t = 0$, they are not able to engage in extra informed trading in the subsequent period ($t = 1$); they either already hold maximum positions if information is unrevealed, or do not have any informational advantage otherwise. Consequently, only those with liquidity shocks reverse their positions, thus, prices do not contain additional information at $t = 1$. 

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11
Further, the price next period, $t = 1$, does not reveal further information, i.e., $\lambda^n(1) = 0$.

Proof. See Appendix A. ■

Because the stock market is only informative in the initial period, $t = 0$, we suppress dependence of $\lambda^n(t)$ on period $t$; henceforth, we denote firm $n$' price informativeness at $t = 0$ by $\lambda^n$ instead of $\lambda^n(0)$.

Now, we move on to the the choice of information acquisition at $t = 0$. The expected trading gains of each stock expressed in Eq. (11) should be equalized across all stocks in equilibrium. If they were different, all informed investors would instead want to gather private information only on those with higher expected trading gains. That is, the indifference condition $J^n_0 = J^m_0$ should be satisfied for any stock $n$ and $m$ in $\mathcal{N}$, or equivalently,

\begin{equation}
(1 - \lambda^n)(1 - \gamma \tau^n) = (1 - \lambda^m)(1 - \gamma \tau^m),
\end{equation}

which describes the equilibrium trade-off between mispricing and duration. Informed investors like mispricing but dislike longer duration; therefore, an increase in duration is compensated for by an increase in mispricing, and vice versa.

Furthermore, because there is one unit mass of informed investors ($\sum_{i=1}^{N} \mu^n = 1$), we also have the following condition in equilibrium, which we call the informational resource constraint:

\begin{equation}
\sum_{n=1}^{N} \lambda^n = \frac{1}{\bar{z}}.
\end{equation}

Using the results so far, we can show that, given maturity choices, there is a unique allocation of information acquisition that satisfies the two constraints.

**Proposition 1. (Financial Market Equilibrium)** Given $\{\tau^n\}_{n \in \mathcal{N}}$, there exists a unique positive solution $\{\lambda^n\}_{n \in \mathcal{N}}$ that satisfies both the indifference condition Eq. (12) and the informational resource constraint Eq. (13). Furthermore, $\lambda^n$ is decreasing and concave in $\tau^n$, and is increasing in $\tau^m$ for $m \in \mathcal{N} \setminus \{n\}$.

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18In real life, prices may be informative every period adding more information over time, but we shut down the channel of this secondary information revelation for simplicity. Under the setup where prices are informative in each period, higher price efficiency creates two confounding effects. On the one hand, higher price efficiency reduces trading benefits by lowering the chance of acquiring the position at dislocated prices. On the other hand, higher price efficiency increases trading benefits by reducing the maturity of investment due to faster convergence of prices to fundamental value. In our paper, we focus on the former effect by shutting down the latter effect because we are interested in exploring competition for informed trading among firms. See Dow, Han, and Sangiorgi (2021) for the analysis on this trade-off.

19This is analogous to the indifference condition for informed investors in Dow, Han, and Sangiorgi (2021) in which informed investors’ preference for shorter duration arises from the opportunity cost of capital. The difference is that here, informed investors like shorter horizons due to the possibility of early liquidation. A similar condition arises in Shleifer and Vishny (1990), but with exogenous duration and in a model without private signals.
Proof. See Appendix A.

The proposition shows that, fixing other firms’ maturity choices, a firm’s price informativeness increases as it shortens its own maturity. This is intuitive in light of investor preferences for shorter maturities. Furthermore, there is a spillover effect because a decrease in one firm’s maturity decreases other firms’ price informativeness. Because the amount of informed investors is fixed, firms compete for price informativeness.

4.2 Optimal Managerial Compensation

In this subsection, we derive the optimal managerial compensation contract of each firm. In case the price reveals the signal of informed investors, managerial compensation depends only on the signal because it is a sufficient statistic for the final payoff (see, for example, Holmstrom (1979) or Shavell (1979)). In case the price does not reveal the signal, managerial compensation depends on the final payoff, if available (because the manager remains until \( t = 2 \), or the manager exits at \( t = 1 \) and the firm’s project also matures at \( t = 1 \)).

Hence, there are only five states relevant for the contract, as follows: (i) the price reveals the signal to be good (\( \omega = G \)), (ii) the price reveals the signal to be bad (\( \omega = B \)), (iii) the price is non-revealing and the manager stays until a successful outcome (\( \omega = S \)), (iv) the price is non-revealing and the manager stays until an unsuccessful outcome (\( \omega = F \)), (v) the price is non-revealing and the manager exits before the outcome is realized (\( \omega = \emptyset \)). A contract will therefore specify non-negative payments corresponding to each of those five states \( \{w^n_G, w^n_B, w^n_S, w^n_F, w^n_\emptyset\} \).

Consider firm \( n \in N \) offering a contract to its manager that induces high managerial effort. We solve the optimal contracting problem taking the maturity choice \( \tau^n \) and price efficiency \( \lambda^n \) as given. The shareholders’ expected wage expense (or the wage bill), denoted by \( E[\tilde{w}^n] \), is given by

\[
E[\tilde{w}^n] = \lambda^n (\sigma_G w^n_G + (1 - \sigma_G) w^n_B) \\
+ (1 - \lambda^n) \left[(1 - \delta^n) (\rho_H w^n_S + (1 - \rho_H) w^n_F) + \delta^n w^n_\emptyset \right]. 
\]

(14)

An optimal contract \( \{w^{*n}_G, w^{*n}_B, w^{*n}_S, w^{*n}_F, w^{*n}_\emptyset\} \) solves the following optimization problem that minimizes the shareholders’ wage bill:

\[
W^n(\tau^n) \equiv \min_{\{w^n_G, w^n_B, w^n_S, w^n_F, w^n_\emptyset\}} E[\tilde{w}^n],
\]

(15)
subject to (i) the manager’s participation constraint (PC):

\[
\left\{ \begin{array}{l}
\lambda^n [\sigma_G u(w^n_G) + (1 - \sigma_G) u(w^n_B)] \\
+ (1 - \lambda^n) [(1 - \delta^n) (\rho_H u(w^n_S) + (1 - \rho_H) u(w^n_F)) + \delta^n u(w^n_\emptyset)]
\end{array} \right\} \geq \bar{u},
\]

(16)

and (ii) the manager’s incentive compatibility constraint (IC):

\[
\lambda^n \Delta \sigma (u(w^n_G) - u(w^n_B)) + (1 - \lambda^n) (1 - \delta^n) \Delta \rho (u(w^n_S) - u(w^n_F)) \geq K,
\]

(17)

and (iii) the limited lability constraint (LL):

\[
w^n_G, w^n_B, w^n_S, w^n_F, w^n_\emptyset \geq 0.
\]

(18)

Then, the solution to the optimization problem in Eqs. (15)-(18) is given by the following lemma:

**Proposition 2.** *(Optimal Managerial Contract)* Given \(\tau^n\), there exists a unique optimal contract. For the optimal contract, \(w^n_B = w^n_F = w^n_\emptyset = 0\) and \(w^n_G > w^n_S > 0\) where \(w^n_G\) and \(w^n_S\) simultaneously solve

\[
\lambda^n \Delta \sigma (u(w^n_G) - u(w^n_B)) + (1 - \lambda^n) (1 - \delta^n) \Delta \rho (u(w^n_S) - u(w^n_F)) = K
\]

(19)

\[
\sigma_G \Delta \rho u'(w^n_G) = \Delta \sigma \rho_H u'(w^n_G).
\]

(20)

Furthermore, the shareholders’ wage bill \(\mathcal{W}^n\) is increasing and concave in \(\tau^n\), and its first-order derivative is given by

\[
\frac{\partial \mathcal{W}^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} [\sigma_G \Psi(w^n_G) - \rho_H (1 - \delta^n) \Psi(w^n_S)] - (1 - \lambda^n) \delta \rho_H \Psi(w^n_S) > 0,
\]

(21)

where \(\Psi(\cdot)\) is a negative, decreasing, weakly concave function such that

\[
\Psi(w) \equiv w - \frac{u(w)}{u'(w)}.
\]

**Proof.** See Appendix B.

Together, Propositions 1 and 2 show that firms with shorter maturity anticipate a lower agency cost. The optimal compensations in state \(\omega = G\) and \(\omega = S\) are determined by the two equations in Eqs. (19)-(20), where Eq. (19) is the IC constraint, and Eq. (20) is the optimality condition that equates the marginal costs across the two states. The RHS of Eq. (21) represents the marginal effect on the wage bill of increased project maturity. The first term is due to the impact of decreased price informativeness (decreased \(\lambda^n\) from the increase in \(\tau^n\) due to Proposition 1). This effect is negative because it is more costly for shareholders to
provide incentives when the price is less informative. The second term is due to the manager’s impatience in case of positive $\delta$. This effect is also negative because it is more costly for shareholders to provide incentives with longer maturity when the manager may exit early.

4.3 Choice of Project Maturity

In this subsection, we solve each firm’s maturity choice problem by embedding endogenous price informativeness in Section 4.1 and the optimal contract in Section 4.2 into it.

Recall that each firm owns a production technology whose final payoff decreases as the firm shortens its project maturity (Eq. (1)). In the financial market equilibrium, price informativeness increases as the firm shortens its project maturity (Proposition 1); informed investors trade off between higher speculative profits and shorter maturities because the possibility of a liquidity shock makes them prefer short-horizon stocks. In the optimal contract that induces managerial effort, the wage bill decreases as the firm shortens its project maturity (Proposition 2).

By the previous results, we can represent the optimization problem of firm $n$’s shareholders in Eq. (2) as

$$\max_{\tau^n \in [0,1]} V^n(\tau^n) - W^n(\tau^n),$$

(22)

where $V^n(\tau^n)$ is the expected value of the final payoff given high managerial effort and maturity choice $\tau^n$ such that

$$V^n(\tau^n) = f(\tau^n) + \rho_H \Delta V,$$

and $W^n(\tau^n)$ is the wage bill under the optimal contract given $\tau^n$ in Eq. (15).

We can now show that there exists a unique choice of maturity that maximizes shareholder value, and the choice is determined by the trade-off between production and agency cost.

**Proposition 3. (Optimal Maturity Choice)** Given the choices of other firms $\{\tau^m\}_{m \in \mathcal{N} \setminus \{n\}}$, there exists a unique interior solution $\tau^{*n}$ for the optimization problem in Eq. (22). Furthermore, $\tau^{*n}$ solves

$$f'(\tau^{*n}) = \frac{\partial \lambda^n}{\partial \tau^n} \left[ \sigma_G \Psi(w^{*n}_G) - \rho_H (1 - \delta \tau^{*n}) \Psi(w^{*n}_S) \right] - (1 - \lambda^n) \delta \rho_H \Psi(w^{*n}_S),$$

(23)

where $w^{*n}_G$ and $w^{*n}_S$ simultaneously solve:

$$\lambda^n \Delta \sigma u(w^{*n}_G) + (1 - \lambda^n) (1 - \delta \tau^{*n}) \Delta \sigma \rho_1 (w^{*n}_S) = K$$

$$\sigma_G \Delta \sigma u'(w^{*n}_G) = \Delta \rho_H u'(w^{*n}_G).$$
Eq. (23) is the first-order condition for the optimization problem (derived from Eqs. (1) and (21)), whose LHS is the marginal change in the firm’s production, and the RHS is the marginal change in the expected cost of compensation. Note that we suppress dependence of $\lambda^n, w^*_G$ and $w^*_H$ on $\tau^*$ to save on notation.

How does a firm’s maturity choice affect other firms? The next proposition provides the answer:

**Proposition 4. (Strategic Complementarities)** A firm’s optimal maturity choice $\tau^*_n$ in Proposition 3 is increasing in other firms’ maturity choices, that is,

$$\frac{\partial \tau^*_n}{\partial \tau_m} > 0 \quad \text{for all } m \in \mathcal{N} \setminus \{n\}.$$ 

The proposition establishes that firms’ maturity choices are strategic complements when one firm chooses a shorter maturity project, the other firms want to do the same. Intuitively, when a firm shortens its project maturity, it increases its price informativeness at the expense of other firms’ price informativeness (Proposition 1). Thus, other firms’ agency cost goes up, increasing their marginal benefit of shortening project maturity to regain price informativeness.

5 Equilibrium

This section describes the equilibrium concept, shows equilibrium existence, and characterizes equilibrium properties.

5.1 Definition and Existence

We define an equilibrium as follows:

**Definition 1.** An equilibrium consists of project maturity choices $\{\tau^n\}_{n \in \mathcal{N}}$, price informativeness $\{\lambda^n\}_{n \in \mathcal{N}}$, and compensation contracts $\{\tilde{w}^n\}_{n \in \mathcal{N}}$ such that,

1. Given the choices of other firms $\{\tau^m\}_{m \in \mathcal{N} \setminus \{n\}}$, shareholders of each firm $n \in \mathcal{N}$ choose maturity $\tau^n$ to maximize firm $n$’s value in Eq. (22).
2. Given $\{\tau^n\}_{n \in \mathcal{N}}$, price informativeness $\{\lambda^n\}_{n \in \mathcal{N}}$ satisfy the indifference condition Eq. (12) and the informational resource constraint Eq. (13).

The proof of Proposition 4 shows that the game played by firms at the maturity choice stage is a supermodular game, i.e., a game of strategic complementarities (Topkis, 1998). In a supermodular game, best responses are increasing.

Although they are determined as part of equilibrium, we drop some less important ingredients for brevity in Definition 1. For example, realizations of prices and order flows are not needed because only price informativeness matters for the equilibrium choice of project maturities.
3. Given \( \tau^n \) and \( \lambda^n \), shareholders of each firm \( n \in \mathcal{N} \) choose contract \( \tilde{w}^n \) to minimize the expected cost of managerial compensation in Eq. (15).

We focus on pure strategy equilibria for our analysis.\(^{22}\) Because payoff functions are symmetric and best responses are increasing (Proposition 4), any pure strategy equilibrium must be a symmetric equilibrium in which all maturities are identical. Then price informativeness should be identical across all firms due to the indifference condition Eq. (12); the informational resource constraint Eq. (13) therefore implies that price informativeness should be equal to

\[
\lambda^n = \frac{1}{N\bar{z}} \quad \text{for all } n \in \mathcal{N}. \tag{24}
\]

If an individual firm increases its project maturity, it loses more informed investors and its price informativeness decreases, as shown in Proposition 1, i.e., \( \partial \lambda^n / \partial \tau^n < 0 \) for all \( n \in \mathcal{N} \). However, if all firms do so by the same quantity, there is no change to informativeness because the total mass of informed investors is fixed (Eq. (24)); attracting informed trade is a zero-sum game.\(^{23}\)

Consider any individual firm \( n \in \mathcal{N} \) choosing its level of maturity \( \tau^n \) when all other firms \( m \) choose the same maturity \( \tau^* \). If \( \tau^n = \tau^* \) satisfies the first-order condition in Eq. (23), \( \tau^* \) is an equilibrium maturity choice. Using the result in Eq. (24) and the intermediate value theorem, we can show that such an equilibrium \( \tau^* \) exists, is unique, and is interior.

**Theorem 1.** There exists a unique equilibrium. The equilibrium is symmetric and interior, and equilibrium maturity choice \( \tau^* \) satisfies

\[
f'(\tau^*) = \Theta(\tau^*) \left[ \sigma_G \Psi(w^*_G) - \rho_H (1 - \delta \tau^*) \Psi(w^*_S) \right] - \left(1 - \frac{1}{N\bar{z}}\right) \delta \rho_H \Psi(w^*_S), \tag{25}\]

where \( \Theta(\tau^*) \), the sensitivity of price informativeness to project maturity, is given by

\[
\Theta(\tau^*) \equiv -\frac{\gamma(N - 1)(N\bar{z} - 1)}{N^2\bar{z}(1 - \gamma \tau^*)} < 0, \tag{26}\]

and \( w^*_G \) and \( w^*_S \) simultaneously solve

\[
\frac{1}{N\bar{z}} \Delta \sigma u(w^*_G) + \left(1 - \frac{1}{N\bar{z}}\right) (1 - \delta \tau^*) \Delta \rho u(w^*_S) = K \tag{27}
\]

\[
\sigma_G \Delta \rho u'(w^*_S) = \Delta \sigma \rho H u'(w^*_G). \tag{28}\]

\(^{22}\)Echenique and Edlin (2004) show that when a game with strategic complementarities has mixed strategy equilibria, these equilibria are unstable. This justifies our focus on pure strategy equilibria.

\(^{23}\)Strictly speaking, since the payoffs are not fixed in total, the game itself is not zero-sum, but the amount of informed trade is fixed so intuitively, if we regard informed trade as the reward, it is a zero sum game.
The shareholder value for each firm is given by

\[ S^* = f(\tau^*) + \rho_H \Delta V - \left[ \frac{1}{Nz} \sigma_G w_G^* + \left( 1 - \frac{1}{Nz} \right) (1 - \delta \tau^*) \rho_H w_S^* \right]. \tag{29} \]

**Proof.** See Appendix D. ■

### 5.2 Benchmark Cases

We study two benchmark cases: (i) no financial market (autarky), (ii) there is a stock market for firms’ shares, but informed investors cannot switch between stocks (exogenous informed trading). To have a meaningful comparison, we focus on parameter values where managers are impatient \((\delta > 0)\).\(^{24}\) Existence and uniqueness of equilibrium as well as their equilibrium characteristics can be trivially proven as special cases of Theorem 1.

#### 5.2.1 Autarky

In case of autarky where the stock market does not exist, firms can incentivize managers only based on the final payoff. This corresponds to the special case of our model where the price is uninformative, and also does not react to firm’s maturity choice. Then, each firm’s maturity choice \(\tau^{Aut}\) should satisfy the first-order condition in Eq. (23) assuming \(\lambda^n = 0\) and \(\partial \lambda^n / \partial \tau^n = 0\), which is equivalent to

\[ f'(\tau^{Aut}) = -\delta \rho_H \Psi(w_S^{Aut}), \tag{30} \]

where \(w_S^{Aut}\) solves

\[ (1 - \delta \tau^{Aut}) \Delta u \left( w_S^{Aut} \right) = K. \tag{31} \]

The shareholder value for each firm is given by

\[ S^{Aut} = f(\tau^{Aut}) + \rho_H \Delta V - (1 - \delta \tau^{Aut}) \rho_H w_S^{Aut}. \tag{32} \]

#### 5.2.2 Exogenous Informed Trading

In our main model, we investigate how informed trade responds to firms’ maturity choices. In contrast, we now compare this to a benchmark in which informed trade in each firm’s stock is just fixed. Assume that informed investors are equally dispersed across all stocks. Then price informativeness is equal across all stocks, and is given by Eq. (24). We now have \(\partial \lambda^n / \partial \tau^n = 0\) for all \(n \in \mathcal{N}\) and each firm’s maturity choice \(\tau^{Ex}\) simply satisfies the first-order condition in

\(^{24}\)Without impatience \((\delta = 0)\), in case of autarky, firms will choose the maximal maturity \(\tau^n = 1\) because there is no benefit of reducing the wage bill.
Eq. (23), which is equivalent to

\[ f'(\tau^E) = - \left(1 - \frac{1}{N\bar{z}}\right) \delta \rho_H \Psi(w^E_S), \]  

(33)

and \( w^E_G \) and \( w^E_S \) simultaneously solve

\[ \frac{1}{N\bar{z}} \Delta \sigma u(w^E_G) + \left(1 - \frac{1}{N\bar{z}}\right) (1 - \delta \tau^E) \Delta \rho u(w^E_S) = K \]  

(34)

\[ \sigma_G \Delta \rho u'(w^E_G) = \Delta \sigma \rho u'(w^E_G). \]  

(35)

The shareholder value for each firm is given by

\[ S^E \equiv f(\tau^E) + \rho_H \Delta V - \left[ \frac{1}{N\bar{z}} \sigma_G w^E_G + \left(1 - \frac{1}{N\bar{z}}\right) (1 - \delta \tau^E) \rho_H w^E_S \right]. \]  

(36)

6 Main Results

6.1 Excessive Short-termism

We now compare equilibrium to the two benchmarks: the case with autarky (Section 5.2.1), and the case with exogenous informed trading (Section 5.2.2). The comparison allows us to understand the interaction among different economic forces. First, using stock prices allows a firm to incentivize the manager more efficiently. Thus, using price signals enhances firm value. Second, price informativeness reacts to maturity choices. Thus, shorter maturity is advantageous because it attracts informed investors, fixing other firms’ maturity choices. Third, other firms also react to the negative spillover effect of reduced price informativeness by shortening their own project maturity. But, this just leads to a race to the bottom where there are no winners, only losers: firms have inefficiently short maturities, but still have the same price informativeness as they would without competition for informed trade.

The first-order conditions (Eqs. (25), (30) and (33)) describe the trade-off between production efficiency and agency cost in the three different cases. In each equation, the LHS captures the marginal change in production with respect to a change in project maturity and the RHS captures the marginal change in agency cost with respect to a change in project maturity.

In the case autarky, the R.H.S in Eq. (30) shows that pursuing a longer term project increases the agency cost when the manager is impatient (\( \delta > 0 \)). In the case with exogenous informed trading, price informativeness dampens this effect. This is illustrated by the coefficient \( 1 - \frac{1}{N\bar{z}} \) in the R.H.S of Eq. (33). With probability \( \lambda^n = 1/(N\bar{z}) \) the price is informative and the manager can be rewarded in the short term. Price informativeness makes the compensation contract more efficient, thereby shrinking the agency cost of longer duration. This allows the firm to pursue longer term projects without impairing incentives, thereby enhancing
value. We call this the “price information effect.” It is similar to the effect that was identified and highlighted in the literature (e.g., Holmstrom and Tirole (1993)): stock prices are useful for monitoring managers; and in our model when it is difficult for wait for final payoff, this allows longer term projects to be implemented successfully.

On the other hand, when we consider endogenous informed trading, the first term on the RHS of Eq. (25) captures the impact on agency costs of competition among firms. In an individual problem, a firm can enhance its value by shortening project maturity, which reduces agency costs (Eq. (23) and Proposition 3). However, this creates a negative spillover effect to other firms, and does not result in any benefit in equilibrium once others’ reactions are endogenized. That is, price informativeness is still at the same level \( \lambda_n = 1/(N \bar{z}) \) for all \( n \in \mathcal{N} \), but project maturities are overly shortened as a result of competition. This leads to a loss in value. We call this the “competition for informativeness effect.”

The following theorem formalizes our results.

**Theorem 2.** (Excessive Short-termism) The case with exogenous informed trading has the longest maturity, i.e.,

\[
\tau^{Ex} > \max (\tau^*, \tau^{Aut}) .
\]

Furthermore, equilibrium with endogenous informed trading has the shortest maturity \( \tau^{Ex} > \tau^{Aut} > \tau^* \) if and only if the competition for informativeness effect dominates the price information effect, i.e.,

\[
\Theta(\tau^*) [\sigma_G \Psi(w^*_G) - \rho_H (1 - \delta \tau^*) \Psi(w^*_S)] > \delta \rho_H \left[ \left( 1 - \frac{1}{N \bar{z}} \right) \Psi(w^*_S) - \Psi(w^*_S^{Aut}) \right] .
\]  

(37)

**Proof.** See Appendix E.

The case with exogenous informed trading has a longer maturity than autarky because of the price information effect: using stock prices, shareholders can lengthen project maturity while still giving good managerial incentives. But in reality, informed trading can switch between firms depending on their project maturities: investors are attracted to shorter term projects. Recognizing this, firms can make their stock prices more informative by choosing projects that are more likely to mature early (the competition for informativeness effect). This offsetting effect may be strong enough that firms choose maturities that are even shorter than if they had no stock market listing. The LHS of Eq. (37) reflects the competition effect (the first term in Eq. (25)), the RHS reflects the price information effect (differentials between Eq. (30) and the second term in Eq. (25)). Figure 1 shows an example.

Shorter term projects in our model have lower final payoffs because \( f(\tau) \) is increasing in \( \tau \). Therefore, we have \( f(\tau^{Ex}) > f(\tau^{Aut}) > f(\tau^*) \) in case \( \tau^{Ex} > \tau^{Aut} > \tau^* \); equilibrium has
Figure 1: Equilibrium Short-termism and Shareholder Value. Parameter values: $\rho_H = .6, \sigma_G = .7, \Delta \rho = .5, \Delta \sigma = .6, K = 1, N = 10, z = 2, \delta = .5, \gamma = .5, \Delta V = 10$. The maturity-sensitive component of a firm’s output is $f(\tau) = \sqrt{1 - (1 - \tau)^2}$, and the utility of a manager given wage $w$ is $u(w) = \sqrt{w}$.

the smallest production compared to the two benchmarks. To address overall efficiency in the economy, however, we need to net off managerial compensation, and compare shareholder value across the three cases.

### 6.2 The Race to the Bottom

In this subsection, we study the (sub)optimality of equilibrium. Specifically, we want to analyze what would happen if firms coordinated on choosing project maturity; in other words, if they internalized any externalities imposed on other firms. To this end, we consider a social planner who chooses the maturity of firms $\tau^s$ uniformly across all firms to maximize the total shareholder value in the economy, but the planner is still constrained by the scarcity of informed trading in the financial market. In this sense, the planner’s choice of $\tau^s$ provides the constrained efficient benchmark. By aggregating shareholder value in Eq. (22) across all firms, we can represent the social planner’s problem as follows:

$$
\max_{\tau^s \in [0, 1]} \sum_{n=1}^{N} \left[ V^n(\tau^s) - W^n(\tau^s) \right],
$$

Because the social planner changes the maturity $\tau^s$ for all firms uniformly, there is no

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25 That is, the planner takes asset price informativeness in Eqs. (12)-(13) as given.
reallocation of informed trading across firms, i.e., the sensitivity of informed trading $\Theta(\tau^s)$ is zero. Then, the first-order condition is identical to the case with exogenous informed trading in Eq. (33), which implies the solution for the social planner, $\tau^{*s}$, is identical to the equilibrium maturity under exogenous informed trading, $\tau^{Ex}$. Thus, the maturity with endogenous informed trading is inefficient, and shareholder value is dominated by the case with exogenous informed trading. Finally, shareholder value under autarky is dominated by that under endogenous informed trading because individual firms can always ignore the share price in the managerial contract, and then choose maturity. Figure 1 shows an example.

The following theorem summarizes the result.

**Theorem 3.** Equilibrium is constrained inefficient. Furthermore, the shareholder value for each firm across different cases is ranked as

$$S^{Aut} \leq S^* < S^{Ex}.$$ 

The theorem shows that there will be an improvement in shareholder value for all firms if they lengthen their maturities in a coordinated manner; shareholder value in equilibrium is suboptimally low due to overcompetition to attract informed trading. This is because firms are competing for a fixed amount of informed trading. They do this by choosing short-term projects. This is similar to the “race to the bottom” described by U.S. Supreme Court Justice Louis Brandeis, in which states designed regulations to compete for firms, which were attracted to incorporate in “states where the cost was lowest and the laws least restrictive ... The race was one not of diligence, but of laxity.” (Liggett Co. v. Lee, 288 U.S. 517, 558-59 (1933), dissenting opinion). It has been used to describe competition among stock exchanges by choice of listing regulations (Chemmanur and Fulghieri (2006)), and competition among jurisdictions by choice of tax rates (Mast (2020)).

This is also parallel with the classical idea of the “tragedy of the commons” (e.g., Hardin (1968), Levhari and Mirman (1980)) where individuals, who have access to a common pool of resource but do not internalize their externalities, end up with a tragic overexploitation of resource (such as fisheries, irrigation systems). In our model, informed trading is the common resource which can be used for more informative managerial compensation schemes. But individual firms do not internalize their externalities, and try to overexploit the informed trading resource with shortened maturities. This overcompetition only creates a suboptimal outcome of extreme short-termism, which leads to the loss of shareholder value in aggregation (which obviously stems from the loss in aggregate production).
6.3 The Impact of Competition on Short-Termism

In this subsection, we study the impact of competition on short-termism using comparative statics with respect to the number of firms $N$.

According to conventional wisdom, competition makes firms leaner, in other words, more efficient and more profitable (e.g., Porter (1990)). In the literature on optimal contracting, however, it has been noted that increased competition may not always lead to an improvement. More competition in product markets may increase agency costs (e.g., Nalebuff and Stiglitz (1983), Scharfstein (1988), Hermalin (1992), Schmidt (1997), Raith (2003)). We also use an agency framework, but we study a different channel for competition. In a highly competitive industry, not only are firms desperate to attract buyers, they are also desperate to attract investors. Firms compete for informed investors who have industry-specific knowledge and limited trading capital.

We can show analytically that more intense competition leads to increased short-termism. In our comparative statics, we fix the product $N\bar{z}$ to be a constant to keep the quantity of informed trade per firm (relative to noise trade) at the same level. Because an increase in $N$ is compensated by a decrease in $\bar{z}$, the equilibrium price informativeness is unchanged regardless of the level of $N$, thus, equals that in Eq. (24).

Proposition 5. (Competition) Fixing $N\bar{z}$, higher competition induces more short-termism and lower shareholder value, i.e., $\tau^*$ and $S^*$ are decreasing in $N$.

Proof. See Appendix F. ■

Figure 2 confirms the result of Theorem 2 by showing that the case with exogenous informed trading features the longest maturities of projects. Equilibrium with endogenous informed trading may have shorter maturities than autarky if competition becomes severe. It also confirms Theorem 3 by showing that shareholder value in equilibrium is lower than the case with exogenous informed trading but greater than autarky. As predicted by Proposition 3, maturities are shortened as the number of firms increases. As equilibrium project maturity decreases further compared to its second-best value, shareholder value decreases.

Our prediction is broadly consistent with empirical findings in the literature. There is some evidence that product market competition can induce short-term pressure (e.g., Aghion, Van Reenen, and Zingales (2013), Acharya and Xu (2017)).

6.4 Other Comparative Statics

In this subsection, we study the impact of various model parameters on equilibrium short-termism. First, we consider investor short-termism.

\footnote{The result is unchanged if we vary $N$ with $\bar{z}$ fixed.}
Parameter values: \( \rho_H = 0.6, \sigma_G = 0.7, \Delta \rho = 0.5, \Delta \sigma = 0.6, K = 1, N = 10, z = 2, \delta = 0.5, \gamma = 0.5, \Delta V = 10. \) The maturity-sensitive component of a firm’s output is 
\[
f(\tau) = \sqrt{1 - (1 - \tau)^2},
\]
and the utility of a manager given wage \( w \) is 
\[
u(w) = \sqrt{w}.
\]

**Proposition 6.** (Investor short-termism) A shift in investor preferences toward early consumption induces more short-termism and lower shareholder value, i.e., \( \tau^* \) and \( S^* \) are decreasing in \( \gamma \).

*Proof.* See Appendix F. ■

When investors become more short-term oriented, they become more responsive to a firm’s decrease in project maturity. As a result, the sensitivity of price informativeness \( \Theta(\tau^*) \) in Eq. (26) becomes more negative as \( \gamma \) increases. That is, the competition for informativeness effect becomes more pronounced, resulting in more excessive corporate short-termism. As equilibrium project maturity decreases further compared to its second-best value, shareholder value decreases.

Next, we consider the impact of agency problem on corporate short-termism.

**Proposition 7.** (Agency problem) An increase in managers’ impatience or effort cost induces more short-termism and lower shareholder value, i.e., \( \tau^* \) and \( S^* \) are decreasing in \( \delta \) and \( K \).

*Proof.* See Appendix F. ■

As the agency problem becomes more severe, equilibrium becomes more short-term. In contrast to the comparative statics results in Propositions 5 and 6 that leave the second best
unaffected, a more severe agency problem decreases project maturity also in the second best. However, because firms’ project maturity choices are strategic complements (Proposition 4), there is an amplification effect in equilibrium that is absent in the second best. That is, equilibrium short-termism is more sensitive to agency cost parameters compared to the second best.

7 Long-Term Investors

Recent trends in investment management aim to pursue long-term value. For example, in their joint statement in March 2020, large public investors, including Japan’s GPIF (Government Pension Investment Fund), the CALSTRS (California State Teachers’ Retirement System) and the UK’s USS Investment Management, write “asset managers that only focus on short-term, explicitly financial measures, and ignore longer-term sustainability-related risks and opportunities are not attractive partners for us.” Despite such an increasing pressure on long-term investing, it is unclear why and whether the increase in long-term investing will be able to curb corporate short-termism, and therefore promote economic efficiency.

Our results in this section suggest that long-term investing is unable to make an impact on curbing short-termism unless it exceeds a critical mass. Even though long-term investors shun short-termist firms, other investors who prefer short-term investment can simply fill that void. Therefore, there is no impact in equilibrium. This is in line with recent empirical findings by Berk and van Binsbergen (2021) who find that the impact of ESG investing on firms’ cost of capital is too small to have any meaningful real impact. If the mass of long-term investors is sufficiently large that they are marginal investors in all firms, however, they can generate a significant effect against the race to the bottom in project maturities. Finally, when the mass of long-term investors is in an intermediate range, there is an equilibrium in which firms choose different project maturities to cater to different investor clientele.

27 More formally, and writing a firm’s first-order-condition in equilibrium in Eq. (25) as \( \hat{\gamma}(\tau^*) = 0 \), by implicit differentiation we have \( \frac{\partial \tau^*}{\partial \delta} = -\frac{\partial Y(\tau^*)}{\partial \gamma(\tau^*)} \). In Appendix F we show that \( \frac{\partial Y(\tau^*)}{\partial \gamma(\tau^*)} \) is negative, and

\[
\frac{\partial \hat{\gamma}(\tau^*)}{\partial \delta} = -\Theta(\tau^*) \frac{\partial \left[ \sigma(\psi(\omega^G_\tau(\tau^*)) - \rho_H (1 - \delta \tau^*) \psi(w^S_\tau(\tau^*)) \right]}{\partial \delta} \\
\quad + \left( 1 - \frac{1}{N^z} \right) \delta \rho_H \psi'(w^S_\tau(\tau^*)) \frac{\partial w^G_\tau(\tau^*)}{\partial \delta} \
\]  

(39)

and both term in the r.h.s. of Eq. (39) are negative. The first term in the r.h.s. of Eq. (39) depends on sensitivity of price informativeness \( \Theta(\tau^*) \) and represents the amplification effect due to the strategic complementarity in project maturity. This term is equal to zero in the second best where price informativeness is independent of project maturity. A similar argument holds for the comparative static in \( K \).

28 The statement is titled “Joint statement on the importance of long-term, sustainable growth”. It can be found in the following link: [https://www.gpif.go.jp/en/investment/Our_Partnership_for_Sustainable_Capital_Markets_Signatories.pdf](https://www.gpif.go.jp/en/investment/Our_Partnership_for_Sustainable_Capital_Markets_Signatories.pdf)
In this section, we consider an extension of our model by introducing a fraction $\mu$ of “long-term investors” who stay in the economy until $t = 2$ (i.e., until all projects pay off). The remaining fraction $1 - \mu$ is “short-term investors” who may exit the economy in $t = 1$ with probability $\gamma$, as in our benchmark model.

We first investigate symmetric equilibria of our extended model. We denote $\tau^{\mu}$ the equilibrium project maturity. The next proposition summarizes our results under a symmetric equilibrium:

**Proposition 8.** (Symmetric equilibrium with long-term investors) (i) There exists $\mu^* \in (0, 1/N)$ such that for $\mu \leq \mu^*$, there is a unique symmetric equilibrium and $\tau^{\mu} = \tau^*$, i.e., equilibrium is identical to the one without long-term investors in Theorem 1; for $\mu \in (\mu^*, 1/N)$, if a symmetric equilibrium exists, it is $\tau^{\mu} = \tau^*$. (ii) For $\mu \geq 1 - 1/N$, there is a unique symmetric equilibrium and $\tau^{\mu} = \tau^{Ex}$, i.e., equilibrium is identical to the one with exogenous informed trading. (iii) For $\mu \in [1/N, 1 - 1/N)$, there is no symmetric equilibrium.

Proposition 8-(i) shows that long-term investors have no impact on equilibrium if their mass is smaller than the threshold $\mu^*$. Intuitively, when the mass of long-term investors is sufficiently small, short-term investors are marginal on all firms. Therefore, a firm’s price informativeness is determined by short-term investors’ indifference condition as in Eq. (12). Hence, the symmetric equilibrium project maturity is the same as in the benchmark case without long-term investors.

Proposition 8-(ii) shows that when the mass of long-term investors is larger than the threshold $1 - 1/N$, their presence eliminates the race to the bottom that is the cause of excessive short-termism. Intuitively, when the mass of long-term investors is sufficiently large, they are the marginal investors. Because long-term investors’ trading profits do not depend on project maturities, a firm’s project maturity has not impact on its price informativeness. As a result, the symmetric equilibrium is the same as in the case with exogenous informed trading, and this equilibrium is constrained efficient (Section 6.2).

The intuition for Proposition 8-(iii) is as follows. Consider a candidate symmetric equilibrium $\tau^{\mu}$. For intermediate values of $\mu$, if firm $n$ deviates to a longer project maturity, there are enough long-term investors to step in and sustain an equal level of price informativeness across all firms even though short-term investors do not invest in firm $n$. Therefore, for all $\tau^{\mu} < \tau^{Ex}$, a firm can profitably lengthen its project maturity without a reduction in price informativeness. At the same time, if firm $n$ deviates to a shorter project maturity, there are enough short-term investors to sustain a higher price informativeness for firm $n$ even though long-term investors do not invest in this firm. Therefore, for all $\tau^{\mu} > \tau^*$, a firm can increase its value by shortening its project maturity and increasing its price informativeness. Because $\tau^* < \tau^{Ex}$ (Theorem 1), there is no symmetric equilibrium in the intermediate region for $\mu$. 

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When the mass of long-term investors is in an intermediate range, however, we can show that a “clientele equilibrium” exists where firms separate into long- and short-term maturities. The next proposition summarizes our results under a clientele equilibrium:

**Proposition 9. (Clientele equilibrium)** For \( 1 - (N - 1) \bar{z} < \mu < 1 - \frac{1}{N} \) there exists a clientele equilibrium in which a fraction \( \alpha_S \) of firms choose maturity \( \tau_S \) and a fraction \( 1 - \alpha_S \) of firms choose maturity \( \tau_L \), where \( \alpha_S, \tau_S, \tau_L \) are such that \( 0 < \alpha_S < 1 \) and \( \tau^* < \tau_S < \tau_L < \tau^{Ex} \). In this equilibrium, short-term investors invest in short-term firms and long-term investors invest in long-term firms. Price efficiency for short- and long-term firms satisfies \( \lambda_L < \frac{1}{N\bar{z}} < \lambda_S \). Equilibrium shareholder value, \( S^{Cl} \), satisfies \( S^* < S^{Cl} < S^{Ex} \).

Note that, for analytical simplicity, the proof of Proposition 9 ignores the integer constraint on the number of firms in each group. When the integer constraint is taken into account, we can establish numerically existence of a clientele equilibrium as shown in Figure (to be added).

The clientele equilibrium in Proposition 9 has the following important features.

First, ex-ante identical firms choose different project maturities to cater to different investor clienteles. Hence, firms become ex-post heterogeneous in equilibrium: long-term firms become more productive than short-term firms, but attract less investor attention. Thus, long-term firms have less informative prices and face higher agency cost compared to short-term firms.

Second, long-term firms choose shorter project maturities and have lower price efficiency compared to the second-best, and short-term firms choose longer project maturities and have higher price efficiency compared to the competitive equilibrium without long-term investors. Intuitively, each firm must have no incentive to deviate to the other type. For this to happen, long-term firms must be sufficiently less valuable than in the second best, and short-term firms must be sufficiently more valuable than in the competitive equilibrium without long-term investors.

### 8 Policy Implications: Salary Cap

It has been suggested that short-termism goes hand in hand with excessive incentive compensation for CEOs (see, for example, Porter (1992)). Limits to CEO compensation have been proposed as a mechanism to improve the management of listed companies. For example, in 1993 the Clinton administration introduced a salary cap on CEO compensation in the form of $1 million deductibility cap (see Murphy (2013) for further details). In the policy debate, critiques of corporate governance often include short termism in a long list of alleged malfunctions, so it seems relevant to study the effect of salary cap in our model. Does a salary cap in our model promote shareholder value by mitigating excessive short-termism? Intuitively, salary is a highly imperfect proxy for project duration. Therefore, it is quite possible that
a salary cap has a number of different effects, and has a net effect in precisely the opposite direction. This turns out to be the case in our model.

We use the same setup of our model as in Section 3 but deviate from it only by assuming that there is an upper bound $\bar{w}$ on managerial compensation in each state. In that case, the optimal contracting problem defined in Eqs. (14)-(18) needs to be augmented by an extra constraint such that

$$w^n_G, w^n_B, w^n_S, w^n_F, w^n_0 \leq \bar{w}. \quad (40)$$

We focus on the range of salary cap $\bar{w}$ that ensures that the incentive compatibility is implementable. We can solve the equilibrium of our model under salary cap in a similar fashion as in Section 5. For notational convenience, we will use a double asterisk notation (**) for the optimal solution under the salary cap, and use a single asterisk notation (*) for the optimal solution without the salary cap.

**Proposition 10.** Suppose that the equilibrium contract is given by $w^*_G, w^*_S$ without salary cap. Then, the equilibrium contract under salary cap is given by

$$w^{**}_G = \min(w^*_G, \bar{w}),$$

$$w^{**}_S = \begin{cases} w^*_S & \text{if } w^*_G < \bar{w} \\ u^{-1} \left( \frac{K - \frac{1}{\tau^*} \Delta u(\bar{w})}{\Delta \rho F(1 - \delta\tau^*)} \right) & \text{otherwise}, \end{cases}$$

where $\tau^{**}$ solves the following first-order condition of each firm’s maturity choice problem under salary cap:

$$f'(\tau^{**}) = \Theta(\tau^{**}) \left[ \sigma_G \left( w^{**}_G - \frac{\Delta \sigma H}{\sigma_G \Delta \rho} \frac{u(w^{**}_G)}{w^{**}_G} \right) - \rho_H (1 - \delta \tau^{**}) \Psi(w^{**}_S) \right] - \left( 1 - \frac{1}{Nz} \right) \delta \rho_H \Psi(w^{**}_S). \quad (41)$$

The maturity becomes shorter (i.e., $\tau^{**} < \tau^*$) when the salary cap just starts binding, and the shareholder value under salary cap is smaller than the case without salary cap.

The equilibrium wage is identical to that without salary cap as long as it does not bind, but the wage in state $S$ needs to be adjusted to satisfy the IC constraint as soon as the constraint starts binding. Theorem 10 shows that the initial impact of a salary cap does not prevent short-termism but rather promotes it. It is because it becomes more expensive to incentivize managers under salary cap at any level of project maturities. Therefore, salary cap forces firms to complement stock-based compensation with short-term compensation. However, pushing salary cap even tighter reduces the sensitivity of stock-based compensation, thereby eventually
preventing short-termism. But this does not help improving shareholder value because it increases wage bill.

In the absence of salary cap, firms can incentivize their managers using high powered contracts. Under salary cap, however, firms respond to the restriction by shortening the maturity of their projects. This actually hurts shareholder value because firms are already engaging excessive short-termism and further shortening of maturities hurt the value even further. Figure 3 demonstrates our results under salary cap.

Figure 3: Equilibrium Short-termism and Shareholder Value under Salary Cap.

In this paper, we explore whether managerial stock-price based compensation leads to excessive short-termism. In previous models, firms’ and managers’ prioritizing of short term results as an individually rational response to short term pressure from the stock market is also collectively rational, in other words it is efficient given the informational constraints that govern managerial incentives and project selection.

In contrast, we study short-termism that is individually rational, but collectively suboptimal. We study an economy with a stock market where informed investors have short-horizons. Regardless of investors’ horizons, stock-based compensation can improve shareholder value of an individual firm by reducing agency cost because stock prices are informative about future cash flows. This allows the firm to pursue longer term projects without impairing incentives.

Because price informativeness is endogenous, however, competition for informed trading can destroy shareholder value as a result of negative externalities to price informativeness of other

9 Conclusion

In this paper, we explore whether managerial stock-price based compensation leads to excessive short-termism. In previous models, firms’ and managers’ prioritizing of short term results as an individually rational response to short term pressure from the stock market is also collectively rational, in other words it is efficient given the informational constraints that govern managerial incentives and project selection.

In contrast, we study short-termism that is individually rational, but collectively suboptimal. We study an economy with a stock market where informed investors have short-horizons. Regardless of investors’ horizons, stock-based compensation can improve shareholder value of an individual firm by reducing agency cost because stock prices are informative about future cash flows. This allows the firm to pursue longer term projects without impairing incentives.

Because price informativeness is endogenous, however, competition for informed trading can destroy shareholder value as a result of negative externalities to price informativeness of other

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firms. Firms compete for informed investors by reducing project maturities because informed investors are short-horizoned. Negative spillover effects arise but firms do not internalize such adverse effects to other firms. Therefore, a short-termism trap arises in equilibrium; firms reduce their maturity excessively, thereby reducing shareholder value. We explore potential policy implications related to short-termism.

This paper is part of a broader research project exploring the impact of limited informed investor capital on stock market performance (Dow and Han (2018), Dow, Han, and Sangiorgi (2021)). Informed trading helps stock markets to perform their economic functions, and shortages of informed capital can disrupt those functions.
 Appendix A

Proof of Lemma 1

Because all stock payoffs and signals are jointly independent, there is no learning across stocks in the market. Therefore, we can analyze market makers’ learning informed investors’ private information for each stock \( n \in \mathcal{N} \) separately.

Let \( g(z^n) \) be the probability density function of noise trading \( z^n \). Because \( z^n \) is uniformly distributed on \([-\bar{z}/2, \bar{z}/2]\), we have \( g(z^n) = 1/\bar{z} \) for \( z^n \in [-\bar{z}/2, \bar{z}/2] \) and \( g(z^n) = 0 \) otherwise. By Bayes’ Rule, market makers’ posterior belief that \( s^n = G \) conditional on aggregate order flow \( X^n(0) \) is given by\(^{29}\)

\[
Pr(s^n = G|X^n(0)) = \frac{\sigma_G g(X^n(0) - \mu^n)}{\sigma_G g(X^n(0) - \mu^n) + (1 - \sigma_G) g(X^n(0) + \mu^n)}.
\]

(A.1)

From Eq. (A.1), it is immediate that

\[
Pr(s^n = G|X^n(0)) = \begin{cases} 
0 & \text{if } -\mu^n - \frac{\bar{z}}{2} \leq X^n(0) < -\mu^n + \frac{\bar{z}}{2}, \\
\sigma_G & \text{if } -\mu^n + \frac{\bar{z}}{2} \leq X^n(0) \leq -\mu^n + \frac{\bar{z}}{2}, \\
1 & \text{if } -\mu^n + \frac{\bar{z}}{2} < X^n(0) \leq \mu^n + \frac{\bar{z}}{2}.
\end{cases}
\]

(A.2)

Given \( \tau^n \) (firm \( n \)'s maturity choice) the posterior belief about the liquidation value conditional on private information \( s^n \) is

\[
E[V^n|X^n(0)] = f(\tau^n) + \sum_{s^n \in \{G,B\}} Pr(R = \Delta V|s^n)Pr(s^n|X^n(0)) \Delta V,
\]

which implies\(^{30}\)

\[
E[V^n|X^n(0)] = \begin{cases} 
 f(\tau^n) + \nu_B \Delta V & \text{if } -\mu^n - \frac{\bar{z}}{2} \leq X^n(0) < -\mu^n + \frac{\bar{z}}{2}, \\
f(\tau^n) + \rho_H \Delta V & \text{if } -\mu^n + \frac{\bar{z}}{2} \leq X^n(0) \leq -\mu^n + \frac{\bar{z}}{2}, \\
f(\tau^n) + \nu_G \Delta V & \text{if } -\mu^n + \frac{\bar{z}}{2} < X^n(0) \leq \mu^n + \frac{\bar{z}}{2}.
\end{cases}
\]

(A.3)

Now, we derive the price informativeness for stock \( n \). From Eq. (A.2), it is clear that prices are informative except when \( \mu^n - \frac{\bar{z}}{2} \leq X^n(0) \leq -\mu^n + \frac{\bar{z}}{2} \). In case \( s^n = H \), we have \( X^n(0) = \mu^n + z^n \). Then, prices are uninformative if \( -\frac{\bar{z}}{2} \leq z^n \leq -2\mu^n + \frac{\bar{z}}{2} \), which occurs with probability \( 1 - \mu^n/\bar{z} \). In case \( s^n = L \), we have \( X^n(0) = \mu^n + z^n \). Then, prices are uninformative

\(^{29}\)See, for example, Lemma 1 in Dow, Han, and Sangiorgi (2021) for a similar analysis with uniformly-distributed noise trading, and also Lemma 4 in Dow and Han (2018) for an analysis with noise trading under general distributions.

\(^{30}\)To see this, \( Pr(R = \Delta V|s^n = G) \times 0 + Pr(R = \Delta V|s^n = B) \times 1 = \nu_B, Pr(R = \Delta V|s^n = G) \times \sigma_G + Pr(R = \Delta V|s^n = B) \times (1 - \sigma_G) = \rho_H \) due to the first equation in Eq. (3), and finally \( Pr(R = \Delta V|s^n = G) \times 1 + Pr(R = \Delta V|s^n = B) \times 0 = \nu_G \).
if \( 2\mu^n - \frac{\bar{z}}{2} \leq z^n \leq \frac{\bar{z}}{2} \), which occurs with probability \( 1 - \mu^n/\bar{z} \). Therefore, prices are informative with probability \( \mu^n/\bar{z} \) regardless of signals.

### Proof of Lemma 2

First, we rewrite the optimization problem in Eq. (10) in a more general form as follows:

\[
J^n_0(s^n) \equiv \max_{x_i^n(0) \in \{-1,0,1\}} -E[P^n(0)|s^n]x_i^n(0) + \gamma \Gamma^n(s^n)x_i^n(0) + (1 - \gamma)E[J^n_1(x_i^n(0), s^n, P^n(0))|s^n],
\]

where

\[
\Gamma^n(s^n) \equiv (1 - \tau^n)E[V^n|s^n] + \tau^nE[P^n(1)|s^n],
\]

and

\[
J^n_1(0, s^n, P^n(0)) \equiv \max_{x_i^n(0) \in \{-1,0,1\}} \tau^nE[(V^n - P^n(1))|s^n, P^n(0)]x_i^n(1)
\]

\[
J^n_1(1, s^n, P^n(0)) \equiv (1 - \tau^n)E[V^n|s^n] + \tau^n \max \{E[V^n|s^n], E[P^n(1)|s^n, P^n(0)]\}
\]

\[
J^n_1(-1, s^n, P^n(0)) \equiv -(1 - \tau^n)E[V^n|s^n] - \tau^n \min \{E[V^n|s^n], E[P^n(1)|s^n, P^n(0)]\}
\]

In this formulation, the value \( J^n_1(x_i^n, s^n, P^n(0)) \) accounts for the possibility that a late-consumer with a non-zero position in \( t = 0 \) may reverse the position in \( t = 1 \) instead of holding the position until \( t = 2 \).

First, we show that a long position conditional on a good signal dominates a long position conditional on a bad signal. This is obvious in \( t = 1 \) since, conditional on \( P^n(0) \) being non-revealing, the expected payoff from a long position given a good signal is

\[
E[(V^n - P^n(1))|s^n = G, P^n(0)] = (1 - \lambda^n(1))E[V^n - P^n_0|s^n = G] > 0,
\]

whereas the expected payoff from a long position given a bad signal is

\[
E[(V^n - P^n(1))|s^n = G, P^n(0)] = (1 - \lambda^n(1))E[V^n - P^n_0|s^n = B] < 0.
\]

In \( t = 0 \), the expected value from a buy order conditional on \( s^n = G \) is

\[
(1 - \lambda^n(0))(1 - \gamma \tau^n(1 - \lambda^n(1)))E[V^n - P^n_0|s^n = G],
\]

whereas the expected value from a buy order conditional on \( s^n = B \) is

\[
(1 - \lambda^n(0))[1 - \gamma \tau^n(1 - \lambda^n(1)) - (1 - \lambda^n(1))(1 - \gamma)] E[V^n - P^n_0|s^n = B].
\]
Subtracting (A.6) from (A.5) gives

\[
(1 - \lambda^n(0))(1 - \gamma \tau^n(1 - \lambda^n(1)))(E[V^n|s^n = G] - E[V^n|s^n = B])
- (1 - \lambda^n(0))(1 - \lambda^n(1))(1 - \gamma)E[P^n_\emptyset - V^n|s^n = B],
\]

which is strictly positive since \(E[V^n|s^n = G] > P^n_\emptyset\) and \((1 - \gamma \tau^n(1 - \lambda^n(1))) > (1 - \lambda^n(1))(1 - \gamma)\). A similar argument shows that a short position conditional on a bad signal dominates a short position conditional on a good signal.

Since informed investors always trade in the direction of their signal, an informed investor with a long position in \(t = 0\) must have received a good signal. If this investor consumes late and \(P^n(0)\) is non-revealing, it is strictly optimal to hold the position until \(t = 2\) because \(E[V^n|s^n = G] > E[P^n(1)|s^n = G, P^n(0)]\). Therefore, \(J^n_1(1, s^n = G, P^n(0)) = E[V^n|s^n = G]\). A similar argument shows that \(J^n_1(-1, s^n = B, P^n(0)) = -E[V^n|s^n = B]\). This shows that if the firm’s project pays off late and \(P^n(0)\) is non-revealing, late-consumers hold the position until \(t = 2\).

Next, we prove that an informed investor prefers trading early. Consider the expected value from a long position at \(t = 0\) conditional on a good signal. Using Lemma 1 and Eq. (10) and simplifying, we can write this value, \(J^n_0\) say, as

\[
\hat{J}^n_0(G) = (1 - \lambda^n(0))(1 - \gamma \tau^n(1 - \lambda^n(1)))(E[V^n|s^n = G] - P^n_\emptyset).
\]

On the other hand, consider the expected value value at \(t = 0\) of trading at \(t = 1\) conditional on a good signal. Using Lemma 1 and Eq. (10) and simplifying, we can write this value, \(J^n_1(G)\) say, as

\[
\hat{J}^n_1(G) = (1 - \lambda^n(0))(1 - \gamma)\tau^n(1 - \lambda^n(1))(E[V^n|s^n = G] - P^n_\emptyset).
\]

In case the signal is good \((s^n = G)\), Eq. (A.3) and Eq. (4) imply that

\[
E[V^n|s^n = G] - P^n_\emptyset = P^n_H - P^n_\emptyset = (\nu_G - \rho_H)\Delta V = \frac{\nu_G - \nu_B}{2}\Delta V > 0,
\]

Therefore, it is immediate that \(\hat{J}^n_0(G) \geq \hat{J}^n_1(G)\), with a strict inequality if \(\tau^n(1 - \lambda^n(1)) < 1\). The same analysis conditional on a bad signal yields \(\hat{J}^n_0(B) = \hat{J}^n_0(G)\) and \(\hat{J}^n_1(B) = \hat{J}^n_1(G)\).

Since it is optimal for informed investors to buy (sell) at \(t = 0\) conditional on a good (bad) signal, there is an equilibrium in which all informed investors trade at \(t = 0\). In this equilibrium, the aggregate order flow at \(t = 1\) is proportional to order flow in the previous period, i.e., \(X^n(1) = -\gamma X^n(0)\). Because \(X^n(0)\) is already known to market makers, there is no new information for market makers in \(X^n(1)\). Therefore, the price is uninformative at \(t = 1\),
that is, $\lambda^n(1) = 0$. In this equilibrium, expected profits equal

$$J^n_0 = \tilde{J}^n_0(G) = \tilde{J}^n_0(B) = (1 - \lambda^n(0))(1 - \gamma \tau^n)\Delta P,$$

as stated in the text of the lemma.

Finally, we prove this is the only trading equilibrium. Consider a candidate equilibrium in which a mass $\eta^n > 0$ of informed investors does not trade at $t = 0$ and waits to trade in $t = 1$. For this trading behaviour to be optimal, it must be $\tilde{J}^n_0(G) \leq \tilde{J}^n_1(G)$, which requires $\tau^n(1 - \lambda^n(1)) = 1$ and therefore $\lambda^n(1) = 0$. However, when a mass $\eta^n(1 - \gamma) > 0$ of informed investors trades at $t = 1$, the order flow at $t = 1$ must be informative, which implies $\lambda^n(1) > 0$, a contradiction. ■

Proof of Proposition 1:

Let $\Lambda > 0$ be

$$\Lambda = (1 - \lambda^n)(1 - \gamma \tau^n) = (1 - \lambda^m)(1 - \gamma \tau^m),$$

for all $n, m \in \mathcal{N}$ (see Eq. (12)). Then, we can write each $\lambda^n$ as

$$\lambda^n = 1 - \frac{\Lambda}{1 - \gamma \tau^n}.$$  \hspace{1cm} (A.7)

By adding Eq. (A.7) for all $n \in \mathcal{N}$, using the informational resource constraint Eq. (13), we can obtain

$$\Lambda = \frac{N \tilde{z} - 1}{\sum_{n=1}^{N} \frac{1}{1 - \gamma \tau^n}}.$$  \hspace{1cm} (A.8)

Therefore, there exists a unique solution for each $\lambda^n$ for all $n \in \mathcal{N}$ given $\{\tau^n\}_{n \in \mathcal{N}}$ from Eqs. (A.7)-(A.8).

Now, we prove that, fixing $\{\tau^m\}_{m \in \mathcal{N} \setminus \{n\}}$, $\lambda^n$ is decreasing and concave in $\tau^n$, where the notation $B \setminus A$ is the set difference, defined as $B \setminus A = \{x \in B | x \notin A\}$. For notational convenience, we represent Eqs. (A.7)-(A.8) as follows:

$$\lambda^n(\tau^n) = 1 - \frac{N \tilde{z} - 1}{\sum_{m \in \mathcal{N} \setminus \{n\}} x(\tau^m)},$$

where $x(\cdot)$ is a positive function such that

$$x(\tau) \equiv \frac{1}{1 - \gamma \tau},$$

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which is increasing in $\tau$ because
\[
\frac{\partial x(\tau)}{\partial \tau} = \frac{\gamma}{(1 - \gamma \tau)^2} = \gamma [x(\tau)]^2 > 0. \tag{A.9}
\]

Because $x(\tau)$ is increasing in $\tau$, $\lambda^n(\tau^n)$ becomes the smallest when $\tau^n = 1$ and $\tau^m = 0$, in which case we have
\[
\lambda^n(1) = 1 - \frac{N\bar{z} - 1}{\frac{1}{1 - \gamma} + N - 1} = 1 - \frac{N - 1}{\gamma + (1 - \gamma)N}.
\]

Therefore, the second inequality in Eq. (5) is sufficient to guarantee that $\lambda^n(1)$ is positive.

The first-order derivative of $\lambda^n$ with respect to $\tau^n$ is given by
\[
\frac{\partial \lambda^n(\tau^n)}{\partial \tau^n} = -A \times \frac{\gamma [x(\tau^n)]^2}{\left( x(\tau^n) + \sum_{m \in N \setminus \{n\}} x(\tau^m) \right)^2} < 0, \tag{A.10}
\]
where $A$ is a positive constant such that
\[
A \equiv \left( \frac{N\bar{z} - 1}{\bar{z}} \right) \sum_{m \in N \setminus \{n\}} x(\tau^m),
\]
which proves that $\lambda^n$ is decreasing in $\tau^n$.

Likewise, the second-order derivative of $\lambda^n$ with respect to $\tau^n$ is
\[
\frac{\partial^2 \lambda^n(\tau^n)}{(\partial \tau^n)^2} = -A \left[ 2\gamma^2 [x(\tau^n)]^3 \left( x(\tau^n) + \sum_{m \in N \setminus \{n\}} x(\tau^m) \right) - 2\gamma^2 [x(\tau^n)]^4 \right] \left( x(\tau^n) + \sum_{m \in N \setminus \{n\}} x(\tau^m) \right)^{-3}
\]
\[
= -A \left[ 2\gamma^2 [x(\tau^n)]^3 \sum_{m \in N \setminus \{n\}} x(\tau^m) \right] \left( x(\tau^n) + \sum_{m \in N \setminus \{n\}} x(\tau^m) \right)^{-3} < 0,
\]
which proves that $\lambda^n$ is concave in $\tau^n$.

Finally, we obtain
\[
\frac{\partial \lambda^n(\tau^n)}{\partial \tau^m} = \left( \frac{N\bar{z} - 1}{\bar{z}} \right) \frac{x(\tau^n) \gamma [x(\tau^m)]^2}{\left[ \sum_m x(\tau^m) \right]^2} > 0. \tag{A.11}
\]
Appendix B.

Proof of Proposition 2:

Because the manager’s outside option $\bar{u}$ is sufficiently low, standard arguments imply that the PC constraint in Eq. (16) does not bind given the LL constraint, and the IC constraint in Eq. (17) must bind. Then, it must be that $w^m_B = w^m_F = w^m_0 = 0$ (i.e., the LL constraint binds) because otherwise shareholders could reduce the wage bill without violating the IC constraint. Hence, an optimal contract solves

$$\min \{w^n_G, w^n_S\} \in \mathbb{R}^2_+ \lambda^n \sigma_G w^n_G + (1 - \lambda^n) (1 - \delta \tau^n) \rho_H w^n_S$$

such that the IC constraint (17) binds,

$$\lambda^n \Delta \sigma u (w^n_G) + (1 - \lambda^n) (1 - \delta \tau^n) \Delta \rho u (w^n_S) = K.$$  

Now, we prove the following lemmas to finish the proof.

Lemma B.3. Eq. (3) implies $\Delta \sigma > \Delta \rho$, $\nu_G > \nu_B$, and $\rho_H / \Delta \rho > \sigma_G / \Delta \sigma$.

Proof. By taking the difference of two equations in Eq. (3), we have

$$\Delta \rho = \Delta \sigma (\nu_G - \nu_B),$$

which implies $\Delta \sigma > \Delta \rho$, and $\nu_G > \nu_B$. Furthermore, Eq. (3) also implies that

$$\frac{\rho_H}{\Delta \rho} = \frac{\sigma_G (\nu_G - \nu_B)}{\Delta \rho} + \frac{\nu_B}{\Delta \rho},$$

which in turn together with Eq. [B.3] implies

$$\frac{\rho_H}{\Delta \rho} = \frac{\sigma_G}{\Delta \sigma} + \frac{\nu_B}{\Delta \rho} > \frac{\sigma_G}{\Delta \sigma}.$$  

Lemma B.4. There exists a unique solution for the optimization problem in Eq. (B.1) such that $w^n_G > w^n_S > 0$ where $w^n_G$ and $w^n_S$ simultaneously solve

$$\lambda^n \Delta \sigma u (w^n_G) + (1 - \lambda^n) (1 - \delta \tau^n) \Delta \rho u (w^n_S) = K$$

$$\sigma_G \Delta \rho u' (w^n_S) = \Delta \sigma \rho_H u' (w^n_G).$$

Furthermore, both $w^n_G$ and $w^n_S$ are increasing in $\tau^n$. 

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Proof. Because of the assumption that \( u'(0) = \infty \), we can rule out any corner solution such that either \( w_G^n = 0 \) or \( w_S^n = 0 \). Therefore, we can drop non-negativity constraints for \( w_G^n, w_S^n \). Then, the Lagrangian is given by

\[
\mathcal{L} = \lambda^n \sigma_G w_G^n + (1 - \lambda^n)(1 - \delta \tau^n) \rho_H w_S^n + \psi \begin{bmatrix} K - \lambda^n \Delta u (w_G^n) \\ -(1 - \lambda^n)(1 - \delta \tau^n) \Delta \rho u (w_S^n) \end{bmatrix},
\]

where \( \psi \) is the Lagrangian multiplier. The first-order conditions with respect to \( w_G^n \) and \( w_S^n \) are given by

\[
\sigma_G - \psi \Delta \sigma u' (w_G^n) = 0, \quad \rho_H - \psi \Delta \rho u' (w_S^n) = 0,
\]

which implies

\[
\frac{\sigma_G}{\rho_H} = \frac{\Delta \sigma u'(w_G^n)}{\Delta \rho u'(w_S^n)}, \quad (B.6)
\]

Therefore, we have \( w_G^n > w_S^n \) because \( u' (\cdot) \) is positive and decreasing (i.e., \( u'(\cdot) > 0, u''(\cdot) < 0 \)), and also \( \rho_H/\Delta \rho > \sigma_G/\Delta \sigma \) from Lemma [B.3].

Using continuity and strict monotonicity of \( u'(\cdot) \), we can obtain \( w_S^n \) as a continuous function of \( w_G^n \) using Eq. (B.6):

\[
w_S^n = W(w_G^n) \equiv u'^{-1} \left( \frac{\rho_H \Delta \sigma}{\sigma_G \Delta \rho} u'(w_G^n) \right), \quad (B.7)
\]

which implies \( w_S^n \) is increasing in \( w_G^n \) because both \( u'(\cdot) \) and \( u'^{-1}(\cdot) \) are decreasing.\(^{31}\) Therefore, we can represent the IC constraint as

\[
\lambda^n \Delta u (w_G^n) + (1 - \lambda^n)(1 - \delta \tau^n) \Delta \rho u (W(w_G^n)) = K. \quad (B.8)
\]

The LHS of Eq. (B.8) is zero at \( w_G^n = 0 \) because \( u(0) = 0 \), it is less than the RHS at the given point. The LHS is greater than \( K \) as \( w_G^n \rightarrow \infty \) because \( \lim_{w \rightarrow \infty} u(w) = \infty \). Because the LHS is an increasing function, the intermediate value theorem implies that there exists a unique solution for \( w_G^n > 0 \), which in turn implies the same for \( w_S^n \) by Eq. (B.7). Furthermore, \( w_G^n \) and \( w_S^n \) simultaneously solve Eqs. (B.4) by construction.

Finally, we prove that both \( w_G^n \) and \( w_S^n \) increase in \( \tau^n \). Note that \( \tau^n \) enters Eq. (B.8)\(^{31}\). Due to strict concavity of \( u(\cdot) \), it is immediate that \( u'(\cdot) \) is decreasing. Likewise, \( u'^{-1}(\cdot) \) is decreasing because

\[
\frac{\partial u'^{-1}(y)}{\partial y} = \frac{1}{u'(u'^{-1}(y))} < 0.
\]

\(^{31}\)Due to strict concavity of \( u(\cdot) \), it is immediate that \( u'(\cdot) \) is decreasing. Likewise, \( u'^{-1}(\cdot) \) is decreasing because
directly but also indirectly through \( \lambda^n \). For the direct effect, the LHS of Eq. (B.8) is decreasing in \( \tau^n \) at any level of \( w^n_G \) (because \( u(\cdot) \) is positive) whereas the RHS is a constant. Therefore, the direct of an increase in \( \tau^n \) on \( w^n_G \) is positive. For the indirect effect, using Eqs. (B.6) and (B.8), we obtain, after some straightforward manipulations,

\[
\frac{\partial w^n_G}{\partial \lambda^n} = \frac{\Delta \sigma u(w^n_G) - (1 - \delta \tau^n) \Delta \rho u(w^n_S)}{\lambda^n \sigma_H w^n_G R(w^n_G)^{-1} + (1 - \lambda^n) \rho_H (1 - \delta \tau^n) w^n_S R(w^n_S)^{-1} w''(w^n_G) \Delta \sigma},
\]

where \( R \) denotes relative risk aversion,

\[
R(x) \equiv -\frac{u''(x)}{u'(x)}.
\]

Since \( w^n_G > w^n_S \) (Lemma B.4) and \( \Delta \sigma > \Delta \rho \) (Lemma B.3), then, Eq. (B.9) is negative. Because \( \lambda^n \) is decreasing in \( \tau^n \) (Proposition 1), also the indirect effect of \( \tau^n \) on \( w^n_G \) is positive. Therefore, the unique solution for \( w^n_G \) should increase in \( \tau^n \). This in turn implies that \( w^n_S \) should also increase in \( \tau^n \) by Eq. (B.7).

Lemma B.5. Under the optimal contract, \( W^n \) is increasing in \( \tau^n \).

Proof. At optimum, the following should be true:

\[
W^n = \lambda^n \sigma_G w^n_G + (1 - \lambda^n) (1 - \delta \tau^n) \rho_H w^n_S + \psi [K - \lambda^n \Delta \sigma u(w^n_G) - (1 - \lambda^n) (1 - \delta \tau^n) \Delta \rho u(w^n_S)].
\]

Then, the Envelope theorem implies

\[
\frac{\partial W^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} \sigma_G w^n_G - \left( \frac{\partial \lambda^n}{\partial \tau^n} (1 - \delta \tau^n) + (1 - \lambda^n) \delta \right) \rho_H w^n_S
- \psi \frac{\partial \lambda^n}{\partial \tau^n} \Delta \sigma u(w^n_G) + \psi \left( \frac{\partial \lambda^n}{\partial \tau^n} (1 - \delta \tau^n) + (1 - \lambda^n) \delta \right) \Delta \rho u(w^n_S).
\]

Substituting the first-order conditions in Eq. (B.5) into Eq. (B.10), we have

\[
\frac{\partial W^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} \sigma_G \Psi(w^n_G) - \left( \frac{\partial \lambda^n}{\partial \tau^n} (1 - \delta \tau^n) + (1 - \lambda^n) \delta \right) \rho_H \Psi(w^n_S)
= \frac{\partial \lambda^n}{\partial \tau^n} [\sigma_G \Psi(w^n_G) - \rho_H (1 - \delta \tau^n) \Psi(w^n_S)] - (1 - \lambda^n) \delta \rho_H \Psi(w^n_S)
\]

where

\[
\Psi(w) \equiv w - \frac{u'(w)}{u''(w)} < 0,
\]

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which is a decreasing function because of the concavity of \( u(\cdot) \):

\[
\Psi'(w) = 1 - \frac{(u'(w))^2 - u(w)u''(w)}{(u'(w))^2} = \frac{u(w)u''(w)}{(u'(w))^2} < 0. \tag{B.13}
\]

Because \( \Psi(\cdot) < 0, \Psi'(\cdot) < 0 \) and \( w^n_G > w^n_S \) (Lemma B.4), we have \( \Psi(w^n_G) < \Psi(w^n_S) < 0 \). It is immediate that \( \sigma_G > (1 - \delta \tau^n) \rho_H \) because \( \sigma_G > \rho_H \). Then, we have

\[
\sigma_G \Psi(w^n_G) - \rho_H (1 - \delta \tau^n) \Psi(w^n_S) < 0. \tag{B.14}
\]

Because \( \partial \lambda^n / \partial \tau^n \) is negative (Proposition 1), Eq. (B.14) implies that the first term in Eq. (B.11) is positive. Because \( \Psi(\cdot) \) is negative (Eq. (B.12)), the second term in Eq. (B.11) is also positive. Therefore \( \partial W^n / \partial \tau^n \) is positive, which proves that \( W^n \) is increasing in \( \tau^n \).

\[\Box\]

Lemma B.6. Under the optimal contract, \( W^n \) is convex in \( \tau^n \).

Proof. From Eq. (B.11), we can obtain the second-order derivative of \( W^n \) with respect to \( \tau^n \) as follows:

\[
\frac{\partial^2 W^n}{(\partial \tau^n)^2} = \frac{\partial^2 \lambda^n}{(\partial \tau^n)^2} \left[ \sigma_G \Psi(w^n_G) - (1 - \delta \tau^n) \rho_H \Psi(w^n_S) \right] + \frac{\partial \lambda^n}{\partial \tau^n} \delta \rho_H \Psi'(w^n_S) \frac{\partial w^n_S}{\partial \tau^n} + \frac{\partial \lambda^n}{\partial \tau^n} \left[ \sigma_G \Psi'(w^n_G) \frac{\partial w^n_G}{\partial \tau^n} - (1 - \delta \tau^n) \rho_H \Psi'(w^n_S) \frac{\partial w^n_S}{\partial \tau^n} \right]. \tag{B.15}
\]

Because \( \partial^2 \lambda^n / (\partial \tau^n)^2 \) is negative (Proposition 1), Eq. (B.14) implies that the first term in Eq. (B.15) is positive. Because \( \partial \lambda^n / \partial \tau^n \) is negative (Proposition 1), and \( \Psi(\cdot) \) is negative (Eq. (B.12)), the second term in Eq. (B.15) is also positive. Because \( \Psi'(\cdot) \) is negative (Eq. (B.13)) and \( \partial w^n_S / \partial \tau^n \) is positive (Lemma B.4), the third term is also positive.

Now, we prove that the fourth term in Eq. (B.15) is also positive. Using implicit differentiation on Eq. (B.6), we can obtain

\[
\frac{\partial w^n_S}{\partial \tau^n} = \frac{\rho_H \Delta \sigma}{\sigma_G \Delta \rho} \frac{\partial \lambda^n}{\partial \tau^n} \frac{w^n_G}{w^n_S} \frac{\partial w^n_G}{\partial \tau^n}. \tag{B.16}
\]

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Then, we have

\[
\sigma \Psi'(w_{G}\tau^n) \frac{\partial w_{G}^n}{\partial \tau^n} - (1 - \delta \tau^n) \rho_H \Psi'(w_{S}^n) \frac{\partial w_{S}^n}{\partial \tau^n} = \frac{\partial w_{G}^n}{\partial \tau^n} \left[ \sigma \Psi'(w_{G}^n) - (1 - \delta \tau^n) \rho_H \Psi'(w_{S}^n) \left( \frac{\rho_H}{\sigma} \Delta \sigma \right) \frac{u''(w_{G}^n)}{u''(w_{S}^n)} \right] = \frac{\sigma}{u'(w_{G}^n)} \frac{u''(w_{G}^n)}{\partial \tau^n} \left[ u(w_{G}^n) - (1 - \delta \tau^n) \left( \frac{\rho_H}{\sigma} \Delta \sigma \right) \frac{u'(w_{G}^n)}{u'(w_{S}^n)} \right]^2 \] (B.17)

where the first equality is due to Eq. (B.16), the second equality is due to Eq. (B.12), and the third equality is due to Eq. (B.6). Because \(u(w_{G}^n) > u(w_{S}^n)\) (Lemma B.4) and \(\Delta \rho/\Delta \sigma < 1\) (Lemma B.3), we have

\[
u(w_{G}^n) - (1 - \delta \tau^n) \left( \frac{\Delta \rho}{\Delta \sigma} \right) u(w_{S}^n) > 0.
\]

Then, the last inequality should hold because \(u''(\tau^n) < 0\) and \(w_{S}^n/\partial \tau^n\) is positive (Lemma B.4).

Finally, because \(\partial \lambda_{\tau^n}/\partial \tau^n\) is negative (Proposition 1), Eq. (B.17) implies that the fourth term in Eq. (B.15) is positive.

Because all four terms in Eq. (B.15) are positive, the second-order derivative of \(\mathcal{W}_{\tau^n}\) with respect to \(\tau^n\) is positive, which finishes the proof of strict convexity of \(\mathcal{W}_{\tau^n}\).

Appendix C.

Proof of Proposition 3:

We define a mapping \(\Upsilon^n : [0, 1] \to R\) as follows:

\[
\Upsilon^n(\tau^n) \equiv \frac{\partial \mathcal{V}^n(\tau^n)}{\partial \tau^n} - \frac{\partial \mathcal{W}^n(\tau^n)}{\partial \tau^n} = f'(\tau^n) - \frac{\partial \mathcal{W}^n(\tau^n)}{\partial \tau^n}.
\]

Then, \(\Upsilon^n(\tau^n) = 0\) is equivalent to the first-order condition in Eq. (23) for the optimization problem in Eq. (22). Because \(f\) is concave and \(\mathcal{W}^n\) is convex in \(\tau^n\) (Proposition 2), \(\Upsilon^n(\tau^n)\) is decreasing in \(\tau^n\), i.e.,

\[
\frac{\partial \Upsilon^n(\tau^n)}{\partial \tau^n} = f''(\tau^n) - \frac{\partial^2 \mathcal{W}^n}{(\partial \tau^n)^2} < 0.
\]
Furthermore, we have

\[ \Upsilon^n(0) = f'(0) - \frac{\partial W^n}{\partial \tau^n} \bigg|_{\tau^n=0} < 0, \quad \text{and} \quad \Upsilon^n(1) = f'(1) - \frac{\partial W^n}{\partial \tau^n} \bigg|_{\tau^n=1} > 0. \]

because \( f'(0) = \infty \) and \( f'(1) = 0 \), and \( \partial W^n / \partial \tau^n \) is positive (lemma B.5) and finite for all \( \tau^n \in [0, 1] \).\(^{32}\)

Therefore the intermediate value theorem implies that the first-order condition is satisfied (i.e., \( \Upsilon^n(\tau^n) = 0 \)) at an interior point \( \tau^n \in (0, 1) \).

We now prove that the firm always prefer incentivizing the manager to provide effort. This requires

\[ f(\tau^*n) + \rho_H \Delta V - W_n(\tau^*n) \geq f(1) + \rho_L \Delta V, \tag{C.1} \]

where the LHS is the maximal value for shareholders in firm \( n \) in case they incentivize the manager and the RHS is the maximal value in case firms do not incentivize the manager (in which case firms choose a long-term project and do not pay the manager). Next, consider the case in which shareholders in firm \( n \) are constrained to set \( \tau^n = 1 \) and to compensate the manager only in case the project succeeds. To induce effort, this compensation, \( \hat{w}_S \) say, must satisfy

\[ (1 - \delta) \Delta \rho u(\hat{w}_S) = K. \tag{C.2} \]

In this case, shareholder value equals

\[ f(1) + \rho_H \Delta V - (1 - \delta) \rho_H \hat{w}_S, \tag{C.3} \]

which is a lower value compared to the unconstrained shareholder value in the LHS of Eq. (C.1). Then, it is immediate that Eq. (C.1) holds whenever

\[ f(1) + \rho_H \Delta V - (1 - \delta) \rho_H \hat{w}_S \geq f(1) + \rho_L \Delta V. \tag{C.4} \]

Using Eq. (C.3) to solve for \( \hat{w}_S \) and simplifying, we can express Eq. (C.4) as

\[ \Delta \rho \Delta V \geq (1 - \delta) \rho_H u^{-1} \left( \frac{K}{(1 - \delta) \Delta \rho} \right), \tag{C.5} \]

or equivalently,

\[ K \leq \bar{K} \equiv (1 - \delta) \Delta \rho u \left( \frac{\Delta \rho \Delta V}{(1 - \delta) \rho_H} \right), \tag{C.6} \]

\(^{32}\)Because the amounts of optimal compensation \( w^*_Gn \) and \( w^*_Sn \) are finite, it is immediate that \( \partial W^n / \partial \tau^n \) is finite from Eqs. (A.10) and (B.11).
Proof of Proposition 4

We prove Proposition 4 with a corollary of the following lemma:

Lemma C.7. (Supermodularity) Consider the simultaneous move game played by the $N$ firms when choosing the project maturity. Each firm chooses $\tau^n \in [0, 1]$ to maximize $\mathcal{V}^n(\tau^n) - \mathcal{W}^n(\tau^n)$, where $\mathcal{V}^n(\tau^n)$ is defined in the text and $\mathcal{W}^n(\tau^n)$ is the wage bill under the optimal contract given $\tau^n$ in Eq. (15). This game is supermodular if either (i) $(N - 1) (1 - \gamma) \geq 1$, or if (ii) the manager has CRRA utility, $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$ and $\alpha \in (\bar{\alpha}, 1)$, for some $\bar{\alpha} \in (0, 1)$.

Proof. By the maximum theorem, $\mathcal{W}^n(\tau^n)$ is continuous in $\tau^n$ and in $\tau^m$ for all $m \in \mathcal{N} \setminus \{n\}$, and so are firms’ objective functions. The strategy space is compact since $\tau^n \in [0, 1]$. Therefore, the game is supermodular if each firm’s objective function has increasing differences in maturity choices, that is, for all $n$ and $m \in \mathcal{N} \setminus \{n\}$,

$$\frac{\partial^2 (\mathcal{V}^n(\tau^n) - \mathcal{W}^n(\tau^n))}{\partial \tau^n \partial \tau^m} \geq 0. \quad (C.7)$$

Since $\mathcal{V}^n(\tau^n)$ is not a function of $\tau^m$ for all $m \in \mathcal{N} \setminus \{n\}$, (C.7) is equivalent to

$$\frac{\partial^2 \mathcal{W}^n(\tau^n)}{\partial \tau^n \partial \tau^m} \leq 0. \quad (C.8)$$

By Eq. (B.11), we have:

$$\frac{\partial^2 \mathcal{W}^n}{\partial \tau^n \partial \tau^m} = \frac{\partial^2 \lambda^n}{\partial \tau^n \partial \tau^m} \left[ \sigma_G \Psi(w^n_G) - \rho_H (1 - \delta \tau^n) \Psi(w^n_S) \right] + \frac{\partial \lambda^n}{\partial \tau^n} \left( \sigma_G \Psi'(w^n_G) - \rho_H (1 - \delta \tau^n) \Psi'(w^n_S) \frac{\partial w^n_S}{\partial w^n_G} \frac{\partial w^n_G}{\partial \lambda^n} \right) \frac{\partial w^n_G}{\partial \lambda^n} \frac{\partial \lambda^n}{\partial \tau^m} \quad (C.9)$$

Using Eq. (B.6) we obtain

$$\frac{\partial w^n_S}{\partial w^n_G} = \frac{w''(w^n_G)}{w''(w^n_S)} \frac{\rho_H \Delta \sigma \Delta \rho}{\sigma_H \Delta \rho}. \quad (C.10)$$

Using Eqs. (B.6) and (B.8), we obtain

$$\left( \sigma_G \Psi'(w^n_G) - \rho_H (1 - \delta \tau^n) \Psi'(w^n_S) \frac{\partial w^n_S}{\partial w^n_G} \right) \frac{\partial w^n_G}{\partial \lambda^n} = \Gamma$$
where we define
\[
\Gamma \equiv \frac{\left( \sigma_G \frac{w_{G}^{\pi}}{u'(w_{G}^{\pi})} - \rho_H (1 - \delta \tau) \frac{u(w_{S}^{\pi})}{u'(w_{S}^{\pi})} \right)^2}{\lambda \sigma_H w_{G}^{\pi} R(w_{G}^{\pi})^{-1} + (1 - \lambda) \rho_H (1 - \delta \tau) w_{S}^{\pi} R(w_{S}^{\pi})^{-1}} > 0.
\]

Since \( \frac{\partial \lambda}{\partial \tau_m} > 0, \Psi(w_{S}^{\pi}) < 0, \Psi'(w_{S}^{\pi}) < 0, \frac{\partial w_{S}^{\pi}}{\partial w_{G}^{\pi}} > 0 \) (Eq. C.10) and \( \frac{\partial w_{S}^{\pi}}{\partial \lambda} < 0 \) (Eq. B.9), the third line in Eq. (C.9) is negative. Therefore, for (C.8) to hold it is sufficient to show that
\[
\frac{\partial^2 \lambda}{\partial \tau_n \partial \tau_m} [\sigma_G \Psi(w_{G}^{\pi}) - \rho_H (1 - \delta \tau) \Psi(w_{S}^{\pi})] + \frac{\partial \lambda}{\partial \tau_n} \frac{\partial \lambda}{\partial \tau_m} \Gamma \leq 0. \tag{C.11}
\]

Since \( \frac{\partial \lambda}{\partial \tau_m} < 0, \frac{\partial \lambda}{\partial \tau_m} > 0, \Gamma > 0, \) and \( [\sigma_G \Psi(w_{G}^{\pi}) - \rho_H (1 - \delta \tau) \Psi(w_{S}^{\pi})] < 0, \) a sufficient condition for (C.11) is that \( \frac{\partial^2 \lambda}{\partial \tau_n \partial \tau_m} \geq 0. \) Using the expression for \( \lambda^n \) in the proof of Proposition 1, we obtain
\[
\frac{\partial^2 \lambda}{\partial \tau_n \partial \tau_m} = \left( \frac{N \bar{z} - 1}{\bar{z}} \right) \frac{\gamma^2 [x(\tau_n)]^2 [x(\tau_m)]^2}{(\sum_{m \in \mathcal{N} \setminus \{ n \}} x(\tau_m) - x(\tau_n))}. \tag{C.12}
\]

Therefore
\[
sign \left( \frac{\partial^2 \lambda}{\partial \tau_n \partial \tau_m} \right) = sign \left( \sum_{m \in \mathcal{N} \setminus \{ n \}} x(\tau_m) - x(\tau_n) \right).
\]

Because \( x(\tau) \) is increasing, we have that \( \sum_{m \in \mathcal{N} \setminus \{ n \}} x(\tau_m) - x(\tau_n) \geq 0 \) if \((N - 1) x(0) \geq x(1),\) which is equivalent to
\[
(N - 1) (1 - \gamma) \geq 1. \tag{C.13}
\]

Hence, for any \( \gamma \in (0, 1), \) the sufficient condition (C.13) is satisfied for \( N \) large enough.

As an alternative sufficient condition that does not depend on investor preferences or the number of firms, we consider the case where the manager has CRRA utility as stated in the lemma. With this assumption, (C.11) can be rewritten as
\[
\frac{w_{G}^{\pi} \eta}{1 - \alpha} \left( \frac{\partial^2 \lambda}{\partial \tau_n \partial \tau_m} + \frac{\partial \lambda}{\partial \tau_n} \frac{\partial \lambda}{\partial \tau_m} (1 - \alpha) (\lambda^D \sigma_H + (1 - \lambda^D) \rho_H (1 - \delta \tau) \xi) \right) \leq 0, \tag{C.14}
\]

where we define
\[
\xi = \left( \frac{\Delta \sigma_H}{\Delta \rho_H} \right)^{-\frac{1}{\alpha}} \quad \eta = \sigma_H - \rho_H (1 - \delta \tau) \xi.
\]
We find that
\[
\lim_{\alpha \to 1^-} \frac{\eta}{(1 - \alpha)(\lambda^n\sigma_H + (1 - \lambda^n) \rho_H (1 - \delta \tau^n) \xi)} = \infty.
\]
Since \(\frac{\partial \lambda^n}{\partial \tau^n} \frac{\partial \lambda^n}{\partial \tau^n} < 0\) and \(\frac{\partial \lambda^n}{\partial \tau^n} \frac{\partial \lambda^n}{\partial \tau^m}, \) and \(\frac{\partial^2 \lambda^n}{\partial \tau^n \partial \tau^m}\) do not depend on the parameter \(\alpha,\) we have
\[
\lim_{\alpha \to 1^-} \frac{\partial^2 \lambda^n}{\partial \tau^n \partial \tau^n} + \frac{\partial \lambda^n}{\partial \tau^n} \frac{\partial \lambda^n}{\partial \tau^m} (1 - \alpha)(\lambda^n\sigma_H + (1 - \lambda^n) \rho_H (1 - \delta \tau^n) \xi) = -\infty.
\]
Since \(\frac{w_G^\alpha \eta}{1 - \alpha} > 0\) for \(\alpha \in (0, 1),\) then \([C.14]\) holds for \(\alpha\) sufficiently close to one. ■

The following corollary provides the proof of Proposition 4:

**Corollary 1.** Under the conditions in Lemma C.7, the best response \(\tau^*\) in Proposition 3 is increasing in other firms’ maturity choices, that is,
\[
\frac{\partial \tau^*}{\partial \tau^m} > 0 \quad \text{for all} \ m \in \mathcal{N} \setminus \{n\}.
\]

**Proof.** Increasing best responses is a standard property for supermodular games (e.g., Topkis (1998)). ■

**Appendix D.**

**Proof of Theorem 1:**

First, we note that payoff functions are symmetric and best responses are increasing (Proposition 1). Therefore, any pure strategy equilibrium must be a symmetric equilibrium. We proceed to show that there exists a unique symmetric equilibrium.

In case of a symmetric equilibrium where \(\tau^m = \tau^*\) for all \(m \in \mathcal{N},\) Eq. A.10 implies that the sensitivity of price informativeness to a decrease of \(\tau^n\), denoted by \(\Theta(\tau^*),\) is given by
\[
\Theta(\tau^*) \equiv \frac{\partial \lambda^n(\tau^n)}{\partial \tau^n}\bigg|_{\tau^n=\tau^*,\tau^m=\tau^*,\forall m \in \mathcal{N}\setminus\{n\}} = \frac{\gamma(N - 1)(N\bar{z} - 1)}{N^2\bar{z}(1 - \gamma \tau^*)} < 0, \quad (D.1)
\]
which is decreasing in \(\tau^*\) because
\[
\Theta'(\tau^*) = -\frac{\gamma^2(N - 1)(N\bar{z} - 1)}{N^2\bar{z}(1 - \gamma \tau^*)^2} < 0. \quad (D.2)
\]

For clarity, throughout this proof, we denote \(w^*_G(\tau^*)\) and \(w^*_S(\tau^*)\) as functions of \(\tau^*\) explicitly, which are the optimal compensation for state \(\omega = G\) and \(\omega = S\) given maturity choice \(\tau^*\)
according to Proposition 2 respectively. We define an equilibrium mapping $$\hat{\Upsilon} : [0, 1] \rightarrow \mathbb{R}$$ as follows:

$$
\hat{\Upsilon}(\tau^*) \equiv f'(\tau^*) - \Theta(\tau^*) [\sigma_G \Psi(w_G^*(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w_S^*(\tau^*))]
+ 
\left( 1 - \frac{1}{N\bar{z}} \right) \delta \rho_H \Psi(w_S^*(\tau^*)).
$$

Then, it is clear that the solution $$\tau^*$$ for $$\hat{\Upsilon}(\tau^*) = 0$$ is the solution for the first-order condition in Eq. (23) under the assumption that $$\tau^m = \tau^*$$ for all $$m \in \mathcal{N} \setminus \{n\}$$, and vice versa. Therefore, it is sufficient to prove that there exists a unique interior solution for the equation $$\hat{\Upsilon}(\tau^*) = 0$$ to finish the proof of the theorem.

The first-order derivative of $$\hat{\Upsilon}(\cdot)$$ with respect to $$\tau^*$$ is given by

$$
\frac{\partial \hat{\Upsilon}(\tau^*)}{\partial \tau^*} = f''(\tau^*) - \Theta'(\tau^*) [\sigma_G \Psi(w_G^*(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w_S^*(\tau^*))]
- \Theta(\tau^*) \left[ \sigma_G \Psi'(w_G^*(\tau^*)) \frac{\partial w_G^*(\tau^n)}{\partial \tau^n} \bigg|_{\tau^n = \tau^*} - \rho_H (1 - \delta \tau^*) \Psi'(w_S^*(\tau^*)) \frac{\partial w_S^*(\tau^n)}{\partial \tau^n} \bigg|_{\tau^n = \tau^*} \right]
+ \left( 1 - \frac{1}{N\bar{z}} \right) \delta \rho_H \Psi'(w_S^*(\tau^*)) \frac{\partial w_S^*(\tau^n)}{\partial \tau^n} \bigg|_{\tau^n = \tau^*}.
$$

The first term is negative because $$f$$ is concave. The second term is negative due to Eqs. (B.14) and (D.2). The third term is negative because $$\Psi(\cdot)$$ is negative (Eq. (B.13)) and $$\Theta(\cdot)$$ is negative (D.1). The fourth term is negative due to Eqs. (B.17) and (D.1). The fifth term is negative because $$1 - 1/N\bar{z}$$ is positive, $$\Psi'(\cdot)$$ is negative (Eq. (B.13)), and $$\partial w_S^*/\partial \tau^n$$ is positive (Lemma B.4). Because all five terms in the RHS is negative, $$\hat{\Upsilon}(\cdot)$$ is decreasing in $$\tau^*$$.

Furthermore, we have

$$
\hat{\Upsilon}(0) = f'(0) - \Theta(0) [\sigma_G \Psi(w_G^*(0)) - \rho_H (1 - \delta) \Psi(w_S^*(0))] + \left( 1 - \frac{1}{N\bar{z}} \right) \delta \rho_H \Psi(w_S^*(0)) > 0,
$$

because $$f'(0) = \infty$$ and the second and the third terms are finite similarly as in the proof of Proposition 3. Likewise, we have

$$
\hat{\Upsilon}(1) = f'(1) - \Theta(1) [\sigma_G \Psi(w_G^*(1)) - \rho_H (1 - \delta) \Psi(w_S^*(1))] + \left( 1 - \frac{1}{N\bar{z}} \right) \delta \rho_H \Psi(w_S^*(1)) < 0,
$$

because $$f'(1) = 0$$ and the second and the third terms are negative. The second term is negative due to Eqs. (B.14) and (D.1). The third term is negative because $$\Psi(\cdot)$$ is negative.

Therefore the intermediate value theorem implies that there exists a unique interior equilibrium.

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33 We note that $$\lambda^*$$ is fixed in a symmetric equilibrium, so the effect of $$\tau^*$$ on $$w_G^*, w_S^*$$ is only the direct effect identified in the proof of Lemma B.4.
Appendix E.

Proof of Theorem 2:

We prove the theorem in several steps.

Lemma E.8. The project maturity is longer in equilibrium with exogenous informed trading than Autarky, i.e., $\tau^E > \tau^A$.

Proof. We prove by contradiction. Suppose that $\tau^E \leq \tau^A$. Because $f'(\cdot) > 0$, $f''(\cdot) < 0$, the first-order conditions in Eqs. (30) and (33) imply that

$$-\left(1 - \frac{1}{Nz}ight)\delta H \Psi(w^E) \geq -\delta H \Psi(w^A) > 0,$$

which implies

$$\Psi(w^E) \leq \Psi(w^A) < 0. \quad (E.1)$$

Because $\Psi(\cdot) < 0$, $\Psi'(\cdot) < 0$, Eq. (E.1) implies $w^E_S \geq w^A_S$. Then, because $w^E_G > w^E_S > 0$ (Proposition 2), it should be the case that $w^E_G > w^E_S \geq w^A_S$.

However, the IC constraints in Eqs. (31) and (34) together with $w^E_G > w^E_S \geq w^A_S$ would imply that

$$\left(1 - \delta \tau^A\right) \Delta u (w^A_S) = \frac{1}{Nz} \Delta \sigma u (w^E_G) + \left(1 - \frac{1}{Nz}\right) \left(1 - \delta \tau^E\right) \Delta \rho u (w^E_S)$$

$$> \left(1 - \delta \tau^E\right) \Delta \rho u (w^E_S), \quad (E.2)$$

where the inequality is true because $u(w^E_G) > u(w^E_S)$ and $\Delta \rho > \Delta \sigma$ (Lemma B.3). Then, Eq. (E.2) implies $\tau^E > \tau^A$. This contradicts.

Lemma E.9. The project maturity is shorter in equilibrium with endogenous informed trading than that with exogenous informed trading, i.e., $\tau^* < \tau^E$.

Proof. We prove by contradiction. Suppose that $\tau^* \geq \tau^E$. Note that Eqs. (27)-(28) are identical to Eqs. (34)-(35) except that $\tau^*$ is different from $\tau^E$ (because $\lambda^n = 1/(Nz)$ for all $n \in \mathcal{N}$ in both cases). Because $w^*_G$ and $w^*_S$ are increasing in $\tau^*$ fixing $\lambda^n$ (Lemma B.4), we have $w^*_G \geq w^E_G$ and $w^*_S \geq w^E_S$. Then, because $\Psi(\cdot) < 0$, $\Psi'(\cdot) < 0$, we have

$$\Psi(w^*_S) \leq \Psi(w^E_S) < 0 \iff \left(1 - \frac{1}{Nz}\right) \delta H \Psi(w^*_S) \leq \left(1 - \frac{1}{Nz}\right) \delta H \Psi(w^E_S) < 0. \quad (E.3)$$
Using Eqs. (B.14) and (E.3), we have
\[
\left\{ \begin{array}{l}
\Theta(\tau^*) \left[ \sigma_G \Psi(w_G^*) - \rho_H (1 - \delta \tau^*) \Psi(w_S^*) \right] \\
- (1 - \frac{1}{N\bar{z}}) \delta \rho_H \Psi(w_S^*) \end{array} \right\} > - \left( 1 - \frac{1}{N\bar{z}} \right) \delta \rho_H \Psi(w_{S}^{Ex}) > 0. \quad (E.4)
\]

Because \( f'(\cdot) > 0, f''(\cdot) < 0 \), however, the first-order conditions in Eqs. (25) and (33) together with Eq. (E.4) imply that \( \tau^* < \tau^{Ex} \), which is a contradiction. \( \blacksquare \)

Using Lemmas E.8 and E.9, we conclude that \( \tau^{Ex} > \max(\tau^*, \tau^{Aut}) \). The inequality in Eq. (37) is immediate from the comparison between the FOCs between the case with endogenous informed trading and autarky. Because \( f'(\cdot) > 0, f''(\cdot) < 0 \), \( \tau^* \) is smaller than \( \tau^{Aut} \) whenever the RHS of Eq. (25) is greater than that of Eq. (30), and vice versa. \( \blacksquare \)

Appendix F.

Proof of Proposition 5:

Recall that, from Eq. (D.3), the equilibrium \( \tau^* \) is derived by solving \( \hat{\Upsilon}(\tau^*) = 0 \). Furthermore, \( \hat{\Upsilon}(\cdot) \) decreases, \( \hat{\Upsilon}(0) > 0 \) and \( \hat{\Upsilon}(1) < 0 \) regardless of the value of \( N \), which implies that there exists a unique solution \( \tau^* \) (Theorem 1). Therefore, if \( \hat{\Upsilon}(\cdot) \) is decreasing in \( N \), an increase in \( N \) will decrease \( \tau^* \), thereby leading to more short-termism. We prove that, fixing \( N\bar{z} \), \( \hat{\Upsilon}(\cdot) \) is decreasing in \( N \) in several steps in the below.

Let \( \zeta \) be a positive constant such that \( N\bar{z} = \zeta \) for any level of \( N \). That is, an increase in \( N \) is compensated by a decrease in \( \bar{z} \) to keep the product of the two at the constant level \( \zeta \). Then, the equilibrium informativeness is unchanged at the level given by Eq. (24). Therefore, Eqs. (27)-(28) imply that the wage is unchanged by an increase in \( N \) fixing \( \tau^* \), i.e.,
\[
\left. \frac{\partial w_G^*(\tau^*)}{\partial N} \right|_{N\bar{z}=\zeta} = \left. \frac{\partial w_S^*(\tau^*)}{\partial N} \right|_{N\bar{z}=\zeta} = 0. \quad (F.1)
\]

Using Eqs. (D.3) and (F.1), we can obtain
\[
\left. \frac{\partial \hat{\Upsilon}(\tau^*)}{\partial N} \right|_{N\bar{z}=\zeta} = - \left. \frac{\partial \Theta(\tau^*)}{\partial N} \right|_{N\bar{z}=\zeta} \left[ \sigma_G \Psi(w_G^*(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w_S^*(\tau^*)) \right]. \quad (F.2)
\]

Eq. (B.14) implies that the term \( \sigma_G \Psi(w_G^*(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w_S^*(\tau^*)) \) is negative. Furthermore, Eq. (26) implies
\[
\left. \frac{\partial \Theta(\tau^*)}{\partial N} \right|_{N\bar{z}=\zeta} = - \frac{\gamma(N\bar{z} - 1)}{N^2\bar{z}(1 - \gamma \tau^*)} < 0. \quad (F.3)
\]

From Eqs. (F.2) and (F.3), we conclude that \( \hat{\Upsilon}(\tau^*) \) is decreasing in \( N \) at any given level of \( \tau^* \).
For the implication for shareholder value, we note that the planner’s problem in Eq. (38) is strictly concave in \( \tau \). This follows from the fact that (i) the production function \( f \) is strictly concave, and (ii) the proof of Lemma B.6 implies that under the optimal contract with fixed \( \lambda^n, W^n \) is convex in \( \tau^n \). Since \( \tau^* < \tau^{Ex} \), it follows that a decrease in \( \tau^* \) is leads to lower shareholder value. We also note that the second-best value \( \tau^{Ex} \) is unaffected by the parameter \( N \). This finishes the proof. 

Proof of Proposition 6:

The proof parallels the proof of Proposition 5. We prove that \( \hat{\Upsilon}(\cdot) \) is decreasing in \( \gamma \). Since price informativeness is independent of \( \gamma \), Eqs. (27)-(28) imply that the wage is unchanged by an increase in \( \gamma \) fixing \( \tau^* \), i.e.,

\[
\frac{\partial w^*_G(\tau^*)}{\partial \gamma} = \frac{\partial w^*_S(\tau^*)}{\partial \gamma} = 0.
\]

Using Eqs. (D.3) and (F.4), we can obtain

\[
\frac{\partial \hat{\Upsilon}(\tau^*)}{\partial \gamma} = -\Theta(\tau^*) \frac{\partial \left[ \sigma_G \Psi(w^*_G(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w^*_S(\tau^*)) \right]}{\partial \gamma}.
\]

Eq. (B.14) implies that the term \( \sigma_G \Psi(w^*_G(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w^*_S(\tau^*)) \) is negative. Furthermore, Eq. (26) implies

\[
\frac{\partial \Theta(\tau^*)}{\partial \gamma} = \frac{(N - 1)(N \bar{z} - 1)}{N^2 \bar{z}(1 - \gamma \tau^*)^2} < 0.
\]

From Eqs. (F.5) and (F.6), we conclude that \( \hat{\Upsilon}(\tau^*) \) is decreasing in \( \gamma \) at any given level of \( \tau^* \). The argument for shareholder value is identical to the proof of Proposition 5 and is omitted. This concludes the proof.

Proof of Proposition 7

The proof parallels the proof of Proposition 5. We prove that \( \hat{\Upsilon}(\cdot) \) is decreasing in \( \delta \) and \( K \).

Eq. (28) implicitly defines \( w^*_S \) as an increasing function of \( w^*_G \), and Eq. (27) implicitly defines \( w^*_G \) as an increasing function of \( \delta \) and \( K \). Therefore, we have

\[
\frac{\partial \hat{\Upsilon}(\tau^*)}{\partial \delta} = -\Theta(\tau^*) \frac{\partial \left[ \sigma_G \Psi(w^*_G(\tau^*)) - \rho_H (1 - \delta \tau^*) \Psi(w^*_S(\tau^*)) \right]}{\partial \delta} + \left( 1 - \frac{1}{N \bar{z}} \right) \delta \rho_H \Psi'(w^*_S(\tau^*)) \frac{\partial w^*_S(\tau^*)}{\partial w^*_G(\tau^*)} \frac{\partial w^*_G(\tau^*)}{\partial \delta}.
\]
We prove that the first term in Eq. (F.7) is negative. Note that $\Theta(\tau^*)$ is negative. Then, omitting explicit dependence on $\tau^*$ to ease notation, we have

$$
\frac{\partial}{\partial \delta} \left[ \sigma_G \Psi(w_G^*) - \rho_H (1 - \delta \tau^*) \Psi(w_S^*) \right]
= \sigma_G \Psi'(w_G^*) \frac{\partial w_G^*}{\partial \delta} - (1 - \delta \tau^n) \rho_H \Psi'(w_S^*) \frac{\partial w_S^*}{\partial \delta}
= \frac{\partial w_G^*}{\partial \delta} \left[ \sigma_G \Psi'(w_G^*) - (1 - \delta \tau^n) \rho_H \Psi'(w_S^*) \left( \frac{\rho^2 H}{\sigma_G^2} \frac{\Delta \sigma}{\Delta \rho} \right) \frac{\partial^2 w_G^*}{\partial w_S^*} \right]
= \sigma_G \frac{\partial w_G^*}{\partial \delta} \left[ u(w_G^*) - (1 - \delta \tau^n) \left( \frac{\partial^2 w_G^*}{\partial w_S^*} \right) \frac{\partial^2 w_G^*}{\partial w_S^*} \right]<0,
$$

where the second equality is due to Eq. (B.16), the third equality is due to Eq. (B.12), and the fourth equality is due to Eq. (B.6). Because $u(w_G^*) > u(w_S^*)$ (Lemma B.4) and $\Delta \rho/\Delta \sigma < 1$ (Lemma B.3), we have

$$
u(w_G^*) - (1 - \delta \tau^n) \left( \frac{\Delta \rho}{\Delta \sigma} \right) u(w_S^*) > 0.
$$

Therefore, the first term in Eq. (F.7) is indeed negative.

Finally, we have $\Psi' < 0$, $\frac{\partial \omega_S^*(\tau^*)}{\partial \omega_G^*(\tau^*)} > 0$ and $\frac{\partial \omega_G^*(\tau^*)}{\partial \delta} > 0$. Therefore, the second term in Eq. (F.7) is also negative. The proof for $K$ is similar and is omitted.

**Appendix G.**

**Proof of Proposition 8**

As a preliminary step, we prove the following results about the financial equilibrium induced by firms’ project maturity choices with long- and short-term investors.

**Lemma G.10.**

(i) Let $\tau^m = \tau$ for all $m \in \mathcal{N} \setminus \{n\}$ and $\tau^n < \tau$. Then, for $\mu \geq 1 - 1/N$ we have $\lambda^n = \frac{1}{N^2}$, whereas for $\mu < 1 - 1/N$ we have $\lambda^n = \min\left\{ \frac{1-\mu}{2}, \lambda^* \right\} > \frac{1}{N^2}$ where $\lambda^*$ solves short-term investors’ indifference condition

$$
(1 - \lambda_n) (1 - \tau^n \gamma) = (1 - \lambda^m) (1 - \tau^\mu) \quad \text{for all } m \in \mathcal{N} \setminus \{n\}.
$$
(ii) Let \( \tau^m = \tau \) for all \( m \in \mathcal{N} \setminus \{n\} \) and \( \tau^n > \tau \). Then, for \( \mu \geq 1/N \) we have \( \lambda^n = \frac{1}{N \bar{z}} \), whereas for \( \mu < 1/N \), we have \( \lambda^n = \max\{\frac{\mu}{z}, \lambda^*\} < \frac{1}{N \bar{z}} \) where \( \lambda^* \) solves short-term investors’ indifference condition

\[
(1 - \lambda_n) (1 - \tau^n \gamma) = (1 - \lambda^m) (1 - \tau \gamma) \text{ for all } m \in \mathcal{N} \setminus \{n\}.
\]

**Proof of Lemma (G.10)-(i):** Let \( \tau^m = \tau \) for all \( m \in \mathcal{N} \setminus \{n\} \) and \( \tau^n < \tau \). We show that for \( \mu \geq 1 - 1/N \) long-term investors are marginal investors for all firms in that \( \lambda^n = \lambda^m \) for all \( m \in \mathcal{N} \setminus \{n\} \). In other words, a firm that deviates from a symmetric maturity choice by lowering its project maturity has no impact on its price informativeness. In this case, all short-term investors choose firm \( n \) because it has informativeness identical to other firms but lower maturity. By contrast, long term investors are indifferent across all firms; \( \varepsilon_L \) unit mass of long-term investors choose firm \( n \), and \( \mu - \varepsilon_L \) unit mass of them are equally distributed over the remaining \( N - 1 \) firms. The condition \( \lambda^n = \lambda^m \) for all \( m \in \mathcal{N} \setminus \{n\} \) requires

\[
\lambda^n = \frac{1 - \mu + \varepsilon_L}{\bar{z}} = \frac{\mu - \varepsilon_L}{(N - 1) \bar{z}} = \lambda^m,
\]

which is equivalent to \( \varepsilon_L = \mu - \frac{N - 1}{N} \). Therefore, there exists \( \varepsilon_L \in [0, \mu] \) such that (G.1) holds if and only if \( \mu \geq 1 - 1/N \).

Next, consider the case where short-term investors are marginal investors for all firms. Since \( \tau^n < \tau \), it must be \( \lambda^n > \lambda^m \) for all \( m \in \mathcal{N} \setminus \{n\} \), which implies that long-term investors do not invest in firm \( n \). Then, \( 1 - \mu - \varepsilon_S \) unit mass of short-term investors invest in firm \( n \), and \( \varepsilon_S \) unit mass of short-term investors are equally distributed over the remaining \( N - 1 \) firms. The condition \( \lambda^n > \lambda^m \) for all \( m \in \mathcal{N} \setminus \{n\} \) requires

\[
\lambda^n = \frac{1 - \mu - \varepsilon_S}{\bar{z}} = \frac{\mu + \varepsilon_S}{(N - 1) \bar{z}} = \lambda^m,
\]

which is equivalent to \( \varepsilon_S < 1 - \mu - 1/N \). Therefore, Eq. (G.2) holds for some \( \varepsilon_S \in [0, 1 - \mu] \) if and only if \( \mu < 1 - 1/N \). Short term-investors are marginal investors for all firms if the following indifference condition holds:

\[
(1 - \lambda^n) (1 - \tau^n \gamma) = (1 - \lambda^m) (1 - \tau \gamma) \text{ for all } m \neq n,
\]

or equivalently,

\[
\left(1 - \frac{1 - \mu - \varepsilon_S}{\bar{z}}\right) (1 - \tau^n \gamma) = \left(1 - \frac{\mu + \varepsilon_S}{(N - 1) \bar{z}}\right) (1 - \tau \gamma).
\]

When \( \tau^n = \tau \), the above equation is solved for \( \varepsilon_S = 1 - \mu - 1/N \). As \( \tau^n \) decreases, \( \varepsilon_S \) must
decrease for the equality to hold. There exists $\varepsilon_S \geq 0$ that solves (G.3) for all $\tau^n \in [0, \tau)$ if $\frac{1-\mu}{\bar{z}} \geq 1$. If, instead, $\frac{1-\mu}{\bar{z}} < 1$, then there exists $t \in [0, \tau)$ such that (G.3) holds for all $\tau^n \in [t, \tau)$, but for all $\tau^n \in [0, t)$ short-term investors are strictly better off investing in firm $n$ and $\lambda^n = \frac{1-\mu}{\bar{z}}$; long-term investors are strictly better off investing in all other firms.

**Proof of Lemma (G.10)-(ii):** Let $\tau^m = \tau$ for all $m \neq n$ and $\tau^n > \tau$. We show that for $\mu \geq 1/N$ long-term investors are marginal investors for all firms such that $\lambda^n = \lambda^m$ for all $m \in \mathcal{N} \setminus \{n\}$. In other words, a firm that deviates from a symmetric maturity choice by increasing its project maturity has no impact on its price informativeness. Since firm $n$ has same price informativeness as other firms but longer maturity, short-term investors do not invest in firm $n$. On the other hand, long-term investors are indifferent across all firms; $\mu - \varepsilon_L$ unit mass of long-term investors choose firm $n$, and $\varepsilon_L$ unit mass of them are equally distributed over the remaining $N-1$ firms. The condition $\lambda^n = \lambda^m$ requires

$$
\lambda_n = \frac{\mu - \varepsilon_L}{\bar{z}} = \frac{1 - \mu + \varepsilon_L}{(N-1) \bar{z}} = \lambda_m, 
$$

or equivalently, $\varepsilon_L = \mu - \frac{1}{N}$. Then, there exists $\varepsilon_L \in [0, \mu]$ solving (G.4) if and only if $\mu \geq 1/N$.

Next, consider the case where short-term investors are marginal across all firms. Since $\tau^n > \tau$, it must be $\lambda^n < \lambda^m$ for all $m \in \mathcal{N} \setminus \{n\}$, which implies that all long-term investors invest in firm $n$ because it has lower price informativeness than other firms. On the other hand, $\varepsilon_S$ unit mass of short-term investors invest in firm $n$, and $1 - \mu - \varepsilon_S$ unit mass of them are equally distributed over the remaining $N-1$ firms. The condition $\lambda^n < \lambda^m$ requires

$$
\lambda_n = \frac{\mu + \varepsilon_S}{\bar{z}} < \frac{1 - \mu - \varepsilon_S}{(N-1) \bar{z}} = \lambda_m, 
$$

or $\varepsilon_S < \frac{1}{N} - \mu$. Then, there exists $\varepsilon_S \in [0, 1 - \mu]$ such that (G.5) holds if and only if $\mu < 1/N$. Furthermore, $\lambda^n$ must satisfy short-term investors’ indifference condition

$$(1 - \lambda^n) (1 - \tau^n \gamma) = (1 - \lambda^m) (1 - \tau \gamma) \text{ for all } m \neq n,$$

or equivalently,

$$
\left(1 - \frac{\mu + \varepsilon_S}{\bar{z}}\right) (1 - \tau^n \gamma) = \left(1 - \frac{1 - \mu - \varepsilon_S}{(N-1) \bar{z}}\right) (1 - \tau \gamma) \text{ for all } m \neq n. 
$$

(G.6)

When $\tau^n = \tau$, the above equation is solved for $\varepsilon_S = 1/N - \mu$. As $\tau^n$ increases, $\varepsilon_S$ must decrease for the equality to hold. There exists $\varepsilon_S \geq 0$ that solves (G.6) for all $\tau^n \in (\tau, 1]$ if $\mu \leq \mu_L (\tau)$,
where \( \mu_L(\tau) \) solves
\[
\left(1 - \frac{\mu_L(\tau)}{\bar{z}}\right)(1 - \gamma) = \left(1 - \frac{1 - \mu_L(\tau)}{(N - 1) \bar{z}}\right)(1 - \tau \gamma), \tag{G.7}
\]
and it is immediate to verify that \( \mu_L(\tau) \in (0, 1/N) \). If, instead, \( \mu \in (\mu_L(\tau), 1/N) \), then there exists \( t' \in (\tau, 1) \) such that \( \text{(G.6)} \) holds for all \( \tau^n \in (\tau, t'] \), but for all \( \tau^n \in (t', 1] \) long-term investors are strictly better off investing in firm \( n \) and \( \lambda^n = \frac{\mu}{\bar{z}} \); short-term investors are strictly better off investing in all other firms.

**Proof of Proposition 8-(i)** Assume all firms choose maturity \( \tau \). For \( \mu < 1/N \), Lemma G.10 implies that when a firm deviates locally to some \( \tau^n \neq \tau \), its price efficiency is determined by the same indifference condition as in the benchmark model without long-term investors. Therefore, by the strict concavity of the firm’s problem established in Appendix C (Lemma B.6), if a symmetric equilibrium exists, it must be equal to the benchmark model, \( \tau^\mu = \tau^* \).

Consider a firm’s deviation to \( \tau^n < \tau^* \). Then, \( \lambda^n \) is at most the value that short-term investors’ indifference condition is satisfied (Lemma G.10-i). Therefore, the firm has no incentive to deviate because its payoff of deviation is less than or equal to the payoff of deviation in the benchmark model.

Consider a firm’s deviation to \( \tau^n > \tau^* \). Then, there are two cases. Define \( \mu^* = \mu_L(\tau^*) \) (see Eq. (G.7)). If \( \mu \leq \mu^* \), the payoff of deviation is identical to the payoff of deviation in the benchmark model (Lemma G.10-ii). Therefore, the firm has no incentive to deviate, which implies choosing \( \tau^* \) is the unique equilibrium. If \( \mu \in (\mu^*, 1/N) \), if a symmetric equilibrium exists, it must be equal to \( \tau^* \) (Lemma G.10-ii).

**Proof of Proposition 8-(ii)** Assume all firms choose maturity \( \tau \). For \( \mu \geq 1 - 1/N \), Lemma G.10 implies that when a firm deviates to some \( \tau^n \neq \tau \), its price efficiency is unchanged and equal to \( 1/(\bar{z}N) \). Therefore, this is the same as the case where informed trading is exogenous and the equilibrium is \( \tau^\text{Ex} \).

**Proof of Proposition 8-(iii)** Suppose that there exists a symmetric equilibrium, and all firms choose maturity \( \tau \). For \( \mu \in [1/N, 1 - 1/N) \), Lemma G.10 implies that when a firm deviates to some \( \tau^n > \tau \), its price efficiency is unchanged and equal to \( 1/(\bar{z}N) \). Therefore, \( \tau^\mu \geq \tau^\text{Ex} \) is necessary for otherwise deviating to \( \tau^n > \tau^\mu \) is profitable. However, when a firm deviates to some \( \tau^n < \tau \), its price efficiency is determined by short-term investors’ indifference condition. Therefore, \( \tau^\mu \leq \tau^* \) is necessary for otherwise deviating to \( \tau^n < \tau^\mu \) is profitable. Since \( \tau^\text{Ex} > \tau^* \), the two necessary conditions cannot be met simultaneously.

This concludes the proof of Proposition 8.

**Proof of Proposition 9**
In a clientele equilibrium, \( N_S \) firms choose maturity \( \tau_S \) and \( N - N_S \) firms choose maturity \( \tau_L \), where \( \tau_S < \tau_L \). Initially we take \( N_S, \tau_S, \tau_L \) as given and derive conditions such that it is optimal for short-term investors to invest in short-term firms and for long-term investors to invest in long-term firms. Let \( N_S \) be the set of short-term firms and \( N_L \) the set of long-term firms. With this allocation of investors across firms, price efficiency for short-term firms, \( \lambda_S \) say, equals
\[
\lambda_S = \frac{1 - \mu}{\bar{z}N_S}.
\]
Similarly, price efficiency for long-term firms equals
\[
\lambda_L = \frac{\mu}{\bar{z}(N - N_S)}.
\]
We denote \( \alpha_S \) the fraction of short-term firms, \( \alpha_S = \frac{N_S}{N} \), and we denote the level of price efficiency in a symmetric equilibrium as \( \bar{\lambda} = \frac{1}{2N} \). With these definitions, we can write
\[
\lambda_S = \frac{(1 - \mu)\bar{\lambda}}{\alpha_S}; \quad \lambda_L = \frac{\mu\bar{\lambda}}{1 - \alpha_S}.
\] (G.8)
Since \( \tau_S < \tau_L \), short-term investors will invest in short-term firms only if \( 1 > \lambda_S > \lambda_L \), and therefore, by Eq. (G.8), we must have
\[
\bar{\alpha} \equiv 1 - \mu > \alpha_S > (1 - \mu)\bar{\lambda} \equiv \bar{\alpha}.
\] (G.9)
Since \( 1 \leq N_S \leq N - 1 \), Eq. (G.9) also implies
\[
1 - (N - 1)\bar{z} < \mu < 1 - \frac{1}{N},
\] (G.10)
Furthermore, for short-term investors to invest in short-term firms, \( \lambda_S, \lambda_L, \tau_S, \tau_L \) must satisfy
\[
(1 - \lambda_S)(1 - \tau_S\gamma) \geq (1 - \lambda_L)(1 - \tau_L\gamma).
\]
Since \( \lambda_S > \lambda_L \), it is optimal for long-term investors to invest in long-term firms.

Next, we define
\[
\nu_S (\tau^n, \tau_S, \alpha_S) \equiv f(\tau^n) - W(\tau^n, \lambda^n),
\]
where \( \lambda^n \) solves short-term investors’ indifference condition
\[
(1 - \lambda^n)(1 - \tau^n\gamma) = (1 - \lambda^n)(1 - \tau_S\gamma), \text{ for all } m \in N_S \setminus \{n\}.
\] (G.11)
Because \( \lambda^m \) in Eq. (G.11) is a function of \( \alpha_S \), \( \lambda^n \) is a function of \( \tau^n, \tau_S, \alpha_S \). Let \( \tau_s (\tau_S; \alpha_S) \) be
the best response
\[ \tau_s(\tau_S; \alpha_S) \in \arg \max_{\tau^n} \nu_S(\tau^n, \tau_S, \alpha_S). \]

By Theorem 5 (with \( N \) replaced by \( N \alpha_S \)) the fixed point \( \tau^*_S = \tau_s(\tau^*_S; \alpha_S) \) exists and is unique. Hence, we denote
\[ \hat{\nu}_S = \nu_S(\tau^*_S, \tau^*_S, \alpha_S). \]

Also, define
\[ \nu_L(\tau^n, \tau_L, \alpha_S) \equiv f(\tau^n) - W(\tau^n, \lambda_L) \]
where \( \lambda_L = \frac{\mu \lambda}{1 - \alpha_S} \) (Eq. (G.8)). Let \( \tau_l(\tau_L; \alpha_S) \) be the best response
\[ \tau_l(\tau_L; \alpha_S) \in \arg \max_{\tau^n} \nu_L(\tau^n, \tau_L, \alpha_L). \]

By Section 5.2.2 (with \( N \) replaced by \( N (1 - \alpha_S) \)), the fixed point \( \tau^*_L = \tau_l(\tau^*_L; \alpha_S) \) exists and is unique. Hence, we denote
\[ \hat{\nu}_L \equiv \nu_L(\tau^*_L, \tau^*_L, \alpha_S). \]

For clarity, in the rest of the proof, we make explicit the dependence of \( \tau^*_L, \tau^*_S, \hat{\nu}_L, \hat{\nu}_S \) on \( \alpha_S \) by writing \( \tau^*_L(\alpha_S), \tau^*_S(\alpha_S), \hat{\nu}_L(\alpha_S), \hat{\nu}_S(\alpha_S) \).

**Lemma G.11.**

(i) \( \lambda_S(\alpha_S) \) is continuous and decreasing in \( \alpha_S \) with \( \lambda_S(\bar{\alpha}) = 1 \) and \( \lambda_S(\alpha) = \bar{\lambda}; \lambda_L(\alpha_S) \) is continuous and increasing in \( \alpha_S \) with \( \lambda_L(\alpha) = \frac{\lambda \mu}{1 - (1 - \mu) \lambda} \) and \( \lambda_L(\bar{\alpha}) = \bar{\lambda}. \)

(ii) \( \tau_S(\alpha_S) \) is continuous and decreasing in \( \alpha_S \) with \( \tau_S(\bar{\alpha}) = 1 \) and \( \tau_S(\alpha) = \tau^*; \tau_L(\alpha_S) \) is continuous and increasing in \( \alpha_S \) with \( \tau_L(\alpha) < \tau_L(\bar{\alpha}) = \tau^{Ex}. \)

(iii) \( \hat{\nu}_S(\alpha_S) \) is continuous and decreasing in \( \alpha_S \) with \( \hat{\nu}_S(\bar{\alpha}) = f(1) - W(1, 1) \) and \( \hat{\nu}_S(\alpha) = f(\tau^*) - W(\tau^*, \bar{\lambda}); \hat{\nu}_L(\alpha_S) \) is continuous and increasing in \( \alpha_S \) with \( \hat{\nu}_L(\alpha) = f(\tau_L(\alpha)) - W(\tau_L(\alpha), 1 - (1 - \mu) \lambda) \) and \( \hat{\nu}_L(\bar{\alpha}) = f(\tau^{Ex}) - W(\tau^{Ex}, \bar{\lambda}). \)

**Proof of Lemma G.11-(i).** This is immediate from Eq. (G.8).

**Proof of Lemma G.11-(ii).** Following the same steps as in Proposition 5 we can show that \( \tau^*_S \) is decreasing in \( \alpha_S \) and that \( \tau^*_L \) is decreasing in \( \alpha_S \). By the implicit function theorem, \( \tau^*_S, \tau^*_L \) are continuous in \( \alpha_S \). Furthermore, since \( \lambda_S(\bar{\alpha}) = 1 \), it is immediate that \( \tau_S(\bar{\alpha}) = 1 \). This is because firms have no incentive to deviate to a shorter maturity to increase price informativeness, and the manager’s compensation only depends on the price realization in \( t = 1 \). Also, since \( \lambda_S(\bar{\alpha}) = \bar{\lambda} \), it is immediate that \( \tau_S(\bar{\alpha}) = \tau^* \) by Theorem 5. Similarly, the analysis in Section 5.2.2 implies that for \( \lambda_L(\bar{\alpha}) = \bar{\lambda} \) we have \( \tau_L(\bar{\alpha}) = \tau^{Ex} \).
Proof of Lemma G.11(iii). We first show that \( \hat{\nu}_L(\alpha_S) \) is increasing and continuous in \( \alpha_S \). This is because \( \lambda_L(\alpha) \) is continuous and increasing in \( \alpha \), and, by the Envelope Theorem, each firm’s wage bill is continuous and decreasing in price informativeness (see the proof of Lemma B.5). Therefore, by the the Envelope Theorem and the planner’s problem in Eq. (38) (with \( N \) replaced by \( N(1 - \alpha_S) \)), \( \hat{\nu}_L(\alpha_S) \) is increasing and continuous in \( \alpha_S \).

Next, we show that \( \hat{\nu}_S(\alpha_S) \) is decreasing in \( \alpha_S \). By contradiction, assume \( \alpha_S' > \alpha_S \) and \( \hat{\nu}_S(\alpha_S') \geq \hat{\nu}_S(\alpha_S) \), or, equivalently

\[
f(\tau_S^*(\alpha_S')) - W(\tau_S^*(\alpha_S'), \lambda_S(\alpha_S')) \geq f(\tau_S^*(\alpha_S)) - W(\tau_S^*(\alpha_S), \lambda_S(\alpha_S)).
\]  

By Lemma G.11(i) and - (ii), we have \( \lambda_S(\alpha_S') < \lambda_S(\alpha_S) \) and \( \tau_S(\alpha_S') < \tau_S(\alpha_S) \). Eq. (G.11) implies that when the fraction of short-term firms is \( \alpha_S \), if firm \( n \) deviates to \( \tau^n = \tau_S^*(\alpha_S') < \tau_S(\alpha_S) \), its price informativeness \( \lambda^n \) is such that \( \lambda^n > \lambda_S(\alpha_S) \). Since \( W \) is decreasing in \( \lambda \), we have

\[
f(\tau_S^*(\alpha_S')) - W(\tau_S^*(\alpha_S'), \lambda^n) > f(\tau_S^*(\alpha_S')) - W(\tau_S^*(\alpha_S'), \lambda_S(\alpha_S')).
\]

By Eq. (G.12), this is a profitable deviation, which contradicts the optimality of \( \tau_S^*(\alpha_S) \). Finally, \( W \) is continuous in \( \tau_S, \lambda_S \), and \( \tau_S, \lambda_S \) are continuous in \( \alpha_S \). Therefore, \( \hat{\nu}_S \) is continuous in \( \alpha_S \).

The values for \( \hat{\nu}_S(\alpha), \hat{\nu}_L(\alpha), \hat{\nu}_S(\bar{\alpha}), \hat{\nu}_L(\bar{\alpha}) \) follow directly from the definitions of \( \hat{\nu}_S, \hat{\nu}_L \) together with Lemma G.11(i) and - (ii).

To conclude the proof of Proposition 9 we observe that, by Lemma G.11(iii), we have \( \hat{\nu}_S(\alpha) > \hat{\nu}_L(\alpha) \) and \( \hat{\nu}_S(\bar{\alpha}) < \hat{\nu}_L(\bar{\alpha}) \). By continuity of \( \hat{\nu}_S, \hat{\nu}_L \), there exists an intermediate value \( \alpha^* \in (\alpha, \bar{\alpha}) \) such that \( \hat{\nu}_S(\alpha^*) = \hat{\nu}_L(\alpha^*) \). Since, by Lemma G.11(i), we have \( \lambda_S(\alpha^*) > \lambda_L(\alpha^*) \), then we can show that \( \hat{\nu}_S(\alpha^*) = \hat{\nu}_L(\alpha^*) \) requires \( \tau_S^*(\alpha^*) < \tau_L^*(\alpha^*) \). Suppose not, i.e., \( \tau_S^*(\alpha^*) \geq \tau_L^*(\alpha^*) \). Then, \( \hat{\nu}_S(\alpha^*) = \hat{\nu}_L(\alpha_S) \) is equivalent to

\[
f(\tau_S^*(\alpha^*)) - f(\tau_L^*(\alpha^*)) = W(\tau_S^*(\alpha^*), \lambda_S(\alpha^*)) - W(\tau_L^*(\alpha^*), \lambda_L(\alpha^*))
\]

Since \( \tau_S^*(\alpha^*) \geq \tau_L^*(\alpha^*) \) and \( f \) is increasing, it must be \( W(\tau_S^*(\alpha^*), \lambda_S(\alpha^*)) \geq W(\tau_L^*(\alpha^*), \lambda_L(\alpha^*)) \). But this is impossible because \( W \) is decreasing in \( \lambda \) and increasing in \( \tau \) (see the proof of Lemma B.5).

Furthermore, by Lemma G.11(ii), we have \( \tau^* < \tau_S^*(\alpha^*) < \tau_L^*(\alpha^*) < \tau^{Ex} \).

Consider a candidate equilibrium number of short-term firms \( N_S \) where \( \alpha_S = \frac{N_S}{N} \) is such that \( \tau_S^*(\alpha^*) < \tau_L^*(\alpha^*) \). Short-term firms do not have an incentive to deviate to a lower \( \tau \) nor to a marginally larger \( \tau \) because a deviating firm’s \( \lambda \) is determined by Eq. (G.11) and the deviation cannot dominate \( \tau_S^*(\alpha_S) \) by construction. Hence, short-term firms do not have an
incentive to deviate if
\[ \hat{\nu}_S (\alpha_S) \geq \max_{\tau^n \geq \tau_S^* (\alpha_S)} f(\tau^n) - W\left( \tau^n, \frac{\mu L}{\bar{z} (1 - \alpha_S + \eta)} \right), \tag{G.13} \]

Where we define \( \eta = 1/N \). Notice that Eq. (G.13) can be equivalently written as
\[ \hat{\nu}_S (\alpha_S) \geq \hat{\nu}_L (\alpha_S - \eta), \tag{G.14} \]

Similarly, a long-term firm does not have an incentive to deviate to a greater \( \tau \) nor to a marginally lower \( \tau \). This is because a deviating firm’s \( \lambda \) is just \( \lambda_L (\alpha_S) \), and the deviation cannot dominate \( \tau_L^* (\alpha_S) \) by construction. Hence, long-term firms do not have an incentive to deviate if
\[ \hat{\nu}_L (\alpha_S) \geq \max_{\tau^n \leq \tau_L^* (\alpha_S)} f(\tau^n) - W(\tau^n, \lambda^n). \tag{G.15} \]

where \( \lambda^n \) solves short-term investors’ indifference condition
\[ (1 - \lambda^n) (1 - \tau^n \gamma) = (1 - \lambda^m) (1 - \tau_S \gamma), \text{ for all } m \in N_S. \]

Notice that Eq. (G.15) can be equivalently written as
\[ \hat{\nu}_L (\alpha_S) \geq \nu_S (\tau_s (\tau_S^* (\alpha_S); \alpha_S + \eta), \tau_S^* (\alpha_S), \alpha_S + \eta). \tag{G.16} \]

Therefore, \( N_S, \tau_S^* (\alpha_S), \tau_L^* (\alpha_S) \) is an equilibrium if both Eq. (G.14) and Eq. (G.16) hold.

Next, we prove that, for \( \alpha_S = \alpha^* \), both Eq. (G.14) and Eq. (G.16) hold. Eq. (G.14) holds because \( \hat{\nu}_S (\alpha^*) = \hat{\nu}_L (\alpha^*) \) and \( \hat{\nu}_S \) is decreasing. Eq. (G.16) holds because
\[ \hat{\nu}_L (\alpha^*) = \hat{\nu}_S (\alpha^*) = \nu_S (\tau_s (\tau_S^* (\alpha^*); \alpha^* + \eta), \tau_S^* (\alpha^*), \alpha^* + \eta) \big|_{\eta=0}, \tag{G.17} \]

and, by the Envelope Theorem and the fact that the wage bill is decreasing in price efficiency, and price efficiency is decreasing in the number of firms, the R.H.S. of Eq. (G.16) is decreasing in \( \eta \).

Finally, consider the case where the integer constraint on \( N_S \) is taken into account. Let \( N_S^* \) be such that \( N_S^*/N < \alpha^* < (N_S^* + 1)/N \) and define \( \alpha^- = N_S^*/N \) and \( \alpha^+ = (N_S^* + 1)/N \). For \( N \) finite but sufficiently large, the distance between \( \alpha^- \) and \( \alpha^+ \) can be made arbitrarily small. We can verify numerically that either \( N_S^* \) or \( N_S^* + 1 \) are an equilibrium.

This concludes the proof. \( \blacksquare \)
Appendix H.

Proof of Proposition 10:

The proof is parallel to that of Proposition 2 except that there are extra constraints due to the salary cap in Eq. (40). Using the same argument as in the proof of Proposition 2, we can find that \( w_B^{**} = w_C^{**} = w_\emptyset^{**} = 0 \), and also drop the non-negativity constraints. Furthermore, because \( w_G^{**} > w_S^{**} \) (Lemma B.4) under the optimal contract without salary cap, it is always the case that the constraint on \( w_G^n \) binds first between the two constraints on \( w_G^n \) and \( w_S^n \). Therefore, to ensure that the incentive compatibility is implementable, it has to be the case that \( w_S^n \leq \bar{w} \) never binds. Then, under such parametric values of \( \bar{w} \), the optimal contracting problem becomes as follows:

\[
\min_{\{w_G^n, w_S^n\} \in \mathbb{R}^2_+} \lambda^n \sigma_G w_G^n + (1 - \lambda^n) (1 - \delta^n) \rho_H w_S^n,
\]

subject to the binding IC constraint (17):

\[
\lambda^n \Delta \sigma u (w_G^n) + (1 - \lambda^n) (1 - \delta^n) \Delta \rho u (w_S^n) = K,
\]

and the salary cap from Eq. (40):

\[
w_G^n \leq \bar{w}.
\]

Then, the Lagrangian is given by

\[
\mathcal{L} = \lambda^n \sigma_G w_G^n + (1 - \lambda^n) (1 - \delta^n) \rho_H w_S^n + \psi_k \left[ K - \lambda^n \Delta \sigma u (w_G^n) \right] - (1 - \lambda^n) (1 - \delta^n) \Delta \rho u (w_S^n) + \psi_w (\bar{w} - w_G^n),
\]

where \( \psi_k, \psi_w \) are the Lagrangian multipliers, which are non-negative. When the constraint in Eq. (G.3) does not bind (\( \psi_w = 0 \) and \( w_G^{**} < \bar{w} \)), the optimization problem degenerates to the same problem in Proposition 2, i.e., \( w_G^{**} = w_G^{**} \) and \( w_S^{**} = w_S^{**} \). When it binds (\( \psi_w > 0 \) and \( w_G^{**} = \bar{w} \)), the solution is given by

\[
w_G^{**} = \bar{w}, \quad w_S^{**} = u^{-1} \left( \frac{K - \lambda^n \Delta \sigma u (\bar{w})}{(1 - \lambda^n) (1 - \delta^n) \Delta \rho} \right).
\]
The first-order conditions derived from Eq. (G.4) become

\[ \lambda^n \sigma_G - \psi_k \lambda^n \Delta \sigma u'(w_G^{*n}) - \psi_w = 0, \]
\[ \rho_H - \psi_k \Delta \rho u'(w_S^{*n}) = 0. \]  

(G.5)

As in Proposition 2, the Envelope theorem implies

\[ \frac{\partial \hat{W}^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} \sigma_G w_G^{*n} - \left( \frac{\partial \lambda^n}{\partial \tau^n} (1 - \delta^n) - (1 - \lambda^n) \delta \right) \rho_H w_S^{*n} \]
\[ - \psi_k \frac{\partial \lambda^n}{\partial \tau^n} \Delta \sigma u(w_G^{*n}) + \psi_k \left( \frac{\partial \lambda^n}{\partial \tau^n} (1 - \delta^n) - (1 - \lambda^n) \delta \right) \Delta \rho u(w_S^{*n}) \]
\[ = \frac{\partial \lambda^n}{\partial \tau^n} \left[ \sigma_G \left( w_G^{*n} - \psi_k \frac{\Delta \sigma}{\sigma_G} u(w_G^{*n}) \right) - \rho_H (1 - \delta^n) \Psi(w_S^{*n}) \right] \]
\[ + (1 - \lambda^n) \delta \rho_H \Psi(w_S^{*n}), \]

(G.6)

where the second equality is due to the first-order conditions in Eq. (G.5). Using Eq. (G.5), we can alternatively represent Eq. (G.6) as

\[ \frac{\partial \hat{W}^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} \left[ \sigma_G \left( w_G^{*n} - \frac{\Delta \sigma \rho_H}{\sigma_G} u(w_G^{*n}) \right) - \rho_H (1 - \delta^n) \Psi(w_S^{*n}) \right] \]
\[ + (1 - \lambda^n) \delta \rho_H \Psi(w_S^{*n}), \]

(G.7)

which is equal to Eq. (B.11) if salary cap does not bind, i.e., Eq. (G.7) becomes

\[ \frac{\partial \hat{W}^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} \left[ \sigma_G \left( w_G^{*n} - \rho_H (1 - \delta^n) \Psi(w_S^{*n}) \right) \right] \]
\[ + (1 - \lambda^n) \delta \rho_H \Psi(w_S^{*n}), \]

(G.8)

Then, similarly as in Eq. (B.15), we can derive

\[ \frac{\partial^2 \hat{W}^n}{(\partial \tau^n)^2} = \frac{\partial^2 \lambda^n}{(\partial \tau^n)^2} \left[ \sigma_G \left( \bar{u} - \frac{\Delta \rho_H}{\sigma_G} \bar{u}'(w_S^{*n}) \right) - (1 - \delta^n) \rho_H \Psi(w_S^{*n}) \right] \]
\[ - \frac{\partial \lambda^n}{\partial \tau^n} \delta \rho_H \Psi(w_S^{*n}) + (1 - \lambda^n) \delta \rho_H \Psi'(w_S^{*n}) \frac{\partial w_S^{*n}}{\partial \tau^n} \]
\[ + \frac{\partial \lambda^n}{\partial \tau^n} \left[ \frac{\Delta \rho_H}{\sigma_G} \bar{u}'(w_S^{*n}) \frac{\partial w_S^{*n}}{\partial \tau^n} - (1 - \delta^n) \rho_H \Psi'(w_S^{*n}) \frac{\partial w_S^{*n}}{\partial \tau^n} \right]. \]

(G.9)

Because \( \frac{\partial^2 \lambda^n}{(\partial \tau^n)^2} \) is negative (Proposition 1), Eq. (B.14) implies that the first term in Eq. (G.9) is positive. Because \( \frac{\partial \lambda^n}{\partial \tau^n} \) is positive (Proposition 1), and \( \Psi(\cdot) \) is negative (Eq. (B.12)), the second term in Eq. (B.15) is also positive. Because \( \Psi'(\cdot) \) is negative (Eq. (B.13)) and \( \frac{\partial w_S^{*n}}{\partial \tau^n} \) is negative (Lemma B.4), the third term is also positive.

In case the constraint binds, \( \psi_w > 0 \), which in turn implies that \( \psi_k \) is greater than the
case the constraint does not bind due to Eq. (G.5). Then, Eq. (G.6) further implies that the
marginal increase in the wage bill \( \hat{W}^n \) with respect to an increase in \( \tau^n \) is greater under salary
cap than the wage bill \( W^n \) without salary cap at any level of maturity \( \tau^n \), i.e.,

\[
\frac{\partial \hat{W}^n}{\partial \tau^n} < \frac{\partial W^n}{\partial \tau^n} = \frac{\partial \lambda^n}{\partial \tau^n} \left[ \sigma_G \Psi(w^*_G) - \rho_H (1 - \delta^n) \Psi(w^*_S) \right] + (1 - \lambda^n) \delta \rho_H \Psi(w^*_S) < 0,
\]

where the equality is due to Eq. (B.11), and the last inequality is due to Lemma B.5. When
\( w^*_G < \bar{w} \), the first inequality holds with equality in Eq. (G.10). When \( w^*_G = \bar{w} \), and the first
inequality holds strictly.

In a symmetric equilibrium defined in Definition 1 but with salary cap, all firms choose the
same contract, denoted by \( w^*_G \) and \( w^*_S \) (i.e., \( w^*_G = w^*_G^n \) and \( w^*_S = w^*_S^n \) for all \( n \in \mathcal{N} \)). They
also choose the same maturity, denoted by \( \tau^* \), and thus, we have \( \lambda^n = 1/(N\bar{z}) \) for all \( n \in \mathcal{N} \)
in equilibrium. Therefore, given the equilibrium choice of \( \tau^* \), Eq. (G.7) should be equal to

\[
\frac{\partial \hat{W}^n}{\partial \tau^n} \bigg|_{\tau^n = \tau^*} = \Theta(\tau^*) \left[ \sigma_G \left( w^*_G - \frac{\Delta \rho_H u(w^*_G)}{\sigma_G \Delta \rho u(w^*_S)} \right) - \rho_H (1 - \delta^n) \Psi(w^*_S) \right] + (1 - \lambda^n) \delta \rho_H \Psi(w^*_S).
\]

Now, we prove that \( \tau^* < \tau^* \) under a symmetric equilibrium when the salary cap binds.
As in Theorem 1 the equilibrium maturity \( \tau^* \) under salary cap is determined by trading off
between production and managerial compensation:

\[
f'(\tau^*) = \frac{\partial \hat{W}^n}{\partial \tau^n} \bigg|_{\tau^n = \tau^*}.
\]

On the other hand, the equilibrium maturity \( \tau^* \) without salary cap is determined by

\[
f'(\tau^*) = \frac{\partial W^n}{\partial \tau^n} \bigg|_{\tau^n = \tau^*}.
\]

Because \( f'(\cdot) \) is negative and increasing and \( \partial \hat{W}^n / \partial \tau^n < \partial W^n / \partial \tau^n < 0 \) whenever \( w^*_G = \bar{w} \),
Eqs. (G.12)-(G.13) imply that \( \tau^* > \tau^* \) whenever the salary cap binds in equilibrium. Note
that \( \tau^* = \tau^* \) when it does not bind.

Finally, it is straightforward to show that, fixing the choice of maturity \( \tau^* = \tau^* \), the
shareholder value is greater for the case without salary cap because the cost of compensation is
smaller or equal to the case under salary cap (recall that the contracting problem under salary
cap features one more constraint in the optimization problem.) Theorem 2 shows that the
equilibrium maturity choice under endogenous choice without salary cap is already excessively

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short-term. Given the concavity of the shareholder value in the social planner’s problem, the shareholder value becomes even lower as the maturity shortens (i.e., $\tau^*$ increases). But our result shows that the choice under salary cap is even more short-term than that without salary cap, which implies that the shareholder value should be lower under the salary cap.

References


